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# The Dimensions of Making Sense: The Understanding of Exponential Functions from an Investigative Activity

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**ABSTRACT**

This research is part of the ongoing Master's dissertation entitled "From the Carbon Atom to the Great Populations: Teaching Exponential Functions under the perspective of Problem Solving" of the Post-Graduation Program in Teaching Natural Sciences and Mathematics of the Regional University of Blumenau (FURB), which sought to verify the contributions of the Teaching-Learning-Assessment Methodology through Problem Solving and the use of GeoGebra software for learning exponential functions. In this specific work, we seek to relate Problem Solving as a theme of the actual current of teaching Mathematics called Making Sense and to verify its implications for the learning of exponential functions. For this purpose, an activity was developed and applied involving Newton's Cooling / Heating Law with the purpose of relating the tabular, algebraic and graphical representations of the exponential functions, validating the results from an experimental practice based on the dimensions of the Making Sense. This activity was applied in a 1st year high school class and it was verified that this problem allowed to contextualize the program content, as well as to promote the learning of new concepts from a practice in which the student becomes the main agent of his learning and the teacher acts as a mediator in this process, encouraging and instigating when necessary. In the end, it was noticed that the students have understood the involved concepts without needing the direct intervention of the teacher.

**Keywords**: Problem Solving. Making Sense. Exponential Function.

## As Dimensões do *Making Sense*: a Compreensão de Funções Exponenciais a partir de uma Atividade Investigativa

**RESUMO**

Esta pesquisa faz parte da dissertação de mestrado em andamento intitulada "Do Átomo de Carbono às Grandes Populações: o Ensino de Funções Exponenciais sob a Perspectiva da Resolução de Problemas" do Programa de Pós-Graduação em Ensino de Ciências Naturais e Matemática da Universidade Regional de Blumenau (FURB), que buscou verificar as contribuições

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da Metodologia de Ensino-Aprendizagem-Avaliação através da Resolução de Problemas aliada ao uso do *software* GeoGebra para a aprendizagem de funções exponenciais. Nesse trabalho, em específico, busca-se relacionar a Resolução de Problemas como uma temática atual da corrente de ensino da Matemática denominada *Making Sense* e verificar as suas implicações para a aprendizagem de funções exponenciais. Para tanto foi desenvolvida e aplicada uma atividade envolvendo a Lei de Resfriamento/Aquecimento de Newton com o objetivo de relacionar as representações tabular, algébrica e gráfica da função exponencial validando os resultados a partir de uma prática experimental baseada nas dimensões do *Making Sense*. Tal atividade foi aplicada em uma turma de 1º ano do Ensino Médio e constatou-se que esse problema permitiu contextualizar o conteúdo, bem como promover a aprendizagem de novos conceitos a partir de uma prática em que o estudante passa a ser o agente principal de sua aprendizagem e o professor atua como mediador nesse processo, incentivando e instigando quando necessário. Ao final, percebeu-se que os estudantes compreenderam os conceitos envolvidos sem a necessidade da intervenção direta do professor.

**Palavras-chave**: Resolução de Problemas. Making Sense. Função Exponencial.

## INTRODUCTION

At present, much has been said about learning mathematics by understanding, but what understanding would that be? Do our schools have not been already teaching students to understand the taught contents? The answer, of course, is yes, but unfortunately a high level of understanding is not achieved by all students. A survey released in 2015 by the movement "Todos pela Educação" showed that only 9.3% of students from Brazil, who finish high school, have the appropriate and necessary understanding of Mathematics. These data show that, unfortunately, only a minority is being contemplated with the current education system.

In order to contribute to the improvement of this scenario, new teaching approaches are necessary to enable the students not only to have an adequate understanding of the contents, but also to prepare them for this new dynamic present in our society, as already indicated in the late 1990s by Hiebert et al.:

> In order to take advantage of new opportunities and to meet the challenges of tomorrow, todays students need flexible approaches for defining and solving problems. They need problem-solving methods that can be adapted to new situations, and they need the know-how to develop new methods for new kinds of problems. Nowhere are such approaches more critical than in the mathematics classroom. Not only is technology making some conventional skills obsolete – such as high levels of speed and efficiency with paper-and-pencil calculations – It is also underscoring the importance of learning new and flexible ways of thinking mathematically. (Hiebert et al., 1997, p.1)

In other words, it is believed that by promoting a teaching practice based on Problem Solving, the teacher encourages students to investigate new situations, which they do not have mechanical resolution methods, stimulating to draw new thinking strategies, questioning and applying their knowledge and skills in new situations for a collective learning environment (Vila & Callejo, 2006).

In this context, this work proposes discussions that enable to the teacher new reflections on teaching practices that have as their motto the Making Sense, which refers to learn mathematics with understanding, with sense. According to Van de Walle (2009) in order to the Mathematics has meaning for the student, it is necessary that advances beyond knowing, to know information, is more than being able to follow a procedure or use an algorithm. One mark of mathematical understanding is that the student has the ability to justify why an answer is correct or why a mathematical rule makes sense.

Based on this perspective of teaching by understanding, the mathematical object that was used in this research refers to the teaching of functions, since this content has a prominent role in Mathematics, due to the need of relating different types of quantities. However, some authors (Willoughby (2000), Candeias (2010), Siqueira (2013)) found that students usually present difficulties in understanding some aspects of this curricular component, whether in the passage of its diverse representations (algebraic, graphical, tabular, etc.), either in the understanding of the concept of function that is usually approached in a strictly algebraic way, or also, by the absence of the use of technological resources, which makes it difficult to see the changes in the parameters of the function in its graphical representation.

Among the various types of functions that are studied in High School, perhaps, the exponential is the one that is present in the most varied range of situations, including outside Mathematics itself. These relations, from exponential functions to the physical world, are emphasized by Oliveira (2014, p.15):

> This connection with other areas of the curriculum and with mathematics itself makes teaching and learning more meaningful, as it creates the opportunity in which the student perceives the importance of the content to be worked on, which makes contextualization an important teaching tool to solve real problems.

So, the problem that defines this research is “What are the implications of a teaching approach based on the assumptions of Making Sense, from the perspective of Problem Solving, for an investigative practice of exponential function?”. This study was based on the analysis of the results of an activity using the above method, whose application occurred in a 1st year high school class.

## THE DIMENSIONS OF MAKING SENSE

As already stated, when proposing the teaching of Mathematics from a premise focused on Making Sense, it is expected, above all, that what the student learns makes sense and that he can use this knowledge not only for a specific activity, but also to a series of new situations that can serve as a source of research. Learning by Making Sense enables the knowledge built in the classroom can be useful outside of it. It means understanding why and for what.

When presenting the main aspects that make learning mathematics by understanding crucial, Hiebert et al. (1997) points out three premises. The first, according to the author, is that when the contents are learned by understanding, they are flexible and can be adapted to the new situations and learning new concepts. The second is that learning mathematics stop being a practice focused on the memorization and application of algorithms, to become an investigative science, enabling the student to see how things work, how they relate to other topics and why they are this way. And finally, learning by understanding is an intellectually satisfying experience that provides confidence and involvement for students.

Van de Walle (2009) exemplifies this change of approach of mathematics teaching from their own activities in the classroom. In the author's view, in traditional teaching the verbs like to listen, to copy, to memorize, to exercise are more abundant. However, when it is sought to provide understanding, new verbs must occupy this place, action verbs that encourage the involvement and the exposition of ideas. The author cites as some examples: to explore, to investigate, to verify, to justify, to build, among others. So, in his point of view, when confronted with this kind of situation, it is virtually impossible for students to behave passively.

> The role of the teacher is to create this spirit of research, confidence and expectation. In this environment, students are invited to do math. Problems are presented and students seek solutions for themselves. The focus is on the students to actively understand things, testing ideas and making conjectures, developing reasoning, and providing explanations. Students work in groups, in pairs or individually, but they are always sharing and discussing their ideas. (Van de Walle, 2009, 33)

In this sense, Hiebert et al. (1997, p.2) propose that a change in the organization of the classes is necessary in order to provide this environment: "We believe that students' understanding is so important that it is worth rethinking how classrooms can be designed to support it ". Faced with this situation, the authors propose that the Mathematics classes be organized considering five different dimensions that permeate the entire process of teaching and learning. These dimensions are summarized in Figure 1 below:

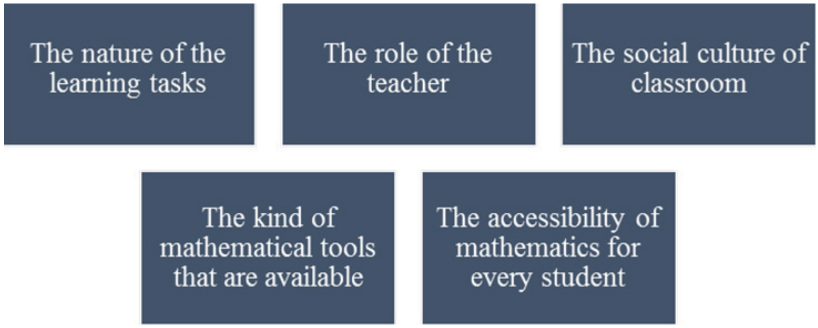


*Figure 1*. The Dimensions of Making Sense (Hiebert et al., 1997).

The first dimension to be considered is the nature of learning tasks. Hiebert et al. (1997) believe that the type of mathematical activity that is offered to students is what defines the learning system. So, for a system to be built based on reflection and communication, there must be real mathematical problems. "These are tasks for which students have no memorized rules, nor for which they perceive there is one right solution method. Rather, the tasks are viewed as opportunities to explore mathematics and come up with reasonable methods for solution" (Hiebert et al., 1997, p.8).

Unfortunately, this kind of problem is not easily found in the mathematical literature, nor in textbooks. Most of the time, it is necessary for the teacher to develop them. However, Van de Walle (2009) establishes three principles that can guide the teacher's work:

a) The problem must begin where the students are;

b) The problematic or surrounding aspect of the problem must be related to the mathematics that students will learn;

c) Mathematical learning should require justification and explanations for the answers and the methods.

Regarding the first principle, Van de Walle (2009) points out that the problem must consider the current stage of student understanding. There is no point in proposing a problem that requires ways of solution whose complexity cannot be reached by the student, nor does it make sense to propose activities that they already have full knowledge of how to solve them, so that there is no new knowledge to be built, nor will it encourage reflection. "They should have the proper ideas to get involved and solve the problem and still find it challenging and interesting. Students should consider the task something that makes sense" (van de walle, 2009, p.57).

In the second principle, Van de Walle (2009) emphasizes that the problems must be aimed at providing an understanding of new knowledge, not being prejudiced in favor of other elements that focus on the activity, such as cut, paste, color, among others. However important these aspects may be, the main purpose of the problem is to develop an understanding of the mathematical topic in question.

And finally, in the third principle, the author specifies that the problem may allow different ways of resolution, but the justification of the path chosen and why an answer is correct or incorrect is responsibility of the students. The teacher has his or her responsibility directed to other moments of learning.

Therefore, the second dimension to be considered in view of Hiebert et al. (1997) it is the role of the teacher in solving these problems. The author points out that, traditionally, in the mathematics teaching, the teacher feels responsible for explaining all the information in detail to the students and then proposes a series of questions so that they can practice what has been seen. However, when teaching by understanding is desired, the teacher changes his role in the classroom.

> Within this work, the role of the teacher changes from communicator of knowledge to that of observer, organizer, consultant, mediator, interventor, controller and facilitator of learning. The teacher launches challenging questions and helps students to lean on each other to get through the difficulties. The teacher mediates, leads the students to think, expects them to think, gives time for it, accompanies their explorations, and resolves, when necessary, secondary problems. (Onuchic, 1999, p.216)

In this context, the teacher does not directly explain all the content, the understanding must result from the context of the problem and the students' joint discussions. During this stage, the teacher has the opportunity to encourage students' thinking without, however, provide direct answers to their questions, which inhibits m the construction of learning. "Carefully, offer appropriate suggestions – but only suggestions based on the ideas of the students and their ways of thinking" (Van de Walle, 2009, p.64). The teacher needs to keep in mind, that the starting point of mathematical learning should not be the definition, but the problem. In this way, Problem Solving should not be an activity to be developed in parallel to other tasks, but rather as an orientation to learning (Onuchic, 1999).

The third dimension proposed by Hiebert et al. (1997) emphasizes the establishment of a social learning culture in the classroom. In general, the author proposes that, in order to develop a community of learners, four main characteristics support the organization of classes.

The first is the socialization of ideas, which in the view of Hiebert et al. (1997), must permeate the entire school environment. Thus, in order to obtain learning communities, it is suggested that the work happens in teams, where each member can suggest and listen to new opinions or methods of solution. From this, the exchanges of ideas and the discussions are encouraged and form the basis of the social interactions of that community (team).

Hiebert et al. (1997) point that students should set their own solution methods and share them with others, helping them also in understanding the argument used. This practice, when not spontaneously implemented, needs to be encouraged by the teacher.

Knowing how to justify the processes and the ways of solution is relevant in Mathematics classes, as stated by Cândido (2001, p.17):

> When it comes to mathematics, whenever we ask a child or a group to say what they did and why they did it, or when we ask them to verbalize the procedures they have adopted, to justify them, or to comment on what they have written, represented or outlined , reporting the steps of his research, are allowing modifying previous knowledge and building new meanings for mathematical ideas. At the same time, students reflect on the concepts and procedures involved in the proposed activity, take ownership of them, review what they did not understand, amplify what they understood, and explain their doubts and difficulties.

This shows that the justification for a resolution brings benefits not only to the other students participating in the discussion, but also to those who are arguing in favor of their ideas. This action provides a unique environment for exploration and mathematical investigation.

The third characteristic, in favor of a classroom social culture based on Making Sense, is concerned with the prerogative that errors made by students should be treated not only for teacher evaluation, but mainly as opportunities to move further towards understanding the subject matter. In this context, an alert is made to the teacher who traditionally points out ways of solving so that the probability of error is less: “As soon as we try to prevent students from making mistakes, we begin specifying the methods they should use. This removes the problematic nature of the task – the foundations of the system” (Hiebert et al., 1997, p.48).

Similarly, Van de Walle (2009, p.50) also highlights the benefits of a classroom culture that does not penalize errors but uses them as opportunities for growth:

> A collective trust should be established with the understanding that it is right to make mistakes. Students have to realize that mistakes are an opportunity for growth when they are discovered and explained. All students should trust that their ideas will be received with the same level of respect, regardless of whether they are right or wrong. Without this collective trust, many ideas will never be shared.

Still in this context, Hiebert et al. (1997) also establish the fourth characteristic for the development of a culture: the correction must be determined from the mathematical logic used by the students themselves. It is necessary to develop a collective confidence to engage students as evaluators of their own methods and solutions obtained by removing from the teacher the role of maximum authority in the classroom regarding the holding of the answers. “Teachers also must help students see that they can, over time and collectively, determine correctness by relying on their own arguments” (Hiebert et al., 1997, p.49).

Likewise, Van de Walle (2009) also supports the idea that the correction itself must reside in the Mathematic. And as for the role of the teacher, the author suggests that he does not need to provide answers to all the students' questions and also makes an alert that when the teacher provides answers of the type "Yes, this is correct" or "No, this is wrong", the students fail to make sense of the ideas involved, harming the discussion and learning in the classroom.

In general, Onuchic and Allevato (2011, p.81, emphasis added) describe the operation of this practice:

> The student analyzes his own methods and solutions obtained for the problems, always aiming at the construction of knowledge. This form of student work is a consequence of his *mathematical thinking*, leading him to elaborate justifications and give meaning to what he does. On the other hand, the teacher assesses what is happening and the results of the process, with a view to reorient classroom practices, when necessary.

Thus, it is believed that, gradually, as these four characteristics are observed in the development of teaching practices, a fertile environment for mathematical investigation begins to develop. Students cease to be passive agents of their learning to act actively before it. However, it is notorious that these practices may lead to some initial questions, but it is hoped that as the lessons move in this format, students will begin to demonstrate confidence in their own knowledge and skills and wish to continue in this learning model.

The fourth dimension, titled "Mathematical tools as learning support, " discusses the benefits of using differentiated resources to aid in the process of developing understanding. These tools can be symbols, words, schemes, software, calculators, posters or other means that enable not only understanding but also the communication of ideas. However, Hiebert et al. (1997) point out that tools alone do not provide understanding, it is developed from the student's interaction with these tools, from the ideas that are developed from the manipulation of the different representations that the same concept can assume. In this way, only making available to students differentiated resources and hoping that, from them, instantaneously, new concepts will be constructed, it will hardly present good results.

In this sense, the teacher needs to analyze what goal he wants to achieve and how a given resource can help in the process of developing new understandings.

> Allowing students to use tools does not guarantee that all students will develop the same meanings for them. Students who use tools as aids for calculating answers are likely to develop different meanings than students who use them to explore alternative solution methods or reflect on the reasons the methods work. (Hiebert et al., 1997, p.55)

In short, the resources can be important mathematical tools in order to assist the learning process, but should not be used alone, or be considered solely responsible for the development of understanding. These features do not replace the role of teacher and cannot suppress communication and creativity in the classroom but can be used in order to provide to all, equal learning opportunities and it is in that look that develops the last dimension of *Making Sense*.

Finally, the fifth dimension refers to "Equity and Accessibility" in the Mathematics classes. This dimension is directly related to the participation of all students in the teaching and learning process. It is hardly possible for a single teaching method can guarantee the learning of the whole class and, due to several reasons that underlie the time, curriculum, lack of preparation or even the absence of new methodologies, the teacher goes on with the programmatic contents, knowing that not all students have acquired the required knowledge.

Comprehension teaching has the premise that all students have the ability to develop any mathematical concepts, since this approach is performed at an appropriate level and that can arouse them the desire to learn. As much as this discourse is easier to implant in theory than in practice, the teacher has at his disposal some artifices that can provide accessible learning for all students.

Towards this, Hiebert et al. (1997) point out that equity is built when the teacher believes that each student can and must learn Mathematics with understanding. In this way, the authors maintain the prerogative that when present in a learning group, or community of learners, students who cannot always immediately absorb a concept or elaborate a resolution strategy have the possibility, from the exchange with others, to build their own bridges towards learning. However, such interactions assume that the tasks must be accessible to all and that each student can be heard during this process. This means that all ideas must be shared and made possible in forums for discussion in the search for consensus.

> It is important for all students to share in this responsibility because all ideas and methods are potential learning sites. Correct methods are appropriate objects of discussion, as are incorrect methods. A variety of ideas are essential for fueling rich discussions. The likelihood that the class, as a group, will get a variety of ideas on the table for discussion and analysis increases as more students find ways to participate. The group is likely to make the most progress when all students participate and offer ideas and methods for discussion. (Hiebert et al., 1997, p.67)

It is therefore assumed that for learning to be accessible it is necessary that everyone is heard and that they can participate in the knowledge construction process. However, for students who are not used to working this methodology it is necessary that the teacher encourages participation in order to ease the insecurity and enable all ideas are heard and discussed.

## METHODOLOGICAL REFERRAL AND RESEARCH SUBJECTS

In this investigative path, regarding the nature of the research, it is classified as qualitative and in relation to the procedures, we opted for the modality action research.

To constitute a qualitative research, Kauark, Manhães and Medeiros (2010) affirm that, from the point of view of approaching the problem, it is necessary to have an inseparable relationship between the real world and the subject who wishes to carry out the research. For this, the researcher bases his analysis on the interpretation of the phenomena, assigning meanings to his questions.

In order to enable this approach, action research was chosen as a research modality. Tripp (2005) defines this modality as being a cycle in which the researcher seeks to improve his practice from his own research. "A change is planned, implemented, described and evaluated for the improvement of its practice, learning more in the course of the process, both regarding practice and research" (Tripp, 2005, p.446).

In order to methodologically organize the process of action research, the author points out as a first step is the identification of a problem. Then, a solution is planned that allows its implementation with the collective involved, in which it monitors and describes the effects of this action to finally evaluate its effectiveness.

In the context of this research, we reiterate the problem that led to the investigative course of this work: "What are the implications of a teaching approach based on the assumptions of Making Sense, from the perspective of Problem Solving, for an investigative practice of exponential function?". In this way, in order to enable the expected improvement in the process of teaching and learning, it was developed a mathematical problem from the theoretical and methodological assumptions studied and it was applied in a class of 36 high school students from a public school in the city of Blumenau (SC). It is emphasized that this research was submitted and approved by the Research Ethics Committee in Human Beings – CEPH[1] from the Regional University of Blumenau.

The methods of data collection were based on the student's audio and image record, the teacher's observation and the scans of the activities developed.

## APPLICATION AND ANALYSIS

In order to carry out this activity, the class was divided into groups of four students, and the time required was five lessons of forty-five minutes each. Initially it was given a problem to the class whose aim was to relate the tabular, algebraic and graphic representations of exponential function validating the results with a practical experience. This practice was constructed from Newton's law of cooling which states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of the surrounding environment (Boyce & Di Prima, 2001).

[1] Document number 2,338,640.

Students were asked how much time was required for a heated or cooled liquid to approach the ambient temperature and, therefore, each team received a cup with hot or cold water and a thermometer to carry out the experiment, promoting an investigative practice that gives meaning to Mathematics, as indicated by Hiebert et al. (1997) in the dimension related to the nature of learning tasks.

Thus, the groups measured the ambient temperature of the classroom at that time and, in a period corresponding to two classes (90 minutes), were responsible for measuring the temperature of the liquid at equal time intervals. The instruments used for measurement were culinary thermometers, and the teams relayed so that each could use the thermometer at the right time to measure the temperature. To time, the students used their mobile applications, as shown in Figure 2.

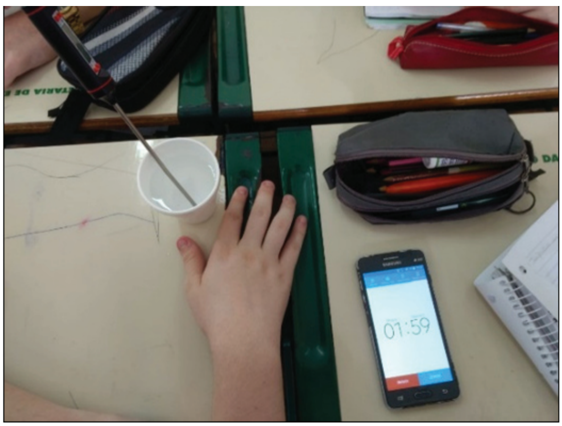


*Figure 2.* Measurement of liquid temperature by students.

The teams had to build a table and note the liquid temperatures at each time interval. The data collected by the groups that accompanied heating liquid can be seen in Figure 3.

| | | | |
|---|---|---|---|
| 00:00 | 3,9°C | 00:33 | 13,7°C |
| 00:03 | 4,9°C | 00:36 | 14,1°C |
| 00:06 | 6,0°C | 00:39 | 14,5°C |
| 00:09 | 7,8°C | 00:42 | 15,1°C |
| 00:12 | 8,4°C | 00:45 | 15,4°C |
| 00:15 | 9,1°C | 00:48 | 15,9°C |
| 00:18 | 9,8°C | 00:51 | 16,6°C |
| 00:21 | 10,7°C | 00:54 | 17,0°C |
| 00:24 | 11,4°C | 00:57 | 17,3°C |
| 00:27 | 12,1°C | 01:00 | 17,5°C |
| 00:30 | 13,1°C | | |

*Figure 3.* Record of measurements collected by students.

In this first stage, the students understood what should be done. The use of the thermometer or the timer did not create doubts. However, in the next step, some difficulties were recorded because the groups should determine a function, following the model proposed by Newton: $T = (T_0 - T_a) \cdot e^{k \cdot t} + T_a$ that fit the values collected. Initially, it was asked whether the collected values increased or decreased steadily in each time interval. All groups found that the temperature difference was not constant, with intervals decreasing/increasing faster than in others, evidencing that a linear model would not describe the phenomenon.

In the next stage, which took place for two 45-minute classes each, students should replace the value of the initial liquid temperature ($T_0$) and the ambient temperature ($T_a$). However, they realized that there was still a constant ($k$) to be determined. At that moment, the groups had difficulties to continue with the question and it was necessary the intervention of the teacher/researcher in order to get them to understand the need to substitute a value referring to the temperature collected by them ($T$) and its respective instant of time ($t$) to determine the value of this constant. It could be verified that this difficulty was the result of the non-comprehension of the concepts of variable and function in which the algebraic expression is not understood as a way of associating two variable quantities with dependence relation. In other words, the prior knowledge needed to construct the content that was intended to be discussed had weaknesses that needed to be overcome. Van de Walle (2009, p.45) in the study of several authors, emphasizes that understanding is the "measure of the quality and quantity of connections that an idea has with existing ones. Understanding is never a proposition 'all or nothing'. It depends on the existence of appropriate ideas and the creation of new connections".

In order to get them to understand this step, questions were asked to instigate the use of the collected data, such as: "*Will we only use the initial liquid temperature to determine the law of function*?", "*Can the other data collected be used to determine the value of the constant?"*. This practice, in fact, is pointed out by Van de Walle (2009, 64) as a recommendation to the teacher's posture during the resolution of the problem: "Give careful suggestions – but only suggestions based on students' ideas and their modes of thought ". Thus, the students recalled that a similar procedure to that had been held in another activity and managed continue with resolution.

After some calculations with the intention of isolating the unknown quantity, the groups again needed support because they no longer remembered that it was necessary to use the logarithms to determine this value. Therefore, together with the whole class an explanation was made to recall the properties of logarithms that allows to solve an equation that presents an unknown in the exponent. This perspective is in line with what Hiebert et al. (1997, p.36) to indicate that the "[…] information can and should be shared as long as it does not solve the problem, does not take away the need for students to reflect on the situation and develop solution methods that they understand".

The Figure 4 shows the calculations performed by a team as well as the corresponding graphic built with the help of GeoGebra application.

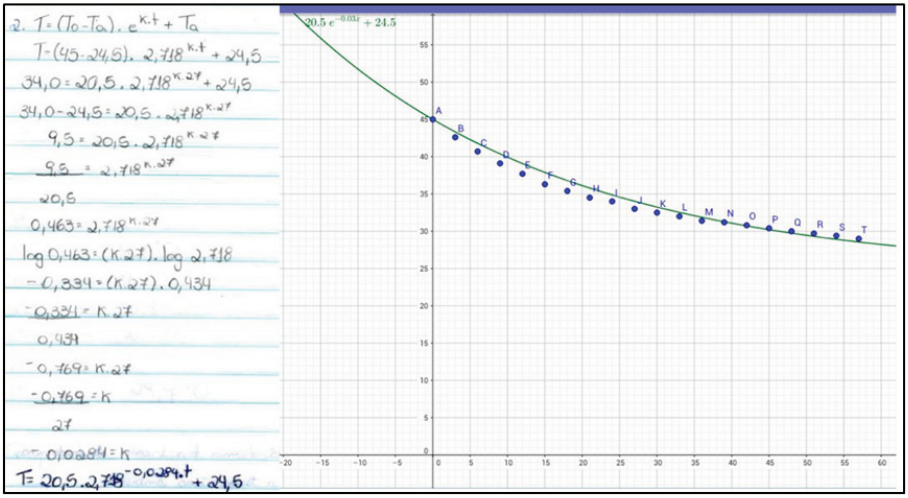


*Figure 4.* Determination of the law of function and graph of the cooling of a liquid.

Not all teams succeeded to find a correct value for the constant and this could be verified when the students built the function graph and it did not adjust to the points measured by them. In the end, during the presentation of the resolution together with the other colleagues, the students of the groups realized that the law of the function found by them did not conform to the measured values and thus found that some stage of the resolution was not correctly performed. When analyzing the calculations made by the groups whose curves did not correspond to the measured temperatures, it was verified that the main causes of the errors involved algebraic passages, manipulation of the logarithms or errors of signals, according to Figure 5.

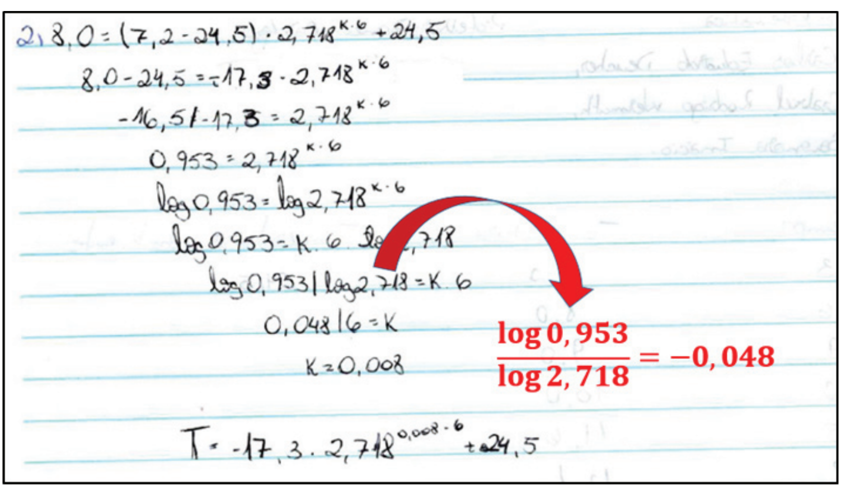


*Figure 5.* Calculation error of Newton's Cooling / Heating Law constant.

Faced with the discussions with the large group, the teams noticed the calculation errors and had the opportunity to redo the activity in search of a more adequate solution for each situation. Next, Figure 6 shows some graphs resulting from this stage of the problem.

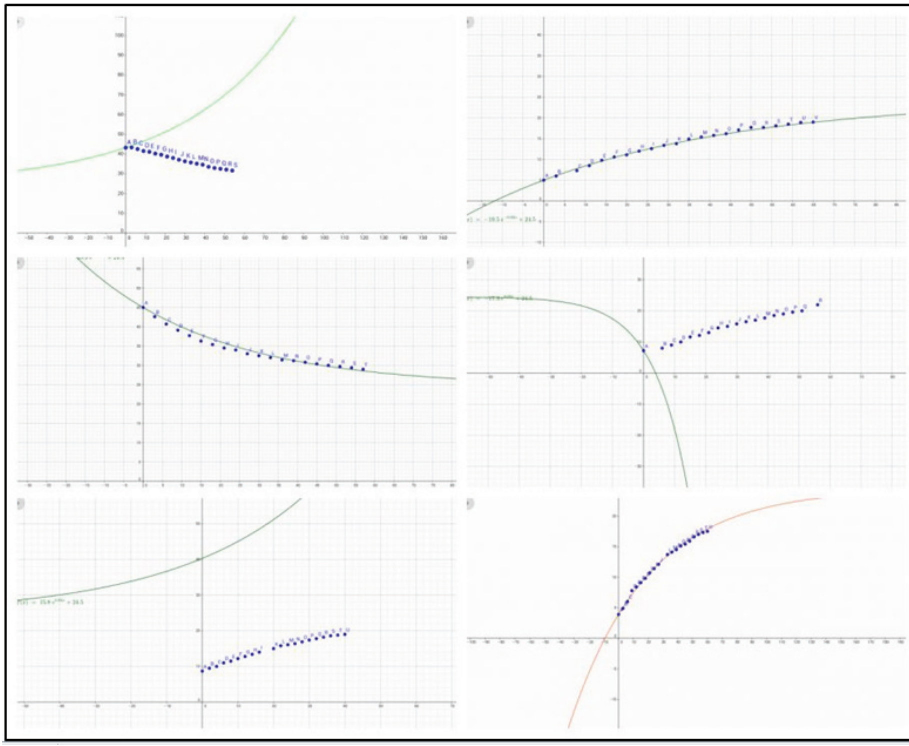

*Figure 6.* Graphs developed by the groups, referring to Newton's Cooling / Heating Law.

It was verified that of the nine teams that performed this activity, six groups were able to find an exponential function whose graph fit approximately to the values obtained experimentally. The others, for presenting incorrect algebraic passages, they did not get a proper fit and had the opportunity to redo.

The next question challenged the students which was the temperature of the liquid after 7 minutes, as to this point in time the measurement of temperature had not been made. All groups answered this question by replacing this time in the law of the function found and performing the calculation to determine the temperature. The groups, whose curve had not adjusted to the points, were questioned as to the veracity of this value within the context of the question, since if the graph of the function was not correct, the same answer also would not make sense. In order to make them understand this idea, they were asked to check the temperature of the liquid at 6 and 9 minutes on the constructed table and compare that value with the result of the function. Thus, the teams found that the calculation of the temperature at 7 minutes was correct for the function found, but the law of the function was not adequate for the measured points.

As an example, the temperatures collected by a group for the instants 6 and 9 minutes, whose values were respectively 8°C and 9°C, are presented in Figure 7. However, in calculating the temperature for the instant 7 minutes the resulting value from the function was 6.3°C. This showed that, as already could be seen from the analysis of the behavior of the function through its graph, the law of the function found was not adequate for the analyzed situation.

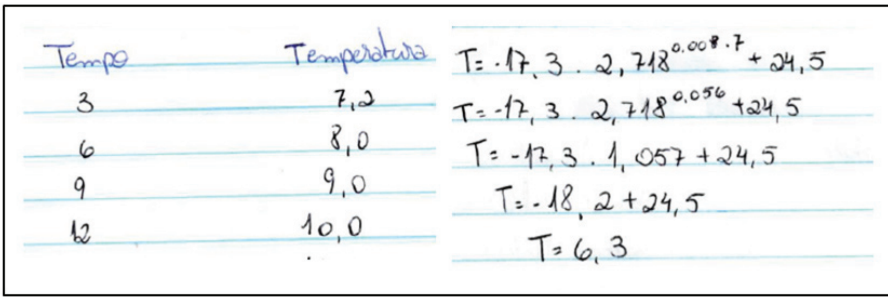


*Figure 7.* Comparison between temperature calculations with values collected experimentally.

Next, the following question was asked of the students: "In your opinion, why does the liquid temperature come close to the ambient temperature?". This question was intended to make them wonder how effectively the liquid temperature increased or decreased to approximate the ambient temperature. Since this question required knowledge related to the physics discipline, the groups were able to consult their cell phones as well as textbooks in search of solutions to this question. Some of the answers can be seen in Figure 8:

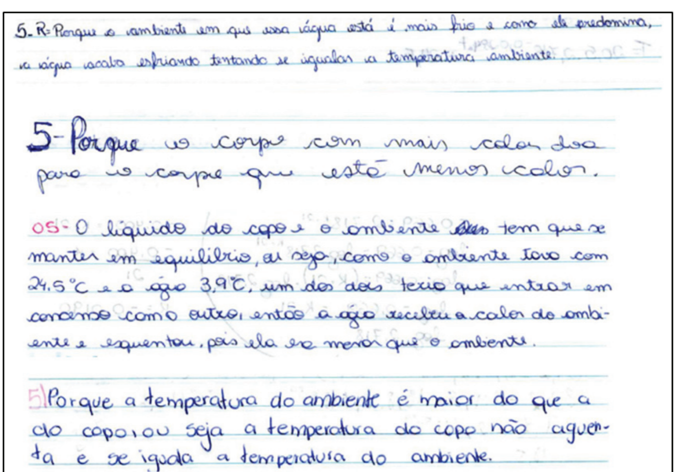


*Figure 8.* The responses of the groups as to the ratio of the temperature of the liquid to that of the environment.

In the end, after reading the solutions presented by the teams, it was noticed that most of them were able to establish the causes of the physical phenomenon observed by them with the heat transference study of Physics, and their answers were constructed from the readings and discussions carried out by them, thus verifying that the activity, besides allowing the application of mathematical contents to a real situation, also allowed the contextualization of this situation with other areas of knowledge, in this case Physics.

It is worth mentioning that the process of modeling the function that was used is compatible with the skills and contents pertinent to a high school class, but it is not the most adequate. The least squares method, which uses derivation concepts, would allow a more meaningful approximation of the data with the modeled function.

However, it is reiterated that this activity also yielded some setbacks to which any teacher who plans a class in which the protagonism is attributed to the student is subject. One of these situations happened when some teams poured out the liquid that they were using, which led to the need to restart the activity. In addition, it is valid to suggest to the teacher that wishes to do this activity at another time, to do only the situation in which the liquid heating occurs because this activity involves only cold water, reducing the risk of some student spilling hot water and causing accidents in the classroom. Fortunately, none of the teams in this incident used the heated liquid, but such a situation could have generated more serious and unpleasant problems. In this way, the teacher is advised not to perform the activity that involves the liquid cooling.

## FINAL CONSIDERATIONS

The experiment used, as well as the problematization discussed from the questions present in the activity allowed to verify the implications of a teaching approach based on the assumptions of Making Sense for an investigative practice of exponential function. Initially, it was soon noticed a greater interest of the students to carry out the activity and this is justified because of the possibility of themselves to perform a data collection, unlike what traditionally occurs in Mathematics classes, where they already receive all the information they will need to solve the activity.

As regards the nature of the learning tasks, it was found that this problem it achieved the intended objective of enabling problematizing mathematics and connecting it to the students' context, as well as allowing the construction of new concepts, which is in line with the dimension related to the nature of the learning activities indicated by Hiebert et al. (1997).

About the role of the teacher during the resolution of the activity, this had a different function from what routinely happens during the lessons. At various times the students questioned and sought direct answers for them. However, the teacher / researcher did not provide ready-made solutions, but instigated, whether from new questions or by suggesting other ways of thinking, to seek within their own collective paths that allow them to get solutions to the problem. Gradually it was being built a social culture of classroom in which the ideas and methods of the students were valued, in which communication happened all the time, prompting the participation of all members. But above all, from a form of work in which mistakes were not penalized, students became not afraid of making mistakes which, in turn, enabled them to attain the desired knowledge. It is reiterated that this classroom culture was not only built in this activity but had already been put into practice from a series of other activities that the teacher/researcher was already doing in the classroom, in the same perspective of approach. In this perspective, Van de Walle (2009, p.50) emphasizes that learning "occurs and is amplified when students involve with each other and engage in the social culture of a community of math learners".

As for the learning support tools, especially the technological resources used, it was found that the students did not have many difficulties in using them and both were not only important to show that doing mathematics goes much beyond the pencil paper, but also provided more dynamicity to learning, since otherwise, the construction of hypotheses and their instant validation would not be possible, evidencing the investigative character of the Mathematics classes.

In addition, another positive factor was that most students were able to understand the need for exponential functions to represent situations in nature, as well as the importance of handling them correctly in order to interpret when a solution makes sense of a mathematical problem , favoring an investigative performance anchored with the use of previous knowledge and with the experiences that each student brings to the classroom.

Finally, regarding the equity and accessibility of the activity – also predicted in the dimensions of the Making Sense – Hiebert et al. (1997) – it was found that it was accessible to the whole class and even with some difficulties of interpretation, after some interventions of the teacher/researcher the students managed to obtain a solution to the problem. During the resolution and presentation of the results, the involvement of all the students was instigated, however, such participation was not fully achieved with the whole class. It is believed that, as this type of methodology is advanced, students will become more and more engaged and secure in the face of mathematical problems.

## AUTHOR CONTRIBUTION STATEMENTS

D.C. developed the theoretical framework, performed the activities, collected and recorded the data. J.P.P. supervised the project, guided data collection and revised the theoretical framework. Both authors analysed the data, discussed the results and contributed to the final version of the manuscript.

**DATA AVAILABILITY STATEMENT**

The data supporting the results of this study will be made available by the corresponding author, D.C., upon reasonable request.

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