The Wittgensteinian Perspective and Ethnomathematics: An Analysis of Language Games and the Rules Governing Their Uses in Certain Work Activities

Luís Tiago Osterberg[a,b]
Isabel Cristina Machado de Lara[b]

[a] Prefeitura Municipal de Gravataí, Secretaria Municipal de Educação, Escola Antônio Aires de Almeida. Gravataí, RS, Brasil.
[b] Pontifícia Universidade Católica do Rio Grande do Sul, Programa de Pós-Graduação em Educação em Ciências e Matemática, Porto Alegre, RS, Brasil.

ABSTRACT

This work, adopting a Wittgensteinian perspective, aims to analyze the language games that involve mathematical concepts present in certain work activities, as well as the rules of use of such concepts, comparing them with the existing rules in School Mathematics. The studies analyzed used Ethnomathematics as a research method to understand the generation, organization and dissemination of mathematical knowledge in certain professions, in particular carpenters, fishermen, farmers and artisans. In considering the language games present in the mathematical practices existing in these professions, it is possible to show that in some games rules are presented that have strong family similarities to the games that make up the School Mathematics when they need a written mathematics, however, the expression of language games orally assume different meanings for terms present in both grammars. In addition, it presents examples of the use of mathematical knowledge without the formalism and rigor present in the language games of School Mathematics. It is a way of doing mathematics generated by another grammar that uses other rules, in this case estimation and rounding, a type of rationality distinct from that which constitutes School Mathematics, but which is effective in that form of use.

Keywords: Ethnomathematics. Language Games. Forms of Life. Rules.

A Perspectiva Wittgensteiniana e a Etnomatemática: uma Análise dos Jogos de Linguagem e as Regras que Regem os Seus Usos em Determinadas Atividades Laborais

RESUMO

Este trabalho, adotando uma perspectiva wittgensteiniana, objetiva analisar os jogos de linguagem que envolvam conceitos matemáticos presentes em determinadas atividades laborais, bem como as regras de uso de tais conceitos comparando-as com as regras existentes na Matemática

Corresponding author: Luís Tiago Osterberg. E-mail: tiagoosterberg@gmail.com

Escolar. Os estudos analisados utilizaram a Etnomatemática como um método de pesquisa para compreender a geração, a organização e a difusão dos saberes matemáticos contidos em determinadas profissões, em particular, marceneiros, pescadores, agricultores e artesãos. Ao considerar os jogos de linguagem presentes nas práticas matemáticas existentes nessas profissões, é possível mostrar que em alguns jogos se fazem presentes regras que possuem fortes semelhanças de família em relação aos jogos que compõe a Matemática Escolar quando essas necessitam de uma matemática escrita, porém as expressões orais de jogos de linguagem assumem sentidos diferentes para termos presentes em ambas as gramáticas. Além disso, apresenta exemplos da utilização de saberes matemáticos sem o formalismo e o rigor presentes nos jogos de linguagem da Matemática Escolar. Trata-se de uma forma de fazer matemática gerada por outra gramática que se utiliza de outras regras, distinta daquela que constitui a Matemática Escolar, mas que é eficaz naquela forma de uso.

Palavras-chave: Etnomatemática. Jogos de Linguagem. Formas de Vida. Regras.

INTRODUCTION

Wittgenstein was one of the great philosophers of the twentieth century who greatly influenced language issues and was given a considerable share of responsibility for the linguistic turn that occurred over the past century.

In analyzing Wittgenstein's works, it is possible to perceive that he is a somewhat peculiar philosopher. His studies are divided into two distinct moments, translated by his main works: Tractatus Logico-Philosophicus; Philosophical Investigations.

In the first one, characterized by many scholars about the philosopher, as his youth phase and called *Early Wittgenstein*, the author seeks to make explicit the logical conditions that language needs to describe facts of the world. In this work he seeks an essence in language, where it should have an end in itself. Moreover, in his first phase, the meaning of a word was in the object itself.

On the other hand, the second phase is characterized as the "maturity" of Wittgenstein, named as the *Later Wittgenstein*. In his work the philosopher, despite taking up numerous questions concerning the language present in his early phase, places himself in a position contrary to his first theory. He asserts that there is no essence in language, but that there are "relationships" in its uses and that we must therefore consider the existence of "languages" (Wittgenstein, 1999, p.52) instead of a universal language, since the meaning of a word is in its use in language.

For this study, the perspective adopted is that presented in the second phase of the author, in which language is directly linked to its uses and the multiplicity of situations involving these uses, defining what Wittgenstein (1999) calls "language-games". Furthermore, the same meaning given to the term *rules* that define the uses of language games is used, and to the set of these rules the author calls "grammar". Thus, grammar describes and studies the rules present in different languages.

In Mathematics Education, the Ethnomathematics aspect stands out in the last decades for the emphasis that it gives to the production of knowledge, in particular, mathematical knowledge, by different cultural groups. D'Ambrosio conceives

Mathematics as "[...] a strategy developed by the human species throughout its history to explain, to understand, to manage and coexist with the sensible, perceptible reality, and with its imaginary, naturally within a natural and cultural context" (2005, p.102), as well as religion, techniques, the arts, and science in general. Therefore, for the author, it is the construction of bodies of knowledge that are "[...] in total symbiosis, within a same temporal and spatial context, which obviously has varied according to the geography and history of individuals and of the various cultural groups to which they belong – families, tribes, societies, civilizations" (D'Ambrosio, 2005, p.102). The emergence of these bodies of knowledge occurs because of the need for survival of these cultural groups in their environment and transcendence, spatial and temporal, to this environment.

In this sense, D'Ambrosio establishes a holistic and transdisciplinary character for the Ethnomathematics Program, creating conditions for "[...] recognition of the impossibility of reaching total and final knowledge" (2005, p.103), placing under suspicion the arrogance of modern science in establishing definite concepts. It is about seeking new ways of understanding the world, recovering the various dimensions of the human being, through the ethics of respect, solidarity and cooperation.

Thus, D'Ambrosio questions the existence of a single mathematics, because historically, different human needs, in different cultural contexts, had different effects of counting, measuring, finally, mathematizing. This means questioning the existence of a single mathematical language, marked by its universality and exactness. In this sense, the Ethnomathematics Program is interested in the narratives and social practices of individuals, which produce particular and specific subjects.

It is from this perspective that the intersection between Ethnomathematics and Wittgenstein's studies can be visualized. Denying the existence of a universal language makes it possible to deny a universal mathematical language. It means considering different mathematics, "[...] generated in different forms of life – which can be understood as language-games that have certain relationship and are not totally incommunicable with each other" (Kroetz & Lara, 2016, p.246).

Thus, with these assumptions, this study seeks to analyze the different language-games involving mathematical concepts present in different labor groups, in particular, fishermen, farmers, joiners and artisans, comparing the rules of use of these mathematical concepts in relation to School Mathematics.

For this, a bibliographical review will be made using Wittgenstein's ideas to give light to the sets of languages and rules existing in these work activities. This is a qualitative research of a theoretical nature, which will analyze a study of multiple cases, since each one of the activities studied in this research have no relation to each other.

In order to carry out the analysis, excerpts were taken from some research conducted by researchers of the "Grupo de Estudos e Pesquisas em Etnomatemática da Pontifícia

Universidade Católica do Rio Grande do Sul – GEPEPUCRS". These investigations were carried out adopting Ethnomathematics as a research method to understand the generation, organization and diffusion of the mathematical knowledge contained in certain cultural groups: Velho (2014), developed with joiners; Saldanha (2015), with fishermen; Kroetz (2015), with farmers; and, Rodrigues (2016), with artisans.

LANGUAGE-GAMES

In the book Philosophical Investigations of Wittgenstein appear numerous concepts referring to the "philosophical problems" of the language. Many of these concepts arise primarily to justify the changes in their conceptions since their first work.

Two key concepts in this change of stance are the forms of *use* and *meaning*, which plays a very important role in the theory of Late Wittgenstein. In this theory "the meaning of a word is its use in language" (Wittgenstein, 1999, p.43). The author abandons the idea that meaning was in the object itself, present in his first work, to give a pragmatic meaning to the meaning of objects.

For Condé (1998), the meaning of a word is determined by its use in different contexts and situations, that is, if we use a word in a given situation, it may not have the same meaning for other people in different contexts. And this is where the importance of the concept of *use* and *meaning* is shown. This is because for Wittgenstein language is intrinsically linked to the contexts of use, that is, the forms of life.

As Wittgenstein himself states (1999, p.35, emphasis added): "Here the term 'language-game' is meant to bring into prominence the fact that the speaking of language is part of an activity, or of a form of life". By referring to language as a part of an activity or a form of life, the author makes clear the diversity of distinct uses that can be made of this phenomenon. In addition, he emphasizes that our language is not, as it were, complete and definitive.

> Our language can be seen as an ancient city: a maze of little streets and squares, of old and new houses, and of houses with additions from various periods; and this surrounded by a multitude of new boroughs with straight and regular streets and uniform houses. (Wittgenstein, 1999, p.32)

It can be seen from this that language is not perfect and finished, and it will never be. Now, an ancient city has never been the same, it has undergone many transformations in the course of its history and in the course of the innovations that the world is undergoing. And so, it will continue to be. So, the comparison between an ancient city and our language. New "languages" are being incorporated into it as our society transforms. New meanings are given to words and objects according to their new uses. This is explained

in §23 of his work, where Wittgenstein (1999) emphasizes the diversity of forms that can present the language:

> There are countless kinds: countless different kinds of use of what we call "symbols", "words," "sentences". And this plurality is nothing something fixed, given once for all; but new types of language, new language-games, as we may say, come into existence, and others become obsolete and get forgotten. (p.35)

This construction creates the conditions to enter into a central concept of Late Wittgenstein's theory: the language-games. In discussing the empirical processes present in language, that is, in affirming that the meaning of a word is in its use, the author approximates this process to a game such as a child learns in his mother tongue and then calls these games "language-games" (Wittgenstein, 1999, p.30). Added to this, it refers to the processes involved in these practices: "I shall also call the whole, consisting of language and the actions into which it is woven, the language-game" (Wittgenstein, 1999, p.30).

Language-games form the basis of Ludwig Wittgenstein's thought in its maturity stage. This is because, in reflecting on his first theory, in which he sought an essence for language in relation to the world by emphasizing its constitutive logic, the philosopher now says that there is no universal language, but a set of different "languages" that are many different forms. There is not something common to the language phenomena, in which we use the same words to describe situations, but situations "*related*" to each other. (Wittgenstein, 1999).

To try to elucidate this question, Wittgenstein uses the concept of game. The word game refers to innumerable distinct thoughts about games. It can be a card game, a board game, a ball game. But in all these examples is there an essence, something common among all of them? Maybe in all of them there is a competition, but if the game is individual? There will be no competition between two or more people. Given this, how to explain to a person what a game is? For Wittgenstein, this is a concept of complex definition, since there is no essence common to all games: "One might say that the concept 'game' is a concept with blurred edges" (1999, p.54).

In relativizing this concept, the philosopher seeks to point out that games do not have a common limit, not even a limit for themselves. One can play a game with certain rules, but that do not limit all possible actions. Tennis, for example, is a well-governed game, "[...] but no more are there any rules for how high one throws the ball in tennis, or how hard" (Wittgenstein, 1999, p.53).

In this sense, Wittgenstein could not have found a better example to bring the philosophical questions of language, especially the concept of language-games, into something concrete. Games, whatever they are, have many common traits among them, but they disappear if compared to others. "Can see how similarities crop up and disappear"

(Wittgenstein, 1999, p.52). To these similarities the author uses the expression "*family resemblances*".

FOLLOW RULES

In *Late Wittgenstein's* theory, rules play a very important role, mainly because, for Wittgenstein, language develops through rules. But despite the importance, the author does not establish a closed concept of rule, but rather presents a great number of examples that aim to express what are rules and the different ways in which we follow a rule.

In §53 of *Philosophical Investigations*, Wittgenstein draws attention to the fact that how a language game can be determined by pre-established rules that have been taught to participants in this language-game, as well as rules of any game. Such rules can be established in different ways. Whether it is through the use of certain signs in the language-game or written in a table where signs correspond to certain elements. This second one, when used for the teaching of language, can be understood as a tool in the use of language.

Moreover, Wittgenstein emphasizes the fact that: "If we call such a table the expression of a rule of the language-game, it can be said that what we call a rule of a language-game may have very different roles in the game" (Wittgenstein, 1999, p.48). With this, the author indicates, so to speak, what role, or roles, that rule will assume in his theory. This is further elaborated in §54:

> The rule may be an aid in teaching the game. The learner is told it and given practice in applying it. – Or it is an instrument of the game itself. – Or a rule is employed neither in the teaching nor in the game itself; nor is it set down in a list of rules. One learns the game by watching how others play. But we say that it is played according to such-and-such rules because an observer can read these rules off from the practice of the game – like a natural law governing the play. (Wittgenstein, 1999, p.48)

However, the main question in Philosophical Investigations does not properly surround the concept of rule, but rather what it is "to follow a rule". But what can be understood by "to follow a rule" or "the rule by which he proceeds?". It may be: "The hypothesis that satisfactorily describes his use of words, which we observe; or the rule which he looks up when he uses signs; or the one which he gives us in reply if we ask him what his rule is?" (Wittgenstein, 1999, p.58).

As with the concept of rule, the author does not stipulate a concept for what is to follow a rule but seeks to elucidate by means of exemplifications that characterize the act of following a rule, presenting it as an indicator of direction (Wittgenstein, 1999). Moreover, for the author, "to follow a rule" has a pragmatic character, that is, it makes

sense when it is used in the practice it proposes. "Where is the connection effected between the sense of the expression 'let's play a game of chess' and all the rules of the game? – Well, in the list of rules of the game, in the teaching of it, in the day-to-day practice of playing" (Wittgenstein, 1999, p.92).

ANALYSIS OF THE LANGUAGE-GAMES PRESENT IN CERTAIN LABOR ACTIVITIES

Considering Wittgenstein's theory one can affirm that the mathematical practices performed by different groups of individuals in their labor activities can be understood as a language-game that are delimited by rules of use in their practices. Considering Mathematics as a cultural product, Condé (2004) states that it can be treated as a language-game. In this sense, Wanderer and Knijnik states that:

> Thus, academic mathematics, school mathematics, peasant mathematics, indigenous mathematics, in short, mathematics generated by specific cultural groups can be understood as language-games associated with different forms of life, aggregating specific rationality criteria. (2008, p.558)

Such rules are explicitly exposed or can be apprehended by observing someone using them and modifying them as you use them. Moreover: "One can also imagine someone's having learnt the game without ever learning or formulating rules. He might have learnt quite simple board-games first, by watching, and have progressed to more complicated ones" (Wittgenstein, 1999, p.38). Analyzing some studies carried out within the scope of GEPEPUCRS, with Wittgensteinian lenses, it is possible to perceive in some forms of life the production of different mathematical knowledge, which, when treated as language-games, presents examples of rules that are followed without a manual , only by observing "other players playing", or passed from father to son, from generation to generation, or through practice from the teachings that took place within the community. These studies show the generation, organization and diffusion of knowledge produced within cultural groups, which, when not being legitimized, as well as School Mathematics, are often marginalized outside their context.

It is noteworthy that, according to Lara (2001):

> Even with the universality of mathematics relativized by the most current discourses in the field, Academic Mathematics – or, if one wishes, Formal Mathematics constituted by modern western discourse – continues to be used as a selection tool for practically all areas of study or work, and not only for access to positions that necessarily require their knowledge as a prerequisite. (p.25)

Using a Foucauldian perspective, Lara (2001) shows that by means of control devices, the Mathematics discipline, that is, School Mathematics produces a certain way of thinking about the student. Modern tradition attributes to the discipline of mathematics "[...] the power to 'develop thought'", instructing teachers to regulate their students by "[...] learning to ration, to think 'correctly', that is, to think mathematically about 'things' [...]" (Lara, 2001, p.29). In this sense, both the present discourse in Academic Mathematics and in School Mathematics were historically constituted with a desire for truth and by means of control devices that make it dominant to this day (Lara, 2001). It is a legitimate discourse that can exclude anyone who does not use its rules.

According to Condé (2004), grammar does not contain an essence in itself, because "[…] the rules that constitute grammar are inserted in social practice. A rule can only be effectively constituted as such by social *praxis*. Grammar is a social product" (p.89). In this sense, rules are created or invented incorporating rationalities that emerge in a form of life, which is not always the same emerging from another form of life. The author states that "[...] Wittgenstein 'proposes' grammar and language-games as a rationality that is forged from social practices in a form of life and no longer rests on ultimate foundations" (Condé, 2004, p.29).

However, when comparing language-games involving mathematical concepts present in these life forms, in particular labor activities, to the games present in School Mathematics it is possible to recognize their grammar and perhaps to identify possible relationship between their rules. Therefore, the following researches of GEPEPUCRS researchers, developed during the course of master's degree in Science and Mathematics Education:

a) Velho (2014) with the research question "How does ethnomathematics used as a teaching method contributes to the learning of geometry?", in particular through the ethnomathematical knowledge of a joiner;

b) Saldanha (2015), "How do the processes of generation, organization and diffusion of the knowledge used by the artisanal fishermen of the Ilha da Pintada occur?";

c) Kroetz (2015), "How did the knowledge of settlers descendants of Germans from Santa Maria do Herval be generated, organized and diffused in their social group?";

d) Rodrigues (2016), "How were the mathematical knowledge involved in the making of marabaixodrums in the Quilombo community of Curiaú, located in Amapá, generated, organized and diffused?"

The studies corroborate the generation of knowledge through observation, experimentation and transmission from father to son. This is evidenced in Saldanha's (2015) research by means of two research subjects: "Fisherman 3: *My father doing what I'm doing here, I was close, I was looking and I was picking and doing*" and "Fisherman 1: *I learned from my father, I did not know, I took the materials for him and he did. I learned by looking*" (p.75).

In the studies by Kroetz (2015), conducted with settlers of German descent from the Vale do Rio dos Sinos region located in the center of the state of Rio Grande do Sul, it is also possible to notice, in the speech of one of the interviewed settlers, this generation of knowledge. in their daily practices and the diffusion of this knowledge passed from father to son:

> Q: But what about school, was it important to you?
>
> EC: Yes, it was. But I learned more by practicing than even the settlers here know?
>
> Q: I know.
>
> EC: In the fields, almost everything is learned in practice. We learned nothing of what we use in the fields at school. At school I learned the basics yes, but I learned to apply what I knew on a daily basis, getting better and learning from my parents. (Kroetz, 2015, p.85)

In Rodrigues (2016) research, developed with artisans from a Quilombo community, whose activity is the production of marabaixo "drums", these examples are evidenced in the speeches of the artisans:

> "I learned to build the 'drums' with my father, many years ago. I started practicing even when I was 25 years old, when I lived in Laguinho". (A06). And "My father learned to make the 'drums' with my grandfather, my grandfather with my great-grandfather, and so it was, this story is a long way off. (A78)". (Rodrigues, 2016, p.58)

In the examples presented it is evident how the knowledge about the production of "drums", from generation to generation, from father to son within the same way of life is acquired and transmitted. In other words, it is perceived that the rules governing the uses, responsible for the way these "drums" are produced, are transmitted from generation to generation, they are communicated to the descendants of the families who will own these rules and will be responsible for transmitting them, to communicate them, to spread them so that, in this way, this form of life and its activities continue to exist.

In the context of this study, what is sought is to highlight the rules of use of mathematical knowledge existing in the language-games of the subjects studied in the studies mentioned previously, since the focus of these studies was to analyze the mathematical knowledge, from a Ethnomathematics perspective, in activities of cultural groups, groups of workers. In this way, we seek to identify family resemblances existing between the Mathematics present in the subjects' practices and the legitimized Mathematics taught in the schools, the School Mathematics, as well as the rules that determine their uses.

As previously mentioned, Wittgenstein introduces the concept of "family resemblances", anchors its exemplification in games, like a card-game, boards, ball, in short, any game, with its own rules and modes of playing. In comparing the immensity of existing games, the author infers that there are many similarities between them, but also, many differences and it is possible "[...] see how similarities crop up and disappear" (Wittgenstein, 1999, p.52).

Approaching the idea of Wittgenstein in the context of this study, one can understand that the various mathematics present in the different daily practices of subjects belonging to the same form of life have their own rules of use, and that they appear to one another and thus , one questions the possibility of appearing with the School Mathematics.

In the studies of Velho (2014), made with joiners from the city of Gramado-RS, the author seeks to evidence the mathematical knowledge present in this profession. These knowledges, if compared to School Mathematics, involve the contents of Geometry.

However, not all workers have a mathematical knowledge produced by School Mathematics. As the author herself comments: "Joiner C is 59 years old, has been working for over 25 years in the profession and studied until the 2nd grade of elementary school, claiming to have only literate" (Vellho, 2014, p.73). But being tasked with producing a prototype of a wardrobe, the worker demonstrates a structured logical reasoning, proving that his knowledge and the rules acquired in practice are worth as much for the practice of his profession as the knowledge acquired at school:

> [...] if the wardrobe is going to be 180 in length, that's the size of it ready, inside it will get smaller, because it will discount the thickness of the wood. If you have breakdown further decreases… 180 minus 4 and a half, because I can divide the wardrobe in half, left 175 and a half, in two divided into about 87 and a half on each side. This for length, at the time also need to think so. And that's where I'm going to calculate shelves, drawers and other things you need to put. (Velho, 2014, p.85)

Still in relation to the prototype design makes the analysis on the doors and shelves that should contain in the wardrobe:

> Then you have to see how these doors look, if you want to make a set of doors or two that open on their own, because the spaces of the rooms change, it will give a room or two, because you have to consider the thickness of the wood that goes into the room. division. Using the 1 and a half will vary the discount on total partitions. So, if it is a set of doors totaling 90 plus 4 and a half of partition, inside the doors would be 87 cm and one go to the open shelves of close to 38 and a half. (Velho, 2014, p.76)

When selecting some language-games used by joiners, there are perceptible rules of uses that do not always have family resemblances to the rules of use of School Mathematics. This can be exemplified in the following talk of one of the joiners, regarding the calculation of materials that would be necessary for the construction of a furniture:

> [...] have to leave a few centimeters left, so that when cutting the wood, stay as long as it has to stay. Everything turns on a fixed dimension, which is the measure of the furniture, and from there on a greater measure you work until you reach it. (Velho, 2014, p.78)

It is noticed that the rule of use necessary to its practice requires that the measure must allow an approximate increase so that the quantity of materials is sufficient. That is to say, while School Mathematics excels in exact contours so that an exact calculation can be made, in this particular form of life the rule is to establish a surplus so that one can go "working" with it until reaching the ideal measure.

Another example is the production of networks, where the fishermen themselves are responsible for producing their instruments of work. There is a family resemblance in the rules of mathematics, as can be seen in the excerpt taken from Saldanha (2015, p.72): "Fisherman 1: *The time the fisherman is there in the river, he puts three fingers and one piece remains... of course it has a bigger finger than the other, but the staff uses that reasoning there*". In this example, according to the author, "It is perceived that even assuming differences in the size of the fingers of each individual, the fisherman demonstrates to be convinced of the validity of the measure once all the fishermen use it" (Saldanha, 2015, p.72).

It is noted that in talking about the production of the fishing net, Fisher 1 notes that not all workers belonging to this form of life have the same measure in network production instruments, in this case their own hands. But the measure "three fingers" gives them a valid approximation for their productions. Unlike School Mathematics that has universal metric systems and that is independent of the subject that is dealing with this measure, it will be the same in any form of life. School Mathematics has precise, indisputable rules that must be known so that it can be used in the form of life established by the school. The rule is there as a "[...] instrument of the game itself" (Wittgenstein, 1999, p.48).

It is possible to notice, in these two examples, that there is no formalism and rigor present in the language-games of School Mathematics. Explaining in the workers speech a mathematics generated by another grammar that uses other rules, in this case estimation and rounding, a type of rationality distinct from that which constitutes the School Mathematics.

In Kroetz's (2015) research, one can perceive language-games proper to that form of life, and thus identify rules of use of these games which, in turn, depend on the practice

performed by them. In an example drawn from the practice of one of the settlers, one notices the use of the concept of proportion:

> EA: Ahh, and for each half bag of fertilizer we had, we would plant a sack of potatoes, so for every 10 sacks of potatoes picked, there would be about 5 sacks of fertilizer. But it depends, it's not always so right.
>
> P: So sometimes it changes?
>
> EA: Yes, it depends on the earth, on time, it is not always the same. (Kroetz, 2015, p.108)

When referring to the relationship between the amount of fertilizer used for the amount of planted potato, the settler (EA) uses the concept of proportion, present in the grammar that operationalizes the School Mathematics, but when used in its practice can assume a character variable. At first glance a strong family resemblance between both rules is seen. When the settler deals only with two variables, be they amount of fertilizer and amount of potato, the rule present in the form of settler use resembles the rule between directly proportional quantities present in the grammar of School Mathematics. However, when he states, "*But it depends, it is not always like that*" and relativizing the rule when in the presence of other variables such as land and time, may be referring to another type of rationality pertaining only to its form of life.

That is, its rules of use vary according to its use and the processes involved in planting. Thus, it is possible to perceive that they are different grammars, although have relationships between some rules.

In the same research, Kroetz (2015) presents another somewhat clearer example of the use of rules, now in the discourse itself, comparing the rules of School Mathematics and mathematical knowledge used in their practices, as suggested by excerpts from settlers EA, EB and EC:

> EA: To write I know how to add one underneath the other, you know? Add and subtract as I learned in school.
>
> Q: When do you write always place one underneath the other?
>
> EA: Yes, if I'm going down and I'm missing a loan, we learned it at school.
>
> But I always had my bills ready in the head before going on sale (market). Prices were always the same at one time or another, it was not like now ... it increases price, it decreases ... everything was always the same price.
>
> EB: We used to do the math in the head, to sell, to buy, we knew it, the father taught it all.
>
> EC: The accounts like this did not have much secrecy, it was in the 'old system of yesteryear.
>
> Q: For example?

EC: It was not so accurate; it was in the eye.

Q: And how did you manage to control everything you earned?

EC: Control was all with me, but I did not need to write down anything, I knew everything in the head. (Kroetz, 2015, p.109)

The operations performed in these three cases vary from person to person, but as you can see, there is a strong family resemblance between the three sets of language when referring to the mental calculation, evidencing that the settlers use a very similar grammar. However, only the games evidenced by settler EA uses rules present in the grammar of School Mathematics.

In his discursive practice, EA settler admits the use of mathematics learned in school, when he needs to write the account. It is possible to see in this case that he uses different rules when he uses writing and when he only uses orality. The rules that compose the grammar of School Mathematics in relation to the resolution of algorithms have been apprehended by the settler EA, in such a way that it is able to repeat them with the same precision that is still taught in schools by some teachers: "one beneath the other"; "If I'm going down and I'm missing, I'll borrow it". But when prices were always the same, they did not vary, had their accounts "ready in your head". Thus, it is possible to have other rules in this case.

In the case of the EB and EC settlers, the calculations were made "in the head", "in the eye", "in the old system of yesteryear", that is, rules are followed in a peculiar way, are proper to that form of life, those practices. The rule used to do the calculation is not explicit, but the rule is followed according to what was learned from the members of that particular form of life, as EB and EC say respectively: "the father taught all this"; "Was in the old system of yesteryear."

According to Kroetz (2015, p.110): "Like the unit of measure used by the settlers interviewed, the examples narrated by them relate to language-games that use rules such as orality, proportional thinking, and approximation in their forms of life". That is, they are rules present in those forms of life that may or may not have family resemblances with School Mathematics.

In the studies of Rodrigues (2016), in the discursive practices of the participating artisans many terms appear related to concepts present in the geometry that is part of School Mathematics. However, according to the researcher it was possible to observe that the mathematical concepts related to geometry, although they are present in the practices, does not mean that the artisans have in mind such concepts. In the words of Rodrigues (2016, page 67):

[...] artisans believe that the mathematics they use is related only to the budget of the material. However, it was possible to verify that there are other concepts

that can be approached, concepts studied in School Mathematics, as well as other knowledge, is part of the "drums" construction process.

That is, they make use of terms that resemble those of School Mathematics, but do not recognize that they have learned it in school. Examples of these terms can be seen in the following statements by the artisans: "*I assemble the cylinder* [...].*"*; "[...] *we use the rules for cylinder mounting*"; "*For the cylinder I have to set the rim, I'll take a ruler like that.* (A41)" (Rodrigues, 2015, pp.63, 64, 65). In pronouncing cylinder although the artisan refers to the shape of the "drum", he is referring to the assembly of the circumferences of its bases. We can see here the use of the same term that assumes very different concepts when compared in different forms of use, be they the making of the "drum" and the teaching of mathematics in school. They are different geometries, the first making use of the term cylinder in a two-dimensional view, while the second defines cylinder as a three-dimensional solid.

The same can be said with respect to the term ruler. The rulers that refer to an exact measuring instrument in the school are called by the artisan as bundles of wood cut into strips without a predefined precision, serving to establish the height of the "drum". It is evident in these discursive practices the use of similar language games constituted by rules that do not present weak family resemblances.

These few similarities can be seen in the final product, since the "drum" is made in cylindrical format. But at the same time as these concepts are exercised in the construction of the "drums", it may be noted that the rules followed by these artisans depend on their practice, and thus, consequently, the rules do not give a precise limit to their uses. That is, the use varies according to the need, the material used, finally, to different factors present in these practices, unlike the School Mathematics that imposes precise limits on the uses of the rules.

This can be seen in the following example which contains the artisan's speech A and the study author's comment: "[...] *this measure I even sought*, [...] (A43). The circular-shaped wood that was removed from crates was an alternative that artisan A found to have a measure of what would be the basis of his 'drum'" (Rodrigues, 2015, p.65).

Another example can be seen in another speech by the artisan A: "*marking the rulers I am doing until complete, no problem if I make a thinner ruler because afterwards, I will seal* [...]" (A84). In this case, the artisan refers to the placement of wood around the arches of the cylindrical box, which is called in the School Mathematics side area of the cylinder. It is noted, through his language-games, that he does not bother to find the side area and complete it completely with the wooden "rulers" since this will be later solved. It is possible, therefore, to identify in these games some rules, in particular the estimation, when cutting the rules, that differ from those found in the grammar that constitutes the School Mathematics, because in that case these rules are variable, they do not have precise limits, they vary according to with the needs encountered by artisans.

SOME CONSIDERATIONS

At the beginning of this study it was established the objective of analyzing language-games involving mathematical concepts present in certain labor activities, as well as the rules of use of such concepts, comparing them with the existing rules in School Mathematics.

Wittgenstein's perspective was based on the analysis of Wittgenstein's work, especially in his later phase, and the introduction of new concepts in the field of philosophy, which serve to avoid the search for an essence, in particular, the language. Concepts such as forms of life, language-games and rules arise to highlight the different ways that human beings can interact with the world and deny the existence of a universal language, creates conditions to analyze studies carried out with an ethnomathematics perspective with new lenses.

Thus, it was possible to bring to light the different mathematical knowledge existing in the discursive practices of different groups of individuals, particularly workers, as well as the language-games practiced by them consisting of rules that form a specific grammar that presented weak or strong similarities of with the grammar that operates the School Mathematics.

Were considered for analysis forms of life studied by researchers of GEPEPUCRS, who use Mathematics in their activities: joiners; fishermen; farmers; artisans.

When considering the language-games existing in the mathematical practices present in these professions, it is possible to show that some games recognize some rules that have strong family resemblances in relation to the games that make up the School Mathematics when they need a written mathematics, farmers when they need to assemble the accounts to calculate, or when they deal with two directly proportional quantities.

However, some professionals, in this case the artisans, when expressing orally their mathematical knowledges use terms present both in their grammar and in the grammar that generates the School Mathematics, but with rules that show a weak relationship between such grammars.

In addition, it presents examples of the use of mathematical knowledge without the formalism and rigor present in the language-games of School Mathematics, it is another way of doing mathematics generated by another grammar that uses other rules, in this case the estimate and rounding, present in the language-games of joiners and farmers.

In most cases, a type of rationality distinct from that which constitutes School Mathematics is evident but is effective in that form of use.

AUTHORS CONTRIBUTIONS STATEMENTS

L.T.O and I.C.M.L. together conceived the idea presented in the present study. L.T.O. was responsible for developing the theory and analysis of the works used for writing the

article. I.C.M.L. was responsible for supervising and guiding the work development. Both authors discussed the results, contributed to the writing and made appropriate adjustments to the finalization of the manuscript.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article because no new data was created or analyzed in this study.

REFERENCES

Condé, M. L. L. (1998). *Wittgenstein linguagem e mundo*. São Paulo: Annablume.

Condé, M. L. L. (2004). *As teias da razão:* Wittgenstein e a crise da racionalidade moderna. Belo Horizonte: Argvmentvm.

D'Ambrosio, U. (2005). Sociedade, cultura, matemática e seu ensino. *Educação e Pesquisa, São Paulo*, *31*(1), 99-120.

Kroetz, K. (2015). *Etnomatemática e relações de poder:* uma análise das narrativas de colonos descendentes de alemães da região do Vale do Rio dos Sinos (155f.). Dissertação de Mestrado, Programa de Pós-Graduação em Educação em Ciências e Matemática – Faculdade de Física, Pontifícia Universidade Católica do Rio Grande do Sul, Porto Alegre.

Kroetz, K., & Lara, I. (2017). Jogos de linguagem e formas de vida: um estudo com colonos alemães do Vale do Rio dos Sinos. *Revista Areté – Revista Amazônica de Ensino de Ciências*, *9*(18), 244-253.

Lara, I. C. M. (2001). *Histórias de um "lobo mau":* a matemática no vestibular da UFRGS (242f.). Dissertação de Mestrado, Programa de Pós-Graduação em Educação – Faculdade de Educação, Universidade Federal do Rio Grande do Sul, Porto Alegre.

Rodrigues, Q. D. F. (2016). *A construção de "caixas" de marabaixo na comunidade quilombola do Curiaú:* uma abordagem Etnomatemática (137f.). Dissertação de Mestrado, Programa de Pós-Graduação em Educação em Ciências e Matemática– Faculdade de Física, Pontifícia Universidade Católica do Rio Grande do Sul, Porto Alegre.

Saldanha, M. A. (2015). *Histórias de pescadores*: uma pesquisa Etnomatemática sobre os saberes da pesca artesanal da Ilha da Pintada-RS (153f.). Dissertação de Mestrado, Programa de Pós-Graduação em Educação em Ciências e Matemática – Faculdade de Física, Pontifícia Universidade Católica do Rio Grande do Sul, Porto Alegre.

Velho, E. M. H. (2014). *Aprendizagem da geometria:* a Etnomatemática como método de ensino (152f.). Dissertação de Mestrado, Programa de Pós-Graduação em Educação em Ciências e Matemática – Faculdade de Física, Pontifícia Universidade Católica do Rio Grande do Sul, Porto Alegre.

Wanderer, F., & Knijnik, G. (2008). Discursos produzidos por colonos do sul do país sobre a Matemática e a escola de seu tempo. *Revista Brasileira de Educação,* 13(39).

Wittgenstein, L. (1999). *Investigações Filosóficas*. Trad. José Carlos Bruni. São Paulo: Nova Cultural.