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Underlying Theories for use of Digital Technologies in Mathematics Education

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ABSTRACT

In this paper, we aim to build on conceptual framework to encourage readers to reflect on the educational scenarios of this digital era, especially in relation to Mathematics Education. We will address research that describes challenges to schools and teachers added by the technological environments. Mathematics Education could employ a wide range of technological resources and researches have shown some positive results with their uses. However, the sole access to these resources does not guarantee meaningful learning by students. We consider that distance learning could allow opportunities for lifelong learning and paths for continuing education to promote a renewal in teaching practices. Therefore, it's important for all teachers to reflect on their competencies to use technologies in their teaching practices. This requires studying and reading about the subject, related theories and papers of which some presented in this paper. We will indicate the theoretical-methodological potentialities in teaching and learning of mathematics in this digital era. We hope that the indications of underlying theories along with the texts can contribute for overcoming such challenges.

Keywords: Mathematics Education, Innovation, Digital Era, Digital Technology, Distance Learning.

Teorias Subjacentes para o uso de Tecnologias Digitais na Educação Matemática

RESUMO

Neste artigo, pretendemos construir um quadro conceitual para incentivar os leitores a refletir sobre os cenários educacionais dessa era digital, especialmente em relação à Educação Matemática. Abordará pesquisas que descreverá desafios para as escolas e professores acrescentados pelos ambientes tecnológicos. A Educação Matemática poderia empregar uma ampla gama de recursos tecnológicos, e pesquisas têm demonstrado alguns resultados positivos com seus usos. No entanto, apenas o acesso a esses recursos não garante a aprendizagem significativa por parte dos alunos. Consideramos que o ensino a distância pode permitir oportunidades para a aprendizagem ao longo da vida, caminhos para a educação continuada e promover uma renovação nas práticas de ensino. Portanto, é importante que todos os professores reflitam sobre suas competências para usar tecnologias em suas práticas de ensino. Isso requer estudar e ler sobre o assunto, teorias relacionadas e trabalhos os quais alguns são apresentados

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neste artigo. Indicaremos as potencialidades teórico-metodológicas para o ensino e para a aprendizagem da Matemática nesta era digital. Esperamos que as indicações de teorias subjacentes, ao longo do texto, possam contribuir para superar estes desafios.

Palavras-chave: Educação Matemática. Inovação. Era Digital. Tecnologia Digital. Educação a distância.

INTRODUCTION

It is important to see that the changes in the so-called Digital Era alter relationships among people, our access to information, and our daily lives. These changes and alternations also started to emerge in Education. It is also expected that power will concentrate in the hands of people who has access to data and information. Each society was developed strategies and rules on how to access information and knowledge and how it is distributed and applied. Technological advances do not necessarily determine access to information, but it is social need for its democratization that determines new categories of time and space, turning them into virtual realities.

> The invention of press also presented opportunities to scholars or, in general, to anyone seeking information. In the Middle Ages, writers suffered due to a lack of books. In the 16th century, on the other hand, the number of books available was large enough to create "information retention" and "information management" problems, with which we are familiar in the Internet Age. In the 1500s, there were around 13 million books in Europe.
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> The existence of printed books made it possible that information could be found more easily as long as the right book was found. An Italian writer complained in 1550 that there were "so many books that we don't even have time to read their titles." (Burke, 2000, p.16)

Similarly, to the knowledge not integrated in books today information can be lost or can be synthesized in such a large scale. The raw material for developing new technologies is data or information, each technological innovation can instantly become the raw material for the next development cycle, contributing to increasing speed of the innovation process. (Kenski, 2007, p.35). However, when it comes to "innovation," regardless of its field, such word can always cause the feeling of something new, raise uncertainty, and, at the same time, offer new possibilities for further innovation. In the relation between technologies and education, we see students need to incorporate information and knowledge as well as become proficient in using technologies to be ready for new jobs in the future.

In the 17th century, according to Burke (2000), the rise of typography created new jobs, such as typographer, cataloguer, editor, among others, causing an intellectual division of work and leading to the fall of the ideal of universal knowledge and learning. Likewise, in the digital era, not integrating technologies into the educational process is disabling opportunities for students in their future job prospects and knowledges in a new era.

It is important to note that the mere existence of technologies that allow the quick access to information and communication does not assure the construction of knowledge and the readiness to be used in society. This depends on people, rather than technologies. It is in this aspect that the role of technologies in education stands out. The use of technology should be directed to build knowledge and assist learning. Thus, the use of technologies in education is not an end, but rather a tool for cognitive development.

It is also key to understand that if technologies are used simply to transmit information that have already been developed, in the learning process, they will be at the service of a traditional educational without enabling any innovation or changes by stakeholders in education.

THE ROLE OF SCHOOLS IN THE DIGITAL ERA

What are the roles of schools in the Digital Era? Schools are places intended to develop citizens who can become valuable members of societies and to contribute improving the quality of their own and other people's lives. Therefore, it is understood that everyone responsible for its creation and existence should provide proper material and human conditions so that school meet the requirements of our era.

The role of educational managers in this context changes and responsibility for creating environments meet these new requirements necessitates the on-going training of teachers. However, integrating new technologies to professional development of teachers so that they know or use technological equipment is not enough. Such integration must be present in their initial and permanent training, a condition that is indispensable for future teacher capacity development. Emerging technologies needs to be learned, either by means of self-initiative or through courses.

In our society, teachers need to engage students in the craft of learning, i.e., lead students towards the operations carried out in the learning processes. Learning to learn is the basic goal of education. To that extent, teachers also need to learn throughout their entire lives. Preparing teachers for such missions necessitates good educational policies, defined for a teacher development that meets such needs. Thus, it is important to promote an incentive to overcoming that failed scenario – both in a theoretical and in a practical point of view – regarding the pedagogical and educational aspects of the use of technologies.

> It is necessary to seek information, attend courses, ask for the assistance of those who are more experienced, at last, use all possible means to learn how to relate to innovation and go even further, to start to create new methods of use and, from that point on, generate other uses. Such new learning, when put into practice redirects all our processes of discoveries, relations, values and behaviours. (Kenski, 2007, p.44)

What are the teaching competences required for such challenges? Changing teacher training are needed in a way as to enable innovation in the digital era, meet social demands, and support the interests of students. School culture is often out of tune with the innovations of a technological society.

European project "Profiles in ICT for Teacher Education", 2002, defined the profile of teachers in Information and Communication Technologies for the 21st century(Table 1):

The proposal below is the result of some initiatives and projects in which Program Nónio-Século XXI was involved (Profiles in ICT for Teacher Education, 2002, p.6).

Table 1
Profile of the Teacher in Information and Communication Technology.

ATTITUDES		Welcome technological innovation. Capacity to adapt/change the role of the teacher. Student-oriented teaching. Teacher as mediator and facilitator in communication.
COMPETENCES	Teaching in general	Teaching methodologies with the ICT. Planning of classes with the ICT. Integration of media. Monitoring/evaluation. Evaluation of ICT contents. Safety, ethics, and legal matters regarding the use of ICT.
	Teaching of subject	Scientific update. Research. Evaluation of resources. Integration in scientific community. Connection to possible partners. Use of materials in other languages. Participation in newsgroups.
	ICT Competences	Update of knowledge in ICT/platforms and tools. Familiarization with tools that are good for: Communicating Collaborating Researching Exploring Collecting data Storing data Expanding knowledge Integrating tools

The Bulletin of Education in Brazil (Fundação Lemann, 2009) – obtained from a broad study on the educational reality in the country – shows that:

> To generate changes in schools and classrooms, it is essential that such professionals be able to identify the problems that prevent their students from mastering certain skills and competences and, based on that, reformulate their teaching practices. One of the paths to reach such goal is providing teachers with technical resources

> – educational materials, curricular guides, training courses – that help them in that goal and that, therefore, are in line with the competences measured by the evaluation system. That is still not present in a systematic fashion in Brazil. (Fundação Lemann, 2009, p.24)
>
> We can observe in the Bulletin that in spite of that huge effort by governments, the impact of the on-going education is not seen in students' performances. (Fundação Lemann, 2009, p.32)

Teachers currently in training were already born in such technological society. Therefore, current challenges are to able to make subjects involved in the school context reflect upon their practices, research, understanding learning styles of their students, and start the use of technologies.

It is important to consider that our society requests changes to the school context and the same knowledge derived from technologies may also lead to changes in teacher education and school programs. Regardless of the type of change suggested, it is important that career plans are focused on improving the learning of students, have the adherence of teachers, and promote a greater balance between the rights and duties of education professionals (Fundação Lemann, 2009, p.35).

One aspect to consider within such changes is the use of technologies to enable access of students at any time and place, e.g., in the classroom itself by means of using tablets or mobile phones with Internet access.

The day is coming when students will create an environment with computers, tablets or phones in their classrooms with no need to go to the lab. That is the ideal access: any time and any place. Teachers will have to be immersed in such new environment and be prepared for that new situation.

Lifelong, on-going teacher education is also indispensable, which means that the conditions that enable it permanently should be structured, guided, socialized, and consolidated as the development continues.

DISTANCE EDUCATION AS AN INNOVATION

It is in this scenario that distance learning has its role not only for the democratization of access to education, but also for offering a life-long teacher education opportunity. With the arrival of technologies, distance education has renewed itself and is currently supported in digital environments with several different approaches.

There is still plenty to investigate regarding the possibilities of distance education in its integration into schools and in the production of knowledge, but it is certain that the possibilities of technologies enable interactivity and dealing with specific and primordial situations for the training of teachers. In addition, it can assist in the development of competences and raised autonomy of teachers.

Thinking about distance education may also be part of the process of considering it as a "new thing" or innovation and cause feelings of unfamiliarity, uncertainty, and possibilities in people.

Writer Everett Rogers (2003), in his theory Diffusion of Innovations, states that an "innovation" is not necessarily something new, but rather something that represents a "new thing" for a person or social system. When it comes to distance education, innovation presents itself as a "new thing" to the extent that everyone involved somehow start or are inserted in the process of teaching such modality whether or not they have some knowledge about it.

Nowadays, distance education may take place in so-called virtual learning environments. "Virtual" because they are accessed via the Internet and computers, and "learning" because they enable uses of several technological tools that may enable the development of knowledge. Such environments enable to offer learning-cantered and constructivist perspectives, where the interaction with others, reflection, and construction of knowledge in a collaborative way are core aspects. If planned, created, and adapted to meet the needs of a local reality, distance education may favour the construction of knowledge in all school and learning levels and a new role to be played by the teacher in such new reality.

We currently have a scenario that aims to assure access to information. Not long-ago technological resources were independent and did not communicate. However, currently interactivity is core of technologies. Interactive digital TV is already a reality and provides access to the Internet, mobile phones enable m-learning[1] and distance education benefits from all those technological opportunities. With such advances, educational possibilities can be integrated into education and maintain the focus on the consistent training of future teachers who are ready to teach and keep learning throughout their teaching careers.

> Whether through virtual or tangible means, whether distance or locally, I am sure that the following concepts should permeate the school of the future: interactivity, collaboration, proximity, and presence (not necessarily physical). Interactive technologies will play a key role in that evolution. (Tori, 2010, p.22)

TRAINING OF TEACHERS IN THE DIGITAL ERA

The training of teachers must be reconsidered: focusing on new methodologies with the use of technologies and support theoretical conceptions and consolidated research. In that aspect, distance education or "no distance" education, by means technologies can bring people together, is an important for on-going training paths that may enable and promote a renewal of teaching. With the convergence of technological tools, distance

[1] m-learning: learning with mobile technological tools.

education could become a path to be more accessible to people and may change current paradigms of on-going education.

> However, it is difficult to quality on-going education courses, since the variation between them is huge: there are programs with pre-established number of hours and structures, but also those intended to only occupy the collective work hours foreseen in a teacher's career. (Fundação Lemann, 2009, p.32)

The current education scenario is way far from the ideal one and shows that even though technology is a part of everyone's everyday life, it is still far from being effectively integrated into school environments – due to the lack of preparation of its managers and teachers. Technology in schools and in classrooms should be something as natural as using the mobile phones that quickly became accessible for a large number of people.

The technological resources found in many Brazilian schools, if any, are not properly managed and are quickly outdated by the rapid development of technologies.

> If, on the one hand, it is important that schools have more power to decide, on the other, to exert such autonomy in a competent fashion, they need to have technical capabilities and adequate responsibilities. In Brazil, such empowerment process is made more difficult by a group of factors which deprive schools of the needed sense of authority and responsibility: legal limitations, little tradition of family participation, lack of technical qualification of the faculty, and principals overloaded with non-educational attributions. (Fundação Lemann, 2009, p26)

It is a never-ending race and only time will bring changes that are both significant and natural. Such changes could only happen with personal and intellectual growth, teachers should become more and more committed not only to their learning processes and the development of their competences, but also to their own learning and the development of their students.

MATHEMATICS EDUCATION IN THE DIGITAL ERA

In the digital era, technological resources are becoming increasingly available to support mathematics education. Interactive software, open learning resources, applets, hypertexts, Internet portals, blogs, podcasts, videos, simulations, games, virtual learning environments, virtual reality, augmented reality, and other resources that enable action, reflection, and interaction are available and at the reach of teachers, parents, and students.

The availability of a wide range of technological resources could result positive effects in classrooms, however, learning environments in schools changing slowly.

Research on teacher training with technology often not reaches classroom and teachers. Even with ample access to such resources, there is no guarantee that significant learning will take place, since that depends on strategies brought by teacher previous practices and experiences.

How can Mathematics Education take advantage from technological resources? How to "speak" or statistically represent on the board graphics of functions using the students' imagination? How to use Mathematical language in a distance learning environment? Those are questions where technology can have a key role as it may allow to overcome methodological barriers to effective learning. Technologies, in addition to enabling situations that simulate reality and a significant and challenging learning could help students to understand mathematics.

According to Dubinsky and Tall (1991), all methods in which computers are used in research are potentially available for teaching and learning advanced mathematics. The authors observed that students may learn to program to deal with certain types of problems or use software as an environment to explore ideas. In their research, Frota and Borges (2004) identified some conceptions of teachers on the use of technology in Mathematics Education influenced by the knowledge, safety, and experience of their use of technology and their personal and teaching experiences.

> The first conception, which we name consuming technology, is related to the arguments that essentially support new Technologies and ICT are powerful resources to teach and learn mathematics. The visions included in the second conception, which we name integrating technology, corroborate that by appropriating new technologies and ICT, turning them into tools and cognitive instruments, teachers and students change the way they do mathematics and change the way of thinking mathematically. Some of the visions that underlie such conception advance by stating that the new technologies and ICT's change the mathematics taught, done, and learned. We add a third conception, not identified in literature and which we name "mathematizing" technology, associated to the ideas that technologies and ICT, in addition to performing the roles of teaching and learning resource, as well as thinking tool and instrument, may become the sources to renew curricular approaches to renowned themes in basic and university Mathematics Education, as well as new sources of new themes for the mathematics program. (our highlighting). (Frota & Borges, 2004, p.3)

We agree with Frota and Borges (2004) when they observe two aspects in the conception of consuming technology: to automate tasks and to change the focus of tasks. They state that any of such conceptions may represent an advance in educational terms, to the extent that the focus of the teaching of mathematics may no longer be operational or procedural to take a more conceptual perspective (Frota & Borges, 2004, p.5).

Looking at the work of teachers, inserted of such conceptions, it is possible to identify actions in classrooms which enable an understanding of technologies in

teaching and ways to integrate technology as a didactic-pedagogical tool in the learning environment with theoretical elements that may subsidize such actions.

The various possibilities in the use of technologies in teaching will configure the conceptions in which teachers are integrated, maybe, such self-knowledge may show their paths for on-going improvement and training that we suggest in the next section.

PATHS FOR OVERCOMING CHALLENGES

For teachers, it is not easy to be aware, clear and have solid foundations of their competence to use technology and also utilise mathematical knowledge that may rise with its uses, as stated by Ponte:

> The deep effects such technologies have had on countless areas of social activity take too long to happen in the educational institution. One must ask, "why"? The main reason is that entering the Information Society implies in a new role for school that it has not fully internalized yet. The school's key role is no longer preparing a small elite for post-secondary studies and provide most people with the minimum requirements for a quick insertion in the job market. Quite the contrary, it now is preparing all youngsters to be inserted in a creative, critical, and intervening fashion in a society that is more and more complex, where the capacity to unveil opportunities, the flexibility of thinking, the adaptation to new situations, the persistence, and the ability to interact and cooperate are key qualities. For Math teachers, such new role has fundamental consequences in two levels: in their view of Math and in their view of what being a teacher is. (Ponte, 1997, p.1-2)

In our experience, with some classes of a subject of the Professional Master's Degree Course in Mathematics Education at PUC/SP[2], we observed that only a few teachers taking their Master's, who see themselves as experts, is surprising. They explained by students:

> I see that regardless of how good a teacher is at a subject, he/she must still expect unusual situations in the classroom, since the use of computers and the Internet require much more from the teacher and, for that reason, he/she should be much more alert in face of so many possibilities. Well, to sum up, this week has been very productive, since I've learned something entirely new, found myself facing new terms and situations that I had not experienced yet. (student1)
>
> That was another subject that surprised me in this course, for it was new for me too. More and more, I realize how much I was chained to the traditional "classroom" classes. (student2)

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Others who regard themselves as digital illiterates in the beginning of the course, were also surprised when they developed innovative proposals for their teaching.

> Uniting the reading of the texts on the use of technologies and the use of software to perform the tasks of the ICT's subject tasks, I have concluded that we need to rethink the way in which we teach Math. Dynamic geometry software provides an approach that is very different from the ones developed traditionally. Therefore, we are facing a new way of doing, learning, and teaching Math. Each software application has a peculiarity that stands out from others, which is why I believe that the teacher's type of activity and educational purpose is what well determine the use of the software. Therefore, I am settling pending issues to carry out lab activities with students in the beginning of the next year. I also intend to show and mobilize the Math teachers in my school regarding the advantages of using software for the effective use of the lab with their students. (student3)

It is important that teachers reflect upon their technological competences to use them in practice. That requires studies and reading in the theme in order to know what already can be explored with their students and which aspects still need improvements. It is certain that with technological advancements teachers needs to convince themselves for future learning and improvements. Their conceptions, beliefs, and knowledge are key factors to integrate technologies.

Another aspect to be considered by teachers is about the theoretical references in relation to the integration of technologies in Mathematics Education. The diversity of technological resources is widely studied, explored, and researched. The literature is broad in that field and shows the theoretical-methodological potentialities in mathematics teaching and learning processes.

> Since 1992, hardware and software have been significantly developed and diversified, with for example calculators, use of the Internet, computers of all types up to everyday technologies like mobile phones and digital cameras. This development of hardware and software has potential implications for teaching and learning mathematics at all levels of education. In parallel, with the development of technologies, research into their uses has evolved in their aims, objectives and orientations, the perspectives have widened, new methodologies have been adopted. The first study was largely centered on mathematics themselves and it is only more recently that a work has developed on multiple actions and feedbacks of technology on teaching and learning Mathematics. The work was particularly interested in the complex process of instrumental genesis, the role of the teacher and the articulation of the use of tools with traditional techniques. New, stronger paradigms emerge to think of the use of tools in teaching mathematics and the 17th ICMI Study aims to make a new step forward in this direction. (Lagrange, 2006) [translation of the authors]

When choosing to use a technological resource, it is important that teachers research results obtained in other experiences that have already been consolidated and think about their teaching approaches for their students. Such resources, researched in dissertations and theses, include software, use of the Internet, calculators, mobile phones, videos, among others, and each of their possibilities, difficulties, and challenges.

The integration of technological resources into teaching may be supported by what Rabardel (2003) regards as the building of the instrument genesis. It is argued that the instrument is the result of a construction process by individuals throughout their activities, from a given artefacts. In the creation, conception, and usability process, humans have central roles, but also change in cognitive and behavioural terms. Thus, an instrument is comprised by the artefact (material component) and by the usage schemes used to perform the task (psychological component); a usage that is not appropriated by the individual spontaneously, but by means of an instrumental genesis process in a double ownership process: instrumentalization and instrumentation.

In instrumentalization, relating to the artefact, individuals customize them according to their needs, which may be considered as a contribution by the user to the conception of the instrument.

Instrumentation relates to subjects with their usage schemes of the potentialities of the artefacts and which condition their actions to solve a certain problem.

Rabardel observes that:

> In such special edition, we focus on appropriation, by the users, of their computers as artefacts. We ask how users carry out their activities and, also, how they adapt their artefacts to the new conditions that the use of such artefacts implies or allows. Understanding such phenomenon and considering it, within a network of artefacts and organizations, requires technical models and empirical research, focused on the activities of people and on their appropriation and development processes. (Rabardel, 2003, p.641)
>
> Such development through usage should be considered, in our opinion, as a characteristic inherent to human activity. The existence of the instrumental genesis is NOT the result of a deficient model, but rather the expression of the concept incorporated by the artefact, which is, in all ways, performed by the user. (Rabardel, 2003, p.643)

Therefore, the integration of technologies into teaching could cause unbalances in the teaching and learning process and requires changes and adaptations by teachers and students according to the specificities and potentialities of the artefacts used.

UNDERLYING THEORIES FOR OVERCOMING CHALLENGES

Reflecting upon the use of technological resources in teaching enables teachers to acknowledge the challenges to be overcome. Each choice implies in different competences for which, in several situations, teachers were not prepared.

Some technological tools provide conditions that favour of teachers and students in the teaching and learning process. In order to such action to happen, teachers need to propose situations where students are able to build on knowledge with the aid of the tools and that such tools allow to identify the paths taken by students, enable the provision of answers to their activities. In addition, feedback relating to their performance beyond the teachers' mediation should be given.

It is key that the teacher has abilities and competences for such mediation and receives a solid training on the contents to be worked on the methodologies to be explored in teaching and, above all, has knowledge on the styles of learning that arise from those who are learning. Added to such context are the technologies that contribute to a better learning and only make sense relating to the methodologies used.

Mishra and Koehller (2006) developed a theory named Technological Pedagogical Content Knowledge (TPCK), which approaches the knowledge needed by teachers to integrate technology into their practices and how such knowledge may be further developed. The authors argue that technology has great potential of changes in everyday lives of humans, including in teaching and learning processes, but what has been taking place is that such scenario has been beneath what reality has shown so far.

The activity of teaching is highly complex and based on several types of knowledges, among which are the knowledge on the development of students' learning and the knowledge on the subject to be taught.

Historically, teachers training programs have focused on the knowledge of content (P) and pedagogical knowledge (P) or general pedagogical practices that would not depend on the content to be taught. When introducing the idea of the pedagogical knowledge of the content (PCK) in 1986, Lee S. Shulman (1986) enabled the discussion on teachers training to move forward. The dichotomy between highlighting content and pedagogy should be replaced with a concern with the relationship between both types of knowledge, i.e., the PCK that represents the knowledge on how certain aspects of the content may be organized, adapted, and represented for its teaching.

Authors Mishra and Koehller (2006) believe that, even though Shulman has not discussed the relationship of technology with pedagogy and content, such discussion is currently necessary due to the wide availability of new technologies, mainly computers, educational software, and the Internet, and the need to apply them in teaching and learning processes.

The TPCK model introduced by Mishra and Koehller (2006) highlights that the content of technology may not be treated without a context and that the teacher needs to understand how technology relates with pedagogy and content. TPCK would be the foundation for good teaching with technology. It requires an understanding on how to represent concepts with technology, pedagogical techniques that use technology, reasons that make a concept difficult or easy to learn and how technology can help, knowledge on epistemological theories and how technologies may be used to help building on from students' previous knowledge. In the model proposed, the following components: content, pedagogy, and technology; exist in a permanent state of dynamic equilibrium, i.e., may not be seen separately and a change to one of them has be "made up for" with changes to the other two.

Mishra and Koehller (2006) also argue that it is important that the pedagogical approach highlights learning, doing what implies in an artefact construction process that allow to integrate theory and practice. In that direction, Kaput (2007) highlights that using a computer as a means in education may satisfy several approaches: games, tutorials, symbolic manipulators, and simulations.

There are countless resources that may be considered as important tools for Mathematics Education, since they enable to create situations to be manipulated and explored by students when checking their hypotheses. Therefore, students are engaged in a context of learning with the proposal of problems, formulation of hypotheses, and making of decisions in a permanent dialogue with reality.

One of said situations that may be proposed is found in Villiers (2002). The author mentions several important functions of the demonstration in Mathematics, mainly with the use of computers, and observes that, initially, the most important function of explanation and discovery must be used to introduce demonstration as a meaningful activity for students.

Among outlined technologies, the Internet stands out with great applicability, since it is a tool of information and assistance to the research, socialization, and interaction for the strengthening of pedagogical practices.

Teacher offered possibilities to use of the Internet, has a wide range of choices for their practices: promote research cantered on their students, on previously selected topics, in order to avoid random navigation; create collaborative work groups in virtual environments, propose activities using mathematical applets or learning objects, being part of communities that may contribute to his/her practice, use the WebQuest technique with their classes, use virtual environments or other tools such as chats, videoconferencing, or blogs to interact with students at moments outside the classroom, create collaborative pages, explore the potentialities of augmented reality, among others.

As well as download demos or free software such as GeoGebra, C&R, Winplot, and Scketchup, which allow to develop proposals of challenging activities to work with

their students. According to Guerra (2001), there are countless opportunities to use the Internet in Mathematics Education:

> In the educational process, such tools can give dynamism to the learning process, change the time spent in acquiring knowledge, stimulating learners to learn by themselves, thus creating new learning and teaching methods, and also deeply changing the traditional role of the interactions that take place between teacher and students in the social space of the physical classroom, and that now may also be a virtual one. (Guerra, 2001, p.1)

It is necessary to pay special attention to other aspects that make it difficult to use the Internet, as Moran (1997) states

> Teach using the Internet requires a great deal of attention by the teacher. In face of so many search possibilities, navigation itself becomes more appealing than the interpretation work needed. Students tend to lose focus in face of so many possible connections, addresses inside other addresses, images and texts that come after one another nonstop. They tend to accumulate to many texts, places, ideas, which are recorded, printed, written down. They place data in sequence more often than confront such data. They copy addresses, articles side by side, without filtering them properly. (Moran, 1997, p.4)

The use of technological tools is intended to provide conditions that favour users in the teaching and learning process. Computer are technological resources that allow, in addition to accessing the Internet, to carry out other activities by means of specific software and which enable several representations of a single mathematical object. By representing graphics of functions on computer screens, other windows could be open to present the corresponding algebraic expression and, sometimes, yet another window with a spreadsheet containing the coordinates of some points belonging to the graphics.

Changes to graphics could become visible immediately in the algebraic window and in the points of spreadsheets. It is the presentation of the dynamism of situations that enable teacher and student to raise conjectures and test hypotheses. These are the possibilities observed, e.g., on GeoGebra.

As to the use of the computer, Balacheff (1994) proposes a theory to rethink its use by the teachers so that it is no longer an element in education, but rather a unique feature that must be very thoroughly studied and evaluated to be used properly. Balacheff considers that:

> The creation of teaching objects is the result of complex process of adapting knowledge to the teaching and learning limitations of educational systems. Such

> of a model and of implementation of computing are added, actually, combined with the limitations of the didactic transposition: limitations of the "computable modeling," limitations of software and of materials for digital support to realization. (Balacheff, 1994, p.4)

When using the computer in teaching and when proposing to look at a single task from a different point of view, the teacher may only be consuming technology, such as, for example, solving the activities of the educational books using specific software. But they may also be going further, integrating the technology into the proposal of activities that change the look upon the mathematics taught and learned.

Teachers and students change the method in which they use mathematics and change the way they think mathematically, as stated by Frota and Borges (2004). Therefore, it is relevant that Mathematics should have the goal of preparing students to successfully perform certain roles, either in schools, lives or work. Thus, competences in the field of Mathematics acquired in school will enable such students to act as successful interlocutors when performing their key roles in the world and, in that aspect, "mathematizing".

CONCLUSIONS

Currently, technology is changing society, economy and lives of all people; this poses important requirements for education to change. To be able to contribute to the needs of economy and society researchers and all stakeholders in education need to contribute to these new developments. In this paper, we outlined theories and their applications to assist educators to better understand and utilise digital technologies in teaching and learning.

We focused on the uses of technologies in mathematics as it has an important role contributing to a wide range of subjects, economical developments, and technology enhancements.

We hope that our contributions will enable more valuable uses in technologies in all levels of education.

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