

Secondary School Mathematics Teachers' Professional Learning in a Lesson Study

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Received for publication on 19 Feb. 2019. Accepted after review on 9 Sep. 2020.

Designed editor: Claudia Lisete Oliveira Groenwald

ABSTRACT

Background: Lesson study is an innovative professional development process, originated from Japan, that is important to know if it may be applicable in other cultural contexts. **Objective:** This paper analyses professional learning of mathematic teachers involved in lesson study. **Design:** Qualitative and interpretative research, using participant observation. **Setting and participants:** The participants are seventeen mathematics teachers of secondary school at the teaching public network of Rio Grande do Sul State. **Data collection and analysis:** Research's empirical material consist of field notes produced in the context of lesson study sessions, teachers' textual productions systematized in their logbooks, audio recordings of the research lesson and debriefing session. **Results:** The results highlight that teachers' participation in lesson study provided them professional learning related to deepening of the curricular content to teach at classroom, as well about teaching resources to teach mathematics topics and also about the collaborative work in the school context, the reflection on professional practices and about the teacher's education approaches prevalent in the educative settings. **Conclusions:** The study suggests that lesson study is a powerful professional development process both regarding mathematics teaching and professional culture.

Keywords: professional learning; lesson study; secondary school; professional development.

Aprendizagens Profissionais de Professores de Matemática do Ensino Médio no Contexto dos Estudos de Aula

RESUMO

Contexto: O estudo de aula é um processo de desenvolvimento profissional inovador, originário do Japão, sendo importante saber se é aplicável noutros contextos culturais. **Objetivo:** O artigo examina as aprendizagens profissionais de professores de matemática envolvidos em estudos de aula. **Design:** Investigação qualitativa e interpretativa, usando

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observação participante. **Ambiente e participantes:** Os participantes são dezessete professores de matemática do ensino médio da rede pública estadual de ensino, pertencentes à 15.^a Coordenadoria Regional de Educação do Estado do Rio Grande do Sul. **Coleta e análise de dados:** O material empírico do estudo constitui-se das notas de campo produzidas no contexto das sessões do estudo de aula, das produções textuais dos professores sistematizada nos seus diários de bordo, dos registros em áudio das aulas de investigação, assim como dos registros em áudio da sessão de reflexão sobre as aulas e avaliação da ação formativa. **Resultados:** Os resultados evidenciam que a participação dos professores no estudo de aula lhes proporcionou aprendizagens profissionais relativas ao aprofundamento do conteúdo curricular a ensinar e das estratégias e recursos para o ensino dos tópicos da matemática e também no que diz respeito ao trabalho colaborativo, à reflexão sobre a prática e às dinâmicas formativas predominantes nos contextos educativos. **Conclusões:** O estudo sugere que o estudo de aula é um promissor processo de desenvolvimento profissional no que diz respeito ao ensino da matemática e, sobretudo, à cultura profissional.

Palavras-chave: aprendizagem profissional; estudo de aula; ensino médio; desenvolvimento profissional.

INTRODUCTION

Teacher professional learning is a relevant and increasingly investigated subject in mathematics education research because it is a central and necessary aspect for the promotion of new classroom practice. Underlying professional learning are the processes and approaches of teacher education and professional development that may favor or promote different learning. Among the approaches to professional development that have interested researchers around the world, lesson study stands out as a reflective and collaborative professional development process focused on teaching practice (Lewis, 2002; Murata, 2011).

In this perspective, our goal is to examine the professional learning of secondary school mathematics teachers in a lesson study, considering the possibilities evidenced by this formative approach to the promotion of professional learning and the implementation of a teaching practice different from conventional practice centered on teacher talk. We chose to approach the learning of secondary school teachers because few national and international studies examine the possibilities of lesson study at this level of education. We chose to work with public school teachers as a way of understanding how lesson study can contribute to the professional learning of these teachers, whose possibilities of experimenting with different formative approaches are influenced by their working conditions.

TEACHER PROFESSIONAL DEVELOPMENT AND PROFESSIONAL LEARNING

Teacher professional development is an important concept that underlies the studies and discussions about teacher education, referring to the development of professional

competence about teaching and non-teaching practices, as well as teachers' autonomy in teaching practice and as members of the school organization (Ponte, 1998). Professional development constitutes a movement from the inside out, from the teacher towards the context in which he/she is involved, combining formal and informal teacher education processes (Nóvoa, 1995; Ponte, 1998). Moreover, Ponte (1998) highlights that professional development might be promoted through teachers' engagement in teacher education processes, which provide opportunities for teachers to reflect, participating in social practices with strong personal involvement and support given by social groups in which they participate.

In this perspective, professional development refers not only to classroom practice, but also to the relations established by the teacher outside the classroom, in sharing thoughts and competencies with peers, thus improving the contribution of the school to students' success (Seco, 2009). Professional development is the total sum of formal and informal learning that is pursued and experienced in an involving learning context on conditions of complexity and dynamic change (Fullan, 1995; Ponte, 2014).

Teacher professional learning is conceived as a dynamic and permanent phenomenon, personally and socially constituted in interaction among teachers and by confrontation and modification of ideas and reinterpretation of experiences (Flores, 2004). Thus, this process involves learning related to a disciplinary field, classroom teaching, management of teaching, elements of the professional context and other processes intrinsic to teaching. Professional practices experienced by the teacher throughout the career contribute to his/her learning process (Zeichner, 1999) and professional development, because they promote learning of distinct nature. This learning involves the disciplinary area, classroom teaching, students' learning process and their difficulties, as well as elements of the social and cultural context in which teacher acts.

Garcia (1999) argues that teachers, as actors of the educational process and of their development, are involved in a professional activity that provides them formal and informal learning situations. From this perspective, Tiezzi (1992) points to the necessity to recognize the teachers as "subjects that learn instead of mere performers or obstacles of/to the change" (p. 19). Thus, it is necessary that research about teacher professional development process "goes on exploring the ways according to which the teachers learn new ways of thinking the teaching and contents learning as well as the conditions which facilitate teachers' learning" (Tiezzi, 1992, p.19).

A theoretical approach to teacher professional learning, based on social learning theory (Bandura, 1977; Rotter, 1954), conceives this learning as a process that is enacted in the interaction among peers and by observation of the environment, of established conducts, and of other colleagues' practice. From this process, the teacher learns news roles, knowledge, action modes and professional practices (Garcia, 1999).

In a study regarding teachers' professional learning, Desimone (2009) highlights five aspects concerning the way these learning influenced the teacher's knowledge, abilities, and practice, namely, focus on content, active learning, coherence, direction, and collective participation. The author adds that the tensions among these characteristics constituted one of the causes by which large scale changes in the teaching process becomes difficult. Thus, to improve teaching promoting changes in practice, professional learning needs to overcome the gap between teacher knowledge and beliefs and knowledge originated from research (Desimone, 2009). Therefore, collective participation can add to this process a collective meaning about content difficulties due to changes in the norms of desirable teaching (Spilane, 2000).

Furthermore, Feiman (2001) indicates that, underlying professional practice as a path of professional development, is the conception that learning about teaching develops in the context of direct experiences, interactions and interlocutions with colleagues and others involved in the educational process about situations related to the classroom practice. Therefore, teacher professional learning is based on everyday experience and involves a structure that is dynamically constituted throughout the teaching career, as well as on the formative activity in which the teacher engages in the profession.

Professional collaboration, an essential context to professional learning, characterizes the interaction, in its more restrict meaning, which is established among teachers insofar as they are involved in a common activity, sharing collective goals, and interacting collectively and cooperatively (Hargreaves, 1998). This author adds that collaboration can play different forms, from co-teaching, going through collaborative planning, work in pairs and collaborative research. From this perspective, collaboration has been pointed as a viable solution to educational problems insofar as it is linked to the process to improve the quality of teaching quality and the development of the school (Hargreaves, 1998; Roldão 2007), constituting an important element of teacher professional development and school improvement (Day, 2001).

Beyond above the mentioned aspects about collaboration, Fullan (1995) indicates that the research focused on school collaborative cultures show that teacher learning develops, especially, in everyday interactions among colleagues who share the concern about continuous learning. In this respect, Borges (2007) considers that a "culture of professional collaboration is that in which everything is shared and discussed, good and bad things, certainty and uncertainty are shared, as well as teachers seek support and assistance to learn" (pp. 370-371). In this direction, the ways of sharing are enacted in professional practice and become significant for teachers' life and work, because the trust that emerges from "sharing and collegial support has led to greater willingness to experiment and to take risks, and with them a commitment of teachers to continuous improvement as part of their professional obligations" (Hargreaves, 1998, p. 209).

Changes in practice refer to the process of resignification and redefinition of the teachers' everyday professional activities in the classroom. However, changes in

teacher practice can reach different levels throughout the career and involve different aspects (Garcia, 1999). In this sense, Guskey (1986) proposes a sequential model of teacher changes that underlies the professional development process, according to which changes in the classroom teaching practice lead to changes in student achievement, and finally, they link to the changes in teacher beliefs and attitudes.

Furthermore, Fullan (1995) considers that changes in teaching and curriculum mostly depends on the teacher. In this direction, Garcia (1995) points that the valorization of contextual and organizational aspects, oriented to the teacher change, is the basal element of teacher professional development. And Day (2001) argues that changes in teacher practice require an efficient professional development process and depend “from his/her past experiences (life histories and career), his/her disposition, his/her intellectual capacities, social convictions and institutional support” (p. 38). Thus, teacher professional development materializes through professional learning which implies change in practice. However, changes in practice are only possible if the teacher has the disposition to make them (Day, 2001; Nóvoa, 1995).

In summary, teacher professional development is realized through professional learning, dynamically and continuously promoted, at the personal and social levels, from the interactions between professionals of a particular group in which ideas are transformed and experiences are reinterpreted (Flores, 2004). Moreover, through this learning, changes of a distinct nature are fostered and grounded. Moreover, this process of professional development presupposes a context of professional collaboration, in which professional learning and changes in practice are favored. In other words, collaboration is fundamental for teachers’ professional development and, thus, for achieving changes in schools (Day, 2001).

LESSON STUDY

Lesson study emerged in Japan at the beginning of the twentieth century, in the Meiji¹ period, at a time in which educational changes become necessary and urgent. This approach was consolidated as a way to prepare teachers to develop their pedagogical practices (Isoda, 2007). Inspired by the teaching model to whole classes (collective teaching), predominant in western countries, it was instituted in the normal Japanese schools as a method to promote collective teaching. Following the publication of a book presenting the collective teaching model experimentally developed in normal schools, which provided comments about didactic resources and instructions to make lesson observations, as well as for conducting critical sessions of observed classes, the Japanese Education Ministry instituted this model throughout the country (Isoda, 2007). Thus, conceived as a reflexive and collaborative approach of teacher professional development,

¹ The Meiji period is the first period of Japanese Empire, between 1868-1912. It was extremely important for the development of Japan, since it made it one of the great world capitalist powers. It was marked by a period of political, economic and social transformations, including the Education Act (1872) that instituted normal schools (Isoda, Arcavi & Mena-Lorca, 2008).

and centered on teaching practice, lesson study became widely practiced in this country (Stigler & Hiebert, 1999; Yoshida, 1999).

The dissemination of lesson study in Western countries took place only from 1990s on, mainly upon the *Teaching Gap* book (Stigler & Hiebert, 1999) that credited the success of the students in mathematic in TIMSS - *Trends in International Mathematics and Science Study*, to the problem solving structure of Japanese lesson study, and especially to the professional development at which all the Japanese teachers are involved at. In his Ph.D. thesis, Yoshida (1999) introduces us deeply in the Japanese lesson study, and therefore his research constitutes a milestone in the dissemination of this approach to the Western world.

Lesson study developed in Japan has a common nuclear structure, which might undergo modifications due to the contexts and goals in which it is carried out. This structure is composed of four stages: definition of goals for the class; planning of the research lesson; teaching of the research lesson; and reflection about this lesson considering the records produced by teacher observers (Lewis, 2002). However, there are experiences of lesson study that present more than one cycle, whereby the research lesson is revised, restructured and taught to another class, repeating the entire process. There also reports of lesson study in initial teacher education (Ponte, 2017).

An important inherent aspect to lesson study refers to the pedagogical approach underlying the research lesson. One of the perspectives highlighted in several works around the world refers to *structured problem solving*, which is at the center of the lesson study approach promoted in Japan. Fujii (2018) points out that the problem solving in Japanese lesson study is a widely valued and widespread perspective, although some aspects are more or less valued. However, in the Japanese approach, four problem-solving phases are highlighted: the teacher presents the problem and the students try to understand it; the students work to solve the problem; the teacher favors the comparison and discussion of student solutions; and the teacher synthesizes the lesson (Fujii, 2018).

From a similar perspective to structured problem solving, one approach that is increasingly evident in lesson study classes is the exploratory approach. Ponte et al. (2014) highlight that the exploratory approach in mathematics teaching enables the teacher to reflect on classroom teaching, while seeking to lead students to face situations for which they do not an immediately applicable solution method (Ponte, 2005), leading them to build or deepen their understanding of mathematical concepts, representations, procedures and ideas.

Similarly, Villegas-Reimers (2003) conceives lesson study as a process that provides all the necessary elements to promote effective professional learning for mathematics teachers. Besides, the author highlights eight aspects of lesson study that make this approach appropriate for promoting different professional learning. Lesson study is *experiential* (stimulated and based on teachers' experiences), *sustained* (involves cycles of planning, anticipation, implementation, and reflection), *grounded* (relies on the teacher's professional practice and adequate resources; context and culture related), *collaborative* (involves networks of teachers and administrators), *supported* (by external

experts and research results), *provocative* (involves both simultaneously pressure and support), *focused* (*attentive to the development of mathematical knowledge of students*). Furthermore, it presupposes *confidence* (teachers are encouraged to communicate their ideas and supported to take risks).

Through an investigation into the possibilities and limitations of the process of developing good mathematics teaching practice for secondary school through lesson study, Marsigit (2007) highlighted the professional learning of a group of eleven Indonesian public school teachers. Teachers stated that they learned to develop teaching methods based on practical activities and daily life using local materials, and that such activity actively involved students in learning and discussion by sharing ideas with their peers. Furthermore, by developing teaching methods, the teachers realized that they needed to plan the teaching setting, plan activities for students, plan teacher role, distribute assignments, develop assessment methods, and monitor progress in student performance (Marsigit, 2007).

In a similar perspective, Verhoef et al. (2014) investigated the changes in the educational goals and teaching strategies by participant teachers in a lesson study. The results evidenced learning related to the development of a new classroom approach. The teachers experienced a focused approach on student understanding, aiming to motivate them and favoring the understanding of mathematical concepts addressed in the research lesson. This approach, named as realistic mathematics, was implemented in substitution to the approach widely developed in the Netherlands, which was supported by established goals in curriculum guidelines and mathematics textbooks, to encourage the students to achieve good results in national evaluations. Besides, the study evidenced professional collaboration related to learning. According to the teachers, the complexity of daily work rarely leads them to talk with colleagues about what they find out concerning teaching and learning, as well as about their practice.

Inoue (2011), when explaining the learning process of a group of elementary education mathematics teachers about the problem solving activities development focused on collective discussions, highlighted the related learning of teachers learning, especially regarding mathematics teaching. The teachers claimed that they began to value the importance of encouraging students to discuss critically and to present problem solving strategies. Other teachers highlighted that they understood what a lesson goal is as they began to see the goal as something that the teacher wants the students to do during the lesson. The teachers added that they learned to observe their colleagues' practices and their practice, as well as to identify what was not in line with the problem solving perspective. By sharing their classroom experiences and changing perspectives, the group agreed that this is important to examine deeply the education goal of the lesson prior to the lesson.

Regarding leading collective discussions in the classroom, Inoue (2011) indicated that teacher learning includes five aspects: knowing what the student is asking (the teacher needs to be clear of what the initial question of a problem wants to know, and then, if it needs to be reformulated, scan it to promote student discussion);

anticipate student responses in the planning phase (this care allows the teacher to think about strategies in advance to help in the students' doubts, misconceptions and difficulties); empower students (give students time to discuss inefficient strategies); allow fluency to collective discussion (when students responses are too fast or inaudible for the whole class, the teacher needs to repeat or rephrase the answers clearly and concisely. Student misconceptions need to be analyzed and explored in the discussion to create consensus, serving as an essential catalyst to develop a deep understanding of the focus topics and student learning. However, this process needs to be contained. If students are confused, do not hesitate to intervene); continue the process (the discussion of creating consensus should not end when the group reaches consensus. Instead, based on the concept formed, propose a new initial question and promote a new cycle) (Inoue, 2011).

The work of Suh e Seshaiyer (2015), which sought to examine the understanding of teachers working in school years and different levels about the progression in mathematical learning by participating in lesson study, also points to teacher professional learning. Through the elaboration of problem solving tasks in algebra, exploring pedagogical strategies, using mathematical tools and technologies, the teachers were able to deepen algebraic thinking, to realize the vertical articulation between grades and teaching levels by sharing ideas and resources, to develop a productive disposition to teach through problem solving and to analyze student learning.

Regarding algebra learning, participation in lesson study provided teachers with significant opportunities to articulate learning in algebraic thinking across levels. Planning in vertical teams, favored by the natural dialogue around learning progressions in algebra and the knowledge of the content developed by teachers, particularly on how they selected and modified problems, allowed them to anticipate students' conceptual misconceptions, strategies, and representations. In addition, the vertical nature of the constituted team and the natural dialogue around curriculum objectives that each teacher was responsible for at the grade level and the grade they were in, led them to know and study algebra beyond those curriculum topics they taught, as well as to know and develop different resources and strategies to address them (Suh e Seshaiyer, 2015).

Lewis and Perry (2017), based on a study that focuses on how developing lesson study supported by the use of fractional teaching resource kits can improve teacher and student knowledge on this topic, highlight professional learning of teachers in relation to the topic of mathematics addressed (fractions) and, mainly, about the students' difficulties and learning process. The teachers' learning refers to changes in knowledge related to the curriculum topic, beliefs about mathematics teaching, in the learning materials due to the use of the didactic resource kit, from which they sought to produce changes in teaching and, therefore, in student learning. The teachers pointed out in the written reflections produced throughout the lesson study that the use of the resource kit allowed them to explore new ideas, such as emphasizing fractions of denominator one using a linear model of fractions and testing these ideas during the research lesson.

In addition, the study highlights changes in the quality of professional learning. The teachers consider that this experience contributed to classroom teaching, especially by focusing on deepening a specific curriculum topic, relying on curriculum resources and relevant research. Regarding the teaching of this topic, the emphasis on the model representation of fraction on the number line constitutes a small but important change from the practice of these teachers, which was strongly influenced by the use of the resource kit for addressing fractions (Lewis & Perry, 2017).

METODOLOGY

The qualitative research we conducted constituted an interpretative and analytical process, supported by procedures such as interviews, conversations, records and field notes (Denzin & Lincoln, 2000), whereby we examine the professional learning of secondary school mathematics teachers who participated in a lesson study carried out in the 2017 school year. Seventeen public school teachers from the Rio Grande do Sul state, Brazil, belonging to the 15th Regional Education Coordination (an organ of the Education Secretariat of that State) participated in the lesson study.

The lesson study was conducted from August to December 2017, under the coordination of a research team from the Grupo de Estudo e Pesquisa em Educação Matemática e Tecnologias (GEP@T), from Universidade Federal da Fronteira Sul. The activities were held fortnightly on the premises of the 15th Regional Education Coordination. The teachers were organized into three groups, taking as a grouping criterion the school year in which they intended to develop the research lesson (third stage of the lesson study). Thus, three teacher groups were formed: Group 1 - lesson for the 1st year of secondary school; Group 2 - lesson for the 2nd year of secondary school; Group 3 - lesson for the 3rd year of high school. The mathematical topics chosen by each group were quadratic functions (G1), systems of linear equations (G2), and rectangular prism (G3).

After the conclusion of each meeting, a detailed description of the activities was carried out, addressing the work done by each group of teachers, highlighting the predominant aspects in the discussions, challenges expressed by teachers, their reflections and impressions concerning the formative approach they were experiencing. These observations and records constitute the field notes, conceived as “written accounts of what the researcher sees, hears, experiences and thinks in the course of collection and reflecting on the data of a qualitative study” (Bogdan & Biklen, 1994; p. 150). In addition, teachers’ productions based on lesson study activities were analyzed, especially the reflections on each of the meetings, as well as the plans for the three research classes planned collaboratively by the three groups. Similarly, the teachers’ reflections and their assessment of the formative experience in the lesson reflection stage were transcribed and incorporated into the empirical material of the study.

These data, therefore, constituted the empirical material of the study, which was analyzed, interpreted and discussed (Erickson, 1986), through procedures of qualitative analysis in mathematics education (Fiorentini & Lorenzato, 2006). We focus on this material considering that it can highlight potential aspects for teachers' professional development, considering the various stages that constituted the lesson study cycle. Such analysis is relevant because it allows us to show the different elements that characterize teachers' professional learning. The analytical process began by reading the empirical material from which we identified the evidence on teachers' professional learning in view of their participation in the lesson study. Finally, we grouped the evidence considering possible convergences, thus constituting the study analysis and discussion categories, which circumscribe two important themes: learning about mathematics teaching and learning related to the professional culture of the participating teachers, as indicated in the table below.

Table 1
Aspects evidenced and categories

Aspects evidenced (subcategories)	Analyses categories
Exploratory approach Use of Dynamics Geometry Software Classroom strategies	<i>Mathematics teaching</i>
Debriefing Teacher education dynamics Professional collaboration	<i>Professional culture</i>

TEACHER'S PROFESSIONAL LEARNING

The analysis of the empirical material showed different aspects related to the participating teachers' professional learning, which we organize in two major themes: Mathematics Teaching and Professional Culture.

Mathematics Teaching

Regarding the mathematics teaching, the analysis pointed to learning related to the exploratory approach to mathematical topics, the use of dynamic geometry software to deepen curriculum issues and the development of classroom work's strategies, specifically for the research lesson.

Exploratory approach

The teachers of the three groups pointed out that the lesson study allowed them to experiment with the ‘exploratory approach to mathematical topics’, an approach that surpassed the historically established tradition of mathematics teaching, in which the results are prioritized over the resolution processes problems or exercises, as Roger points out:

Maybe the mistake is in the way we work with them [in the classroom]. We work with them that way, that question like that, you work [always trying to find a] result. You have to find an exact result. (Roger, G3, Nov. 2017).

Alternatively, this teacher suggested approaches to mathematics teaching that value the way in which results are achieved, from which students have the possibility to find amazing conclusions, as happened in the research lesson in which the students presented conclusions beyond what the team had considered at the planning stage:

I believe that our work could be different. We might propose more open classes, with more open exercises. [And from our experience] I stress the emphasis of the [of mathematical investigation] work, in which the students gave us [very interesting] answers, surprising us with the results, as they went beyond the expected. They also suggested a similar activity with other disciplines. (Roger, G3, Nov. 2017).

Indicating the specificity of the mathematical activities designed to promote the exploratory approach to the curriculum topic studied in each group, Jade and Joana emphasized the various ways students can follow to solve them:

In the whole group, we discussed and concluded that mathematical activities have several ways to reach a solution. However, we, mathematics teachers, when exploring the activities, turned lot to Algebra which shows that we have difficulties in presenting strategies without using algebraic expressions and consequently, we transmit it to our students, they also resort directly to algebra. (Jade, G1, Dec. 2017).

In the socialization of questions, we realized that the same question can be solved in different ways. We also concluded that we teachers use algebra a lot and that sometimes we can employ simpler and more practical ways. (Joana, G1, Nov.2017).

In addition to the possibilities of promoting approaches focused on research, discussion and justification of strategies to solve mathematical activities, teachers Julia,

Marina, Alice and Sofia stressed the challenges that the elaboration of exploratory tasks brought to the group:

Each group was developing the first activities of the research lesson plan. At first, it seemed easy, but when we were writing the problems, questions arose where we asked ourselves: “Will the student understand this way? Are we making clear what the problem requests? From the form the question was elaborated, is it possible to understand what the approached content is?” These are some of the questions we made to ourselves [...]. (Júlia, G1, Dec. 2017).

Julia highlighted the questions that permeated the group discussions at the planning stage of activities in such a way to promote the exploratory approach to the topic studied by the group. Marina, for her part, indicated the principles established by the group for the elaboration of activities focused on this approach:

As we began to elaborate the activities, following the content and related objectives, we realized that it was not easy, because we had a tendency to develop questions that since the beginning demanded an exact result. However, after we overcome this, we realized that it was possible to introduce items that lead to reason and motivate in relation to functions, work with basic concepts such as coefficients and verification of a solution, prove that there are real situations that fall into the study of such equation, deduce and apply the quadratic equation formula (Bhaskara's), and explore the representation of a parable, table (graph of a quadratic function) in the Cartesian plane. (Marina, G2, Dec. 2017).

Marina's reflection added that although to elaborate exploratory activities requires other teacher knowledge, they can promote the approach of curriculum topics, such as the quadratic function, coherently in relation to the properties of the topic and its applications, as well as in terms of the different representations (algebraic, tabular and graphical) of the mathematical concepts involved, favoring the deepening of mathematics. In this same direction, this teacher and Julia also stress the exploratory nature of the activities as an aspect that favors students' learning of mathematics:

We could observe that the study of a particular topic through various forms/methods allows a more meaningful learning from students. We also observe that when we deal with mathematical themes from a problem situation, involving the daily life of society, the activities become more pleasant and interesting. We also used the “Geogebra software” to construct the graph (parabola) for the purpose of not only interpreting the graph but also of exploring the relationship between table values and the graph and for mathematical calculations. (Marina, G2, Dec. 2017).

With this learning, it is possible to realize that you can teach mathematics in simpler way, not only with the use of algebraic notation, but with the reasoning and research of how to reach the result of each question. It was possible to realize that each student has a different difficulty, but with the support of the closest teacher, the problem becomes simple [due] to the direct relationship between teacher and student. (Júlia, G1, Dec. 2017).

In addition to the aspects already mentioned, Marina and Sandy added that the exploratory approach favors students' personal growth and autonomy, highlighting their understanding of the teachers' changing roles and students in the exploratory approach:

We learn from teacher [educators] that, by providing a problem for the student, we need to focus on a wide variety of tasks and develop problem situations involving applications of a set of mathematical ideas, favoring personal progress and student autonomy, while also allowing the understanding of one's own walk in relation to knowledge. (Marina, G2, Dec. 2017).

I believe it was a "different" experience [...], we had to look in a different way at the content worked in classes, we made our students think and draw their own conclusions and relationships. I think an investigative lesson can be considered "interdisciplinary", thereby facilitating learning. (Sandy, G3, Dec. 2017).

Lastly, Mateus refers to the learning related to the role taken by the teacher in the context of the exploratory approach, in which he needs to stimulate discussion, the formulation of conjectures and, above all, the communication between teacher and student, stimulating trust, confidence and overcoming students' difficulties:

[In the research lesson] at no time did we make a judgment. Our position was also different. We were not there to say if what they were doing was right or wrong. Of course we tried to control ourselves [...] The fact that they always listen to you, that either you are right or you are wrong, that is, there is always true and false, discourages students. There we were not saying what was true or false. Even if [the students initially said something wrong], somehow we would talk to them and let them come to their own conclusion, but he did not hear from anyone that he made a mistake, that he didn't know. I think this is the important part of this activity, we have to make them comfortable. Because [the student] may be wrong at this point, but when he gets better, when he improves his knowledge, looks at it a little better, he will realize that that initial statement was not completely correct or incorrect and will reformulate it. This I noticed in the groups. They said something and then began to argue otherwise, because they already realized that the path they were following was not appropriate. Whether it was wrong or not, it was not the most important. The most important is that they realized that there are other ways, other possibilities. That was a point that caught my attention in this approach. (Mateus, G2, Nov. 2017)

The analysis shows that the experience in the lesson study provided the teachers important learning in relation to the exploratory approach to curriculum topics in mathematics. According to the reports, this learning involves the elaboration of exploratory activities, a process that begins with the definition of the objectives to be achieved and which needs to be discussed, formulated and reformulated in the group. It also involves the possibility of promoting differentiated approaches to curriculum topics by developing specific teaching materials for these topics, as well as by modifying roles, status and classroom relationships.

Use of dynamic geometry software

Another aspect stressed by the participant teachers in relation to the teaching of mathematics refers to the ‘use of dynamic geometry software’, namely GeoGebra, to promote the deepening of the topics covered by the groups and for the elaboration of mathematical activities in the planning stage, as well as for students to develop these activities in the research lesson. The words of Sofia and Joana point to these aspects:

[In the research lesson planning]. If we use GeoGebra, we will be able to explore this further [quadratic function], because students will be able to represent functions and analyze properties. They can compare function graphs with different coefficients. Moreover, they may come to some conclusions. (Sofia, G1, Oct. 2017).

We seek to familiarize ourselves with the GeoGebra software, since in our planning we chose to use it to represent the parable and to analyze the proposed graphic variations. (Joana, G2, Dec.2017).

Rubia, for her part, considered that the group’s decision to use GeoGebra in the planning stages and in the research lesson allowed them to learn about the software, and how to use it to conduct mathematical investigations:

In our 1st year group, the greatest difficulty was in the activities that should be developed using the GeoGebra software, since none of the participants had a hold in the use of the software. One of our goals was to highlight the minimum and maximum points in the function graph, but we did not know how to do it. With the help [of the teacher educators] of this teacher education process, we were able to carry out the activity and this was another learning from so many that this teacher education opportunity provided us. (Rubia, G1, Dec. 2017).

Another member of Group 1, Caio, points out that during the planning phase of the research lesson, specifically in the elaboration of the exploratory activities, they were

able to deepen their knowledge about this software and this was important, because this resource favors the visualization and research of function properties:

Our group [with the assistance of the teacher educators, explored GeoGebra] where it was possible to construct the parable and to identify its remarkable points by demonstrating the variation of a function from its graph, thus contributing to the lesson study when it is applied to students and they can have a better understanding of the quadratic equation and the graph it represents. GeoGebra software is an important tool that allows the visualization of what happens with the parable from the moment that the coefficients of the quadratic equation are modified, helping to understand them. (Caio, G1, Nov. 2017).

Therefore, in the different stages of the lesson study, the use of GeoGebra favored teacher professional learning. The teachers' familiarity with the features of this software favored the learning about how to explore it for mathematical investigations and, especially, for the deepening of curriculum topics as they could establish relationships between the different representations of the mathematical concepts approached.

Classroom strategies

Another important aspect highlighted by the teachers concerns 'classroom strategies', especially the opportunity to learn about strategies to promote classroom discussion and the possibilities of such communication for students' learning. Caio points out that this learning led them to look at students and what they do while solving mathematics problems in class:

The lesson study contributed to our growth as teachers, and assisted students in understanding the subjects worked during the research lesson. Most of the students showed interest and participated in the classes, even though at first they were a little apprehensive like the whole group of teachers in the room. This experience has shown us how important it is for students to work in teams so that we can see how they organize their own ideas to solve problem situations and build their knowledge independently. When debating with all the students in class, we noticed the richness of information that emerged, as each student can expose their ideas which helped in understanding the content of the lesson (Caio, G1, Dec.2017).

Caio also adds that the strategy of getting students to work in groups, besides modifying the way the teacher teaches, favors the student's protagonist role in the process. Melissa restates this aspect:

The research lesson allowed us to observe how important it is to rethink our teaching practice, as it is necessary to make students become protagonists of

their knowledge construction, and we have to provide this to them, modifying the approach of our classes, making them more attractive as these research classes were. (Caio, G1, Dec. 2017)

By prioritizing the construction of knowledge by the student's doing and thinking, the teacher's role is more of guiding, stimulating and encouraging learning. It is up to the teacher to develop the students' autonomy, urging them to reflect, investigate and discover, creating in the classroom an atmosphere of search and cooperation, where dialogue and the exchange of ideas is constant both between teacher and student, as well as between student and student. (Melissa, G2, Dec. 2017)

Nalu corroborates this view and considers that different classroom strategies, such as the one put into practice in the research lesson, promote students' learning:

[The lesson study allowed us] to learn new ways to work with students, developing new techniques for elaboration and application of activities that encourage students to solve them. The teacher educators who taught the course presented us with proposals for activities that help students learning by holding their attention in the development of their works. (Nalu, G2, Dec. 2017).

In terms of learning about the classroom strategies, the teachers considered that the lesson study provided a different experience to the extent that the students took on more active roles in the mathematics learning process. Through the experience in the lesson study, the teachers promoted group work in the classroom as a way to foster discussion among students, communication, argumentation and justification of mathematical ideas, favoring students' learning.

Synthesis

The analysis highlights aspects associated to learning about mathematics teaching, especially covering the exploratory approach to mathematics, which led teachers to change their beliefs about mathematics teaching, the roles assumed by the teacher in the mathematics class, to mobilize knowledge about planning classes and elaborating of exploratory activities. This learning led teachers to experiment with a different way of approaching mathematical curriculum topics. Finally, the lesson study allowed them to develop knowledge about the GeoGebra software and the possibilities to explore it to deepen knowledge about these topics and to develop mathematical activities. Finally, the teachers learned about the possibilities arising from the implementation of classroom strategies centered on collective work and mathematical discussion. These lessons, as a whole, favored the professional growth of the participating teachers, as they influenced their knowledge and practice in the research lesson.

Professional Culture

Participation in the lesson study also allowed teachers to learn about professional culture, with emphasis on reflection on classroom practice, on the specificity of the dynamics of the teacher education process (lesson study) that they experienced and, also, on professional collaboration.

Reflection

Regarding the ‘reflection on classroom practice’, the teachers considered that their experience in the lesson study led them to look at their daily work in a different, more reflective and critical way, especially in relation to the aims that guide their practices, as indicated by Jade and Marina:

I believe that this work [in the lesson study] contributed to the professional development of each mathematics teacher who took the course, especially by stimulating a reflection on their pedagogical practice. (Jade, G1, Dec. 2017).

The course was very productive and interesting, [the teacher educators] enabled us to rethink mathematics education, [provided us with] a reflective work of our daily classroom practices, using students’ ideas as a basis for growing their knowledge and help them to reflect on their learning by considering the importance of [peer] interaction for learning. The organization of the classes should propose moments of discussion in small groups and moments of discussion in the large group. (Marina, G2, Dec. 2017).

Marina’s reflection focuses on aspects that have to do with the way students are involved in mathematics learning, an aspect that suggests a modification in terms of the teacher’s beliefs and predispositions to rethink the roles assumed by the students in this process. Alice corroborated this aspect and added the group’s growth in terms of knowledge due to the possibility of interaction between professionals from the same area:

The Teacher Education Course offered to us helped to reflect on our pedagogical practice. The meetings were of great value, as they added more knowledge to those we already had. During the meetings, we realized that we always have something to improve and the exchange of experiences and ideas enriches our knowledge. The very interaction between teachers of the area was very important in the development of the process. (Alice, G1, Dec. 2017).

Therefore, the experience in the lesson study added value to the professional culture of the participating teachers, as it provided them with moments of reflection about their

professional practice and, especially, about what they seek to achieve in their work with students. In addition, this learning led them to value moments of collective work and share with peers.

Education Dynamics

Alice's emphasis on the possibility of interlocution with professionals in the same area shows that the lesson study, due to its specificities and development dynamics, allowed the group to learn about the 'teacher education dynamics' predominant in the education systems. They were able to experiment with a classroom-centered approach to professional development, in their immediate professional needs, and their knowledge and education level. The reports of Estela, Jade and Júlia point in this direction:

Teacher education in our area of training is very important, because when we talk and discuss something about mathematics with people who have studied and understand the subject; it enables much more learning and the construction of knowledge. The meetings and discussions led us to rethink our pedagogical practice. The well-conducted education by the teacher educators really made us realize the importance of good planning. (Estela, G3, Dec. 2017).

For a long time, it was expected for a specific continuing education course in the area of mathematics. This involvement with dialogue, with research, a participation in discussions about content and methodologies, it is here to strengthen the educator as an agent of the teaching process, and provides conditions for reevaluating our practice. Sometimes, the abstract way of teaching and the lack of relationship with everyday life makes it difficult for students to understand the contents of mathematics, not achieving good results. (Jade, G1, Dec. 2017).

It was a different teacher education, first because it was specific to mathematics. In general, teacher education that we have are for all areas together. Moreover, what I found most interesting is that we all had to do a part of the work, at the same time that everybody was doing this same part. For each activity, we discussed, redid, discussed and redid again, until it was good. (Julia, G1, Oct. 2017).

Julia calls the attention to the appreciation of teacher planning, an activity marked by group work and especially focused on addressing the mathematics curriculum topic. In this sense, promoting teacher education with emphasis on planning is a way of articulating theory and practice, as Marina points out:

The realization of the Teacher Education Course provided us with greater knowledge about our pedagogical practice, bringing it to the school context. We want to highlight here that, the activities developed sought to link theoretical

and practical aspects, we realized that the good progress of the course made us to reflect our educational practice in the face of the reality and from it, so that we can seek a quality education, which is guaranteed in Law (LDN). (Marina, G2, Dec.2017).

Therefore, for the participating teachers, the specificity of the dynamics of development of the lesson study led them to bring classroom and practice closer together through careful planning and shared practice. In the same way, this approach allowed the participants to take teacher education into the classroom, achieving professional collaboration also in the daily practice and changing the relationships between colleagues and with students, because together they experienced and evaluated the developed practice. Furthermore, the experience positively marked the group by taking on more dynamic roles in the educational process.

Collaboration

Beyond these aspects, our analysis pointed to learning about ‘collaborative work’ as the teachers experienced a different experience of planning and promoting mathematics teaching. A special emphasis was given to the process of conducting lesson planning in a collective and collaborative manner, as stressed by Marina and Ana:

Before, I have never had done such a teacher education course where everyone had to do something and where everyone could give their opinions, or criticize and be criticized. I felt myself valued. (Marina, G2, Dec. 2017).

From Marina’s perspective, the appreciation of points of view and ideas favored the group’s collaboration. Ana draws attention to the good communication and cooperation that existed in the group, aspects that are essential for the achievement of a professional collaboration:

The course was important, it aggregated knowledge, provided the opportunity to exchange experience between different mathematics teachers at our region. We managed to maintain good communication throughout the course, exchanged ideas, always cooperating to carry out the activities. The new always frightens us in some way, but life is made of challenges, so we need to be always innovating, especially when it comes to education. Courses like this renew our practice, treading new paths, as well as allowing our students to see mathematics with other eyes and even enabling the love for it. (Ana, G3, Dec. 2017).

Nalu, for her part, highlights two important aspects: the process of negotiating ideas and paths to follow in the elaboration of mathematical activities; and the exchange, sharing points of view and ways of working topics in the classroom:

The elaboration of this activity was important, despite the difficulty of choosing the task, build it and group members' divergence of ideas. I had a good learning about the aspects to consider when developing of a more elaborated task to apply in the classroom. The exchange in the whole group was important, everybody putting their own ideas on how some content proposed by the groups could work. This course should continue, but with teachers from other areas, because through it we can improve in learning. (Nalu, G2, Dec.2017).

Sofia shows that her initial expectations and impressions were not very positive, but she was surprised by the dynamics of the lesson study as the teachers were able to work in groups and collaboratively planned and realized the research lesson:

Initially I thought it would be a kind of boring course. When the texts came, I thought the discussions would be the same as the other courses I already took. However, as we started preparing the activities we were going to do in class, discussing what we were going to work on, and preparing the activities in the group, everything was different. I found it very interesting. (Sofia, G1, Dec.2017).

For Mateus, the experience in the lesson study led him to understand that the teacher's work need not be an isolated, individualized and that the teacher has greater opportunities to grow professionally in collaborative contexts:

The way as the planning of the activities went on surprised me, because all the teachers in the group worked collaboratively, there was no right or wrong. All the opinions and ideas presented were discussed by the group and then defined if it would be incorporated to the activities or if they did not meet the proposed objectives. It was a moment when everyone could express their ideas and reflect with the large group, including the presentation of their pedagogical strategies used in the classroom. I realized that we created a network of sharing and cooperation between us, members of the group, which ended up producing a feeling of trust among everybody, to expose their ideas and reflections. (Mateus, G2, Dec.2017).

According to the teachers, the dynamics of the lesson study led them to work collaboratively as they engaged in working together; focusing on classroom practice,

negotiating the whole process, sharing ideas and experiences, and seeking to promote a better teaching, which took place in the research lesson.

Synthesis

The teacher education process in which the teachers were involved in the lesson study favored professional learning in relation to professional culture. In effect, the specificity of the lesson study dynamic allowed teachers to experience collaborative work situations, which allows the teacher's professional growth through dialogue and exchange of experiences between peers. Likewise, the lesson study through the collaborative and reflective planning of the research lesson and later the reflection on this lesson led them to reflect on their own practice, questioning aspects culturally instituted in school contexts and their beliefs and predispositions. Finally, they were able to learn a teacher education approach, which differs substantially from the predominant approaches in schools and state education systems.

DISCUSSION

Participation in the lesson study provided the teachers with professional learning concerning their roles, knowledge, behaviors and professional practices, promoting peer interaction and observation of the classroom environment, including established behavior and peer professional practice (Garcia, 1999), covering the teaching of mathematics and professional culture. These aspects were favored by the collaborative and reflective nature of lesson study (Lewis, 2002; Murata, 2011; Ponte et al., 2014) and by the fact that this process involved teachers from the same area of knowledge and teaching level, focusing on their immediate professional needs.

Regarding **mathematics teaching**, an aspect highlighted refers to the professional practice (Garcia, 1999) that underpinned the research lesson, which was planned and conducted from an exploratory approach perspective of mathematics teaching (Fujji, 2018; Ponte, 2005; Ponte et al., 2014), which presupposes a modification in relation of the educational objectives promoted by the teacher (Inoue, 2011; Verhoef et al., 2014) in the classroom, focusing on the students' difficulties and ways of overcoming them. According to the participating teachers, the exploratory approach they tried in the research lesson allowed them to think about their daily teaching, at the same time leading students to face situations for which they did not have an immediately applicable solution method (Ponte, 2005). They added that the exploratory approach allowed them to promote the teaching of mathematics centered on students' understanding and as a way to motivate them to learn, as well as seeking to favor the understanding of curriculum topics addressed in the research lesson (Verhoef et al., 2014; Suh e Seshaiyer, 2015). Beyond these aspects, this approach favored the modification of teachers' beliefs and predispositions (Desimone, 2009), leading them

to rethink the roles and attitudes assumed by teachers and students in the classroom, as well as to promote mathematics teaching with emphasis in the process, and not only in the results. In this process, they modified the way to see and address students' mistakes, assuming them as a resource and context for mathematical discussion.

Beyond that, from the research lesson the teachers experienced different *classroom strategies* (Garcia, 1999; Inoue, 2011; Suh e Seshaiyer, 2015). The analysis indicates that teachers, by promoting collective work with students and especially mathematical discussions, were able to observe and understand students' difficulties and learning, especially when they were asked to communicate their mathematical ideas, solution strategies, and conclusions.

It was also stressed the learning about the *use of dynamic geometry software*. In addition to developing basic knowledge about the features of the GeoGebra software, the teachers were able to use this software to explore the properties of curriculum topics, deepening their and their students' knowledge about these topics (Lewis & Perry, 2017) in the planning stages and materializing it in the research lesson. They were also able to propose a new practice through this resource as a way to experiment with curriculum innovations (Lewis, 2016), especially in this case, the use of GeoGebra to approach mathematics in the classroom. The participants stressed at various times that the use of the software allowed them to explore new ideas, such as emphasis on different representations of quadratic equations and linear systems, and to test these ideas during the research lesson.

Regarding **professional culture** (Hargreaves, 2002; Fullan, 2001), the study points out that teachers' professional development can be promoted through their involvement in teacher education processes that provide opportunities for *reflection in practice* (Ponte, 1998). A point highlighted by teachers concerning their practice refers to a change in the way students conceive their participation in learning mathematics, a change verified by observing the practice of colleagues when teaching the research lesson and the teaching process, contrasting this experience with the work they do in the classroom. This aspect points to a modification in terms of teachers' beliefs and predisposition (Desimone, 2009) to rethink the roles that teachers and students assume in mathematics education. The reflection on practice led them to promote changes in teaching, specifically in the research lesson, as the intense and prolonged planning in the group allowed them to mobilize personal knowledge and beliefs, with knowledge from research (Desimone, 2009), articulating theory and practice.

In terms of the opportunity to transcend culturally instituted *teacher education dynamics*, the teachers point out that they were able to experiment with a teacher education approach focused on their professional development (Day, 2001; Flores, 2004; Ponte, 1998), that was distinct as it favored the collective and active work of teachers and, above all, by including professionals from the same area and level of education, contemplating their most specific professional needs in relation to teaching content (Desimone, 2009). In this sense, the lesson study allowed the teachers to experiment with a teacher education approach in which they worked collectively on the planning of

the research lesson, designed to situate students' particular learning problems (Cajkler et al., 2015). The overcoming of the formative tradition became possible as the teachers experienced forms of communication, work, and discussion in a process of teacher education centered on professional practice in the classroom. They also considered that the experience led the teacher education process into the classroom, also making professional collaboration possible in daily practice and changing the relationships between colleagues and students because together they experimented and reflected on the practice developed.

The teachers also add that in the lesson study they were able to enact collaborative work, and that the collaborative environment favored the growth of the group as they strengthened the relationship between them and the teacher educators. One aspect highlighted by the teachers was the negotiation of decisions at the planning stage and the exchange of experiences, which represented an important form of sharing. From this sharing, the confidence and support that encouraged them to try a new practice and take risks (Hargreaves, 1998) emerged by assuming the task of teaching the research lesson. Therefore, collaborative work was highly valued by teachers (Flores, 2004), favored by the dynamics of the lesson study and by the close interaction that was established in the group, providing the exchange of experiences and negotiation of the process. The collaborative work also permeated the relations with the members of the teacher education team as they participated in the teaching of the research lesson, accomplishing the shared teaching.

CONCLUSION

The present study supports the perspective that professional practice, properly integrated into teacher education contexts, constitutes a way for teacher development. In fact, the study shows how learning about how to teach and the development of a teaching culture takes place in the context of direct experiences, interactions and interlocutions with colleagues and teacher educators about situations related to classroom practice (Feiman, 2001). Therefore, teachers' professional learning is a social process (Bandura, 1977; Rotter, 1954), which is enhanced by the structure and mode of operation of the lesson study, providing significant learning about the teaching of mathematics and also the development of important aspects of professional culture.

ACKNOWLEDGMENTS

We thank especially the seventeen mathematics teachers of Rio Grande do Sul State Public Schools for accepting our challenge and truly engaging in the lesson study. We also thank the management team at the time of the 15th Regional Education Coordination of that State for contributing to the dissemination and conduction of the lesson study.

CONTRIBUTIONS FROM AUTHORS

A.R. designed the presented idea. A.R. developed the theory. M.L.T. adapted the methodology to this context, created the models, performed the activities, and collected the data. A.R. analyzed the data. All authors discussed the results and contributed to the final version of the manuscript.

DECLARATION OF DATA AVAILABILITY

The data of this research will be made available by the authors upon reasonable request.

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