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"*Arithmetical Complement of a Number*": A Mathematics Knowledge to Teach

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ABSTRACT

This article aims to present a theoretical-methodological discussion about *mathematics to teach*, a theoretical category developed by Valente (2017). This concept is mobilised from studies in the perspective of the History of Mathematics Education that intends to search the history of the professional knowledge of teaching and training of teachers, in the scope of mathematics teaching. These studies have used the conceptual categories of *knowledge to teach* and *knowledge for teaching*, developed by the Research Team on Social History of Education, as a theoretical-methodological tool for their investigations, which are the basis for the theoretical construction of the concept *mathematic to teach*. To achieve our objective, we tried to perform an articulation between the conceptions of Charlot (2000) and Hofstetter e Schneuwly (2017) regarding the presentation of the knowledge in the form of an object. Lastly, we applied the concept of mathematics to teach, in our perspective, to point the movement that has turned the *arithmetical complement* in knowledge in the form of an object, that is, in a *mathematics to teach*.

Keywords: Knowledge to Teach; Mathematics to Teach; Arithmetical complement; History of Mathematics Education.

"*Complemento Aritmético de um Número*": um Saber Matemático a Ensinar

RESUMO

Este artigo tem por objetivo apresentar uma discussão teórico-metodológica acerca da *matemática a ensinar,* categoria teórica desenvolvida por Valente (2017). Este conceito é mobilizado a partir de estudos na perspectiva da História da Educação Matemática que buscam investigar a história dos saberes profissionais de ensino e formação dos professores, no âmbito do ensino de matemática. Esses estudos, também, têm utilizado as categorias conceituais *saberes a ensinar* e *saberes para ensinar*, desenvolvidas pela Equipe de Pesquisa em História Social da Educação, como um ferramental teórico-metodológico para suas investigações, que são as bases para a construção teórica da *matemática a ensinar*. Para alcançar nosso objetivo, procuramos realizar uma articulação entre as concepções de Charlot (2000) e Hofstetter e Schneuwly (2017) quanto a apresentação do saber na forma de objeto. Por fim, mobilizamos a categoria *matemática a ensinar*, na perspectiva de nossa interpretação, para apontar o movimento

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que tornou o *complemento arithmético* em um saber na forma de um objeto, ou seja, em uma *matemática a ensinar*.

Palavras-chave: Saberes a Ensinar; Matemática a Ensinar; Complemento Aritmético; História da Educação Matemática.

INTRODUCTION

When it comes to the study of school knowledge, authors such as Dominique Julia (2001) and Jean-Claude Forquin (1992) advocate, for example, that such knowledge constitutes a school creation for school. This knowledge emerge as a product that is established in the school environment, the fruit of a denominated and specific school culture. To Valle (2014), the culture that is transmitted by the school, that is, the school culture is presented in an objective and unquestionable way. It is of a social nature, instituting itself from disputes and socio-political tensions. In this sense, many research works seek to apprehend the social status of different pieces of knowledge that are taught at school

In a defence that school knowledge is distinguished from other types of knowledge, we refer to another reference. Vincent (2008), who addresses the distinction between school knowledge, knowledge of science and knowledge of practice from the works Geneviève Delbos and Pascal Jorion (1984), states that:

> The **knowledge provided to the school**, even the contemporary school and whatever the epithet that accompanies the word school, **is not a scientific knowledge in the sense of knowledge of science. It is propositional knowledge.** It is closer to common sense than knowledge of science, more archaic (school physics is Aristotelian and not Galilean). Moreover, above all, it is not theoretical. **In fact, it summarises the knowledge in the form of propositions that are not logically connected and are content to declare contents.** For example, instead of a theory of multiplication as can be found in a book on sets theory, the school teaches the multiplication table, following propositions that affirm the "true" content, but that does not have logical connections between each other.[1] (Vincent, 2008, p.54, translation, our emphasis)

For Vincent (2008), school knowledge distances themselves from scientific knowledge from the perspective of their constitution. The school knowledge, according

[1] Le savoir dispensé par l'école, même de l'école contemporaine et quelle que soit l'épithète qui accompagne le mot école, n'est pas un savoir scientifique au sens de savoir de la science. C'est un «savoir propositionnel ». Il est plus proche du sens commun que le savoir de la science, plus archaïque (la physique scolaire est aristotélicienne bien plus que galiléenne). Et surtout, il n'est pas théorique. De fait, il résume le savoir sous forme de propositions non logiquement connectées et qui se contentent d'énoncer des contenus. Par exemple, en lieu et place d'une théorie de la multiplication telle qu'on pourrait la trouver dans un ouvrage sur la théorie des ensembles, l'école fait apprendre la table de multiplication, suite de propositions qui énoncent des contenus « vrais » mais qui n'ont pas de connexions logiques entre elles. D'où la nécessité ressentie par les élèves d'« apprendre par cœur », de « retenir la chanson » (Vincent, 2008, p.54).

to the author, are propositions that in sets will constitute a representation of intellectual content such as arithmetic, geometry, etc. These propositions, in turn, do not represent a theory in its complete form; they are closer to common sense. Taking the same example, the multiplication within the theory of sets is understood as a function over the numerical sets. In addition, one can associate a series of properties on this operation. However, the teaching of this operation, in some historical contexts, trails a distant path from the theoretical one. In the period of the First Republic in Brazil, some books as, for example, by the author Antônio Trajano, present the multiplication tables that synthesise the teaching of operations. However, there is still the presence of the Parker's Cards, which constitute a set of images that assist the teacher in the teaching of arithmetic, including the operations. Thus, the multiplication is seen from a resource that requires the student's senses, bringing this operation closer to its practical use, that is, closer to common sense.

In the wake of school knowledge, the mathematics that is taught at school can be thought of as a product within a school context. This mathematics, in general, presents itself as mathematics that will serve a type of student, teacher and school institution. Santos and Lins (2016), from his study on the ways of looking at mathematics in the initial formation of mathematics teachers, present two readings with respect to mathematics: one that argues in favour of the existence of a single mathematics and another that argues about the existence of different mathematics. Regarding the existence of different mathematics, the authors bring the School Mathematics to the scenario of debates. For the authors, this mathematics differs from Academic Mathematics, since the first (School Mathematics) refers to an object of the teacher and the second (Academic Mathematics) an object of the mathematician. Furthermore, they present argumentations that indicate differences between these two mathematics, which focus on the contents, in the purposes of each of them and the scope of semantics. For the contents, they consider that those that teachers mobilise in their professional practice are not the same produced in the academic environment. Regarding the purposes, the authors present the argument that school mathematics has educational purposes, that is, to instruct the student mathematically; Academic mathematics has scientific purposes, that is, to produce knowledge. From this exposition, the authors seek to instigate and broaden the conceptual discussions about school mathematics. In this sense, we understand that the contents that compose the school mathematics can be studied in the epistemic aspect, constituted of new knowledge, by showing this autonomy of the school contents in relation to the academic content.

We are interested in addressing the mathematics that deserves our attention, school mathematics. This has been the role of some researches that are developed in GHEMAT. The Master's dissertation by the author Alana Godoy Lacava, defended in 2017, in the Graduate Program in Scientific and Technological Education, conducts a study on the different approaches to the checking of calculations by casting out nines in textbooks. According to the author, this content is associated with other content such as arithmetic operations and divisibility. In his work, she presents a demonstration that validates such practice. However, between the lines of this work, we understand that this is a strict school content, which composes school practices, in which it is used as a resource capable of providing students with elements to verify their operations. Thus, we are faced with a content that should be taught to the student and that does not have direct references to the academic bosom. Therefore, the checking of calculations by casting out nines is established as know-how that must be taught in the school for the purpose of being used in their practices. Its legitimation as knowledge is found in the various books that address this content, as well described and analysed in the work of Lavaca (2017).

Valente (2017) brings this discussion to the scope of the history of mathematics education. He seeks to discuss school mathematics as an object that constitutes a teaching profession and, therefore, a work object of the teacher who teaches mathematics. For their discussions, they have mobilised the concepts of *knowledge to teach* and *knowledge for teaching*,[2] which present elements of a theoretical object consisting of the results of extensive studies carried out by the Research Team in Social History of Education (ERHISE).[3] These are two conceptual categories in relation to the professional knowledge present in the teaching and training professions. Thus, the studies of the ERHISE group have subsidized the researches of the Group of History of Mathematics Education (GHEMAT), which use the *knowledge to teach* and *knowledge for teaching* as theoretical-methodological contribution to their studies conducted in thematic projects. To establish a vectorised study in the context of the history of Mathematics Education, Valente (2017) establishes the categories *mathematics to teach* and *mathematics for teaching*, which rely on the conceptions of *knowledge to teach* and *knowledge for teaching*, but the proposal of the first two leads to studies that unroll on mathematical knowledge.

We seek in this article to present a path of interpretation for the category *mathematics to teach*, proposing a theoretical discussion about the *knowledge to teach* that enables us to broaden the mobilisation of its use for the analysis of the subjects present in the teaching of Mathematics. So, as an example, we propose the study and socio-historical analysis of the constitution of the "Arithmetical complement of a number" as an example of *mathematics to teach*.

THEORETICAL-METHODOLOGICAL FRAMEWORK: *MATHEMATICS TO TEACH* AS A THEORETICAL CATEGORY FOR THE STUDY OF SCHOOL MATHEMATICS IN TEACHING

For the authors Hofstetter and Schneuwly (2017), the knowledge exerts a privileged position in the education and training institutions; as a consequence, it also confers attributions to the professionals who work in teaching and training. For this reason, the authors assume that knowledge is integrated into the action. However, unlike other literature that addresses knowledge from its mobilisation in practice, they consider the knowledge formalised at the centre of their discussions, which are created and/or

[2] Because they are expressions that carry definitions and conceptions of their own, established by the studies of the Swiss group of Research in History of Education Sciences (ERHISE), the authors of this paper choose to highlight in italics for the two terms. Likewise, it occurs with the expressions mathematics to teach and mathematics for teaching developed in researches in GHEMAT in the perspective of the History of Mathematics Education.

[3] Acronym of *Équipe de Recherche en HIstoire Sociale de l'Éducation*.

established for/by the institutions of education and training, or their jurisdictions. For the authors, the formalised knowledge is of an objectified nature. For this reason, they adopt Barbier's postulate (1996) on objectified knowledge, which relates this knowledge:

> The realities with the status of representations [...] giving way to propositional utterances and being the object of a social appreciation sanctioned by a transmission-communication activity. They, these representations, therefore, have a distinct existence of those who enunciate them or those who appropriated them. They are conservable, cumulative, and appropriable. (Barbier, 1996, p.9 as cited in Hofstetter & Schneuwly, 2017, p.131)

The *knowledge to teach* and *knowledge for teaching* are presented as categories of objectified knowledge and which are constitutive of the profession of teaching and training (Hofstetter & Schneuwly, 2017). In this case, we understand that they can be observed in the actions of these professions, that is, in education and training. Without a deepening for now, the social valorisation, which allows the transmission and communication of this knowledge, will confer to it, with their respective proportions, an identity of formalised knowledge in the institutions of education and training, appearing, for example, in its normative and prescriptive documents. We have no intention of saying that this is the only way we can identify the *knowledge to teach* and the *knowledge for teaching*. Its social nature, seen as a result of objectification, drives research around this subject beyond normative and prescribed. In addition, its characterisation as propositional utterances goes to meet what was described by Vincent (2008), in which he places the school knowledge as propositional knowledge. This shows us a potential study from the epistemic point of view of this knowledge. These are the potentialities of application that make us perceive these two bodies of knowledge as categories for the analysis and interpretation of the knowledge present in the educational processes of education and training.

Hofstetter and Schneuwly (2017) define this knowledge as "the knowledge that are **objects** of their work" (p.131-132, emphasis added). As the authors attribute to the teaching profession the function of forming the other from the teaching of certain knowledge, described by the authors, as "knowledge to which to form" (p.132). In this sense, we understand that *knowledge to teach* is also an object of teaching. However, there is one question: what can we define as objects of the teacher's work? The use of the word object gives a conceptual breadth to the knowledge to teach since the etymological dictionaries refer to the meaning of object as being anything material that can be perceived by the senses, which for us is not enough to define knowledge as objects. In search of a better understanding and a deepening of the discussion about *knowledge to teach* treated as objects, we resorted to Charlot (2000), who in his work seeks to explore issues in the perspective of the relationship with knowledge and school. Thus, from a theoretical framework, from a sociological and psychoanalytic perspective, it clarifies the concept of relationship with knowledge and proposes a definition for this relationship. To do so, he conceptualises knowledge-object understood as the "very knowledge, while objectified", that is, when it presents itself as an intellectual object, as “the referent of a content of thought” (Charlot, 2000, p.75, note 10).

For Charlot (2000), learning is a process that has in its centre the acquisition of knowledge, which can be a knowledge in its broad sense, i.e., of intellectual content such as mathematics, history, geography, etc. or a knowledge that associates itself with an object or an activity (reading, writing, counting etc.). Thus, it is possible to enunciate two types of knowledge in the process of learning: knowledge, which binds to an intellectual and theoretical product, and know-how, which is associated with an activity. Given this, the author wonders: What is the knowledge? For him, knowing is “under the primacy of objectivity” equally as information, which according to the author, “is understood as being an external data to the subject, which can be stored, stocked, even in a database” (Charlot, 2000, p.61).

Charlot’s conceptualisation of knowledge (2000), from the activity of learning, is appropriate for our purpose that is to establish a discussion that implies a better understanding of knowledge to teach taken as objects. For this author, knowledge is constructed from “methodological frameworks” and a product of confrontation between subjects, which validate and share it. In the perspective of making this knowledge a product communicable and available to others, one can think of it in the order of the object, because as it states:

> Knowledge is presented in the form of “objects”, of decontextualized utterances that appear to be autonomous, to have existence, meaning and value by themselves and as such. [...] Knowledge is built on a collective history that is that of the human mind and the activities of man and is subjected to collective processes of validation, capitalisation and transmission. (Charlot, 2000, p.63-68)

To know whether to present in the form of object needs to go through a process of decontextualization, which consists of “disassociate it” from the activities and context that gave it origin, making it a generic product, with general properties, allowing to enunciate it From a normative set, approaching an algorithm, so that it can only refer to a given activity. Will be attributed a sense and a value in itself (Charlot, 2000). To achieve autonomy, we understand that knowledge goes through the process of depersonalization, in which its existence is separated from a subject, its emotions and perceptions, and can be analysed and interpreted only by its “algorithm” that provides it. This process of decontextualization and depersonalization, we understand that contribute to the objected knowledge to be enunciated in the form of propositions and conferring a self-existence, autonomous and independent. Also note that the conception given by the author about knowing converges with the proposed by Barbier on objected knowledge.

For Hofstetter and Schneuwly (2017) knowledge can be understood from two senses: in its broad sense, when it comes to a knowledge of an abstract nature, linked to intellectual content, or as know-how, which is associated with the practice. Therefore, these two senses seem to meet what Charlot proposes (2000) that knowledge is constructed from the human mind (knowing in its broad sense) and the activity of man (know-how). What this comparison places us is the existence of "two" forms of knowledge about the learning activity: one with an intellectual sense and the other with a practical sense. Here it is worth distinguishing between practical knowledge and knowledge of the practice. For Charlot (2000), learning a certain practice should not be regarded as knowledge. Practice will mobilise knowledge. In this sense, it is more correct to say that there is knowledge in practice. The practice mobilises tools and with it will produce a process of learning. However, this learning, which is the domain of a situation, is not of the same nature, nor in its process, nor its product, that the enunciable knowledge as knowledge-object.

Construction of knowledge, as quoted, is not individual, is a product of a "collective history", that is, it cannot be analysed and interpreted as an isolated result. It is the product of the interaction between the subject and his world, of how he knows him, and with other subjects. Therefore, for Charlot (2000), from the perspective of learning, knowledge is a relationship, called relations of knowledge, considered as social relations. It is in this scenario, of the knowledge seen as a social relationship, in the activity of learning that other processes occur on the knowledge: the validation, capitalisation and transmission. It is from the interpersonal confrontation between the subjects that knowledge will gain its validation in a social and scientific context, becoming this way legitimate. It may be capitalised; in other words, to be accumulated along with other knowledge, and transmitted, thus gaining a status of durability, since the transmission tends to pass to the following generations the knowledge that is considered relevant.

To this moment, what we have is a movement that presents us the knowledge in the form of an "object" in the context of discourse and ideas, called by Charlot (2000), a virtual object. It may be necessary now to discuss how knowledge can be presented materially. In this case, the author indicates that knowledge must be incarnate in the empirical object, i.e., books, manuals, etc. To do so, "[...] knowledge can only take the form of an object through language; even better, the written language, which gives it an existence apparently independent from a subject "(Lahire, 1993a and 1993b apud Charlot, 2000, p.68). Therefore, we understand that knowledge can only take the shape of an object (virtual and material) by writing.

So far, we have observed that the process of objectification appears in the theoretical frameworks of Hofstetter and Schneuwly (2017) and Charlot (2000). Therefore, we need to problematize this process. Valiant when discussing the objectification of knowledge, points us a path of how to identify this movement, because as the author states

> When everyone goes to "say the same thing (there is an establishment of consensus, through its circulation and appropriation by different actors, researchers, teachers, trainers, etc.) gives itself the objectification, that is, naturalisation of the" object "occurs the legitimation of objectification (through publications, courses, seminars, congresses, etc.) [...]. (Valente, 2017, p.20)

From the affirmation of Valente, we can think of the objectification from two perspectives: as circulation and appropriation of thoughts or even as something put (legitimised). As regards the objectification of knowledge, we understand that the first perspective describes the movement of objectification of knowledge, in which it is in the process of establishing itself. The second one presents a consolidated knowledge, or, in the "final" phase of consolidation, since it has undergone the initial process of objectification (circulation and appropriation) and that is, at a given moment, as a legitimised knowledge that constitutes an object of transmission. Given the source to be analysed, we can identify one of these perspectives. For example, a normative document that establishes a curricular grid presents us with the knowledge that is placed; that is, we are faced with objectified knowledge. The book adopted by a teaching system, or also of great circulation, can be thought of as support of an already objectified knowledge. Of course, these documents also serve to problematize the movement of consolidation of such knowledge, but it is not worth thinking about these sources if such knowledge is or is not objectified. The same cannot be thought about the school notebooks; what is put in these documents can be problematized in both perspectives, from a posted knowledge, which was prescribed by a normative document, even as the appropriation of a knowledge that can be in circulation. Thus, with regard to the type of source, objectification can be thought from the scope, jurisdiction, political power of documents, etc. Thus, the objectification of the knowledge put in normative documents, textbooks and school notebooks are distinct.

To this point, Charlot (2000) puts the discussion of knowledge in the perspective of learning without associating it with teaching. This link is important, since learning is not exclusive to teaching. For the author to learn is considered as an exercise of an activity in a place, at a time of its history and in different time condition, with the help of people who help to learn. In this perspective, the author expresses more clearly that learning is not an a-historical activity. Thus, each historical moment will determine qualities, properties, traits for the activity of learning. Thus, learning is an activity that serves its time, understood here as a historical context. In this way, we can dialogue with the understanding of Hofstetter and Schneuwly (2017) with regard to learning. The authors describe it as an intentional activity, of transforming the subject itself, in which it will develop the ability to transform in knowledge the resources it mobilizers in its action. Therefore, for all authors, learning is a specific and intentional activity that will specialise throughout history, implying the creation of specific places, which dedicate themselves to this activity, and of knowledge proper to it.

The sites destined for the activity of learning is an important point to be discussed. Charlot (2000) states that "the places in which the child learns have different statutes from the point of view of learning" (p.67). Between these statutes, it is possible to assign the function of educating, instructing and forming. For the author, instructing will be the central function of the school. For Hofstetter and Schneuwly (2017) is no different; the authors also understand, or even associate, learning to school spaces. It is in the school space that education is instituted, understood here as an activity that has the purpose of forming the other and defining the teaching profession. In this sense, while the learning activity falls on the student, teaching is a "responsibility" activity of the teacher. Among the teaching and learning activities, school knowledge represents a common element.

Therefore, the knowledge to teach compose a set of knowledge of an objected nature, which have undergone a process of validation and capitalisation, making them conservable over time. This process implies a process of social appreciation that puts it on a level of knowledge that deserves to be appropriate, soon transmitted. It is for this reason that the documents that relate to this process of transmission of knowledge, such as normative documents, prescriptive and textbooks, become so important to capture the knowledge to teach. Moreover, it has the epistemic aspect of these bodies of knowledge, in which they present themselves in the form of propositional, decontextualized and depersonalised statements, with their own meaning and autonomous existence, without being linked to other meanings.

Therefore, *mathematics to teach* is presented as a conceptual category for the studies of school knowledge, in particular, those focused on the teaching of mathematics, from a historical perspective. As this category relies on *knowledge to teach*, a theoretical-methodological framework is established that allows us to analyse and classify a given subject with *mathematics to teach*.

"*COMPLEMENT OF A NUMBER*" AS AN EXAMPLE OF A MATHEMATICS TO TEACH

The book *Tratado Elementar de Arithmetica* was written by José Adelino Serrasqueiro, having its first edition published in 1869.[4] In the pre-textual elements of the work, we found information about the author's formation: Bachelor of Philosophy from the University of Coimbra, his professional practice was being a professor of Mathematics at the Lyceu Central de Coimbra. In addition to this work, the author wrote others as, for example, *Tratado de Geometria Elementar* (first ed. 1879), Treatise of Elementary Algebra (first ed. 1878), all geared towards secondary education.

The pre-textual information indicates that the *Tratado Elementar de Arithmetica* was elaborated according to the program of Lyceu Central. Thus, the choice of the subjects that compose the book, as well as its organisation, probably follows the logic of the Mathematics teaching program of Lyceu Central. According to Guerra (2008), Portuguese high schools are created in 1836 and had the purpose of offering secondary education. The creation of this teaching network is inserted in a context of secondary education reform that occurred in Europe in this period, based on liberal thoughts that defended the preparation of the educating for useful work. The author also states that

> […] the positivist influence on the liberal regime was felt in defence of disciplines more connected to the new **Industrial society** and contributing to the preparation

[4] This information is on the website of the Portuguese Association of Mathematics Teachers. Retrieved 11 November 2018, from http://www.apm.pt/files/05.pdf.

> of the **Modern Man**, facilitating its integration into a changing world. A **set of disciplines linked to science and practice begin to be evident in the whole curriculum, contributing to the close relationship between learning in classes and the world of work.** Emerge in the 1st curriculum of teaching high school disciplines such as: "Principles of physics, chemistry, and mechanics applied to the arts, and crafts", "Principles of Natural History of the Three Kingdoms of Nature applied to the Arts and Crafts", "Principles of Political Economics, of Public and Trade Administration "and "**Arithmetic and Algebra, Geometry, Trigonometry and Drawing**". Through the analysis of the curriculum, it can be perceived that it was intended to **give young people a preparation not only of the part of humanities but also at the technical level, allowing them to access a set of professions in the areas of trade and industry**. (Guerra, 2008, p.29, our emphasis)

From the quotation, we note that the liceal teaching, offered by the Lyceums, was based on liberal thoughts that established a "new" conception of society, the Industrial one, of man, considered modern, integrated to change. So, in the liberal regime, the modern man is seen as an individual integrated into an industrial society, and it is necessary to inculcate in young people this new thought. To this end, teaching should be linked to the world of work, allowing this young man to access the universe of this industrial society. With this, the high school teaching structured its curriculum in order to contribute to this purpose.

It is possible to note that in the curricular structure of the high school education, there are a set of disciplines geared to the teaching of mathematics such as Arithmetic, Algebra, Trigonometry and Drawing. According to Hofstetter and Schneuwly (2017), the "training and teaching institutions are defined by the knowledge to teach that they specify" (p.137). Thus, the *knowledge to teach*, present in the Lyceums, should contribute to the formation of the modern man, so they must be constituted of knowledge that is related to the world of work. It will not be different with *mathematics to teach*, present in the disciplines of Arithmetic, Algebra, Trigonometry and Drawing, present in these educational institutions. Therefore, the topics set out in the book *Tratado Elementar de Arithmetica* should be regarded as knowledge that was legitimate and in agreement with the world of work since it was selected according to the Lyceu Central program. Therefore, we are faced with an objectified knowledge, first because it is the knowledge that is put by a teaching program, secondly, because they are materialised in the book, which can be understood as an instrument that has the role of transmitting this knowledge. Then, the work of Serrasqueiro for teaching arithmetic is a privileged source to capture *mathematics to teach*.

Through the pages of the book, we face the "arithmetical complement of a number". This issue presents itself as a resource that assists in the subtraction operations of more than two numbers, but in the book of Serrasqueiro it appears as *mathematics to teach*, that is, as a knowledge to be taught. In fact, if in the period of the nineteenth Century there was a shortage of technological apparatus that helped people with operations, so it was necessary to establish a knowledge that could provide elements that would instrumentalise people to act in various situations involving large operations as, for example, in commercial transactions. Therefore, knowing "instruments" was important, what gives a status of knowledge this type of knowledge.

Valente (1999) says that "the comparative analysis of his Arithmetic (Serrasqueiro) with that of Bertrand shows that certainly the Portuguese was based largely on the French text to construct his didactic" (p.160). Louis François Bertrand (1822 – 1900), was a mathematician who served as a professor at the *École Normale Supérieure* and the *École Polytechnique de Paris*. His book *Traité d'arithmétique* was reissued twelve times and considered one of the best of his time (Valente, 1999). Here we already perceive a concern of Serrasqueiro in seeking for a knowledge that had a social appreciation, since, according to Valente, his book has its contents based on the work of Bertrand. However, as we confront the issues that are established in the two works, we do not find "arithmetical complement of a number", or something similar, in the Arithmetic treatise written by Bertrand, which leads us to question the presence of this knowledge in the work of Sancheti. Are we facing an example of *mathematics to teach*?

In Bertrand's *Traité d'Arithmétique*, we did not find references to the "arithmetical complement". In search of this subject in other works of the same period, we find "arithmetical complement" in the book *Éléments d'Arithmétique*,[5] of 1847, of the author Bézout. According to the pre-textual elements of the book, it was in agreement with the norms of the Ministry of Public Education for French teaching; that is, we are facing a book intended for use for educational purposes. Although we do not wish to lengthen the discussion of the achievement of Bézout's works in French teaching, it is important to position ourselves in the choice of this work rather than others. Bézout was a widely disseminated author in the early days of Brazilian education, being adopted, for example, by the *Academia de Artilharia, Fortificações e Desenho* of Rio de Janeiro, in 1792. According to Valente (1999), Bézout's work is not committed to mathematical rigour, for this reason, it is not a work with scientific bias; that is, it contains new knowledge for science. On the contrary, it is regarded as a didactic manual, with texts for teaching and intended for students; for this reason, it is appointed as a diffuser of mathematical knowledge. Therefore, we are facing an author, whose mathematical knowledge is widely disseminated, including internationally.

In the book *Éléments d'Arithmétique*, we observed that the author intended a section to discuss the arithmetical complement, called "*Des compléments arithmétiques*". In this section, the arithmetical complement is presented as a means of simplifying operations with subtractions and also makes it clear that it is an application of logarithms. The way to get the arithmetical complement of a number is presented, and then an example is proposed to illustrate its application in the subtraction operation, in which it transforms it into an addition. In other words, in the *Éléments d'Arithmétique*, the arithmetical

[5] The digitized version of this work is available in the Gallica repository, which is associated with the National Library of France. Available at: https://gallica.bnf.fr/ark:/12148/bpt6k6211492w?rk=85837;2 Accessed on: 25 Nov. 2018.

complement is associated with the study of logarithms, presented as a resource for simplifying subtraction operations.

We do not intend to discuss the mathematical aspects of this subject, even if it has potential for future research. In addition, we do not intend to establish comparative analyses between the works of Serrasqueiro and Bézout as to the arithmetical complement of a number. What we care about is that in Serrasqueiro's work, the arithmetical complement has undergone a depersonalization and decontextualization, that is, its origin that rests in the studies of the logarithms, was disregarded in the book *Tratado Elementar de Arithmetica*, in which it is presented only as an arithmetic topic, without any association to the logarithms.

The depersonalization and decontextualization process contributes to making this knowledge an object. However, this is just not enough, because, according to Charlot (2000), a knowledge becomes an object by writing. In the book *Tratado Elementar de Arithmetica*, page 24, the "arithmetical complement of a number" is put:

> **28.** *Complemento arithmetico de um numero* é a quantidade que, sommada com elle, dá em resultado a unidade seguida de tantos zeros, quantos são os seus algarismos.
>
> Para achar o complemento arithmetico de um numero, subtrahe-se de nove cada um dos seus algarismos, excepto o ultimo algarismo significativo á direita, que se subtrahe de dez. Assim o complemento arithmetico de **7568** é **2432**.

Figure 1. Proposition 28 that of the arithmetical complement of a number (Serrasqueiro, 1926, p.24).

In this work, this subject appears composing the set of propositions of the session that deals with subtraction. This statement is placed independently of the logarithms, which gives it an autonomous character, of its own existence, different from that which originated it. Thus, in consonance with Charlot, we have that this subject assumes a status of knowledge in the form of "object", since, it is enunciated in the book in the form of a decontextualized proposition, even if it has an origin in another context. In this work, the arithmetical complement of a number is presented as an arithmetical knowledge.

After the enunciation, there is an example of how to get the arithmetical complement of a number. We can notice that the example mobilises a knowledge that stems from the proposition. Considering a one-digit number, for example, 7 we will have to complement it 3 because 7 + 3 = 10. In this example, the sum implied in the unit (1) followed by so many zeros equal to the number of digits of the number involved in the problem, in our example, it was a number formed by only one digit so the unit will be followed by only one zero. Another example would be to think of the complement of number 23 that will be 77, since 23 + 77 = 100. However, for cases of numbers with more than two digits, the book presents a simplified method. It indicates that one should think about the complement of each digit, that is, in the case of 23, we should think about the complement of 2 and 3. Following this logic, we will obtain 8 as a complement of 2 and 7 as a complement of

3; but 23 + 87 = 110 and it is for this reason that, in the example indicated in the book, it is indicated to take 9 of the digits that occupy the different order of the units, since these larger orders always will inherit the unit the previous order, since the arithmetical complement is what is missing a number to reach the immediately higher decimal unit.

The following topic, number 29, present on page 24 of the book *Tratado Elementar de Arithmetica*, presents the application of the arithmetical complement of a number:

29. Por meio do complemento arithmetico podemos reduzir a subtracção á somma. *Para isso, somma-se o diminuendo com o complemento arithmetico do diminuidor, e do resultado subtrahe-se a unidade immediatamente superior ao diminuidor.* Porque supponhamos a differença 3425 — 648: ajunctando ao diminuendo e ao diminuidor o complemento d'este ultimo, não alteramos o resto e vem

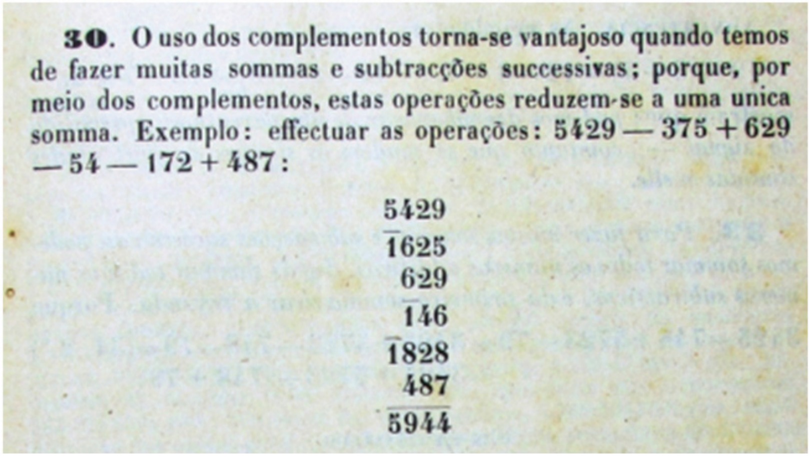
$$3425 - 648 = 3425 + \text{cl. } 648 - (648 + \text{cl. } 648)$$
$$= 3425 + \text{cl. } 648 - 1000.$$

Advertencia. Na practica, costuma-se escrever á esquerda do complemento arithmetico a unidade, que temos de subtrahir, affectada com o signal —. Exemplo: effectuar a subtracção 76498 — 57309:

$$\begin{array}{r} 76498 \\ \overline{1}42691 \\ \hline 19189 \end{array}$$

Figure 2. Example of the application of the arithmetical complement of a number (Serrasqueiro, 1926, p.24).

We note that this point seeks to give an example of the practical use of the knowledge put. Let us consider that the practice of subtracting now mobilised the learned knowledge, that is, the arithmetical complement of a number. According to what Charlot claims (2000), learning practice should not be understood as learning a knowledge, but the knowledge will be in what practice mobilises for its application. This example illustrates this idea, in which we have the arithmetical complement of a number being mobilised in a practice. Therefore, we understand that this matter is a knowledge that distances itself from practice, which assigns it an intellectual and autonomous character. Thus, it establishes the arithmetical complement as a knowledge to be taught, that is, *mathematics to teach*. Point 30, present on page 25 of the arithmetic treaty by Serrasqueiro, reinforces this idea of employing an intellectual knowledge in a practical activity:



30. O uso dos complementos torna-se vantajoso quando temos de fazer muitas sommas e subtracções successivas; porque, por meio dos complementos, estas operações reduzem-se a uma unica somma. Exemplo: effectuar as operações: 5429 — 375 + 629 — 54 — 172 + 487:

$$\begin{array}{r} 5429 \\ \bar{1}625 \\ 629 \\ \bar{1}46 \\ \bar{1}828 \\ 487 \\ \hline 5944 \end{array}$$

Figure 3. Example of the use of the arithmetical complement of a number in the practice of subtractions with more than two numbers (Serrasqueiro, 1926, p.25).

According to what was already been said, within the epistemic aspect, we seek to explore the application of the points that establish a *knowledge to teach*. In the example presented, we observed that the arithmetical complement went through a process of decontextualization, in which the author Serrasqueiro presents the subject in a detached form of his origin – from the studies of logarithms – and bestows him a strict arithmetic character, transforming it into an arithmetic knowledge. It then establishes a propositional writing that confers an existence and meaning in itself, no longer having the possibility of associating the utterance with the origins of this subject, that is, the logarithms. With this, Serrasqueiro prints on the "arithmetical complement of a number" a sense of *mathematics to teach*.

Thus, we seek to describe, albeit in a limited way, the constitution of an example of *mathematics to teach*, which constitutes a school knowledge. Although we cannot disregard that its origin is in scientific mathematics, the meaning that this knowledge acquires in the work of Serrasqueiro is strictly a school one. Its creation takes place to serve the purpose of instruction. Then, a rupture is created that distances the "arithmetical complement of a number" from its mathematical (academic) origin, the logarithms. This makes this knowledge make sense in itself, thus making it an example of *mathematics to teach*.

FINAL CONSIDERATIONS

School mathematics consists of a set of pieces of knowledge that differ from scientific knowledge. This mathematical knowledge constitutes objects that are created to fulfil their purpose towards teaching. We understand that *mathematics to teach* is constituted of these objects, which are described as objectified knowledge, which is a knowledge that undergoes a process of valorisation, legitimation and institutionalisation that end up printing meanings within the context of the school culture. For this reason, we understand that *mathematics to teach* presents itself as a theoretical-methodological resource to discuss such objects since they theoretically define a set of objects that constitute the professions of teaching and mathematical training. In the methodological sense, it offers us a path to follow and observe from some ideas, such as, for example, the decontextualization, objectification and bookkeeping of knowledge, the apparatus used to disseminate this knowledge, which allows us to discuss and analyse the production, or, the transformation of knowledge in the form of an object.

Given what has been put, we understand that the "arithmetical complement of a number", present in Serrasqueiro's *Tratado Elementar de Arithmetica*, is presented as an object to be taught. To do so, we seek to point out traces that allow us to identify it as *mathematics to teach*. We begin by characterising this knowledge as an objectified knowledge since it is present in a work of wide circulation that conforms to the norms of teaching, which allows us to read it as a knowledge that possesses a certain validation and legitimation. Then, we note that the arithmetical complement is a mathematical knowledge that is associated with the study of logarithms; however, in the work of Serrasqueiro, to which everything indicates, there was a decontextualization, in which the author sought to distance this subject from the logarithms. With this, it is presented to the student as an independent arithmetic knowledge. For this, the author was concerned to write it in the form of a proposition, attributing an autonomous and independent character to the arithmetical complement. Its consolidation, as a knowledge in the form of an object, gains relevance when gaining space within a work that aims to disseminate mathematical knowledge in teaching, as is the case of the *Tratado Elementar de Arithmetica*, which is configured as a textbook.

We understand that the analysis described is neither complete nor finalised. However, we hope that this discussion will contribute to instigating new work that focuses on the study of *mathematics to teach*, that new works seek to deepen in the points of the transformation of knowledge in the form of object

AUTHORS CONTRIBUTIONS STATEMENTS

C.S.B. and D.A.C. conceived the idea presented and developed the theory. Both authors analysed the data, discussed the results, and contributed to the final version of the manuscript.

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