

Algebra in Teacher Training in the Normal School of São Paulo: The First Teaching Programs of this Discipline

Ana Maria Basei ^a
 Wagner Rodrigues Valente ^a

^a Universidade Federal de São Paulo (UNIFESP), Escola de Filosofia, Letras e Ciências Humanas,
 Departamento de Educação, Guarulhos, SP, Brasil.

ABSTRACT

This text presents partial results of a research in development that aims to identify the purposes of Algebra in the formation of teachers in the Normal School of São Paulo, from 1880 to 1930. In this text, in particular, it is considered a specific period of time frame of research: mid-1890s; and, in order to analyse the first programs of this discipline for the training of primary teachers, documents such as legislation, a compendium of Algebra and programs of teaching of the normal and secondary course are mobilized. It was found that, in this period, the teaching of Algebra for the future teachers was more restricted than that offered in the secondary course. This reduced format of the Normal School program may be associated with the duration of normal course studies, shorter than the secondary course, but mainly, for the different purposes of the courses.

Keywords: teacher training; normal school; algebra; teaching program; compendium.

A Álgebra na Formação de Professores na Escola Normal de São Paulo: Os primeiros Programas de Ensino dessa Disciplina

RESUMO

Este texto apresenta resultados parciais de uma pesquisa em desenvolvimento que visa identificar as finalidades da Álgebra na formação de professores na Escola Normal de São Paulo, no período de 1880 a 1930. Neste texto, em particular, considera-se um período específico do marco temporal da investigação: meados da década de 1890; e, com o objetivo de analisar os primeiros programas dessa disciplina para a formação de professores primários, mobilizam-se documentos como legislação, um compêndio de Álgebra e programas de ensino do curso normal e do secundário. Verificou-se que nesse período o ensino de Álgebra para o futuro professor era mais restrito do que o oferecido no curso secundário. Esse formato reduzido do programa da escola normal pode estar associado à duração dos estudos do curso normal, inferior ao secundário, mas principalmente, às distintas finalidades dos cursos.

Palavras-chave: Formação de professores. Escola Normal. Álgebra. Programa de ensino. Compêndios.

Corresponding author: Ana Maria Basei. E-mail: anambasei@gmail.com

INITIAL CONSIDERATIONS

This text brings partial results of a research that aims to analyse the purposes of Algebra in the formation of teachers in the Normal School of São Paulo, in the interval between the 1880s and 1930s. In this text, in particular, it is considered a specific period of the research timeframe: mid-1890s. Documents such as legislation, an algebra compendium and normal and secondary course syllabi are mobilized, with the objective of analysing the first programs of the discipline for the training of teachers of primary education.

It should also be mentioned that the current research integrates a broader research given by the thematic project *Mathematics in teacher training and teaching: processes and dynamics of production of professional knowledge, 1880-1990*, which analyses, in historical perspective, the constitution of the professional knowledge of the teacher who teaches mathematics in the first years of school. Thus, the analysis of the empirical material of the investigation is carried out taking into account the theoretical-methodological references that guide this thematic project and that will be presented in the following.

THE TEACHER'S THEMATIC PROJECT AND PROFESSIONAL KNOWLEDGE

The theme teacher training is not new. In Brazil, it appears officially since the Law of Schools of First Letters, promulgated on October 15, 1827. This law, in establishing the mutual method as a method of instruction, required teachers to seek training in the capitals of the provinces (Saviani, 2009). Since then, the discussion about the nature of the specific knowledge to form the teacher has become present both in society in general and in academic research¹. In addition, in the last decades, this subject has gained important space also in the area of Mathematics Education, as shown in the studies of Fiorentini, Passos, and Lima (2016).²

Taking into account syntheses of the state of knowledge of the topic of teacher education,³ Valente, Bertini, Pinto and Moraes (2017) observe that “since the 1990s there is an emphasis on subjectivity in the way of dealing with the knowledge of the teaching profession, they are considered within specific contexts and situated” (Valente et al. 2017,

¹ There are several examples that could be cited revealing how the theme has surpassed the academic scope. Mention should be made, as a highlight, of the articles *Após pressão, formação de professor terá menos teoria e mais aula prática* published in the *Folha de São Paulo* newspaper, October 28, 2015, available at: <https://www1.folha.uol.com.br/educacao/2015/10/1699449-apos-pressao-formacao-de-professor-tera-menos-teoria-e-mais-aula-pratica.shtml>; and *Professores não são preparados para ensinar*, published in the IG Portal on April 23, 2012, available at: <https://ultimosegundo.ig.com.br/educacao/2012-04-23/professores-nao-sao-preparados-para-ensinar.html>

² The study reveals that between 2001 and 2012, 858 academic papers were developed that include master's dissertations and doctoral theses related to the training of teachers who teach Mathematics.

³ André (2011), Gatti (2014), Xavier (2014) and Cericato (2016) are examples of syntheses of the state of knowledge on the subject of teacher education.

p.15). The studies also reveal the difficulty of incorporating this knowledge in the initial teacher training. Being contextual, localized, proper to the subjects that are in action in the pedagogical practice, such knowledge has difficult systematization in the constitution of academic disciplines of the initial formation of teachers.

The analysis of the syntheses of the knowledge state also revealed Valente et al. (2017) “that the difficulty of professional characterization is linked to the better characterization of the teacher’s knowledge” (Cericato, 2016 as cited in Valente et al., 2017, p.15). That is, the difficulty of specifically specifying the knowledge of teaching reflects on their own recognition as a profession.

Distancing itself from the prevailing approach in current scientific research, which considers the subjective point of view of practice, from its mobilization in the making, the thematic project under development conceives the professional knowledge as “formalized forms of knowledge [...] that make it possible to construct a systematization and conceptualize its role in the professions of education and training” (Valente et al., 2017, p.10). This way of conceiving the professional knowledge of the teacher who teaches mathematics relies on theoretical-methodological references elaborated by *Équipe de Recherche en Histoire Sociale de l’Éducation* (ERHISE), working group at the University of Geneva, Switzerland.

From the studies of this team, to approach the professional knowledge of teaching, the thematic project considers the *objectified knowledge*. Such knowledge refers

[...] to realities with the status of representations [...] giving rise to propositional statements and being the object of a social valorisation sanctioned by an activity of transmission – communication. They, these representations, consequently have an existence distinct from those who enunciate them or from those who appropriate them. They are conservable, cumulative, appropriable. (Barbier, 1996, p.9, as quoted in Hofstetter & Schneuwly, 2017, p 131)

It is understood that this knowledge is separated from specific contexts, outside the situations of mobilization, separated from the subjects, disembodied, allowing them to be conservable and cumulative.

Thus, researches aligned with the thematic project consider the knowledge objectified to deal with the professional knowledge of the teacher.

It is imperative to characterize here what is being called the professional knowledge of the teacher. According to the research, this expression refers to the knowledge of teacher formation given by the articulation between the *knowledge to teach* and the *knowledge for teaching*. The former are “objects of his [the teacher’s] work; and the *knowledge for teaching*, [...] are the tools of his work “(Hofstetter & Schneuwly, 2017, pp.131-132). The *knowledge to teach* involves the school contents, in the case of the thematic project, the mathematics present in the school. On the other hand, the *knowledge for teaching*

have the specificity of teaching, are linked to those for the exercise of the profession of teacher, according to Hofstetter and Schneuwly (2017, 134).

They are mainly knowledge about “the object” of teaching and training work (about the knowledge to teach and about the student, the adult, their knowledge, their development, the ways of learning, etc.), about the practices (methods, procedures, devices, choice of knowledge to teach, organizational and management modalities) and the institution that defines its field of professional activity (study plans, instructions, purposes, administrative structures of organization and policies, etc.) (Hofstetter & Schneuwly, 2017, p.134)

The studies carried out by the Geneva team also show that the objectified knowledge is a result of long historical processes involving disputes between multiple social actors such as associations/unions, school administration, universities, among others (Outier, Passeron, & Revel, 2006; Hofstetter & Schneuwly, 2014 as cited in Valente et al., 2017, p.19). Thus, a historical research may reveal the processes of elaboration of the teacher’s professional knowledge, in particular, of the teacher who teaches mathematics in the first years of school (Valente et al., 2017, p.19).

PROFESSIONAL KNOWLEDGE OF PROFESSOR AND THE NORMAL SCHOOLS

Considering the interest in the constitution of the professional knowledge of the teacher who teaches mathematics, together with the contributions of ERHISE, works like that of Nóvoa (2014) help the investigation. This author, in an analysis of the historical process of teacher professionalization in Portugal, highlights the role of normal schools – institutions created from the 19th century onwards to train teachers for primary education – in the process of production and reproduction of knowledge of the profession:

The training institutions occupy a central place in the production and reproduction of the *body of knowledge* and the *system of norms* of the teaching profession, playing a crucial role in the elaboration of *pedagogical knowledge* and a *common ideology*. More than training teachers (at an individual level), normal schools produce the teaching profession (at a collective level), contributing to the socialization of its members and to the creation of a professional culture. (Nóvoa, 2014, p.18)

Corroborating with Nóvoa (2014), it is understood that the normal schools participated in the historical processes of constitution of the professional knowledge of the teacher, mainly with respect to the *knowledge for teaching*, the specific knowledge for the teaching profession. Thus, studies of documents relating to these schools, such

as laws, curricula, teaching programs, etc. can contribute to characterize the process of elaborating the professional knowledge of the teacher who teaches mathematics.

The institutionalization of primary teacher education occurs in Brazil, as well as elsewhere in the Western world, during the nineteenth Century (Tanuri, 2000; Lussi Borer, 2017, p.173). In Brazil, the first normal school was created in 1835, in the province of Rio de Janeiro, and, in the following years, several provinces established schools for the formation of primary teachers. A process of creation and extinction of normal schools in the provinces begins: due to the maintenance costs and the low number of graduates, the existence of these schools was questioned. Only after the years 1870, “when the liberal ideas of democratization and compulsory primary education are consolidated, as well as freedom of education”, emphasizes the need to better train the teacher for the first years of schooling, and the model of teacher training, via normal schools, gradually establishes itself (Tanuri, 2000, p.64, 66). In 1890, with the reform of the third⁴ Normal School of São Paulo, it is established the “standard of organization and functioning” to be expanded to cities in the interior of the state of São Paulo, and reference to other states of the country (Saviani, 2009, p.145). Thus, due to the representativeness of the model of the Normal School of São Paulo, it was decided to delimit the doctoral research to this school.

Specifically on the knowledge that was part of the training of future teachers during the nineteenth century, studies of the history of education indicate that the earliest curricula were limited to the curriculum of primary schools, in the case of mathematics, the four operations and proportions. It was about assuring the teacher the *knowledge to teach*. As for the pedagogical formation, it was in charge of a discipline – Pedagogy or Teaching Processes – of a prescriptive character. A scenario that has persisted for much of the nineteenth century. When the model of teacher training through normal schools is consolidated, a sophistication of the teacher training curriculum occurs, bringing it closer to the secondary education, while the pedagogical training is reduced to at most two disciplines – pedagogy and/or methodology and legislation and educational administration (Tanuri, 2000, p.67). According to Tanuri (2000, p.67), “As a rule, the normal schools did not reach the level of the secondary course, being inferior to this either in the content or in the duration of the studies”.

This approach of the teacher training curriculum with the secondary and reduced pedagogical training refers to the model of training called normal in the works of Lussi Borer (2017): in this model, offered by the institutions of secondary education to which future teachers ascend after primary education; a general training is offered, which includes a set of disciplines taught at a secondary level and reduced vocational training, mainly by the school director. According to Lussi Borer (2017),⁵ throughout

⁴ The first Normal School of the province of São Paulo was created in the year 1846 and extinguished in 1867; the second was installed in 1875 and worked until 1878. In 1880, the third Normal School was created, which, unlike the previous ones, did not have its existence interrupted.

⁵ According to Lussi Borer (2017), in terms of specific knowledge for teaching, historically we have two models: the *normal* and the *superior*. The *superior* model distinguishes between general and vocational training institutions: the general training courses are taught in the secondary course and vocational training takes place in higher establishments called Pedagogical Studies (Lussi Borer, 2017).

the nineteenth Century, school authorities progressively increase the duration of training courses, ranging from two or three years to four or five years, and try to approach the level of education offered in upper secondary courses; the knowledge of general formation predominates while increasing the knowledge specific to teaching, offered mainly at the end of the studies.

Specifically in relation to mathematical knowledge, Valente's (2011) study deals with the mathematical formation given to future teachers in the Normal School of São Paulo from 1875 to 1930. In this research, Valente (2011) articulates a wide variety of sources such as legislation, records of teachers' classes, tests of Normal School students, textbooks, etc. and focuses in particular on the Arithmetic and Geometry headings. The author notes that in the last quarter of the nineteenth century, the mathematics of the future teacher is aligned with the curriculum of the *Colégio Pedro II*,⁶ model for secondary education during the nineteenth Century.

Based on the findings of Valente (2011), and considering the interest of the research that tries to characterize the Algebra of teacher training, the present text confronts two programs of teaching of the Normal School of São Paulo with programs of the *Colégio Pedro II* referring to a specific period: mid-1890s. In this comparison, we analyse the programs in order to search for the specificities of Algebra in the context of teacher education.

ALGEBRA OF NORMAL SCHOOL VERSUS ALGEBRA OF COLLEGE PEDRO II

As for the knowledge present in the formation of future teachers, studies such as Tanuri (1979), Almeida (1995), Reis Filho (1995) and Monarcha (1999) present the curricula⁷ established for the Normal School of São Paulo during the nineteenth Century. Looking at the sets of planned disciplines, it is verified that the inclusion of the heading Algebra in the Normal School occurs, it seems, for the first time in the year 1890. A late inclusion in comparison to other mathematical rubrics, since Arithmetic and Geometry are present in the program of studies of the teachers already in the first foundation of the school, in 1846.⁸

During the search for documents of the Normal School of São Paulo referring to the 1890s, it was possible to locate two Algebra programs – one for the year 1894 and

⁶ Pedro II College was founded in 1837 in Rio de Janeiro and became a reference for similar schools, public and private, through programs of "preparatory exams" for entrance to higher education.

⁷ Education historians such as Tanuri (1979), Almeida (1995), Reis Filho (1995), and Monarcha (1999) bring the curricula of the Normal School from 1846, 1874, 1880, 1884, 1887 and 1890. However, Golombek (2016: 53) mentions a reorganization of the curriculum in 1882 with the inclusion of Algebra between disciplines. It seems that this is a reform proposed by a commission charged with reformulating public instruction and presented in 1881 (Brito, 1881, pp.11-14). The reformulation of the Public Instruction was carried out for seven years in the Provincial Assembly and was approved in 1887. As for teacher training in the Normal School of São Paulo, the only change in the course curriculum was the addition of Calligraphy and Drawing (Tanuri, 1979, p.38).

⁸ Geometry is absent in the curriculum of the second phase of the Normal School in 1874.

the other for 1895. The first program appears in the report of the director of the Normal School published in 1894, referring to the previous year. This document, prepared by the then director of the School, Gabriel Prestes, is available⁹ on the website of the Public Archive of the State of São Paulo (APESP) and gathers data such as: name of school teachers, teaching program of the disciplines, adopted textbooks, number of enrolments, budget etc..

According to the report, the professor of the chair of Arithmetic and Algebra was Joaquim José Azevedo Soares, named on May 1, 1889. The book *Professores da Escola Normal de São Paulo –1846-1890 – a história não escrita*, authored by Marcia Dias Hilsdorf, allowed locating information about this teacher.

Joaquim José de Azevedo Soares was born in 1852, in the city of Maricá, in the interior of the province of Rio de Janeiro, and held his studies at *Colégio Pedro II*. He began his teaching career at the Köpke College in Petrópolis and, in 1881, founded the Azevedo Soares College in the city of Amparo. In this college, institution of primary and secondary education, Azevedo Soares held the position of director and taught classes in Arithmetic, Algebra and Geometry. He joined the faculty of the Normal School of São Paulo in 1889, after tendering for the chair of Arithmetic and Geometry. Approved, he took office in May of 1889 and remained as teacher of the school until 1921 (Dias, 2002, p.208).

Moreover, what did teachers studied in Algebra classes of Professor Azevedo Soares? The report presents the programs¹⁰ of the normal school seats to be executed in the year 1894. In that year, due to Law no. 169 of August 7, 1893, the duration of the normal course increased from three to four years. The discipline¹¹ of Arithmetic and Algebra was located in the first year of the course and distributed as follows: in the first part of the academic year (1st grade) only Arithmetic was taught; and, in the second part (2nd series), Algebra and continuation of Arithmetic. For each of the subjects there were three classes per week (Prestes, 1894).

⁹ Available in: http://200.144.6.120/uploads/acervo/periodicos/relatorios_educacao/RDRSP1894.pdf

¹⁰ The programs were organized by the professors of the chairs and later analyzed by a committee composed of five professors (José Feliciano, Godofredo Furtado, Carlos Reis, Dr. Canuto do Val, Thomaz R. de Lima). After amendments, they were approved in November of 1893, coming into force from the following year, "save the modifications that at the end of each year the teachers judge of convenience to propose in view of the necessities indicated by the practice" (Prestes, 1894, p.43).

¹¹ By Decree nº 144-B, of December 30, 1892, Arithmetic and Algebra included the 10th chair; then, with Decree No. 218, of November 27, 1893, they moved to the 6th chair.

The expected contents for the Algebra discipline are shown in Figure 1:

ALGEBRA.	
1. ^ª PARTE.	
	1. Noções geraes. Redução dos termos semelhantes.
	2. Adição e subtração algebraica.
	3. Multiplicação algebraica, emprego dos signaes e formulas.
	4. Divisão algebraica, emprego dos signaes e formulas.
minador.	5. Theoria das fracções algebraicas. Redução ao mesmo deno-
	6. Theoria do maximo common divisor.
	7. Operações sobre as fracções algebraicas.
	8. Noções sobre as equações e suas classificações.
	9. Equações e problemas de 1. ^o grau a uma incognita.
minação.	10. Theoria elementar da eliminação. Diversos methodos de eli-
	11. Equações e problemas do 1. ^o grau a duas e mais incognitas.
equações de 1. ^o grau.	12. Formulas geraes para a resolução de um systema qualquer de
	13. Solução negativa. Theoria das quantidades negativas.
	14. Discussão das equações e problemas do primeiro grau.
	15. Problema dos correios. Sua discussão.
	16. Problemas indeterminados.
2. ^ª PARTE.	
	17. Quadrado e raiz quadrada das quantidades algebraicas.
	18. Equações do segundo grau a uma incognita.
	19. Equações irracionais.
	20. Equações simultaneas do segundo grau.

Figure 1. Algebra Program –1894 (Prestes, 1894, p.50).

Also according to the report, for Algebra classes, we chose the compendium *Elementos de Álgebra*, by Cristiano Benedito Ottoni. The compendium was used in the superior courses of the Empire and in the *Colégio Pedro II* between the years of 1856 and 1870, becoming reference for the secondary courses during much of the second half of the nineteenth Century (Vechia & Lorenz, p.54, 59).

In general, the eighth edition¹² of Ottoni's *Álgebra* presents the following contents distributed in five chapters and an appendix:

¹² For the writing of this text it was possible only the editions of 1856, 1879, 1893 and 1894. As announced in the online bookstore Traça.com (<https://www.traca.com.br/livro/948849/>) the contents of the 1892 edition are the same as those of the 1893 edition.

Table 1

Topics presented in Ottoni's Algebra – 1893.

Chapter II	Algebraic symbols; Classification of algebraic expressions; Similar terms – reduction; Algebraic fractions. Smaller common multiple; Larger common divisor; Notions about the theory of functions – classification.
Chapter II	Preliminary notions about equations; Equations and problems of the first degree to one, two or more unknowns; Discussion of problems and equations of 1st grade – General formulas.
Chapter III	Undetermined problems.
Chapter IV	Equations and problems of the second degree to an unknown; Composition of the equation of the second degree; Properties of the 2nd degree trinomial; Equations and problems of the second degree to two or more unknowns; Inequalities; Biquadratic Equations.
Chapter V	Powers and roots of all degrees; Newton's binomial; Powers and roots of algebraic quantities; Calculation of radicals; Method of indeterminate coefficients.
Appendix to Chapter IV	Properties of the 2nd degree trinomial. Maximum and minimum. Binomial equations. Irrational equations. Progressions for differences and quotient; Theory of exponentials and logarithms; Compound interest; Notions about the series; Elementary theory of derivatives; Elementary theory of differences.

Source: Prepared by the authors from Ottoni (1893).

Comparing the teaching program and Ottoni's *Algebra*, it is verified that the course taught by Professor Azevedo Soares contemplated the part of Ottoni's work restricted to the equations of first and second degree. Specifically in the second part of the program, there are two aspects that suggest the use of another reference for the program besides the Ottoni compendium: different titles indicate the systems of 2nd degree equations – in the program there are simultaneous equations of 2nd degree and in the book Equations and problems of the second degree to two or more unknowns; the topic Irrational equations was the only one chosen in a set of topics listed in the appendix of the book.

Due to the alignment of the formation of the normalist with the secondary course previously explained, we looked for Algebra programs of the *Colégio Pedro II*. A search of academic papers about the *Colégio* and about the secondary course resulted in locating the master dissertation of Elaine Cristina Luiz, which contains the programs of the years of 1892, 1893, 1895 and 1898.¹³ For this text, it was decided to take the programs of 1892, 1893 and 1895, by the proximity to the dates of the normal school programs located so far – from the years of 1894 and 1895.

According to Luiz (2014: 190), in 1892 the study of Arithmetic and Elementary Algebra was planned in the first year of the course and continuation of the study of Algebra in the third year:

¹³ In that decade, *the Tratado de Álgebra Elementar*, by José Adelino Serrasqueiro as reference (from 1892 to 1914 and from 1926 to 1928) was adopted at *Colégio Pedro II*.

	(Arithmetica – estudo completo; Algebra Elementar – estudo completo)
Primeiro Anno 1ª Cadeira	<ol style="list-style-type: none"> 1. Quantidade, unidade e numero. Numeração e consideração sobre os signaes. 2. Operações sobre numeros inteiros e decimais (seis operações). 3. Operações sobre fracções ordinarias e numeros mixtos (seis operações). 4. Divisibilidade; suas consequencias: restos e provas. 5. Maximo comum divisor e menor multiplo comum. Simplificação e redução de fracções ao mesmo denominador. 6. Conversões: fracções periodicas e continuas. Metrologia. 7. Igualdade. Razões e proporções. Regras de tres, de juros simples, de desconto, e de companhia ou das partes proporcionaes. 8. Estudo sobre a composição do polynomio. 9. Multiplicação. Divisão. Quadrado e raiz quadrada dos polynomios. 10. Da função e da equação. 11. Resolução da equação do 1º grão e uma incognita – Discussão. 12. Da eliminação na resolução dos differentes systemas de equações do 1º grão. 13. Resolução e composição da equação do 2º grão – Discussão, tudo a uma incognita. 14. Equações reductiveis ao 2º grão. 15. Analyse indeterminada do 1º grão. 16. Progressão – Logarithmos. Regra de juro composto e anuidade. 17. Fórmula do binomio – Fórmulas de Cramer – Discussão geral das equações do 1º grão. <p>Todos os pontos deste programma serão seguidos de exercicios e problemas.</p> <p>Por ultimo: consideração geral sobre o estudo da arithmetica e algebra, precisando suas differenças e acompanhando suas evoluções.</p>
	Algebra
Terceiro Anno	<p>Emprego dos sinaes algebricos e suas consequencias principaes. Estudo comparativo das operações fundamentais, bem assim das potencias e raizes, que se referem ao 2º grão. Propriedades geraes dos numeros. Equações do 1º e 2º grãos a uma incognita. Da eliminação das equações do 1º grão a muitas incontinitas. Analyse indeterminada do 1º grão entre duas variaveis. Discussão dos problemas e equações do 1º e 2º grão a uma incognita. Problemas. Exercicios sobre calculo algebrico.</p>

Figure 2. Colégio Pedro II. Algebra Program – 1892 (Luiz, 2014, p.90).

The study of algebra began in the chair of Arithmetic and continued in that of Algebra. As for the topic of *irrational equations*, which is constant in teacher training, it is also part of the secondary course, within the study of *equations irreducible to the second degree*.

In 1893, the study of algebra was carried out in the second year, in the chair of Elemental Algebra, with a complete study and arithmetic revision:

1. Estudo sobre o monomio e o polynomio.
2. Adição e subtracção.
3. Multiplicação.
4. Divisão.
5. Potenciação.
6. Radiciação.
7. Da função e da equação.
8. Resolução e discussão da equação do 1º gráo a uma incognita.
9. Da eliminação nos diferentes systemas de equações do 1º gráo.
10. Resolução, composição e discussão da equação do 2º gráo a uma incognita.
11. Equações reductives ao 2º gráo.
12. Analyse indeterminada do 1º gráo.
13. Formula do binomio de Newton.
14. Formula de Cramer – Discussão geral das equações do 1º gráo.
15. Progressões e logarithmos.
16. Equações exponential. Consideração geral sobre a arithmetica e a algebra.

Todos os pontos deste programma serão seguidos de exercicios e problemas.

Figure 3. *Colégio Pedro II*. Algebra Program – 1893 (Luiz, 2014, p.190).

Comparing these two programs of *Colégio Pedro II* with that of the Normal School of São Paulo for the year 1894, it is observed that the sequence of contents studied by the teachers was more restricted than in the secondary course: it did not comprise study of functions, binomial of Newton and exponential equations. Instead, the contents of progressions and logarithms, in the case of future teachers, were offered in the discipline of Arithmetic (Prestes, 1894, p.50). On the other hand, in *Colégio Pedro II*, the study of the equations of second degree was restricted to an unknown, while in the Normal School studies of equations of second degree with more than one incognito.

In the continuity of the research, it was possible to locate the report of the director of the Normal School for the year 1895. This document contains the programs for each chair and the name of the teacher responsible. However, unlike the document of the year 1894, it does not contain information on the compendia used.

In 1895, Joaquim J. de Azevedo Soares remains responsible for the subjects of Arithmetic and Algebra. As stated in the report, at the meeting of the congregation held on July 2, the teacher indicates the joint teaching of the two disciplines:

[...] by indication of the respective teacher, the Congregation decided that from now on, the teaching of Arithmetic, instead of being given only in the first Series, it would also extend to the second, along with that of Algebra, with three classes of each one of these subjects per week in each section. (Prestes, 1895, p.71)

The curricula of both disciplines were presented in the Report as follows:

Table 2

Arithmetic and Algebra Chair Program – 1895.

August	From 1 to 10	<i>Arithmetic:</i> Proportion. - Fundamental property and its consequences. - General properties and their corollaries. - Exercises on equidifference. <i>Algebra:</i> Preliminary notions. - Addition. - Subtraction. - Multiplication. Notable multiplication facts.
	From 12 to 31	<i>Arithmetic:</i> -Theory of progressions - Progression by difference. - Properties and formulas. - Progression by quotient. - Properties and formulas. -Logarithms. -General properties of logarithms. - Briggs system. -Tables of Callet. - Given an integer or fractional number, find its logarithm. - Given a log, find the corresponding number. <i>Algebra:</i> - Notable cases of divisibility. - Operations on algebraic fractions. - Maximum common divisor. - Of the equations in general. - Classification of equations. - Equations of the first degree to an unknown. - Problems of the first degree to an unknown.
September	De 2 a 14	<i>Arithmetic:</i> - Three simple and compound rule. - Problems. -Reasonable and commercial discount rule. -Problems. -Complete rule. <i>Algebra:</i> Elementary elimination theory. - Elimination methods: addition and subtraction, substitution, comparison and Bézout method. - Principles concerning simultaneous equations. -Systems of simultaneous equations.
	From 16 to 30	<i>Arithmetic:</i> - Simple and compound company rule. - Annuity rule. - False position rule. -Problems. <i>Algebra:</i> Negative solutions. -Theory of negative quantities. -General interpretation. - Descartes' principle. -Discussion of the equations and problems of the first degree to one and two unknowns.
October	From 1 to 8	<i>Arithmetic:</i> - Recall: Theory of logarithms. -Rule of three. - Simple and compound interest rate. -Company rules. <i>Algebra:</i> -Problem of the post office. - Positive solutions. - Negative solutions. - Infinite solutions. - Undetermined solutions. -Discussion of undetermined problems.
	From 10 to 31	<i>Arithmetic:</i> - Recall: -Divisibility of numbers. -Theory of the common maximum divisor. -Theory of prime numbers. -Common fractions. - Continuous fractions. <i>Algebra:</i> -Formation of the square and root extraction of algebraic quantities. -Records of the second degree. Calculation of second-degree radicals. - Equations and problems of the second degree.

Source: Retrieved from Prestes (1895, p.42).

Despite the format of the presentation of the contents of Arithmetic and Algebra, the distribution of the contents of the subjects leaves doubts as to whether teacher Azevedo Soares articulated the two teachings, as indicated by the September provision, for example.

When considering the programmatic contents, the title, *Principle of Descartes*, is found among the topics. This principle, used to interpret negative solutions to problems,

does not appear in the 1894 program nor in Ottoni's *Algebra*, which again suggests that prof. Joaquim J. de Azevedo Soares used other references besides Ottoni. Returning to the programs of the *Colégio Pedro II*, it is also noted that the title is not in the *Colégio*.

<p>Second Year 1st discipline</p>	<p>Preliminaries. Addition, subtraction, multiplication, and algebraic division. Divisibility by $x \pm a$.</p> <p>Newton's binomial. Powers and roots of algebraic expressions.</p> <p>Algebraic fractions and their simplification.</p> <p>Functions, equations, and their respective classification.</p> <p>Resolution and discussion of the equations of the first degree with one or more variables. Elimination processes.</p> <p>Undetermined calculation of the first degree.</p> <p>Resolution, composition and discussion of the equation of the second degree with a variable.</p> <p>Reducible equations to the second degree. Notions about exponential calculation.</p> <p>Progressions by quotient. Algebraic theory of logarithms.</p> <p>Problems of 1st and 2nd grades. Compound interest and annuities.</p> <p>Considerations on Arithmetic and Algebra: Their Fundamental Differences.</p>
---	---

Figure 4. Algebra Program of the *Colégio Pedro II* – 1895 (Luiz, 2014, p.191).

Another observation to be made is that the contents of functions, Newton's binomial and exponential equations, which remain in the *Colégio*, were not incorporated into the formation of future teachers in 1895.

CONSIDERATIONS

Comparing the programs of Algebra of the Normal School of São Paulo with those of the *Colégio Pedro II*, it is verified that, at least until the year 1895, the formation of the teacher in the Normal School of São Paulo, with respect to Algebra, was more restricted than that offered at the secondary level, since it did not include the study of functions, binomial of Newton and exponential equations, being limited to the equations of second degree. This reduced format of the normal school program may be associated with the duration of normal course, lower to the secondary one, but mainly with the different purposes of these courses: secondary education was prepared for higher education, while normal education¹⁴ for primary education, at that time divided into preliminary

¹⁴ Article 261 of Decree 218, dated November 27, 1893, establishes: The purpose of these establishments is to provide the theoretical and practical education necessary for those who are destined to the career of the primary teaching profession as preliminary, complementary teachers or their deputy (Article 23 of Law No 88).

and complementary.¹⁵ Among mathematics¹⁶ to be taught in the complementary course, it was included, for the second year, *Elements of algebra up to equations of the second degree, included*. Apparently, in that period, the presence of Algebra in the normal course was associated with the intention to approach the knowledge of teacher training and those taught by the teachers in their professional work. In this case, it is a question of making the teacher to acquire the *knowledge to teach* in the complementary school. Thus, the knowledge of teacher training should be kept to the limit of what would be taught in the courses. With this conception of formation, it would not fit the development of a complete elementary course of algebra. It should be noted that among the subjects of the complementary course there were the “Elements of algebra up to equations of second degree” integrating part of the planned contents for the second year of the complementary course, according to the decree 218, of November 27, 1893. It seems that, in that period, the presence of Algebra in teacher training was associated with the approximation between the knowledge of teacher training and the knowledge to be taught in his or her craft, with the *knowledge to be taught* in the complementary school.

However, the analysis of the presence of Algebra in teacher training and its purposes show, with the data obtained up to the present, that references to the Normal School are closely linked to their teachers. Specialists and recognized in their *status* as *professors* had the power to build references for teaching, in addition to greater institutional discussions. This role is reinforced, moreover, by the long duration exercised in the teaching of the school by a certain teacher. Thus, this study leads us to investigate, in future works, the course of the Normal School teachers in charge of the mathematics taught to future teachers.

AUTHORS' CONTRIBUTIONS STATEMENTS

Both authors conceived the idea presented, developed the theory, collected and analysed the data, discussed the results, and contributed to the final version of the manuscript.

REFERENCES

Almeida, J. S de (1995). Currículos da Escola Normal Paulista (1846-1920): revendo uma trajetória. *Revista Brasileira de Estudos Pedagógicos*, 76(184), 665-689.

¹⁵ Law no. 88, of September 8, 1892, divided primary education into two courses: the preliminary one, compulsory for children from 7 to 12 years old, and the supplementary, destined for students qualified in the subjects of the preliminary course; conceived as a complement to primary education, parallel to secondary education, aimed primarily at those who were not to continue schooling. Both lasted four years.

¹⁶ The curriculum of the complementary course was established in law no. 88 and contained: Elementary arithmetic and algebra elements to 2nd degree equations, inclusive, Flat and space geometry, Trigonometry and mechanics notions, aiming their applications to the simplest machines.

André, M. E. D. A. (2009) A produção acadêmica sobre formação de professores: um estudo comparativo das dissertações e teses defendidas nos anos de 1990 e 2000. *Revista Brasileira de Pesquisa sobre Formação de Docente*, 1(01), 41-56. Disponível em: <http://formacaodocente.autenticaeditora.com.br/artigo/exibir/1/7/3>. Acesso em: 25 mar. 2017.

Barbier, J. (1996) (Ed) *Savoirs théoriques et savoirs d'action*. Paris: PUF.

Cericato, I. L. (2016) A profissão docente em análise no Brasil: uma revisão bibliográfica. *Revista Brasileira de Estudos Pedagógicos*, 97(246), 273-289. Disponível em: <http://dx.doi.org/10.1590/S2176-6681/373714647>.

Decreto n.º 144B, de 30 de dezembro de 1892 (1892). Aprova o regulamento da instrução pública. Retirado de 20 de agosto, 2018, de <http://www.al.sp.gov.br/repositorio/legislacao/decreto/1892/decreto-144B-30.12.1892.html>.

Decreto n.º 218, de 27 de novembro de 1893 (1893). Aprova o Regulamento da Instrução para execução das leis n. 88, de 8 de Setembro de 1892, e 169, de 7 de Agosto de 1893. Assembleia Legislativa do Estado de São Paulo, 1893a. Retirado em 20 de agosto, 2018, de <https://www.al.sp.gov.br/repositorio/legislacao/decreto/1893/decreto-218-27.11.1893.html>.

Dias, M. H. (2002) *Professores da Escola Normal de São Paulo – 1846-1890 – a história não escrita*. São Paulo: FEUSP.

Fiorentini, D., Passos, C. L. B. & Lima, R. C. R. (Orgs.). (2016). Mapeamento da pesquisa acadêmica brasileira sobre o professor que ensina matemática: período 2001-2012. Campinas: FE/UNICAMP. Retirado em 14 de julho, 2018, de <https://www.fe.unicamp.br/pf-fe/pf/subportais/biblioteca/fev-2017/e-book-mapeamento-pesquisa-pem.pdf>.

Gatti, B. A. (2014) Formação inicial de professores para a educação básica: pesquisas e políticas educacionais. *Est. Aval. Educ.*, São Paulo, 25(57), 24-54. Disponível em: <http://www.fcc.org.br/pesquisa/publicacoes/eae/arquivos/1899/1899.pdf>.

Golombek, P. (2016). *Caetano de Campos: a escola que mudou o Brasil*. São Paulo: EDUSP.

Hofstetter, R. & Schneuwly, B. (2017). Saberes: um tema central para as profissões do ensino e da formação. In R. Hofstetter & W. R. Valente (Orgs.). *Saberes em (trans) formação: tema central da formação de professores* (pp.113-172). São Paulo: Editora Livraria da Física.

Hofstetter, R. & Schneuwly, B. (2014). Disciplinarisation et disciplination consubstantiellement liées. Deux exemples prototypiques sous la loupe : les sciences de l'éducation et des didactiques des disciplines. In: Balz Engler (Orgs.). *Disziplin-Discipline*. Fribourg: Academic Press (pp.27-46).

Lei n.º 88, de 8 de Setembro de 1892 (1892). Reforma a instrução pública do Estado. Retirado em 14 de julho, 2018, de <https://www.al.sp.gov.br/repositorio/legislacao/lei/1892/lei-88-08.09.1892.html>.

Lei n.º 169, de 7 de agosto de 1893. (1893). Addita diversas disposições á lei n. 88, de 8 de Setembro de 1892. Retirado em 14 de julho, 2018, de <https://www.al.sp.gov.br/repositorio/legislacao/lei/1893/lei-169-07.08.1893.html>.

Luiz, E. C. (2014). *Os compêndios de Cristiano Benedito Ottoni e José Adelino Serraqueiro para o Ensino de Álgebra no Colégio Pedro II (1856- 1928)*. Dissertação

de Mestrado em Educação, Universidade Estadual de Mato Grosso do Sul, Unidade Universitária de Paranaíba, Parnaíba.

Lussi Borer, V. (2017). Saberes: uma questão crucial para a institucionalização da formação de professores. In: Hofstetter & W. R. Valente (Orgs.), *Saberes em (trans) formação: tema central da formação de professores* (pp.173-199). São Paulo: Editora Livraria da Física.

Monarcha, C. (1999). *A Escola Normal da Praça – o lado noturno das luzes*. São Paulo: Editora da Unicamp.

Nóvoa, A. (1999). O passado e o presente dos professores. In: Nóvoa, A. *Profissão professor*. (2a ed.). (pp.13 – 34). Porto: Porto Editora.

Outier, J., Passeron, J. & C.; Revel, J. (2006). *Qu'est-ce qu'une discipline?* Paris: Editions EHESS.

Otoni, C. B. (1879). *Elementos de Álgebra* (8a ed.). Aumentada com muitas notas no texto. Rio de Janeiro: Clássica de Alves & Comp. (Obra original publicada em 1852). Retirado em 14 de julho, 2018, de: <https://app.uff.br/riuff/handle/1/762>.

Prestes, G. (1894). Relatório da Escola Normal apresentado ao sr. dr. Cesário Motta Junior, Secretario dos Negócios do Interior, por Gabriel Prestes, diretor da Escola Normal em 1894. In Relatório apresentado ao Presidente do Estado de São Paulo por Cesário Motta Junior, Secretário dos Negócios do Interior em 28.3. 1894. São Paulo: Typographia a Vapor de Vanorden & Comp. Retirado em 14 de julho, 2018, de http://200.144.6.120/uploads/acervo/periodicos/relatorios_educacao/RDRSP1894.pdf.

Prestes, G. (1895). Relatório da Escola Normal apresentado ao sr. dr. Cesário Motta Junior, Secretario dos Negócios do Interior, por Gabriel Prestes, diretor da Escola Normal em 1895. São Paulo: Typ. do Diário Oficial.

Reis Filho, C. dos (1995). *A educação e a ilusão liberal: origens do ensino público paulista*. Campinas: Autores Associados.

Saviani, D. (2009). Formação de professores: aspectos históricos e teóricos do problema no contexto brasileiro. *Revista Brasileira da Educação*, 14(40),143-155. Disponível em: <http://www.scielo.br/pdf/rbedu/v14n40/v14n40a12.pdf>.

Brito, L. A. de (1881). Relatório apresentado à Assembleia Legislativa Provincial de São Paulo pelo presidente da província, Laurindo Abelardo de Brito, no dia 13 de janeiro de 1881. Santos, Typ. a Vapor do Diario de Santos, 1881. Retirado em 13 de setembro, 2018, de <http://brazil.crl.edu/bsd/bsd/1025/000009.html>.

Tanuri, L. M. (1979). *O ensino normal no Estado de São Paulo: 1890-1930*. São Paulo: USP.

Tanuri, L. M. (2000, maio-agosto). História da formação de professores. *Revista Brasileira de Educação*, (14), 61-88.

Valente, W. R. (2011). *A Matemática na Formação do Professor do Ensino Primário: São Paulo (1875-1930)*. São Paulo: Annablume: Fapesp.

Valente, W. R., Bertini, L. de F., Pinto, N. B., & Moraes, R. dos S. (2017) *A Matemática na Formação de Professores e no Ensino: processos e dinâmicas de produção de um saber profissional, 1890-1990*. Projeto de Pesquisa. São Paulo: FAPESP. Disponível em: <http://bv.fapesp.br/pt/auxilios/98879/a-matematica-na-formacao-de-professores-e-no-ensino-processos-e-dinamicas-de-producao-de-um-saber-p/?q=17/15751-2>.

Vechia, A. & Lorenz, K. M. (2004, abril). Os Livros Didáticos de Matemática na Escola Secundária Brasileira no Século XIX. *História da Educação*, ASPHE/FaE/UFPel, Pelotas, (15), 53-72.

Xavier, L. N. (2014) A construção social e histórica da profissão docente: uma síntese necessária. *Revista Brasileira de Educação*, 19(59), 827-849, 2014. Disponível em: <http://www.scielo.br/pdf/rbedu/v19n59/02.pdf>.