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Investigating Relationships Between Combinatorial and Probabilistic Reasonings in Youth and Adult Education

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ABSTRACT

We present the main findings of a master's dissertation study that investigated the contributions that the exploration of combinatorial problems can bring to probabilistic reasoning and vice versa. In the light of the theoretical reference adopted (the theory of conceptual fields), we considered the different situations that give meaning to combinatorics and probability, their invariants and the symbolic representations/strategies used in the solving of the problems proposed. The relations established between combinatorial and probabilistic reasoning were the focus of the study. Data was collected with 24 students of Youth and Adult Education who were attending different phases of basic education. The influence of schooling, of the types of problems and of the order of presentation of these problems in the performance of the students was also analysed. Contributions to the investigated reasoning that emerged from the resolution of combinatorial and probabilistic problems were perceived. Therefore, the teaching of these areas of knowledge in an articulated way is recommended.

Keywords: Combinatorics; Probability; Youth and Adult Education.

Investigando Relações entre os Raciocínios Combinatório e Probabilístico na Educação de Jovens e Adultos

RESUMO

São apresentados os principais achados de um estudo de dissertação que investigou as contribuições que a exploração de problemas combinatórios pode proporcionar ao raciocínio probabilístico e vice-versa. À luz do aporte teórico adotado (a Teoria dos Campos Conceituais), foram consideradas as diferentes situações que atribuem sentido à Combinatória e à Probabilidade, seus respectivos invariantes e as representações simbólicas/estratégias utilizadas na resolução dos problemas propostos, sendo as relações que se estabelecem entre os raciocínios combinatório e probabilístico o foco central do estudo. Os dados foram coletados com 24 estudantes da Educação de Jovens e Adultos, cursando diferentes momentos da Educação Básica. Observou-se a influência

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da escolarização, dos tipos de problema e da ordem de apresentação destes nos desempenhos apresentados. Foram percebidas contribuições aos raciocínios investigados que surgem a partir da resolução de problemas combinatórios e probabilísticos. Desse modo, recomenda-se que o ensino dessas áreas do conhecimento seja realizado de maneira articulada.

Palavras-chave: Combinatória; Probabilidade; Educação de Jovens e Adultos.

INTRODUCTION

It is important that mathematics teaching aims at more than the mere appropriation of various concepts by providing the development of mathematical and hypothetical-deductive logical reasoning. This is essential for students of different stages and teaching modalities to be able to apply their mathematical knowledge to solve problems - including those that demand a gathering of possibilities.

Combinatorial and probabilistic reasoning are ways of thinking that structure logical-mathematical reasoning that provide tools to relate sets of elements, think proportions and understand random events. Given their importance for the understanding of everyday or school problems, different authors argue that we should work with concepts related to combinatorics and probability throughout basic education in a progressive way, aiming to the full development of such reasoning (Fischbein, 1975, Borba, 2016, Campos & Carvalho, 2016).

In the light of the theory of conceptual fields (Vergnaud, 1986, 1996), concepts related to combinatorics and probability are inserted in the same conceptual field – the multiplicative structures - and, given that a conceptual field is a heterogeneous set of interconnected problems, situations and concepts, it is essential that the relationships between such concepts are explored. Based on such a theoretical contribution, the present study aimed to investigate the contributions that the exploration of problems related to combinatorics can bring to probabilistic reasoning and vice versa.

We chose to carry out this study with EJA students, given the incipient number of studies carried out with adults and their vast background. These students' learnings come from every day and social experiences that can be a starting point for the development of their mathematical knowledge at school.

The theoretical contributions, the objectives and the method used and the main results obtained are presented in the following sections.

THE CONCEPTUAL FIELD OF MULTIPLICATIVE STRUCTURES

Gèrard Vergnaud adopts a developmental approach to knowledge, turning the interest not only to the overall construction of knowledge, but also the process of conceptualization by the subjects and, thus, the theory of conceptual fields (Vergnaud, 1986, 1996) assigns an essential role to the very mathematical concepts. In this way, such theory supports an in-depth look at the concepts and their articulations with each other

- articulations that lead to the constitution of different conceptual fields, defined by the theorist as "a set of situations, whose mastery requires a variety of concepts, procedures and symbolic representations in close connection" (Vergnaud, 1986, p. 10).

Vergnaud (1986) further states that "a concept can, in effect, be defined as a tripod of three sets" (p. 9). The three sets are: *situations* (which give meaning to the concept - S), *invariants* (properties and constant relations in different situations - I) and *symbolic representations* (used to represent the concepts - R).

In particular, the conceptual field of multiplicative structures concerns "the set of situations that require a multiplication, a division or a combination of these two operations" (Vergnaud, 1996, p. 167). It therefore encompasses concepts such as the rational number, proportionality, functions and concepts related to combinatorics and probability.

Therefore, the present study turned its attention to the field of multiplicative structures, especially to combinatorics and probability. We sought to investigate the understanding of the *invariants* related to the different *situations* that give meaning to the concepts investigated and the *symbolic representations* used by the participants when solving the problems proposed.

In the next section, the references adopted regarding combinatorics and probability and the *situations* that give meaning to the concepts related to such areas of mathematics (and their respective *invariants*) are presented. Based on the theoretical contributions used, the *symbolic representations* utilized during the resolution of the proposed combinatorial and probabilistic problems were also considered for the data analysis.

COMBINATORIC, PROBABILITY AND ITS RELATIONSHIPS

Combinatory Analysis[1] is defined by Morgado, Pitombeira de Carvalho, Pinto de Carvalho and Fernandez (1991) as "the part of Mathematics that analyses discrete structures and relationships" (p. 1). These authors highlight the two most frequent types of problem in its study, related to "1. demonstrate the existence of subsets of elements of a given finite set and that satisfy certain conditions; 2. count or classify the subsets of a finite set and that satisfy certain given conditions" (p. 2).

Hence, combinatorics studies the discrete sets and configurations that can be obtained from certain transformations in the structure of the composition of their elements. The use of combinatorial knowledge in solving problems of this nature, therefore, makes it unnecessary to list or enumerate all the elements that form a set, in order to determine the total number of elements that compose it.

This research adopted the classification of *situations* that give meaning to combinatorics in particular, proposed by Pessoa and Borba (2009). It integrates four types of combinatorial problems in the same categorization (*Cartesian product, combination,*

[1] Term considered synonymous with Combinatorics in the present study.

permutation and *arrangement*). Such problems differ from each other depending on the nature of their *invariants of order* and *choice* (Borba, 2016).

The problems categorized as a *Cartesian product* concern working with more than one set, in which the order of the elements does not imply different possibilities. In turn, the problems of *combination, arrangement* and *permutation* relate to situations in which the choice occurs within the same set, so that in the situations of *arrangement* the change in the order of the elements constitutes new possibilities, in those of *combination* this change does not form new possibilities, and in the *permutation* problems all the elements of the set are used and the different possibilities to be explored are built from the modification of the positions of its elements.

About probability, Morgado *et al.* (1991) define it as "the branch of mathematics that creates, develops and in general researches models that can be used to study experiments or random phenomena" (p. 119). The term 'probability' has different uses within and outside the academic context. According to these authors, "the definition of probability as a quotient of the number of 'favourable cases' over the number of 'possible cases' was the first formal definition of probability" (p. 119), and that conception (known as *classical* or *Laplacian*) is adopted in the present study given its central focus - as it is the most strongly related to combinatorics, since it demands the gathering of all the possibilities that constitute the sample space.

According to Bryant and Nunes (2012), probability is a complex concept that demands the development of four cognitive requirements for its broad understanding, namely: 1) *understanding the notion of randomness*, 2) *forming and categorizing sample spaces*, 3) *comparing and quantifying probabilities* and 4) *understanding correlations* (relationships between events).

The first cognitive requirement is related to understanding the nature of non-deterministic events, that is, random events. Randomness is very present in everyday life and it plays an important role, as its understanding is essential to distinguish a random event from a non-random sequence of events.

The second cognitive requirement is intrinsically based on combinatorial thinking: determining the *sample space* of a given problem is important not only for calculating probabilities, but it is also essential for the understanding of the nature of randomness, since probabilistic problems "are always about a set of possible, but uncertain, events [...], we need to know precisely what are all possible events" (Bryant & Nunes, 2012, p. 29, our translation).

In turn, the third cognitive requirement refers to the ability to *compare and quantify probabilities*. Since the probability is an intensive quantity, its calculation requires us to understand its proportional character, since "the calculation of the probability of the occurrence of an event or a class of events must be based on the total amount of the sample space and not only on the quantity of events that we want to predict" (p. 46, our translation).

Finally, the fourth cognitive requirement pointed out concerns the identification of dependent and independent events, since the association between two events can happen randomly or represent a genuine relationship. In this case, since "the purpose of analysing the correlation between two events is to determine whether they co-occur more often than expected to occur at random" (Bryant & Nunes, 2012, p. 67, our translation), the most important ability is to distinguish a random from a non-random event.

Besides investigating several reasonings, this study focused on the relationships that are established between them. Such relationships are pointed out by different authors (Piaget & Inhelder, 1951 *apud* Navarro-Pelayo, Batanero & Godino, 1996, Santos, 2015), who emphasize that combinatorial reasoning is essential for the understanding of the idea of probability, since it allows the subject to understand random experiments, from the most elementary to the most elaborate ones. It is also worth noting that probabilistic concepts (including the concept of sample space) are an important tool for solving combinatorial problems.

Thus, we advocated a teaching that allows the articulation and communication of ideas between those areas of mathematics (which involve the gathering of possibilities and the understanding of non-deterministic situations). In this sense, this research work results from our interest in investigating these relationships from the resolution of combinatorial and probabilistic problems articulated through revisits, which consisted of proposing new perspectives on the problems, from the exploration of their different aspects.

METHOD

The study was carried out with 24 adult students from public schools EJA located in the countryside of Pernambuco[2]. They attended three distinct groups: Module II, Module IV and Middle EJA 3 (schooling periods equivalent to the termination of the initial years and final years of elementary school and high school respectively).

Data collection consisted of conducting individual recorded audio clinical interviews, as we sought to monitor closely the combinatorial and probabilistic reasoning of the involved in the study. This method was chosen considering that "reasoning [...] tends to be reflected in actions, in the choices that a subject makes, for example, when solving a problem" (Carraher, 1998, p. 1). It also responded to the suggestion given by Lima (2010) -who investigated how much EJA students understood multiplicative problems (with a focus on combinatorics)- that, in later studies carried out with students of this teaching modality, the use of methods such as the Piaget's clinic interview could provide a better comprehension of the processes used by the participants when solving the problems proposed.

[2] The data presented and discussed here were collected, with the due consent of the institutions involved, from students who volunteered to participate in the study. The collection took place in a context very similar to that which normally occurs in the classroom - solving mathematical problems with statements appropriate to the target audience - and approval by the Ethics Committee was not necessary. It should be noted, however, that Acta Scientiae is not responsible for any consequences and/or damage resulting to students participating in the research that originated the present work.

During clinical interviews, half of the participants in each group solved one type of test (Test 1) and the other half solved a second type of test (Test 2). Both collection instruments were composed of four combinatorial problems (*Cartesian product*, *combination, permutation* and *arrangement*) and 16 probabilistic problems (four related to each of the cognitive requirements of probability: *sample space, correlation, randomness* and *comparison of probabilities*). The participants had a pencil/pen, paper, printed test and calculator to solve the problems, which were the same in both types of tests. The tests differed according to the order of presentation of the problems: in Test 1, each of the combinatorial problems was revisited under the perspective of probability (from problems related to the different cognitive demands considered), while in Test 2 the order was the opposite, that is, the various probabilistic problems were presented first and revisited under the perspective of the combinatorics. This structure is illustrated in Figure 1.

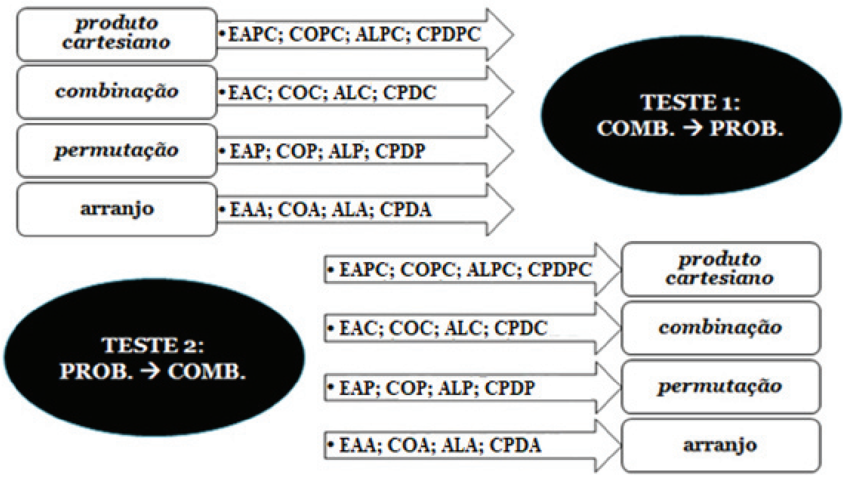

Figure 1. Structure of the collection instruments used. (Lima, 2018).
EA: *sample space*; CO: *correlation*; AL: *randomness*.
CPD: *comparison of different probabilities.*

The problems regarding the combinatorial *situations* considered in the present study have a similar number of steps of choice and order of magnitude: their results are between 6 and 12 possibilities. The elaboration of such problems took into account the diversity of educational levels of the participants, since problems with a small number of possibilities can be easily solved using *symbolic representations* and varied strategies - even the simplest/informal ones such as oral enumeration and non-systematic listing. Figure 2 shows the proposed combinatorial problems.

With regard to the proposed probabilistic problems, the block of problems that explore the *construction of sample space,* the *investigation of correlations,* the *understanding of randomness* and the *comparison of probabilities* regarding the *combinatorial situation* of the *Cartesian product* is presented as an example (Figure 3). The other blocks of probabilistic problems (related to combinatorial *situations* of *combination*, *permutation* and *arrangement*) were constructed in a similar way, using the same context as the combinatorial problems presented above and expanding their understanding.

(PC) Carlos começou a trabalhar em uma rede de supermercados e acabou de receber o seu fardamento: 4 camisetas em cores diferentes com o logo da empresa e 2 calças. Quantos conjuntos de uniforme diferentes Carlos pode formar com as peças recebidas?

(P) Maria gosta muito de literatura brasileira e seu autor favorito é José de Alencar. Ela ganhou 3 livros desse autor de presente de aniversário e ainda não decidiu em que ordem irá lê-los. Quantas ordens de leitura diferentes são possíveis?

(C) Sara tem 5 primos e quer escolher 3 deles para acompanhá-la no aniversário de uma amiga. De quantas maneiras diferentes ela pode fazer essa escolha?

(A) 4 rapazes desejam participar de uma 'pelada' com seus amigos e querem definir um atacante e um goleiro. De quantas formas diferentes os rapazes podem se organizar para ocupar as posições citadas?

Figure 2. Combinatorial problems proposed. (Lima, 2018).
PC: Cartesian product, two stages of choice, 8 possibilities.
C: combination, three stages of choice, 10 possibilities.
P: permutation, three stages of choice, 6 possibilities.
A: arrangement, two stages of choice, 12 possibilities.

(EAPC) Liste todos os conjuntos de calça e camiseta que Carlos pode formar com as peças de roupa recebidas.

(COPC) Carlos decidiu usar a calça de cor marrom. Na escolha da camiseta para completar seu uniforme, todas as camisetas têm a mesma chance de serem escolhidas ou alguma camiseta tem mais chance? Por quê?

(ALPC) Carlos guarda as camisetas do seu uniforme lado a lado penduradas em cabides. No segundo dia de trabalho, Carlos acordou apressado e pegou uma das camisetas sem olhá-las. Todas as camisetas têm a mesma probabilidade de terem sido pegas ou alguma camiseta tem mais chance? Explique.

(CPDPC) João e Mário trabalham na mesma empresa que Carlos. Eles possuem tempos de serviço diferentes e, ao longo dos anos de trabalho, João recebeu 3 camisetas vermelhas, 2 azuis, 2 pretas e 1 laranja. Mário recebeu 2 camisetas vermelhas, 1 azul e 1 laranja. Se os dois escolherem ao acaso a camiseta que vão usar, é mais provável que João ou Mário use uma camiseta na cor vermelha? Justifique.

Figure 3. Probabilistic problems regarding the situation of Cartesian product. (Lima, 2018)
EAPC: Cartesian product sample space. COPC: Cartesian product correlation.
ALPC: Cartesian product randomness.

CPDPC: comparison of different probabilities of Cartesian product..

Each *sample space* problem required a written listing of the possibilities regarding the corresponding combinatorial *situations*. Such *symbolic representation*/strategy can be used spontaneously when solving any combinatorial problem, however, from the proposition of this type of probabilistic problem, we sought to ensure that all participants used the list to indicate the possibilities of each problem (whether in the revisits - Test 1 - or the first contact with those *situations* - Test 2).

The proposed *correlation* problems sought to investigate the participants' ability to perceive the independence between the given events. In turn, the *randomness* problems proposed demanded, besides the participants' understanding of the random character of the situations in question, the judgment of the equiprobability of the given events. Finally, the probabilistic problems of *comparing different probabilities* explored the comparison of probabilities of different events, making it necessary for participants to consider the proportional character intrinsic to the calculation of probabilities.

Quantitative analyses of the participants' performances were carried out using the Statistical Package for the Social Sciences (SPSS) software, considering the variables: types of problems, schooling and type of test. Qualitative analyses, in turn, allowed us to identify the invariants understood by the students, the *symbolic representations*/strategies used - their limitations and effectiveness - and to raise the relationships established between combinatorial and probabilistic reasoning.

With regard to combinatorial problems and problems of the *sample space*, zero (0) points were assigned when less than half of the possibilities were considered, one (1) point when half or more of the possibilities were considered and two (2) points when there was exhaustion, that is, a total success. In those problems, the *symbolic representations*/strategies used by the participants were categorized *a posteriori*, aiming to facilitate the qualitative analyses carried out.

In turn, with regard to the other probabilistic problems (*correlation*, *randomness* and *comparison of different probabilities*) zero (0) points were attributed when there was an error, one (1) point when there was a correct answer, but the justification presented was inappropriate or absent and two (2) points for correct answers with appropriate justifications.

From the analyses carried out, the main findings of the exploratory study conducted are presented and discussed below.

PRESENTATION AND DISCUSSION OF RESULTS

Given the scores attributed to the combinatorial and probabilistic problems proposed, the total performance could reach a maximum of 40 points. The participants' overall performance ranged from 2 to 23 points, with an average performance equal to 17.16 points. Such data suggests students' poor understanding of combinatorics and probability.

It is important to highlight that this general performance was influenced by the Group variable, since the performance tended to increase according to the level of education of the participants: Group 1 presented an average performance of 8.6 points, while the average performance of Group 2 was 18.8 points and that of Group 3 was 24.1 points. This influence of schooling on the participants' general performance proved to be statistically significant ($F (2, 23) = 8.862$; $p = 0.002$). However, such an advance in performance was only significant when comparing Group 1 (students in the beginning of schooling) with the others, with no significant advance when comparing the performances of EJA students attending the equivalent to the conclusion of elementary school (Group 2) and high school (Group 3)[3].

This finding corroborates what was observed in Lima's study (2010), also carried out with EJA students solving problems of multiplicative structures. The author observes that, as the level of schooling advanced, a better understanding of the *invariants* of the problems addressed and the use of *symbolic representations* and strategies more appropriate to their resolutions was perceived.

It is noteworthy, based on the results obtained, that schooling alone provided advances in the performance of the participants in this study when solving the proposed combinatorial and probabilistic problems. Such an advance, however, may not owe to the specific study of combinatorics and probability, since better performances by students of EJA Médio 3 (high school) were expected, given the learning expectations shown in the Curriculum Parameters for Basic Education of Pernambuco (Pernambuco, 2012), in which it is possible to observe the existence of greater emphasis on knowledge related to combinatorics and probability.

Another central variable of this study (type of test) refers to the order of presentation of the proposed combinatorial and probabilistic problems. It is paramount to observe whether and how this variable influenced the performances presented, since it reflects the nature of the articulations between combinatorics and probability proposed in each test (from the revisits present in them).

In this sense, the average performance for Test 1 was 18.2 points, while for Test 2 it was slightly lower, being 16.2 points. This difference in performance, however, was not statistically significant ($t (22) = 0.497$; $p = 0.625$). Therefore, the order of presentation of the problems and their respective revisits did not influence quantitatively the overall performance of the participants in the study[4]. However, this order of presentation had a direct influence on the choice of *symbolic representations* and strategies used by the participants when solving the different combinatorial problems and when explaining their respective *sample spaces*, consisting of qualitative advances in performance (most evident in Test 1). This discussion is deepened further below.

[3] According to post hoc Tukey, we have: Group 1 x Group 2 → $p = 0.034$; Group 1 x Group 3 → $p = 0.001$ and Group 2 x Group 3 → $p = 0.340$.

[4] The type of test also did not influence significantly the performances of students in the same group, even though the performances related to Test 1 tended to be slightly higher. We have: Group 1 → $t(6) = 1.608$; $p = 0.159$; Group 2 → $t(6) = -0.139$; $p = 0.894$ e Group 3 → $t(6) = 0.392$; $p = 0.708$.

Figure 4 shows the average performances for each type of combinatorial problem and the problems of building *sample spaces* for each combinatorial *situation*. The analyses related to the other probabilistic problems proposed will be presented below.

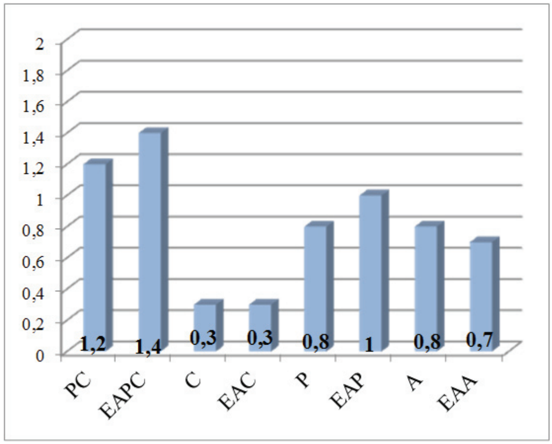

Figure 4. Average performance by type of combinatorial problem and respective constructions of sample spaces (maximum score 2 points per type of problem). (Lima, 2018).

PC: *Cartesian product*; EAPC: *Cartesian product sample space*.

C: *combination*; EAC: *combinatorics sample space*.

P: *permutation*. EAP: *permutation sample space*. A: *arrangement.* EAA: *arrangement sample space.*

The combinatorial problem and the *sample space* problem related to the *Cartesian product situation* were those in which the participants had better average performances. On the other hand, the poorest performances were observed regarding the *situation* of *combination* (both in the combinatorial and probabilistic problems)[5]. This influence of the type of problem on the performance presented by students when solving the proposed combinatorics problems and when building their *sample spaces* reinforces the central role of *situations* in conceptual development (Vergnaud, 1986, 1996), since students did not understand equally the different types of combinatorics and probabilistic *situations*.

Such results corroborate previous studies such as those of Pessoa (2009), Lima (2010) and Azevedo (2013), carried out with different basic education attendees, which indicate that among the combinatorics problems, the ones involving *Cartesian product* were the simplest, whereas *combination* problems were the most difficult ones to be resolved. A possible explanation for the higher performance in the *situation* of *Cartesian product* is that it is more worked since the beginning of schooling. In turn,

[5] There was a statistically significant difference in terms of combinatorial problems (except between the *arrangement* and *permutation* problems): PC x C → $t(23) = 6.868$; $p < 0.001$; PC x P → $t(23) = 2.318$; $p = 0.030$; PC x A → $t(23) = 2.460$; $p = 0.022$; C x P → $t(23) = -2.937$; $p = 0.007$; C x A → $t(23) = -3.140$; $p = 0.005$ e P x A → $t(23) = 0.000$; $p = 1$. The same was observed when dealing with *sample space* problems: EAPC x EAC → $t(23) = 8.177$; $p < 0.001$; EAPC x EAP → $t(23) = 3.191$; $p = 0.004$; EAPC x EAA → $t(23) = 4.290$; $p < 0.001$; EAC x EAP → $t(23) = -5.127$; $p < 0.001$; EAC x EAA → $t(23) = -2.387$; $p = 0.026$ e EAP x EAA → $t(23) = 2.070$; $p = 0.050$.

the low performance related to *combination* reinforces the existence of difficulties in the understanding of the *invariants* of that type of combinatorial *situation* (mainly the *order invariant*, being necessary to realize that the change of order in the presentation of elements does not constitute new possibilities).

Regarding the other probabilistic problems proposed, Figure 5 shows the average performances for the problems of *correlation*, *randomness* and *comparison of different probabilities*[6].

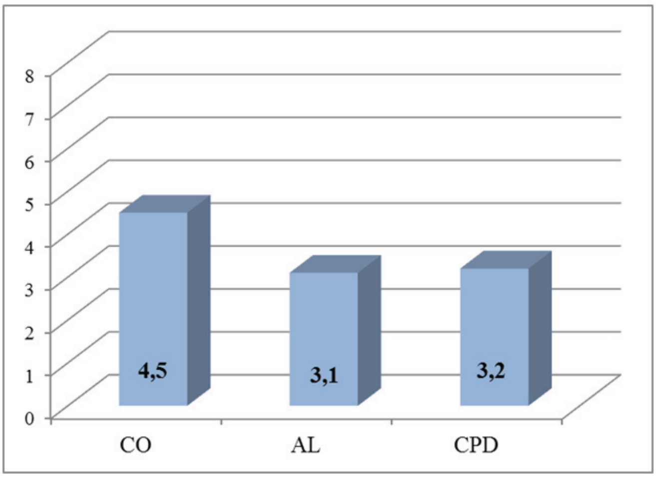

Figure 5. Average performance on correlation, randomness and comparison of different probabilities (maximum score 8 points per type of problem). Lima (2018).
CO: summation of the *correlation* problems. AL: summation of *randomness* problems.
CPD: summation of the problems of *comparing different probabilities.*

Significant differences in performance were found only when comparing the *correlation* problems with the others: CO x AL → $t(23) = 3.725$; $p = 0.001$; CO x CPD → $t(23) = 2.397$; $p = 0.025$ e AL x CPD → $t(23) = -0.081$; $p = 0.936$. The influence of the type of probabilistic problem on the performances presented thus reinforces that the different *invariants* of the probabilistic *situations* proposed are not understood by the students equally.

Regarding the resolution of the combinatorial problems, in general, the participants of this study used spontaneous simpler *representations*/strategies. Mostly, *oral enumeration* was used: the indication of different possibilities regarding the combinatorial *situations* in question, without a written record of each one. In turn, the *written listing* was the second most used *symbolic representation*/strategy (with low frequency).

[6] There were no significant differences in performance in the probabilistic problems of *correlation*, *randomness* and *comparison of different probabilities* regarding different combinatorial *situations*. Thus, the summation of the four probabilistic problems of each type are presented.

It is noteworthy, therefore, that the main errors associated with such problems consisted of *non-systematic listing errors* (omission of cases caused by the use of a list without systematization) and *order errors* (consideration of repeated cases or non-consideration of possibilities based on modification of order of elements when necessary). These are the main types of errors in solving combinatorial problems also pointed out by Navarro-Pelayo, Batanero and Godino (1996).

It is also worth noting that even though the type of test did not have a quantitative influence on performances, the order of presentation of the problems (solving the combinatorial before or after the probabilistic problems- including those of *sample spaces*) influenced the *symbolic representations*/strategies used by the participants to solve the combinatorial problems proposed. In Test 1, *oral enumeration* was widely used (more than 65% of the cases in all combinatorial problems)[7]. In turn, the data related to Test 2 point to an important result: even though *oral enumeration* was also widely used, a large part of the students who solved this test tended not to revisit the combinatorial problems, that is, both for having already solved the respective problems of *sample spaces* and, as they do not have a repertoire of more refined *representations*/strategies to improve their answers, they chose to only repeat the results given previously. This lack of revisiting occurred in approximately 25% of the cases for the *Cartesian product* problem, 58% for the *combination* problem, 42% for the *permutation* problem and 58% for the *arrangement* problem.

The revisits proposed in the two tests consisted of rich moments of reevaluation of the problems, allowing students to better analyse the *invariants of order* and *choice* considered, to check if any possibility had not been considered or if there were repeated cases. It also made possible for students who had solved Test 1 (who used written records only a few times), from revisiting with explanation of *sample space*, to register and control the cases considered; and, for students who had solved Test 2 (producing from such listing from the beginning), to revisit the records made or make use of new *representations*/strategies to check their answers.

With regard to the problems of *correlation*, *randomness* and *comparison of different probabilities*, it was possible to perceive that the main difficulties presented by the participants were based on superficial understandings of the probability, evidenced from the justifications that were requested in the problems. Thus, sometimes, even in problems where there were successes, the justifications presented were inappropriate. Such difficulty was observed mainly in the problems of *randomness*[8], in which the equiprobable character, necessary so that the events considered had the same chance of occurring, was not evidenced in the justifications. The probabilistic problem in which the highest percentage of errors was presented was the *comparison of different probabilities* (56%)[9]:

[7] Approximately 75% in the *Cartesian product* problem, 75% in the *combination* problem, 67% in the *permutation* problem and 83% in the *arrangement* problem.

[8] Approximate percentage of correct answers with inadequate/absent rationale: 20% on *correlation* problems, 49% on *randomness* problems and 8% on problems on *comparison of probabilities.*

[9] Approximate percentage of errors: 33% in the problems of *correlation*, 37% in the problems of *randomness* and 56% in the problems of *comparison of different probabilities*.

this low performance shows the non-consideration of the proportional character intrinsic to the probabilistic problems, that is, a non-use of combinatorial reasoning to compare the respective *sample spaces* and not just the absolute number of favourable cases - such difficulty was also pointed out in previous studies such as Batista and Francisco's (2015), Santos's (2015) and Lima e Silva's (2017).

Throughout this study, we could also observe relations that are established between combinatorial and probabilistic reasoning. In particular, in Test 1, the exploration of the *sample space* provided the discovery of new possibilities in the combinatorial problems, since the revisiting and written record of the possibilities related to these problems allowed the participants to evaluate/modify the *symbolic representations* and strategies used, having the chance to reflect, also, on the *invariants of order* and *choice* of each type of combinatorial *situation*, reevaluating and refining the answers given to the problems. On the other hand, presenting a way of thinking proper to combinatorial reasoning made possible the use of an approach more focused on school mathematics during the resolution of probabilistic problems (Test 2) and detached from personal preferences, since these involve the gathering of possibilities (being important that the entire *sample space* is considered so that the probabilities are evaluated and/or compared in school problems).

Thus, we argue that the articulation between combinatorial and probabilistic reasoning can help their development in EJA - given the contributions that arise between knowledge of Combinatorics and Probability from the resolution of problems that relate both reasonings.

FINAL CONSIDERATIONS

From the exploratory study carried out, the relationships established between the knowledge of combinatorics and probability were investigated with EJA students. The analysis revealed that schooling influenced the performance of the participants, impacting on how they solve problems, how they think hypothetically and raise possibilities, even when there is no focus on the specific teaching of such areas of mathematics. Schooling alone, however, is not enough for the full development of the reasonings in question (Fischbein, 1975), and it is important that there is specific instruction aimed at contact with different *situations*, that is, with various types of problems, the understanding of their respective *invariants* and the expansion of the repertoire of *symbolic representations* and strategies, even from the articulation between combinatorics and probability.

The type of test, that is, the order of presentation of the combinatorial and probabilistic problems proposed, did not influence quantitatively the performances presented, but played an important role in the choice of *symbolic representations* and strategies, also affecting the potential to contribute to the development of the reasonings investigated from the proposed revisits. Therefore, based on those contributions, we advocate that the teaching of combinatorics and probability in an articulated manner can favour the development of both reasonings in EJA and, most likely, in other teaching modalities.

With this study, we hope to contribute with reflections on the teaching of combinatorics and probability and the possibilities of articulations between them (whether in the EJA or not). Subsequent studies can deepen the investigation of the relationships between those reasonings also in regular education, using different - exploratory or interventional - approaches. It is also important to research whether and how such relationships are present in materials that can influence the teacher's approach to combinatorics and probability in the classroom, such as curriculum guidelines and textbooks.

AUTHOR'S STATEMENT OF CONTRIBUTIONS

E.T.L. and R.E.S.R.B. conceived the idea of the research presented, which was developed in the context of E.T.L's master's degree, under the supervision of professor Dr. R.E.S.R.B. Both authors contributed to the construction of this text from the data and analyses of the original research.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author, E.T.L., upon reasonable request.

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