The Committee of Fifteen and the First Movements about the Teaching of Algebra in the Brazilian Primary School

Jeremias Stein Rodriguês [a,b]
David Antonio da Costa [b]

[a] Instituto Federal de Santa Catarina, Departamento Acadêmico de Linguagem, Tecnologia, Educação e Ciência, Florianópolis, SC, Brasil.
[b] Universidade Federal de Santa Catarina, Programa de Pós-Graduação em Educação Científica e Tecnológica, Florianópolis, SC, Brasil.

ABSTRACT

At the end of the XIX century, movements that propose the reform of the primary school in the United States are started. The aim of this paper is to find out what changes proposed to the American teaching of mathematics are, specifically about the teaching of algebra and the knowledge related to it, while seeking indications of the circulation of those ideas in Brazil. To perform an historiographic research, which aims to the writing of a historic narrative, we use the theoretical contributions of the Cultural History based on Burke and Chartier and the studies about the circulation and appropriation of ideas by Oliveira and Warde. As sources of the research are the reports of those movements, textbooks on algebra and arithmetic teaching, notes from a conference given by Othello S. Reis about the insertion of algebra in the primary school, and other documents. It was found that a Committee of fifteen was created to propose changes in the U.S. primary school because of a previous study, done by the Committee of ten. One of the proposals of both committees can be highlighted: the teaching of algebra topics in the two last years of the primary school. The ideas of the committee about algebra were brought to Brazil, in an explicit way, in one conference presented by Othello S. Reis, and it was revealed that those ideas had already been presented in Antonio Trajano's textbooks, in editions that preceded the committee's study.

Keywords: history of mathematics education; teaching of algebra; primary school; committee of fifteen.

A Comissão dos Quinze e os Primeiros Movimentos Acerca do Ensino da Álgebra na Escola Primária Brasileira

RESUMO

Ao final do século XIX iniciam-se movimentos que propõem reformas para o ensino primário nos Estados Unidos. Neste artigo, tem-se como objetivo analisar as mudanças propostas para o ensino de matemática estadunidense, mais especificamente no que refere ao ensino de álgebra e os saberes vinculados a este ensino, do mesmo modo que se procuram indícios da circulação de tais

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Corresponding author: Jeremias Stein Rodriguês. E-mail: jeremias.stein@ifsc.edu.br

ideias no Brasil. Para a realização de uma pesquisa historiográfica, que objetiva a elaboração de uma narrativa histórica, utilizam-se aportes teóricos da História Cultural de Burke e Chartier, assim como os estudos acerca da circulação e apropriação de ideias de Oliveira e Warde. São tomados como fontes de pesquisa os relatórios de tais movimentos, livros didáticos referentes ao ensino de álgebra e aritmética, apontamentos de uma conferência de Othello S. Reis sobre a inserção da álgebra no ensino primário, bem como outros documentos. Constata-se que uma Comissão dos quinze é constituída para propor mudanças no ensino primário estadunidense em consequência de um estudo anterior, realizado pela Comissão dos dez. Exara-se uma proposta dessas duas comissões: a inserção de conteúdos de álgebra nos últimos dois anos do ensino primário. As ideias da comissão acerca da álgebra são trazidas para o Brasil, de forma explícita, pela conferência realizada por Othello S. Reis, mas também já estavam presentes em livros de Antonio Trajano, em edições que antecedem o trabalho da comissão.

Palavras-chave: história da educação matemática; ensino de álgebra; ensino primário; comissão dos quinze.

INTRODUCTION

Throughout the 19th century, the expansion of primary education programs accompanied the expansion of the social purposes attributed to popular education in the United States. Elementary knowledge - reading, writing and calculating - became insufficient to prepare new generations to live in modern, urbanized and industrialized societies. In view of the widening of cultural selection, it was necessary to create new devices for curriculum ordering. (Souza, 2016, p.40)

The movement in the United States regarding the expansion of primary education purposes and programs Souza observed culminated, in the late 19th century, in the elaboration of committees that should study and propose changes not only for primary education, but also for other levels of education. Such actions echoed on the dissemination of research on the "correlation of studies" or studies on children. Other aspects, such as curriculum proposals, or more specifically, the implementation of algebra content in primary education, can be observed. In this sense, there has been traces of algebra contents in Brazilian primary education arithmetic textbooks since the beginning of the 20th century (Rodriguês & Costa, 2019).

The first committee was the "Committee of Ten," which outlined proposals, initially for the US secondary education, having its final report published in 1894: "Report of the committee of ten on secondary school studies" (NEA, 1894). This report pointed out the need for changes in primary education, among other results. After its publication, a second committee was formed to focus on the education al proposals for the primary level. This was called the "Committee of Fifteen", which produced its final report in 1895: "Report of the committee of fifteen on elementary education"[1] (Draper, Powell, Poland, Seaver & Lane, 1895, Harris, Greenwood, Gilbert, Jones & Maxwell, 1895, Tarbell, Brooks, Balliet, Dougherty & Cooper, 1895).

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[1] Report of the Committee of Fifteen on Elementary Education, elaborated in three parts.

Works presenting relationships between the Committee of Fifteen and the field of algebra have been carried out in the context of research in the history of mathematics education. Valente (2017, p. 9) and Basei (2017, p.1) intend to answer to the following questions, respectively: "How can the entrance of Algebra in the education of primary school teachers be justified?"; and "What is the attribution of Algebra in the education of teachers of the first school years in the period between 1890 and 1970?" Hence, the authors analyze the report of the Committee of Fifteen; Antonio Trajano's books; and "Álgebra - Primeiros Passos", by Othello de Souza Reis, among other materials. Valente (2017) concludes that Trajano plays an important role in the scenario of the appropriation[2] of US ideas for the insertion of algebra in primary education. Valente and Basei point out, that such algebra was linked to solving complex arithmetic problems, either in basic education or in teacher training. Rocha (2019) discusses how the US proposals approach the use of algebra to solve arithmetic problems. In her research, the author uses as sources Reis's book and the report of the Committee of Fifteen, concluding that algebra in teacher education would not be an algebra to be taught, "but an algebra that assists arithmetic problems to resolve more difficult problems" (Rocha, 2019, p.1).

However, from these authors' works it is not possible to identify whether the proposals presented by the Committee of Fifteen regarding the teaching of mathematics address only an algebra aimed at solving arithmetic problems or whether a series of algebraic knowledges designed for primary education is proposed. Likewise, it is not possible to understand what the rationale behind the constitution of this committee, the execution of its work, and the possible appropriations of works prior to that of the Committee of Fifteen were.

Thus, considering the circulation of US ideas in Brazil a complex matter regarding the insertion of algebra in the curriculum of primary education of the 20th century, we defined as the main objective of this work: to analyse the changes proposed to the teaching of US mathematics, more specifically regarding the teaching of algebra and the knowledges related to it, and find evidence of the circulation of those ideas in Brazil. Thus, we seek to answer the following guiding questions: What changes are proposed in the teaching of mathematics, particularly those concerning the teaching of algebra, in the discussions promoted by these committees about curriculum and teaching in primary school? What evidence can be observed regarding the circulation of US ideas in Brazil? What algebraic knowledge can be observed in the US proposal and in the Brazilian movement?

For those discussions, this article was structured aiming to address sequentially the following themes: a) Theoretical and methodological considerations of the research; b) The study movement to reformulate the US education; c) The algebra present in the report of the Commission of Fifteen; d) The repercussion and circulation of such ideas in Brazil.

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[2] We took the idea of appropriation from the perspective of Chartier (2002), which could be understood as a process of interpretation, of giving meaning to something with what you have contact.

THEORETICAL AND METHODOLOGICAL CONSIDERATIONS

Chartier (2010), saying that one can "hear the dead with one's eyes", points out that one knows the past from the traces left behind. According to the author, those traces allow the production of representations of something that is not present, such as a distant past (Chartier, 2002). Thus, "the historian's work takes place in the process of questioning the traces left by the past, which are led to the position of research sources" by the questions asked by the historian, "aiming at constructing historical facts, represented by the answers to the sources" (Valente, 2007, p.39).

In this sense, for Certeau (2013), "the historical operation refers to the combination of a social *place*, of "scientific" *practices* and of a *writing*" (p. 47, emphasis added). Based on Certeau, Valente (2007) then proposes that we should "*think history as a production*" (p. 34, emphasis added). This writing cited by Certeau (2013) is the result of the historiographical operation and constitutes a (historical) narrative that, according to Burke (2016, p.110), "produces knowledge by revealing the connections and, therefore, making the experience understandable". Burke (2016) contributes for the understanding that historical narratives are consequences of a process that involves the collection and analysis of some information.

Thus, this work is developed under this bias of cultural history. This narrative, presented in the next headings, is the result of a process of collecting and analyzing information found in the documents about the movement of insertion of algebra in Brazilian and US primary education in the late 19th and early 20th centuries. To construct the narrative, we sought relations that could highlight aspects of a circulation of ideas between these two countries.

Warde (2000, p.37) advocates that the construction of the Brazilian identity as a nation was based on references outside our continent, "Europe and later the United States of America were very soon the mirror where Brazil had to look at".

> From the mid-19th century, thus only two to three decades after the declaration of independence, theses - according to which the chances of Brazil to progress were in mirroring no longer in the Old World, but in the New World, that is, in the United States - began to circulate in Brazil, particularly in and from the urban centres of the Southeast (especially São Paulo). (Warde, 2000, p.37)

However, even if the presence of elements of an external movement in Brazil is observed, it is not easy to determine their origin. Similarly, it is difficult to establish where the beginning of a movement occurs, as Warde exemplifies:

> the [...] Americanism as US hegemony over the outside world is the result of the internal hegemony of some of the projects in dispute, as is the incorporation of

cultural projects and patterns born outside the social boundaries of the United States. (Warde, 2001, p. 43, apud Schneider & Neto, 2008, p.138)[3]

In this sense, how could one determine whether the circulation of the US ideals of a given movement actually occurred elsewhere? That is, which elements mark a process of appropriation of the US thinking? According to Souza and Garnica (2016),

> This theme can easily be linked to the theme of importing models, as it can be easily verified if we consider, for example, the discussions on national curricula which, in Brazil, sometimes take as parameters the guidelines from one country, sometimes from another. [...] Historically, from an educational point of view, importing models seems to characterize, in fact, national public practices and policies: our primary education, for example, was created inspired by the US education, and our textbooks have the indelible mark of the French production. (p. 414)

Oliveira (2018) chooses not to present a closed definition of "circulation", since, by doing so, he would limit the concept and disregard distinctions of each case, preferring only to expose forms of characterization of the circulation process. Burke (2016) does the same by identifying several processes that can be observed in what the author calls dissemination of knowledge. Chartier (2002) approaches this movement from the perspective of those who appropriate a set of ideas, that is, the process of making sense of what is circulating. According to the author, appropriation "aims at a social history of interpretations, referred to their fundamental determinations [...] and inscribed in the specific practices that produce them" (p. 26). Similarly, Burke (2016, p.113), when talking about the dissemination of knowledge, indicates that "we need to remember that the knowledge received is not the same as the knowledge given, because of misunderstandings[4] [...] and deliberated adaptations or cultural translations". However, the author states that such reinterpretations and translations are what make sense of the knowledge it appropriates.

Thus, based on Chartier (2002) and Burke (2016), we understand that appropriation and dissemination are elements of a circulation of ideas and are linked to the existence of contrasts, since it is not necessary to appropriate something just like your own, so that it becomes essential, and possible, to interpret a set of ideas or knowledges. Thus, the circulation of ideas is characterized by the existence and action of two poles: one that seeks to disseminate a series of ideas; and another, that appropriates the disseminated ideas, making them meaningful.

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[3] Warde, M. J. (2001). *Cultura e educação: o americanismo e a fabricação do homem*. [Research Project]. Postgraduate Program in Education of the Pontifícia Universidade Católica de São Paulo, São Paulo.
[4] In this sense, Bourdieu (2002) points out that a circulation of texts that disregards the context of the work's elaboration causes them to be reinterpreted in the places of reception, which ends up generating "colossal misunderstandings" (p.VI). Thus, in the author's view, when a work circulates in foreign lands, it is common for those places to determine its meaning rather than its origin, since the meaning attributed in its country of production can be ignored. Thus, we can understand that Bourdieu's (2002) perspective meets Chartier's (2002) conception of appropriation.

However, according to Oliveira (2018), for a set of ideas or even a movement to circulate in a specific historical period, it is necessary, first, that what will circulate exists. Such an existence is not only characterized by its superficial sense, as the existence of the movement itself, in which case the existence of something to circulate also depends on having an audience that can receive it through an appropriation process.

But why then turn to the late 19ᵗʰ century US movement? Resuming Souza (2016),

> [...] going deeper on the theme is paramount for historians of education in Brazil, since the US educational reference was widely spread in this country in the transition from the 19th to the 20th century, justifying educational reforms in the states and serving as an argument for pedagogical renewal. Thus, looking into the US education contributes to a better understanding of the circulation and appropriation of educational models (2016, p. 38 - 39)

About the circulation of a series of ideas, the process of diffusion occurs through the various "means of communication available and accessible in each epoch and in each context" (Oliveira, 2018, p.16). According to Burke (2016) and Oliveira (2018), books, magazines, news and even an individual (through speech, performance etc.) are means for such dissemination, as they can make circulate a series of ideas related to them. We seek, in this way, the elements that help characterize the circulation of ideas, since it "demands an effective publicization, that is, a movement to make public what is intended to be recognizable to - and recognized by - a community" (Souza & Garnica, 2016, p. 436). Therefore,

> [...] it is reasonable to *indicate that the means of communication, as they characterize the message diffusion processes, are elements that inject some impact on the reception and appropriation of the message propagated* [5]. (Oliveira, 2018, p. 28)

Thus, to analyse how the proposals for change in US primary education took place, especially with regard to the field of algebra, and to search for - even if initial - evidence of its circulation in Brazil, the report developed by the Committee of Fifteen and its possible repercussions were examined in detail. To this end, we founded this work on the research carried out by GHEMAT-BRAZIL[6], from the perspective of cultural history, to promote

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[5] According to the author, the translation is an example, since no translation can carry everything that is intended to be transmitted in the original language, causing a different impact from what is proposed in its original form. Burke (2016) also contributes in this regard by indicating that "what was often, though not always," lost in translation "was what turned that information into knowledge […]" (p. 129).
[6] Associated Group of Studies and Research on History of Mathematics Education, it is an association linked to research groups focused on the area of History of Mathematics Education. For better details: https://www.ghemat-bras.com/.

a dialogue between the committee's report, its impacts, the circulation of its ideas and relations, particularly on the history of the teaching of algebra at the elementary level.

THE SOURCES OF THE RESEARCH

Our main source was the report presented by the Committee of Fifteen. The report, divided in three parts, was published in 1895 in the Educational Review journal. The titles were respectively: "Report of the sub-committee on the correlation of studies in elementary education" (Harris et al., 1895); "Report of the sub-committee on the training of teachers" (Tarbell et al., 1895); and "Report of the sub-committee on the organization of city school systems" (Draper et al., 1895). Each of these parts was prepared by subcommittees of the Committee of Fifteen. The first part of the report discusses the correlation of studies, seeking to discuss how the primary curriculum, its disciplines and the sequence of its contents should be. The second part of the report focus on teacher education, addressing topics such as the level of training deemed necessary for the profession, and knowledge and practices in teacher professional education. The last part of the report presents a discussion on how the school organization in cities should be done, covering topics such as the management of school spaces and the hiring of professionals linked to education. After a thorough reading of the entire report, we observed that only the first part refers to algebra teaching, what made us decide not to discuss the other parts in this paper.

Other historical documents were also consulted for article, such as the "Report of the committee of ten on secondary school studies" (NEA, 1894) and a publication on a conference given by Reis (1918a, 1918b). Textbooks by Walsh (1911) and Trajano (1905; n.d.)[7] and an article by Button (1965) were also analysed.

The report of the Committee of Ten, as anticipated in the introduction, was produced prior to the work of the Committee of Fifteen. They had their report published in 1894, suggesting proposals for the change of US secondary education. The report of the Committee of Ten also proposes changes in primary education, as they would be necessary to enable the proposals made for secondary education.

At a lecture by Othello de Souza Reis, in 1918, published by the journal "A escola primaria" in 1918, and in the preface to his book "Algebra – primeiros passos" in 1919, held in Rio de Janeiro, Reis beckons that the suggestions by the Commission of Fifteen for the teaching of algebra in the last two years of arithmetic in primary education should be integrated in Brazil. During his lecture, Reis also cited some authors who supported his ideas. Also here are some contributions by John Henry Walsh, one of the authors cited by Reis, from his book "Practical methods in arithmetic".

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[7] Trajano's book does not explicit the year of this edition, but there is indication that it is a copy of the 68th edition. See at: https://repositorio.ufsc.br/handle/123456789/104081. For reference, a scan of the 92nd edition of the same title, dated 1922, is available at: https://repositorio.ufsc.br/handle/123456789/105107.

In his book "Algebra elementar", Trajano (1905), in the preface to the book, points out that the insertion of algebra in primary education is a movement that had already occurred in countries such as England, France, Germany and the United States. In the textbook "Arithmetica Elementary Illustrada", also by Trajano (nd), we observe the teaching of knowledge related to algebra in an arithmetic book aimed at primary education. It is important to emphasize that Trajano was the author of textbooks on subjects such as arithmetic and algebra from the late 19th and early 20th centuries, whose works were disseminated by Brazilian schools. According to Oliveira (2019), Trajano was an author of textbooks which were widely disseminated in Brazil, having played a relevant role in this period for the circulation of ideas in the country.

Finally, there is Burton's work (1965), entitled "Committee of Fifteen", in which he discusses the history of that committee, and how its work was developed, considering the its members' performance. This work provided access to a subsequent view of the committee's report, to the criticism it received at the time, and later, the consequences of the report in the country.

THE SEARCH FOR CHANGE IN US PRIMARY EDUCATION

Until the early 19th century, the notion that subjects such as grammar, arithmetic, and spelling were at the core of the primary curriculum in the United States prevailed. Gradually, in the beginning of the 19th century, studies such as literature, history, geography and the natural sciences migrated from secondary and higher education, gaining space in primary education (Monroe, 1935, p. 529)[8].

According to Monroe (1935), in those times the Pestalozzian ideas had a great influence on the educational leaders of the country, introducing the study through simple objects and phenomena of nature, but, by the end of the century, the presence of Herbatian literatures could also be noticed[9]. It was during this century that the movement of the public primary and secondary schools spread throughout the United States. This movement was followed by one that sought to exchange small district schools for more centralized and better-maintained municipal schools, seeking better teacher preparation, the establishment of regular schools - teacher training schools, and curriculum enrichment.

> For US scholars, the 1890s were a milestone in the country's history. The period is highlighted by the unprecedented intensification of economic, social and scientific development. [...] In this period, we highlight the professionalization of scientific knowledge in the area of Human Sciences and the how this knowledge was used

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[8] According to Monroe, teaching reading, writing and arithmetic are considered the most important. Ordinary schools offered only reading, spelling, and English grammar, while more prestigious schools also offered writing, arithmetic, geography, and history (Monroe, 1935, p. 701).

[9] The German Johann Friedrich Herbart, 1776 - 1841, was an educator and pedagogical thinker, author of such literatures, who developed mainly the idea of educational instruction (Hilgenheger, 2010).

by the public power and the middle class to implement social reform projects. (Souza, 2016, p.37 - 38)

At the end of the 19th century, secondary education in the United States suffered from a dichotomy between teaching based on educating students for life or preparing them for higher education (Monroe & Herriot, 1928). However,

> [...] only 3 per cent. Of our high school pupils enter our colleges. It follows, therefore, that the best possible provision for secondary education, particularly in our high schools, should be made, if we would send into the world with the fullest equipment for citizenship the 97 per cent of high school pupils who do not enter college. (Mackenzie, 1894, p. 149)

By seeking to satisfy both objectives, the result presented high dropout rate in secondary education and low rates of entry into higher education, highlighting the inefficiency of the education at the time and need for restructuring it. Aiming at studying and suggesting changes in the secondary education system in the United States, the National Education Association (NEA[10]) created the Committee of Ten, in July 1892. This committee was composed of ten members that should organize conferences to discuss those changes in secondary education, according to the broad areas of knowledge of the curricula, which they presented in their report in 1893. "Had the Committee not been limited in time, doubtless fuller discussion would have resulted in modifying some statements embodied in the report" (NEA, 1894, p.b56).

As mentioned above, the Committee of Ten stressed in their report that the reform of the secondary education would require the reform of the country's primary education. The committee presented a program that would be necessary and sufficient to enable the execution of their proposal for secondary education. However,

> Regarding primary education, different societal projects were also in dispute, but the debate focused mainly on the principles of curriculum ordering, involving emerging scientific approaches to education and new conceptions on children, teaching and curriculum. (Souza, 2016, p.37)

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[10] An association made up mainly of teachers, school administrators and correspondent teachers from other countries. Composed in 1895 of fifteen departments related to levels, systems and some school fields, the association held meetings to discuss the educational issues related to its various departments (NEA, 1895). Being one of his "concentrating the wisdom and power of numerous minds, and by distributing among all the accumulated experiences of all" (NEA, 1892).

As stated by Souza (2016, p. 38), based on the work of Charles De Garmo[11],

> [...] what was at stake in the definition of the primary school curriculum was to show how children's knowledge could be used. Therefore, it was important to specify the nature, the time required and the order to be established in the organization of the teaching contents.

At a meeting of the NEA, attended by the Department of Superintendence, which took place in early July 1893, Francis Weyland Parker[12] suggests the organization of a committee, which would later be called the Committee of Fifteen, to study primary education in the country. "Parker hoped that the Committee of Fifteen would revise the elementary curriculum as the Committee of Ten was revising the high school curriculum" (Button, 1965, p. 253).

> *Resolved;* That a committee of ten be appointed by the committee on nominations, to investigate the organization of school systems, the co-ordination of studies in primary and grammar schools, and the training of teachers, [...] and to report the results of their investigations and deliberations [...]. (NEA, 1895, p.232, author's emphasis)

Nevertheless, after a motion by William H. Maxwell, five other members of the nominating committee were added to the committee already set up, forming a committee of fifteen members, who were to present a proposal to reformulate the primary education. Parker was not to join the committee, taking over as chairman Maxwell, a school superintendent[13] in Brooklyn, New York, and as a member, William T. Harris, who was also a member of the Committee of Ten and country's education commissioner. There were also school superintendents Horace S. Tarbell, Edward Brooks, Thomas M. Balliet, Newton C. Dougherty, Oscar H. Cooper, Charles B. Gilbert, James M. Greenwood, Lewis H. Jones, W. B. Powell, Edwin P. Seaver and Albert G. Lane. Andrew S. Draper, president of Illinois State University, and Addison B. Poland, state superintendent of Public Instruction were also committee members.

The first meeting of the Committee of Fifteen took place in 1894, where they decided to submit questionnaires to "all persons throughout the country whose opinions

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[11] From Garmo, C. (1895). *Herbart and the herbartians*. New York: Charles Scribner"s Sons.

[12] US citizen, also known for being the author of the "Cartas de Parker", "A device that brought a modern proposal for the teaching of arithmetic in the early years of schooling, from the intuitive teaching, a method that opposed the traditional memorization, repetition and abstraction, very common practices since the school of the first letters, from imperial times" (Portela, M. S. As cartas de Parker na matemática da escola primária paranaense na primeira metade do século XX: circulação e apropriação de um dispositivo didático. Thesis (PhD in Education). PUC / Paraná, 2014. Available at: https://repositorio.ufsc.br/handle/123456789/128465).

[13] Position of the hierarchy of the US teaching, in charge of executive functions, how to make reports about the conditions of the school system, participating in discussions of the school subjects etc. (NEA, 1895).

might be considered as of value" (NEA, 1895, p.233). The methodology was similar to that used by the Committee of Ten, which contacted several high schools to respond to a survey aimed to check which subjects were taught and how many teaching hours during the week, questions that were included in the concerns of the Committee of Fifteen.

The questions would be related to the topics that should be part of the committee's research: correlation of studies, teacher education and organization of the school systems. The questionnaires were prepared with 17 questions for correlation of studies, 18 questions for teacher education and 19 questions for the organization of school systems. However, no data were found to show how many people answered such questions.

> It was further decided that all papers written in answer to these lists of questions were to be placed in the hands of the chairmen of the sub-committees not later than Nov. 1, 1894, and that the chairmen should prepare reports to be submitted to the full committee **[of fifteen]** at a meeting to be held in December of that year. (NEA, 1895, p.235, our emphasis)

As indicated by the committee, the three themes presented led to the constitution of three reports, each of which would be prepared by a five-member subcommittee, so that the division of subjects was: Correlation of Studies by Harris (chair), Greenwood Gilbert, Jones and Maxwell; Teacher Education by Tarbell (President), Brooks, Balliet, Dougherty and Cooper; Organization of city school systems elaborated by Draper (President), Seaver, Lane, Poland and Powell.

A second committee meeting took place in mid-1894, and in December of that year the Committee of Fifteen met once again for four days to discuss the reports of the conferences held on the issues that were raised. The three reports were presented to the Department of Superintendence at a meeting held in February 1895 in Cleveland. Regarding the publication of the report, the committee, without funding and "wishing to spread the report before the public at once" (NEA, 1895, p.237), then decides:

> *Resolved,* That the reports of the three sub-committees be read by their respective chairmen before the Department of Superintendence at Cleveland, in February, 1895, and published in the "Educational Review" for March, 1895; provided, that the publishers of the "Review" agree to furnish to each member of the Committee of Fifteen, and also to each person appointed to discuss the report before the Cleveland meeting, a printed copy of the report; and immediately after the meeting to send to each educational journal desiring it such printed copy, with the request that it be published in as nearly complete a form as possible. *(NEA, 1895, p.237, author's emphasis)*

The terms were accepted, and a copy was sent to other publishers as requested, namely: American Book Company, New England Publishing Company and Public-School

Publishing Company. Similarly, each member of the Department of Superintendence who had expressed interest also received a hard copy of the report. Thus, it can be seen that the circulation or, more specifically, the proposals and material produced by the Commission of Fifteen were widely disseminated across the United States.

ALGEBRA PRESENT IN THE CORRELATION OF STUDIES

> The branches to be studied, and the extent to which they are studied, will be determined mainly by the demands of one's civilization. These will prescribe what is most useful to make the individual acquainted with physical nature and with human nature, so as to fit him as an individual to perform his duties in the several institutions – family, civil society, the state, and the church. (NEA, 1895, p.289)

The questions sent to those whose "opinion mattered" addressed various topics (NEA, 1895, p. 233-234) and, with their responses, served as the basis for the elaboration of the subcommittee's report. Some of these questions addressed, among other topics were whether the fields of knowledge had distinct pedagogical values; whether new subjects should be included in primary education and, if so, in which grade; what the development of content in each subject should be; which topics should be addressed; and what the best teaching method in a given subject is.

Regarding the teaching of arithmetic and algebra in primary education, according to the subcommittee, the student who masters the numbers "divide and conquer", "makes possible all the other sciences of nature which depend on exact measurement and exact record of phenom" (ibid, p. 296), as well as other aspects, all precisely defined by numbers. The educational value of the arithmetic would be highlighted as it becomes indispensable for the other sciences of nature. Psychological aspects would also be linked to its importance, since the counting process, for example, would be linked to a quantitative but also qualitative perspective, because, to perform the counting process, the subject must set aside qualitative aspects of the objects, as distinctions could not be considered in this process.

This arithmetic would consist of two types of examples, (1) those in which there is a direct application of simple numbers, fractions, and powers, and (2) those involving operations to achieve numerical solutions through indirect statements. In the second type of example, according to the subcommittee, there would be a difficulty linked to this process, not in the arithmetic development of the question, but in the transformation of the data, since this process would belong to the field of algebra.

> They hold, therefore, that arithmetic pure and simple should be abridged and elementary algebra introduced after the numerical operations in powers, fractions,

and simple numbers have been mastered, together with their applications to the tables of weights and measures and to percentage and interest. (Harris et al., 1895, p.245)

Such algebra would not be the same rigorous secondary school algebra, stated the subcommittee (Harris et al., 1895, p. 248). The primary school algebra would use letters for unknown quantities and would keep the numerical form of the known quantities, doing the opposite only when seeking to show a general form or a rule. As indicated by Valente (2017), Basei (2017) and Rocha (2019), this algebra would solve the most advanced problems of arithmetic, called "conundrums"[14] by Parker, but at the same time would be an introductory algebra for secondary school algebra, thus having a propaedeutic character. However, no indication is given of how a correlation of studies involving arithmetic or algebraic contents could be carried out.

With this, the committee sought to solve two problems of education:

> [...] first to aid the pupils in the elementary school to solve, by a higher method, the more difficult problems that now find place in advanced arithmetic; and secondly, to prepare the pupil for a thorough course in pure algebra in the secondary school. (Harris et al., 1895, p.248)

More specifically, in the 7th grade students should learn 1st degree equations, the arithmetic problem solving that "fall under proportion or the so-called "rule of three," together with other problems containing complicated conditions" (NEA, 1895, p.299). In the 8th grade of primary school, 2nd degree equations could be taught, as well as "other problems of higher arithmetic solved in a more satisfactory manner than by numerical methods" (NEA, 1895, p.299).

More generally, teaching should be concentrated in the areas of literature, grammar, arithmetic, geography and history. However, other areas could also seek space during primary school, such as the natural sciences, drawing etc., each developing distinct characteristics in the student. It would also be the primary school's role to teach morals and manners. The subcommittee also agreed that a one-size-fits-all training would be best, since education would be an introduction of the students to the "art of learning".

Contrary to one of the possible changes proposed by the Committee of Ten, the Committee of Fifteen believed that primary education should not less than eight years. However, they suggested that Algebra and Latin should be included in the program, which, according to the subcommittee's report (Harris et al., 1895, p. 285), would be an

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[14] The term could be translated as "riddles" or "problems", thus being difficult problems of arithmetic or problems of more advanced arithmetic.

appropriate transition to secondary education, something that had also been proposed by the Committee of Ten (NEA, 1894).

Given the format of the report, in which proposals for changes in the educational system are presented from the perspective of the committee based on the questionnaires, opposition were expected to arise. At the end of subcommittee's report on the correlation of studies, a "discussion" section is presented (NEA, 1895, p. 343-350), in which NEA members involved show their views on the subcommittee's theme and report. The committee's proposal on the teaching of arithmetic and algebra was not criticized, however, according to NEA (1895), it was criticized for not having presented a solution to the problem they had been established to solve. In one statement, Parker indicates that:

> When, in Boston, two years ago, I moved that a committee be appointed to report on correlation, I had in mind a full and complete treatment of the subject. I had confidence in the ability of the committee. I supposed they would make a careful study of every phase of the subject; that they would make themselves familiar with Herbart, Ziller, Stoy, and Rein. They have ignored the very subject which they were intended to treat. The report is a grand restatement of facts long known to all of us. But it is like the play of "Hamlet" with Hamlet left out; or, as I might better say, with Hamlet kicked out […]. I shall accept this report respectfully. (NEA, 1895, p. 344)

In other words, the report defended the current *status quo*, thus, it pointed to setbacks rather than advances in this area. Hence, De Garmo (NEA, 1895, p.344) states that, based on that outcome, people could think that the reform of primary education was not necessary, since it already had what was being proposed. Charles A. McMurry's[15] biggest objection to the report is that "it will be a tower of strength to those who still hold to the old formalism in education" (NEA, 1895, p. 346). In Button's (1965) view, they reached their objective, since the primary school curriculum remained unchanged for the next decade or so.

> This is not to say that the "establishment" **[the committee]** was made up of opportunists. The circumstances and spirit of the times brought to prominent places conservatively inclined individuals who acted and thought and spoke in keeping with the conservative temper of the times, and with credibility. (Button, 1965, p.261-262, our emphasis)

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[15] During this time he was director of the training department of the Illinois State Normal School. He would be the author of several books on methodologies in various areas of education.

THE REPERCUSSION AND CIRCULATION OF US IDEAS IN BRAZIL

Can one then observe the circulation of the late 19th-century US movement, its set of ideas, ideals or knowledges, in the beginning of the following century in Brazil? In this sense, the first steps of the analysis unfold over the history of the movement and its repercussions, both in the US and in Brazil.

According to Valente (2017, p. 9), research shows the presence of US models in Brazil since the 19th century and their role in the teaching of mathematics in Brazilian primary school. The author also points out that those studies highlight the importance of the role of the American School[16] for the state of São Paulo. In this sense, in the preface of his work Trajano (1905) argues that a reform in public education in São Paulo made compulsory the teaching of algebra in primary schools. According to Valente (2017, p. 10), a major reform of normal education in the state occurred two years after the publication of the first edition of Trajano's book, in 1888. This evidence reveals that some movements for the insertion of algebra in Brazilian primary education occurred before the work of the Committee of Ten and the Committee of Fifteen. Valente (2017) also emphasizes that

> Trajano is considered a privileged author for reading the current discussions in the US about early school curricula, whether due to his condition of belonging to the American School, or to having had his Arithmetic books already adopted and spread throughout Brazilian schools when his Algebra work was launched. The US model of thinking the primary education, will be more and more reaffirmed in daily school after Trajano. (p.12)

However, on September 12, 1918, in Brazil, Professor Othello Reis came to give a lecture entitled "Os dois ultimos annos de arithmetica, na escola primaria, segundo a Comissão dos quinze", at the National Library Hall. This conference was later divided and publish in two issues of the journal "A escola primaria", in 1918. According to Valente, "[Reis] proves to be enthusiastic about the US model for teaching mathematics. Thus, he is dedicated to studying the proposals of the Committee of Fifteen" (2017, p. 11). On the commission, Reis then indicates that:

> It is certainly not necessary to say that the Report of the Commission of Fifteen, on the subject of primary education, is the great compendium, the bible of modern American pedagogy. In this little blue book, in this two-hundred-page booklet, whose translation and dissemination should be undertaken by the Directorate of Public Instruction, all the guiding principles, all the advanced standards of elementary education of the United States are admirably condensed. (Reis, 1918a, p.11)

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[16] According to Oliveira (2019), Trajano was a school teacher, thus having greater contact with US methods and models.

Therefore, it is worth remembering that the conference held by Reis was also published in the preface of his book "Algebra – primeiros passos", in 1919. In conjunction with the previous citation, Reis affirms to support and validate foreign work regarding the insertion of algebra in primary education, in addition to seeking to disseminate the US movement and its appropriations throughout Brazil.

At the beginning of his conference, Reis sought, from some boundaries drawn by the Commission of Fifteen, to present how algebraic knowledge could be inserted into Brazilian primary education. In this sense, Reis states that:

> I therefore have no doubt as to whether or not, whether or not to introduce rudimentary-algebraic study into primary school, enough to solve certain problems. I concede that we should not get to the equations of the second degree soon, but those of the first may, without difficulty, be presented. [...] What I want is to show how, without any previous preparation other than knowledge of the four operations on whole numbers and fractions, a child can perform this africa[17] of solving the equations. (Reis, 1918a, p. 12)

The author already gives evidence that perhaps this algebra has a character of its own, by seeing it as a rudimentary study of another, perhaps more advanced, algebra, and by indicating a proper meaning in such teaching, that is, the resolution of problems. Regarding content, Reis (1918a, p. 12) recommends that the first step in the insertion of algebra in primary education would be the student's familiarization with the use of letters to represent unknown quantities and that would satisfy the conditions of a given problem.

> To intuitively teach the students how to deal with this symbol x, let us have them represent x as an object whose name they do not know, and then count numerous such objects. They will count: one object, two objects, three ... fifteen objects, or things, or anyway: one x, two x, three x ... fifteen x. (Reis, 1918a, p.12)

According to Reis, the students could, then, write 15x just as they write "15 pencils" or "15 objects". In a very similar way, it indicates how such terms would be added, suggesting that, if there were 15x and 6x in one place (such as 15 pencils and 6 pencils), any child would say how many objects x there are in this place, making the sum $15x + 6x = 21x$. Similarly, Reis mentions that any child would tell, if asked, how many objects would be left "if we take out, of the 12 we own, 7 objects of the same kind" (Reis, 1918a, p. 13).

> We are thus gradually entering the algebra. It will not be a big step to imagine that a person has 10 objects of species x, gets 6 more, gives 3, buys 8 more, loses 5, which represents:

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[17] Here the author uses the term "africa" as a feminine noun, synonymous with "a difficult goal to achieve".

10x+6x-3x+8x-5x

Any student, even of the elementary class, knows how to look for with how many objects, after all, the person remains, adding what he owns and acquires, adding what he subtracts, and evaluating the difference. (Reis, 1918a, p.13)

The aspects signaled by Reis for the introduction of the concept of the unknown, or even the operations with the unknown, refer to the intuitive teaching, as Reis quotes (1918a, p. 12). It is noteworthy that, according to Oliveira (2019, p.15), the "modern pedagogy of intuitive teaching had as its basic characteristic to offer data sensitive to students' perception and observation".

According to Reis (1918a, p. 14), could any student from the "1st grades" solve such expressions as "3+?=7" ou "5×?= 30", which would be known from Parker's letters on. John H. Walsh, author mentioned by Reis (1918a), devoted, in his book, an appendix to the theme, entitled "Equational Arithmetic". In this chapter, Walsh (1911, p. 349) signalizes that there was a growing movement in the tendency to introduce equations in the last two years of the arithmetic course, replacing "many of the useless applications of percentage and for some obsolete topics". In this sense, Walsh (1911, p. 350) points out that the teaching of equations should not start with definitions or unknowns, but with the resolution of questions, as indicated by Reis.

Walsh holds the same position as the Committee of Ten and the Committee of Fifteen, that such algebra should not be complementary to the content of the secondary education, that is, it would not aim to reduce the amount of algebra content of this teaching. According to the author, in the operations, teaching should focus on initial principles and simple operations with 1st degree equations, as presented by Reis (1918a, 1918b). After the equations has been taught, the students should not be required to use them to solve a problem but use whichever (algebraic or arithmetic) method they wanted. In addition, the most skilled students should be encouraged to omit steps in their solutions, demonstrating better their knowledge.

Finally, Reis discusses that the students should, then, be presented a way to understand what an equation is. For this, the teacher brings the idea of the scales, since the students know what it is for and how to operate it. For the author, one could "tell the student, without giving a formal definition, that this expression of equilibrium is an equation, that the left part is the first member, and the right part is the second member" (Reis, 1918a, p. 14). From this aspect, as well as a perspective of an intuitive teaching, it is possible to perceive characteristics of Reis's appropriation of the US work, since the committee's report does not present discussions on the introduction, interpretation and understanding of concepts related to equation.

In the second part of his conference, Reis begins by saying that in problem solving, they should obtain equations, rather than just make use of abstract exercises. In this sense, the author states that

> All problems present, usually with too many words, several relationships between the incognita and known quantities. Having caught and expressed one of these relationships, we will have an equation, which should solve the problem. Clarifying this relationship is what is called putting the problem into equation. Rule to *put the problem into equation*? None. Some problems will suffice as follows: "A dealer bought a flock of sheep at three different prices. He paid 1/3 of the herd at the rate of 21 francs a head; 2/5 at the rate of 19 francs and the rest at the rate of 15 francs. Resold the entire herd for 1,674 francs, earning 1/5 of the purchase price. How many sheep did the flock have?" (Reis, 1918b, p. 41, emphasis added)

According to Reis, no one would be able to "make a clear, easy, current reasoning" in solving such a problem just by using arithmetic knowledge, but that the process would be easy as long as the equation was used. This, according to the author, would be an example of "conundrum". However, he reminds us that the subcommittee of correlation of studies indicates that, in solving those problems, algebra would serve as a "superior method", while preparing the student for secondary education.

> Their difficulty **[of the problems]** is not found in the strictly arithmetical part of the process of the solution […], but rather in the transformation of the quantitative function given into the function that can readily be calculated numerically. (NEA, 1895, p.298, our emphasis)

With his conference, Reis proposes to make circulate in the Brazilian territory, more precisely starting in the city of Rio de Janeiro, the discussion about the insertion of algebra teaching in the primary school curriculum from the perspective of the Committee of Fifteen, that is, for solving 1st degree equations and their use for solving advanced arithmetic problems. However, Reis (1918b, p.42-43) still addresses, in his conference, the resolution of what he calls "systema" (linear systems), which would be problems involving equations with more than one unknown, something that is not addressed by the US movement. The author also points out that after approaching systems of order two, that is, with two equations and two unknowns, it would be easy to teach systems of order three, four orders and so on. From another perspective, it was not possible to observe, during Reis's conference, the view that equations of the 2nd degree were part of the algebraic knowledge of primary education, as proposed by the Committee of Fifteen. These aspects highlight a possible appropriation by Reis regarding the proposals of the Committee of Fifteen, giving a proper meaning to the insertion of algebra contents in the primary curriculum.

It is noteworthy that the author, at the end of his lecture, also indicates that the insertion of such knowledge would not be the same as the insertion of an algebraic method:

> I have not come to advertise for the introduction of the algebraic method in your schools. I have just come to show you that algebra is not one of those disciplines that you may or may not learn, and are never useful, and that you should not be surprised if you find it reduced to very simple elements, in teaching programs. (Reis, 1918b, p. 43)

Based on what is proposed by the report of the subcommittee of correlation of studies, it can also be observed that Trajano (nd) seems to seek to disseminate a proposal that complements what is approached by Reis for the teaching of algebra knowledge in the country's primary school. This can be observed by noting that the author addresses, in the 68th edition of his work "Arithmetica Elementar Illustrada", prior to 1922, the content of proportion[18]. In the work, the author presents, for example, how to "find any term of a proportion", which he calls "incognita", knowledge that refers to algebraic knowledge, because they are related to the solving of 1st degree equations, something proposed by the committee.

However, Trajano (n.d.) does not keep solving the problems of advanced arithmetic or discussing the resolution of equations[19], as Reis does and as indicated by the Committee of Fifteen as one of the purposes of such algebraic knowledge. Reis, on the other hand, does not mention the resolution of proportions or problems involving the rule of three.

It seems that Trajano (nd) may have appropriated certain ideas from the US movement, not as a whole, but that he adapted them to arithmetic teaching, making it circulate through his book. In this perspective, Trajano and Reis, as agents of circulation within intellectual and educational networks, should not be seen as passive subjects in such process, since their appropriation of the thinking of the US movement on the insertion of algebra in primary education is clear. Called "creative agents" by Oliveira (2018), subjects like Trajano also contribute to what circulates from their appropriations, since, as the author reveals, "the news came to and fro, gaining other garments, or rather, new verses that intersected with old melodies" (p. 26).

FINAL CONSIDERATIONS

Initially, we questioned which changes had been proposed by the Committee of Fifteen for the mathematics teaching, particularly in the field of algebra, in discussions

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[18] Trajano also addresses the content of rule of three in his book, as he mentions the Committee of Fifteen.
[19] The author shows how to solve proportions and rules of three, but does not use knowledge such as inverse operations based on the idea of the scale. He approaches such content from a more practical perspective, distancing himself from a proposal of algebraic knowledge (RODRIGUÊS & COSTA, 2019).

about curriculum and teaching in primary school. The analysis made clear that the algebra proposed by the Committee of Fifteen, focused on primary education, was not the same algebra of secondary education at the time, but a series of knowledges from the algebraic field that would become tools for problem solving of a more advanced arithmetic. The committee also predicted the teaching of 1st and 2nd degree equations in the last two grades of primary education, as well as their methods of solution. Such knowledge would not only be taught aiming at offering the student a tool to solve arithmetic problems in a simpler way, but, above all, in order to prepare the student to enter high school, that is, they assumed a propaedeutic character.

As for the circulation of the US ideas in Brazil, the evidence observed shows that some of the ideas and ideals presented by the Commission of Fifteen regarding the knowledge related to algebra in the late 19th century circulated in Brazil at the beginning of the following century. However, we found, through the work of Trajano (1905) and the works by Valente (2017) and Basei (2017), that the movements of insertion of algebraic knowledges in primary education were already circulating throughout Brazil when the work of the committees was developed and thus they could have revived or strengthened them. Reis's conference indicates that the US discussion has come to Brazil derived from his appropriations of the US thinking. Through his book, the author sought to disseminate in the Brazilian educational field the ideas about the insertion of an algebra proper of primary education, focused on solving "difficult problems, or whose solution is difficult to explain in words" (Reis, 1918b, p. 43). Although Reis (1918a) showed he had contact with the material prepared by the Commission of Fifteen and said that report should be translated and should circulate in Brazil, it was not possible to determine if this actually occurred.

Reis (1918b) demonstrates that the appropriation of the US movement is performed in a transformative way (Chartier, 2002) regarding this algebra that should be part of primary education, since the author proposes an introduction to the teaching of algebra under aspects of an intuitive methodology, presents conceptions about equation, and contents as systems, elements that are not mentioned by the US committees.

Beyond the results already presented by Valente (2017), Basei (2017) and Rocha (2019), we could note that the US proposal to insert algebraic knowledge in primary education would not only be linked to solving complex problems, but had specific content and time for student learning. We also noted that Reis's proposal (1918a, 1918b) varies from what was presented by the Committee of Fifteen, since it does not indicate a perspective of necessarily propaedeutic teaching and presents differences between algebraic contents and knowledge.

Finally, we should note that the report prepared by the committee was criticized, whether for its content or for its approach to the various topics it proposed to discuss. However, we must remember that the material produced by the Committee of Fifteen had no legislative or normative purpose, it was only a study based on issues raised initially. Still, other works can focus on how such propositions affected the US primary

education system and its subsequent reforms, as well as other Brazilian appropriations made from it.

AUTHOR CONTRIBUTION STATEMENT

JSR and DAC conceived the idea and developed the discussions presented. Both authors contributed with the writing and the necessary adjustments to finalize the manuscript. DAC was also responsible for overseeing and guiding the development of this work.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article because no new data was created or analyzed in this study.

REFERENCES

Basei, A. M. (2017). Álgebra na formação de professores que ensinam matemática (1890-1970) análise de um debate. *Anais do XV Seminário Temático*, Pelotas, 1-10.

Button, H. W. (1965). Committee of fifteen. *History of Education Quarterly, 5*(4), 253-263. Recuperado em 10 outubro, 2019, de http://www.jstor.org/stable/367584.

Bourdieu, P. (2002). As condições sociais da circulação internacional das idéias. *ENFOQUES – Revista Eletrônica*, *1*(1), IV-XV.

Burke, P. (2016)**.** *O que é história do conhecimento?* Tradução Claudia Freire. 1ª ed. São Paulo: Ed. Unesp.

Certeau, M. de (2013). *A escrita da história*. Tradução de: Maria de lourdes Menezes. 3. ed. Rio de Janeiro: Forense Universitária.

Chartier, R. (2002). *A história cultural entre práticas e representações*. Tradução de: Maria Manuela Galhardo. Rio de Janeiro: Berthand do Brasil.

Chartier, R. (2010). "Escutar os mortos com os olhos". *Estudos Avançados*, *24*(69), 6-30. Recuperado em 31 de outubro, 2019, de http://www.revistas.usp.br/eav/article/view/10510.

Draper, A. S., Powell, W. B., Poland, A. B., Seaver, E. P. & Lane, A. G. (1895). Report of the sub-committee on the organization of city school systems. *Educational Review, 9*, 304-322. Recuperado em 10 outubro, 2019, de https://archive.org/details/educationalrevie09newyuoft/page/n8.

Harris, W. T., Greenwood, J., M., Gilbert, C. B., Jones, L. H. & Maxwell, W. H. (1895). Report of the sub-committee on the correlation of studies in elementary education. *Educational Review, 9*, 230-303. Recuperado em 10 outubro, 2019, de https://archive.org/details/educationalrevie09newyuoft/page/n8.

Hilgenheger, N. (2010). *Johann Herbart*. Tradução e organização: José Eustáquio Romão. Recife: Fundação Joaquim Nabuco, editora Massangana.

Mackenzie, J. C. (1894). Report of the committee of ten. *School Review, 2*, 146-55. Recuperado em 10 outubro, 2019, de https://archive.org/details/jstor-1074830/page/n1.

Monroe, P. (1935). *A text-book in the history of education*. Nova York: MacMillan & Co.

Monroe, W. S. & Herriot, M. E. (1928). *Reconstruction of the secondary-school curriculum its meaning and trends*. Urbana: University of Illinois.

NEA. (1895). *Journal of proceedings and addresses: session of the year 1895 held at Denver, Colorado*. Saint Paul: NEA. Recuperado em 10 outubro, 2019, de https://babel.hathitrust.org/cgi/pt?id=miua.0677752.1895.001&view=1up&seq=5.

NEA. (1894). *Report of the committee of ten on secondary school studies*. Nova York: American Book Company. Recuperado em 10 outubro, 2019, de https://archive.org/details/reportofcomtens00natirich/page/n5.

NEA. (1892). *History of the national educational association of the United States: 1857 - 1891*. Washington: NEA. Recuperado em 10 outubro, 2019, de https://babel.hathitrust.org/cgi/pt?id=miua.0677752.1892.001&view=1up&seq=5.

Oliveira, M. A. (2019). Antonio Bandeira Trajano e a renovação pedagógica lida em livros escolares: ensinar aritmética de modo intuitivo (final do século xix). *Revista História da Educação, 23*, 1-41. Recuperado em 10 outubro, 2019, de https://seer.ufrgs.br/asphe/article/view/79977.

Oliveira, M. A. (2018). *Volume 1: circulação*. São Paulo: Livraria da Física.

Reis, O. S. (1918a). Os dois ultimos annos de arithmetica, na escola primaria, segundo a comissão dos quinze. *A Escola Primaria, 3,* (1), 11-15. Recuperado em 10 outubro, 2019, de http://memoria.bn.br/DocReader/DocReader.aspx?bib=097497&PagFis=769&Pesq=%22ultimos%20annos%20de%20arithmetica%22.

Reis, O. S. (1918b). Os dois ultimos annos de arithmetica, na escola primaria, segundo a comissão dos quinze (continuação). *A Escola Primaria, 3*(2-3), 41-43. Recuperado em 10 outubro, 2019, de http://memoria.bn.br/DocReader/docreader.aspx?bib=097497&pasta=ano%20191&pesq=intellectuaes%20se%20comparam.

Rocha, I. L. (2019). O método algébrico estadunidense e os problemas aritméticos na proposta de Otelo de Souza Reis em 1919. *XVII Seminário Temático*, Aracaju, 1-14.

Rodriguês, J. S. & Costa, D. A. da (2019). Saberes algébricos sob uma perspectiva aritmética: um olhar sobre estes saberes a ensinar nos livros de aritmética de Ruy de Lima e Silva e Antônio Trajano. *XVII Seminário Temático*, Aracaju, 1 - 15.

Schneider, O. & Neto, A. F. (2008). Americanismo e a fabricação do "homem novo": circulação e apropriação de modelos culturais na revista Educação Physica (1932-1945). *Movimento, 14*(1), 135-159. Recuperado em 10 outubro, 2019, de https://seer.ufrgs.br/Movimento/article/view/2236.

Souza, C. A. & Garnica, A. V. M (2016). Sobre a Dinâmica de Circulação de Ideias (em Educação Matemática). *Perspectivas da Educação Matemática*, *9*(20), 413-446. Recuperado em 31 outubro, 2019, de https://periodicos.ufms.br/index.php/pedmat/article/view/2880.

Souza, R. F. (2016). As disputas pelo currículo e a renovação da escola primária nos estados unidos na transição do século 19 para o século 20. *História da Educação, 20*(48), 35-53. Recuperado em 10 outubro, 2019, de https://seer.ufrgs.br/asphe/article/view/56892.

Tarbell, H. S., Brooks, E., Balliet, T. M., Dougherty, N. C. & Cooper, O. H. (1895). Report of the sub-committee on the training of teachers. *Educational Review, 9,* 209-229. Recuperado em 10 outubro, 2019, de https://archive.org/details/educationalrevie09newyuoft/page/n8.

Trajano, A. (1905). *Algebra elementar*. 5. ed. Rio de Janeiro: Companhia Typographica do Brasil. Recuperado em 10 outubro, 2019, de https://repositorio.ufsc.br/handle/123456789/160598.

Trajano, A. (n.d.). Arithmetica elementar illustrada. Rio de Janeiro: Martins de Araujo & C. Recuperado em 10 outubro, 2019, de https://repositorio.ufsc.br/handle/123456789/104081.

Valente, W. R. (2017). A matemática para o professor dos primeiros anos escolares – a álgebra entre a cultura enciclopédica e a formação profissional. *Jornal Internacional de Estudos em Educação Matemática, 10*(1), 8-14.

Valente, W. R. (2007). História da Educação Matemática: interrogações metodológicas. *REVEMAT*, *2*(2), 28-49. Recuperado em 30 outubro, 2019, de https://periodicos.ufsc.br/index.php/revemat/article/view/12990.

Walsh, J. H. Practical methods in arithmetic. Nova York: D.C. Heath & Co., 1911. Recuperado em 10 outubro, 2019, de https://archive.org/details/practicalmethods00walsuoft/page/n3.

Warde, M. J. (2000). Americanismo e educação: um ensaio no espelho. *São Paulo em Perspectiva, 14*(2), 37-43. Recuperado em 10 outubro, 2019, de http://www.scielo.br/pdf/spp/v14n2/9786.pdf.