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Kuhnian Analysis of Why, even after Euler's Contributions, the Fundamental Principle of Motion is still "Newton's Second Law"

Camila Maria Sitko[a, b, c]
Marcos Rodrigues da Silva[d]

[a] Universidade Federal do Paraná, Departamento de Física, Curitiba, PR, Brasil
[b] Universidade Federal do Sul e Sudeste do Pará, Programa de Pós Graduação em Educação em Ciências e Matemática, Marabá, PA, Brasil
[c] Instituto de Estudos Superiores de Fafe, Centro de Investigação, Desenvolvimento e Inovação, Fafe, Portugal
[d] Universidade Estadual de Londrina, Programa de Pós Graduação em Filosofia, Departamento de Filosofia, Londrina, PR, Brasil

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ABSTRACT

Background: The F = ma principle was produced by Euler, between 1752 and 1776, and not by Newton in 1687, as is usually brought up in the physics manuals. It took over sixty years of conceptual and mathematical developments to develop this fundamental principle of mechanics. Still, after all this time, the principle continues to be called "Newton's law". **Objectives:** This paper seeks to discuss the possible reasons that led to the omission of Euler's contributions to the elaboration of the fundamental principle of mechanics. **Design:** The study fits as documentary analysis, followed by a philosophical analysis of the researched material. **Data collection and analysis**: Historical research was carried out, using primary and secondary sources, regarding the reasons that led to such omission. After that, four main hypotheses were listed. Thomas Kuhn's philosophical structure was applied to these hypotheses to support the explanation of the historical omission. **Results**: From the Kuhnian analysis, the Newtonian paradigm was presented and discussed, of which the principle formulated by Euler is part. **Conclusions:** The principle remains "Newton's," due to being within the Newtonian paradigm; and the conclusion that the principle ought to remain Newtonian matches the image of science within the baselines of the field of science teaching.

Keywords: Second law of motion; Newton's second law; $F = ma$; Leonhard Euler; Thomas Kuhn.

Corresponding author: Camila Maria Sitko. Email: camilasitko@yahoo.com.br

Análise kuhniana de por que, mesmo após as contribuições de Euler, o princípio fundamental do movimento ainda é “segunda lei de Newton”

RESUMO

Contexto: O princípio $F = ma$ foi produzido por Euler, entre 1752 e 1776, e não por Newton, em 1687, como em geral é trazido nos manuais de Física. Foram necessários mais de sessenta anos de desenvolvimentos conceituais e matemáticos para a elaboração desse princípio fundamental da mecânica. Ainda assim, depois de todo esse tempo, o princípio continua a ser denominado “lei de Newton”. **Objetivos:** Este trabalho busca discutir os possíveis motivos que levaram à omissão das contribuições de Euler para a elaboração do princípio fundamental da mecânica. **Metodologia:** O estudo enquadra-se como uma análise documental, seguido de análise filosófica sobre o material pesquisado. **Coleta e análise dos dados:** Foi realizada uma pesquisa histórica, utilizando fontes primárias e secundárias, com relação aos motivos que levaram a tal omissão. Após isso, foram elencadas quatro hipóteses principais. A estrutura filosófica de Thomas Kuhn foi aplicada sobre essas hipóteses, para embasar a explicação da omissão histórica. **Resultados:** A partir da análise kuhniana, foi apresentado e discutido o paradigma newtoniano, do qual o princípio formulado por Euler faz parte. **Conclusões:** O princípio permanece sendo “de Newton”, devido a estar dentro do paradigma newtoniano; e a conclusão de que o princípio é newtoniano está de acordo com uma imagem de ciência dentro dos parâmetros do campo de ensino de ciências.

Palavras-chave: Segunda lei do movimento; Segunda lei de Newton; $F = ma$; Leonhard Euler; Thomas Kuhn.

INTRODUCTION

In academic circles, the thesis that science is a community activity and involves many participants is already commonplace; some histories of science, however, due to some type of orientation (intellectual, editorial, etc.), omit several participants in a scientific construction. An example of this is the so-called “Second Law of Movement” by Isaac Newton, whose original formula differs from the formula taught ($F = ma$). Such a formula, proposed by Euler, is presented pedagogically by omitting the name of Euler.

At first, a science education researcher (or a historiographically well-educated physics teacher) might feel uncomfortable with such an omission; however, this is not the central point of this article. Instead, the omission (conceptually speaking) reveals something structural about the construction of scientific knowledge, as we will discuss below.

Let us suppose that a physics teacher, when teaching the Second Law of Movement, stated that Euler formulated it. This teacher could do it in two ways: i) simply by saying that Euler produced it; ii) showing that Euler developed a conception (which resulted in the second law as we know it today) *from an existing conceptual structure*.

In choosing (i), the teacher simply added a name to the history of science; when opting for (ii), however, the teacher presented a fundamental trait of scientific knowledge: scientists, like Euler, can propose conceptual novelties such as "$F = ma$", because they operate within a scientific universe already in progress. Thus, when choosing (ii), the teacher conveyed to his students not only information, but also revealed a characteristic feature of the nature of science: its form of development.

In addition, such a form of development is quite frequent in science, as we see that as soon as Euler enunciated the Second Law of Movement, in 1752, in the modern form ($F = ma$), it was immediately accepted and used by the community, and there are no controversies and doubts about its use. Euler had an incredible capacity for systematisation and generalisation, and perhaps that was what allowed him to take mechanics to the relatively definitive form that we know today (Gautschi, 2008). The easily accessible language he uses, such as that which appears in the work in which he wrote the letters to a German princess (Euler, 1823), may have facilitated the dissemination of Euler's work.

However, as stated by Maltese (1992), Truesdell (1975), and other historians, the law, in the form $F = ma$, was taken for granted, since all the necessary tools were already in Euler's hands, to the point of one might think that there had never been a time when this law was not Newton's.

It is difficult to see today that the application of Newton's law of motion to types of systems with many degrees of freedom was not trivial, and in some cases, it was not possible to carry out. An important detail for this perception is the use of intrinsic coordinates in the 17th century. Cannon and Dostrovsky (1981 *apud* Maltese, 1992, p. 32) share this opinion, and still state that the neglect of mechanics in the 18th century is precisely due to this "*great leap in complexity*", which shortly after the construction of $F = ma$, it was said as obvious.

However, in fact, it was necessary to understand the generalisation of the law, as well as the abandonment of collateral principles. In this period, before Euler, the Second Law of Movement was not adopted as a fundamental principle but as one more law among many others. Euler (and others, such as the Bernoullis and d'Alembert) was responsible for the image we have of Newton today (Sitko, 2019b).

Today, all the hard work of decades and dozens of scientists and the development of new concepts have been ignored. Everyone accepts the law, but just calls it Newton's law.

Maltese believes (1992) that it may have been the difficulty in understanding the *Principia* that caused Euler's mathematisations to be included as Newton's, to make the content of the work more "*intelligible*," as if everything Euler elaborated was contained in Newton's work, but that needed a clearer "explanation." However, we believe that the explanation is not so trivial, as shown by Sitko (2019a, 2019b).

Other scientists had already written the modern form, like Varignon, in 1703, when writing about falling bodies; like Taylor, who in 1715 had studied the frequency of vibration of a string, using the second law in modern form for such a problem; like Hermann, who had written the modern form in 1716; as well as Johann Bernoulli, who had used orthogonal coordinates to solve mechanical problems, in 1742.

That is, the use of the differential format had already been made, so why do we say that Euler was the first? Because again, we argue that it is not a change in mathematical characters that has been made[1], but a conceptual change and a physical and mathematical generalisation.

The fact is that the law proposed by Newton is not the same proposed by Euler (according to Sitko 2019a), which we currently call by the name "Newton's law." For some reason, or more than one, the history of mechanics told in the manuals practically ignores Euler's contributions and all post-Newtonian conceptual development. Thus, the question arises as to the reasons for this omission (or distortion) in the history of the construction of the fundamental principle of mechanics.

Thus, four main hypotheses are highlighted in this work, which we believe have influenced the scientific community and the general public, over the years, to defend the idea that there was no conceptual production after Newton, which is already known not to be true, according to Sitko (2019a, 2019b). The hypotheses listed are:

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[1] Already dealt with in Sitko 2019a; 2019b.

1) The strong influence of Newtonianism in Europe, right after the creation of the *Principia*;

2) The repercussion of the Jesuit Edition (JE) of *Principia* (published between 1739 and 1742) in Europe, in which Newton's law appears in a more analytical form (but still with a geometric view);

3) The influence of Lagrange's *Mécanique Analytique*, 1788, in which he omits Euler's work in the production of the law of motion and claims Newton as the last producer of concepts in mechanics;

4) The work and vision of the influential physicist and philosopher Ernst Mach, already in the 19th century, who also argues that after Newton, there were only mathematical reformulations, but not the creation of new concepts in mechanics.

We believe that science is a construction that depends on context, alliances, disclosures, conceptual developments, scientists, nature, in short, countless different elements. Because of this, the four hypotheses go in that direction, of merging influences, scientific developments, and contexts, which will corroborate for the emergence of "Newton's second law."

In this text, each of the four hypotheses is discussed in more detail, as well as the reasons why we believe that these influenced the omission of Euler's contributions to the elaboration of the fundamental principle and reduced all conceptual developments to the pages of the *Principia*. After detailing the hypotheses, Kuhn's theoretical framework is presented; then, this framework is used to explain the historical omission. In conclusion, we point out the importance of the historical discussion presented here for science teaching.

FIRST HYPOTHESIS: THE NEWTONIANISM

Analytical Mechanics was known for dealing with pure mathematics problems, without worrying about the reality of the problems, while Newtonian mechanics dealt with the world (or at least that is what most of those who read understood). Is it possible that the Newtonian appeal to deal with the real problems made the production of Euler's fundamental principle an appropriation of Newton's, since Euler's proposal was a construction that worked for the world and not only for mathematics, as was usual for analytical mathematicians? Newton was a strong character in this story. He could certainly bear the name of the production of the new principle, not only because it deals with real problems, but for other reasons, which will be mentioned below, to support why this hypothesis was established.

The *Principia* brought a new vision to Europe. Newton's followers (such as the astronomer Edmond Halley, for example, who was an opinion maker) were so excited by the Newtonian writings that they placed him above other philosophers of the time. However, this work was hardly read, possibly due to its complexity, but still much cited; his fame comes precisely from these few able readers' enthusiasm. The non-mathematical thinkers who defended Newton's ideas did not even understand the technical part of the book, as is the case with John Locke and Voltaire (2015). These and many others disseminated the messages of Newton's work to the public (Dominiczak, 2012), based on accessible memories and treatises they themselves had elaborated. The content was extracted from what the few scientists who had read the work understood and disseminated to these philosophers (it is already clear that some erroneous interpretations could be made, whether on purpose or not).

The work was then quickly disseminated and expanded throughout Europe. Many wished to put the mechanics described in the *Principia* in easier terms. Not only are the scientists who made this "translation" responsible for the popularisation of Newtonian mechanics, but they are also responsible for transforming its nature (Snobelen, 1998). This simple translation did not happen because together with it, there was the elaboration of new concepts for the elucidation of mechanics problems in general because Newton's concepts were insufficient for a more general class of problems.

Thus, the Newtonian work soon became known (even if many elements of the text were not from Newton), and its importance was recognised, especially at popular levels. At the philosophers' level, it was a little different; from them, the *Principia* were a reference, however, not the only work available in mechanics. When Newton's work hit the streets, popularised, without mathematics, it ceased to be a philosophical and mathematical text to become a more practical and enjoyable form of knowledge. Thus, those responsible for Newton's figure were then his advocates of ideas, the popularisers (Snobelen, 1998).

After the *Principia*, another very famous Newtonian work is *Opticks*, in which Newton describes the experimental method in a clear and detailed way. However, it was not only the productions of those two works that defined Newtonianism, but also the interpretations and adaptations of his works to various intellectual means; and even more, a mixture of scientific, political, religious ideas, and it is important to emphasise, they only partly refer to the original Newtonian ideas, because again, many who supported Newton did not even know his scientific theories, just used the opinions of a few connoisseurs of the works.

Much of Newton's cultural influence did not come from his works but from the inspiration they brought due to the new way of approaching thought. The development of this new way of thinking about physics brought well-defined reflexes in the construction of machinery, in technical improvements, in geographic discoveries, in the capitalist economy. The *Principia* were both a foundational and innovative work (Bussotti & Pisano, 2014). The French considered Newton a hero because he established that the movement of the planets obeyed the same terrestrial laws in enunciating the law of universal gravitation and because England was known as the place of freedom of thought, according to Hankins (1985). If Newton were in France, perhaps his achievements would not have had the impact and support they had in England. Voltaire, for example, associated English social, cultural, and freedom of thought with the Newtonian ideas and was largely responsible for spreading them in Europe (Barra, 2012).

Newton's influence on religion was also strong, however, his religious ideals were interpreted by all types of people, according to their beliefs, even if this did not refer (which happened quite often) to Newton's original ideas.

His subjection to the role of God in his theories was also a decisive factor for its acceptance and continuity: the Newtonian system was theological-scientific. At the time, Newton was just acting in this transition of thinking of individuals (Dominiczak, 2012). The fact that he did not eliminate the deities completely counted in his favour because of this transition. However, until about 1740, France still did not use this mechanics. As some thinkers like Maupertuis and Laplace worked with Newtonian celestial mechanics[2], they soon transformed this study into a sophisticated system of celestial mechanics free from Newton's religious ideas. After Darwin, religion became superfluous to support scientific ideas, and the Newtonian precepts were no longer followed.

What Newton did was to replace Aristotelian mechanics and change the way of looking at the universe, a step forward for modern science. Bussotti and Pisano show (2014) how the birth of this modern science, its contradictions, developments, also represent a cultural phenomenon.

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[2] It is important to remember that this sophisticated system has the contributions of Euler and his contemporaries, with concepts not available in Newton's time.

For the Newtonian mathematician Colin MacLaurin, it was Newton's methodology that made his tradition perpetuate, "*paved the way for future research, which could confirm and expand his doctrines, but never refute them*" (MacLaurin, 1742, p. 10). His work *Treatise of fluxions* is known as one of the greatest examples of Newtonian vision (Maronne & Panza, 2014). Also, with his *Account of Sir Isaac Newton's Philosophical Discoveries* (1748), MacLaurin had great responsibility in the reception of Newtonianism.

The diffusion of Newton's work is not only due to his scientific work, but also to a mix between him, his personality, his influence (since he was the president of the Royal Society and the mint), and the culture of the time. The context in which Newton worked had a lot to do with the socioeconomic issues of the time, which were Astronomy, Physics, Geometry and Calculus. Newton was also known for his improvement of telescopes and other optical instruments for navigation; as for the physics he developed, it was totally different from Aristotelian. The *Principia* were a theoretical work. However, as Bussotti and Pisano (2014) stressed, Newton always mentioned the practical importance of his elaborations.

Newtonianism also comes from the ideas of instrumentalism, of motion determined by the accelerating forces, and of the continuous mathematics of calculus. Thus, others have also used it for their own endeavours. From that, many of his ideas were extended, and many topics that today we deal with as Newtonians, in fact, are not. The mathematical and conceptual reformulations of the laws of motion underwent reductionisms, which lay publicists possibly did not realise, and Newton's idea as the sole producer of $F = ma$ was perpetuated.

The first most accessible version of the *Principia* was created by Richard Bentley in 1692 (Snobelen, 1998, p. 160), and then by William Whiston in 1707, followed by many others. Thus, his method soon disappeared and was replaced by the analytical one, and in this transition, many things were misunderstood. The Geneva Edition (GE) of the *Principia* (or also known as the Jesuit Edition (JE)) then appears as a great system of explanatory notes that helps in the understanding of Newton's ideas, techniques, and methodologies. The work tries to explain Newton's propositions more clearly, to translate them in a more analytical way. The work explains developments in Physics based on Newton's discoveries. Due to its explanatory character, the JE became very well known in Europe in the 18th century, and it is our second hypothesis for Euler's omission in contributions to the second law.

From what has been exposed here, we can see how much Newtonianism influenced Europe in the late 17th and early 18th centuries, from the replacement of Aristotelian mechanics, the way of thinking about phenomena, religious beliefs and their role in science, the culture established from all that, etc. Newton's power in influencing people in such diverse ways contributed significantly to the whole post-Newtonian conceptual development being considered as mathematical dismemberments of its mechanics.

SECOND HYPOTHESIS: THE JESUIT EDITION OF PRINCIPIA

Varignon's (1703) approach to the second law was improved by Johann Bernoulli and was the basis of Euler's *Mechanica*. In addition, his formulation was used in the footnotes of the Jesuit Edition (JE) of the *Principia* (Panza, 2002). This edition was prepared in 4 volumes, based on the 3rd edition of the Newtonian work, and published between 1739 and 1742, by Thomas Le Seur (1703-1770), François Jacquier (1711-1788), French friars, and Jean-Louis Calandrini (1703 -1758), a Swiss mathematician. The edition contains several footnotes with explanations, comments and even reinterpretations of the Newtonian work. In this version, the processes adopted by Newton, which in general were difficult to understand, were simplified and placed more analytically, more accessible to the public. The differential equation method used by Euler in *Mechanica* was used as the basis for these analytical reformulations of the *Principia* (Rocha, 2017). Without Euler's method, this reformulation would not be possible. The JE was reissued three times, the last one being in 1822, which is the version analysed by Bussotti and Pisano and the one used in this work.

This is an important version, studied by experts and used as a source of explanations of aspects of Newtonian mechanics to the public in general (Pisano & Bussotti, 2016). Not only important, but as Rocha (2017) states, there is no more meticulous nor more classic approach in the 18th century about *Principia*. In addition, with the work, it is possible to observe the development of ideas in Physics in the decades following the *Principia*, as well as the difference between Newton's and the analytical approach.

In the explanatory footnote regarding the second law in the JE, specifically in note 31[3], there is a fragment in which the authors comment on the accelerated movement and bring the equations $G\,T = 2\,S{:}T$, and $G\,T^2 = 2\,S$, both describing the accelerating force. From this part, we can see that the force is called G, the time T and S is the distance covered. From the equation brought by Le Seur and Jacquier, if G = F is replaced and the S / T division is equal to speed, a direct relationship between force and acceleration will be obtained ($F \propto a$).

As already mentioned, this more analytical form was achieved based on the material of Varignon and of Euler, which is a reinterpretation of Newtonian law, however, it is not the law proposed by Newton. Moreover, it is also not the second law of motion proposed by Euler, since the concept used in the notes of the JE is still geometric and does not share the conceptual advances necessary for the construction of the principle as general, which would only emerge in 1752.

Despite all this, the format explained in the notes is very similar to what is used today, and due to the great size of JE in the dissemination of Newtonian mechanics in the 18th century, in addition to the publication of Euler's new principle a few years later, it is quite plausible that many (scientists and the general public) let themselves be carried away by the easy access of the notes, in order to look for a clearer explanation of the subject, forgetting that the content could have been modified by others[4], and ignoring the limit between the Newtonian essence and new construction.

We believe that due to what was discussed about the Newtonian influence and the complexity of the original work, the popular versions of *Principia* gained strength, without comparing their content and essentiality. Thus, the JE and its footnotes may have been considered by many to be a work written entirely by Newton.

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[3] "Coroll.3 .... celeritas B D, motu uniformiter accelerato acquisita, est semper (5) ut duplum spatium percursum 2 S K, applicatum ad tempus T B, quo percurritur, seu ut 2 S K: T B. quare si vis accelatrix constans dicatur G; spatium percursum S; tempus quo percurritur T; erit $G\,T = 2\,S : T$ (13) adeóque $G\,T^2 = 2\,S$, seu vis acceleratrix constans in quadratum temporis ducta, est ut duplum spatium eodem tempore vis illius actione descriptum" (NEWTON, 1822, p. 17).

[4] The numbered notes, such as note 31, correspond to commentators' interpretations and annexes, and not to direct passages made or thought by Newton.

The hypothesis launched in this work is that when Euler published the new principle, the Newtonian popularised and the general public took it as something aesthetically equal (however, we know that it is not conceptually equal) to what appears in note 31 of the JE, which in turn, was understood as a work entirely by Newton, and, instantly, $F = ma$ and the generality that this law brought came to be the result of Newton's thoughts and elaborations.

THIRD HYPOTHESIS: LAGRANGE'S MÉCANIQUE ANALYTIQUE

In the 17th century, there were many attempts to create a coherent system of mathematical principles in natural philosophy; obviously, Newton's work was a beautiful attempt, however, his work was not recognised as "revolutionary" at the time, as is now believed (Pulte, 2001). What was new was the use of laws to explain the planetary system, but even the proposition of laws was considered new due to previous works by names like Huygens and Descartes, which were an inspiration for Newton. The idea that he would have created laws that would solve all the problems of mechanics was created by his followers, as is the case of Lagrange, in *Mécanique Analytique*, of 1788.

Still following these attempts, the second half of the 18th century was inflated with principles, as Pulte (2001) comments. These principles were not deduced from phenomena or from larger principles but from the practice of mathematical physics alone; they did not have a scientific metaphysics but were relevant due to their explanatory power. However, this hodgepodge of principles was not tolerated, and it was important to find fundamental principles that would solve all classes of problems. At this stage, there was also no longer a concern with the existence of entities, the nature of space, of time, but with technical issues. A change in the concept of science appears then, due to the lack of methodology or foundational metaphysics, without philosophical reflections.

As an example of the change in concern with the nature of phenomena to the techniques of describing them, it is the case of Newton's attractive force, which had a confused ontological basis, as presented in Sitko (2019a). 18th-century mathematicians preferred to avoid the work of dealing with this type of confusion and the need for metaphysics; thus, they wanted to place Newton's laws on a kinetic basis, i.e., to just work with energies, without worrying about the metaphysics of the concept of force. Because of this, in the 18th century, mathematical physics became increasingly independent of philosophical foundations, so that deductive power was more important than empirical, and formality was more important than material truth, that is, the idea is that if we have true axioms, we will not have to worry about the source of that truth (Pulte, 2001). Furthermore, it is not enough to have specific and evident axioms, but all knowledge in mechanics must fit under these axioms. The 18th century, therefore, had a lot of work in the construction of this body of knowledge, having elements from different programs, but being better known for the transformation of Newton's laws by Euler.

Until the transformation of the law by Euler, insofar as the problems were solved, Euler and his collaborators, like Maupertuis, for example, realised that the Minimum Action Principle[5] could be used as an organising principle of all mechanics, that is, from which laws of motion could be deduced. So much so that Maupertuis, a staunch Newtonian, wanted to replace Newton's concept of force with principles of minimal action. Maupertuis based his statics on the principle of rest and the dynamics on the Principle of Least Action, as quoted by Dias (2006), on a metaphysical basis. For Pulte (2001), it is not the empirical success that explains this issue, but the practice of mathematical physics, in evidence in the 18th century.

From these bases and seeking to establish a deductive-axiomatic basis independent of metaphysical issues, Lagrange writes the *Mécanique Analytique*, published in 1788, entirely analytical, in contrast to Newton's geometric method. In addition, another contrast is that Lagrange's work deals with a much wider range of problems, such as connected systems, rigid bodies, continuous medium, etc. In general, we can say that after Newton, mechanics moved to the continent, and after Euler, especially to France (Grattan-Guinness, 1990), where Lagrange was.

Euler and his contemporaries believed in the novelty of the principles of their time; however, just some time after that, Lagrange no longer expressed the same opinion, writing in *Mécanique* that the knowledge of accelerative force after Newton was only his translation into the analytical form (Lagrange, 1811). For many, Lagrange's work completes the development of Analytical Mechanics.

Lagrange had different principles of different problems at hand, and had good reason to accept them as valid, because they described different

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[5] Non-Newtonian variational principle.

classes of problems so well. Lagrange’s goal was the deductive organisation of laws, not their discovery, by reducing them to generalisation.

Unlike Euler’s work, *Mécanique* does not allow the construction of physical quantities as a linear moment, a centre of mass; it is just about energy. It is a treatise that has no images and schemes, only purely algebraic reasoning. Although it does not contain any discovery, there are unprecedented results from Lagrange. It is a theory of differential equations. In this work, Lagrange renews the principles of natural philosophy, with calculus as the foundational basis.

Lagrange began his mechanics with analytical principles. He used Euler’s Principle of Least Action as the universal principle, but with another formalism, and from it, he derived Newton’s (or Euler’s) equations of motion for conservative forces. It was the first work of mechanics that did not need an *a priori* concept of strength (Pulte, 2001).

Lagrange wanted a coherent deductive system of laws of rest and motion, an analytical system, seeking “order in science”, according to what was needed at the time. Geometry remained important to him in the context of discovery, but it could not appear for presentation and justification (Pulte, 2001), as well as philosophical foundations, which also did not appear in his purely mathematical mechanics. His mechanics is thus known as mathematical instrumentalism. For Pulte (2001), Lagrange’s mechanics is a logical consequence and, at the same time, a dissolution of Euclidianism: logical coherence in place of material truth.

To maintain the order and unity of science, Analytical Mechanics seeks abstract mathematical tools and techniques, a process that ends with Lagrange, who writes from formal axioms, which is no longer “laws of nature” to be a deductive structure- axiomatic.

However, such a structure was built from a solid base of knowledge already established by others, as already mentioned. But when we come across Lagrange’s work, what draws attention, and is relevant in this work, is the omission regarding Euler’s contribution to the laws of motion, and we wonder why it happened. In *Mécanique Analytique*, Lagrange comments:

> But it was reserved for Newton to take this new step and complete the science of the varied movements and the accelerated forces that can generate them. This science now consists only of a few very simple differential formulas; but Newton constantly used the simplified geometric method by

> considering the first and last reasons, and if he sometimes used analytical calculus, it was only the series method he himself used, which must be distinguished from the differential method, although it is easy to assemble them and remember them at the same principle.[6] (Lagrange, 1811, p. 225)

In this excerpt, we can see that Lagrange states that Newton completed the mechanics even though he used geometric methods, and that today, if this science is described by differential elements, such description can be forwarded to what Newton elaborated, i.e., Lagrange says that Newton was the last to describe new concepts and theories in mechanics. In the sequence, he also points out that his successors only translated their productions into the differential format:

> The geometers who, after Newton, dealt with the theory of accelerating forces, almost all were content to generalise their theorems and translate them into differential expressions. Hence the different formulas of the central forces found in various works of Mechanics, but which are no longer used, because they only apply to curves that should be written under a single force tending to a centre, and that we now have general formulas for determining movements produced by any forces.[7] (Lagrange, 1811, p. 225)

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[6] Mais il était réservé à Newton de faire ce nouveau pas et de compléter la science des mouvemens variés et des forces accélératrices qui peuvent les engendrer. Cette science ne consiste maintenant que dans quelques formules différentielles très-simples; mais Newton a constamment fait usage de la méthode géométrique simplifiée par la considération des premières et dernières raisons, et s'il s'est quelquefois servi du calcul analytique, c'est uniquement la méthode des séries qu'il a employée, laquelle doit être distinguée de la méthode différen tielle, quoiqu'il soit facile de les rapprocher et de les rappeler à un même principe (p. 225).

[7] Les géomètres qui ont traité, après Newton, la théorie des forces accélératrices, se sont presque tous contentés de généraliser ses théorèmes, et de les traduire en expressions différentielles. De là les différentes formules des forces centrales qu'on trouve dans plusieurs ouvrages de Mécanique, mais dont on ne fait plus guère usage, parce qu'elles ne s'appliquent qu'aux courbes qu'on suppose dé crites en vertu d'une force unique tendante vers un centre, et qu'on a maintenant des formules générales pour déterminer les mouve mens produits par des forces quelconques.

However, the general formulas mentioned, and which take into account any types of strength, come precisely from the work of new conceptualisations and elaborations by Euler, Bernoulli, d'Alembert, and others.

Euler was also responsible for the insertion of the Leibnizian analytical formalism in the resolution of the problems of mechanics and the introduction of the concept of function, which expanded the reach of the Newtonian principle, changing its substantiality when solving problems that were not previously possible. Lagrange describes this process in detail, but as if such elaborations were nothing more than a simple translation of Newtonian thought, and thus, without having to mention those responsible for the new view of mechanics.

> [...] the effect of the accelerating force consisting only of altering the speed of the body, this must be measured by the ratio between the increase or decrease in speed during any unspecified moment, and the duration of this instant, that is, by the differential speed divided by time; and as the speed itself is expressed in the various movements, by the differential of the space, divided by the time, it follows that the force in question will be measured by the second differential of the space divided by the square of the first differential of the assumed time constant. So, also the second differential of space [...] will express the accelerating force whose body must be moved in the same direction, and must, therefore, be equal to the current force that must act in that direction. This constitutes the well-known principle of accelerated forces.[8] (Lagrange, 1811, p. 226)

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[8] l'eflèt de la force accélératrice ne consistant qu'à altérer la vitesse du corps , cette force doit être mesurée par le rapport entre l'accroissement ou le décroissement de la vitesse pendant un ins tant quelconque, et la durée de cet instant, c'est-à-dire, par la différentielle de la vitesse divisée par celle du temps ; et comme la vitesse elle-même est exprimée dans les mouvemens varies, par la différentielle de l'espace, divisée par celle du temps , il s'ensuit que la force dont il s'agit sera mesurée par la différentielle seconde de l'espace, divisée par le carré de la différentielle première du temps supposée constante. Donc aussi la différentielle seconde de l'espace que le corps (...) exprimera la force accélératrice dont le corps doit être animé suivant cette même direction, et devra par conséquent être égalée à la force actuelle qui est supposée agir dans cette direction. C'est ce qui constitue le principe si connu des forces accélératrices.

Johann Bernoulli, in the 1742 work, in addition to the generalisation described for the problem of compound oscillations and the writing of the respective equations of motion, took another big step in mechanics, which was the introduction of the use of orthogonal Cartesian coordinates in general (Maltese, 1992). Euler then established their use to solve mechanical problems by the decomposition of forces, and making contributions as independent from each other. Euler was the first to express the Newtonian second law in Cartesian format, in the 1747 work "*Recherches sur le mouvement des corps celestes en général*" (published in 1749), which, however, was not yet the general principle.

However, for some unknown reason, perhaps because of the Newtonian vision and great defence shared between MacLaurin and Lagrange and brought up in Treatise, or perhaps because of the lack of separation of geometry by Varignon during the analytical process in this work (Grabiner, 2004), Lagrange put MacLaurin as being the first to use this new way of resolution, according to the following excerpt: *"(...) it seems that MacLaurin was the first to use it in his Traité des Fluxions, which appeared in English in 1742; it is now universally adopted"*[9](Lagrange, 1811, p. 227).

Truesdell disagreed, with good arguments (1960b), pointing to Johann Bernoulli (1742) as being the first to use the coordinates in the solution of a mechanical problem, that of the vibrating rope with two punctual masses (Maltese, 2000). For Truesdell (1968) and for Maltese (1992), it was Lagrange's lack of citation and his fallacious argument that MacLaurin would be the precursor in the use of orthogonal coordinates that contributed strongly to the erroneous image of mechanics passed on to the history of mechanics literature.

Anyway, in countless parts in *Mécanique* Lagrange discussed, expressed, and defended concepts created after Newton, however, always with the conviction that all of that was Newton's work, and those that were not just translations of Newtonian feats into the analytical form.

Lagrange's work had a reputation throughout Europe at the end of the 18th century due to its accessible character and the summarised thinking in mechanics he developed over many decades. Thus, we believe that the previous history was ignored simply because Lagrange's material was a proper summary of the earlier events.

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[9] Il paraît que Maclaurin est le premier qui l'ait employée dans son Traité des Fluxions, qui a paru en anglais en 174a; elle est maintenant universellement adoptée.

Who would doubt Lagrange and retrace the paths that led him to the *Mécanique*? Or rather, who would doubt the path that would take him back to the elaboration of the *Principia*? And why would anyone do it, if a new and practical formalism emerged with Lagrange? It may be that Lagrange did not follow the correct path without realising it when he wrote *Mécanique*.

FOURTH HYPOTHESIS: ERNST MACH

With the publication, in 1788, of *Mécanique Analytique*, scientists practically forgot the knowledge and principles previously produced, adopting the work as the final product of mechanics. What Lagrange does is to defend the idea that developments after Newton were purely mathematical. Ernst Mach, a physicist and philosopher who formed ideas in the 19th century, and other historians used those pages as references for their historical reconstructions.

Mach became known for encouraging the teaching of the history of physics, besides promoting science. From 1887 on, he started to publish Physics textbooks for schools, and easily accessible materials, soon translated into German, Italian, and Russian, and quickly disseminated (Hiebert, 1970). He had a very strong influence throughout Europe in the 19th and 20th centuries.

In his work *The Science of Mechanics*, issued first in 1883, Mach stated that:

> The merits of Newton concerning our subject were twofold. First, he significantly extended the range of mechanical physics by his Discovery of universal gravitation. Second, he completed the formal enunciation of the mechanical principles now generally accepted. Since his time, no essentially new principle has been stated. All that has been accomplished in mechanics since his day has been a deductive, formal, and mathematical development of mechanics on the basis of Newton's laws.

We must disagree with Mach about this citation, based on Sitko's (2019a, 2019b) discussion and what we have already exposed in this work. For a long time, the fundamental laws of dynamics were unquestionably credited to Newton. Until the beginning of the 20th century, neither historians of science nor scientists were concerned with Mach's ideas, whether they were correct or distorting events that occurred in the 17th to 19th centuries. In this way, they agreed with his ideas without any questions, just a summary of the history and the technical conceptual content for teaching. However, the mechanics that is taught today in the classroom is not Newton's primitive one, but the one developed by Bernoullis, Euler and others. From this historical and conceptual confusion, some historians started to question the debatable bases of this conception that was strongly defended by Mach. Among them, one of the main names is Clifford Truesdell.

Truesdell (and the authors of this paper) wonders where this mechanics between Newton and Euler comes from and how it was built. To answer questions like these, Truesdell proposed the program to rediscover the rational mechanics of the Enlightenment (1960a).

Truesdell disagrees with Mach's view that Newton's mechanics is a complete system and that no new conceptual developments have occurred after his own, only those deductive and mathematical (Truesdell, 1960a). Truesdell and Hankins started the review from a Machian point of view, indicating several documented proofs of the conceptual insufficiency of mechanics in the first half of the 18th century (Maltese, 1992), as was done in Sitko (2019a). Maltese (1992) and Gaukroger (1982) also criticised Mach's position, claiming that much more effort was needed to understand mechanics at that time than mere formalisms.

However, even after all of Truesdell's exposition, not many adhered to his vision, and the difficulty was precisely in accepting that the mathematical procedure was not only formal, technical, but in a way, conceptual. Many find it difficult to accept this view even today, and it is aiming to further clarifying this historical episode that we have elaborated this work.

Mach's vision hides all the search and analysis of concepts, making his readers think of mechanics as a science that emerged from experimentation. For Truesdell, mechanics is a mathematical science (Truesdell, 1960a) of problems whose solutions need new principles and methods, which, in turn, are used in new problems, i.e., they are reduced and generalised. Thus, the idea that Newtonian methods dominated the 18th century is erroneous, according to Grattan-Guinness (1990), due to the participation of many contemporaries and successors who extended and modified his work, and others that brought parallel and alternative approaches to Newton's methods, as well as generalisations, as was the case with variational mechanics.

According to Mach, Newton's mechanics was sufficient to solve all classes of problems. However, to determine the motion of fluids, for example, whenever this type of problem was attacked, it was not through the principles of mechanics (Truesdell, 1960). From the moment that Euler achieved this conceptual advance, the law was immediately treated as obviously from Newton, since Euler was a mathematician who apparently (and unfairly was so treated) did not care about the actual physics of the problem.

In the 19th century, there was a phase of scientific utilitarianism, no longer focused on metaphysical issues. This utilitarianism changed how the foundations of scientific thought during the 17th-century scientific revolution were perceived. This is the positivist current defended by Mach, who supports that the metaphysical questions should be hidden and only the pure description of the technical content should be made.

At the end of the 19th century, Mach presented his criticisms of Newtonian work and his positivist view of science, making great use of the Principle of the Economy of Thought, which argues that laws and theories should be used to save the scientist's time (Fitas, 1998). According to this principle, a good scientific theory must be written by mathematical formulations, without any relation with the senses, with the causal explanation of the phenomena, or with nature itself (Fisette, 2009). For Mach, every general principle involves an economy of thought, and, in fact, this is the basis of science[10], which is why in *The Science of Mechanics*, Mach commented (1919, p. 467) on Lagrange's massive contribution to the Principle of the Economy, by incorporating in his work many possible concepts in a single formula.

To present his criticisms to the science and to Newton, Mach wrote in *The Science of Mechanics* on the mechanics of the 17th and 18th centuries, in which Newton's idea (already taken up by Lagrange) appeared again as the one which concluded the conceptual developments of the subject. This view has been carried in the physics manuals until today and is at the heart of our discussion: a historical-philosophical approach to the subject would not make the study of mechanics more understandable and with a more motivating character, from the construction tracking of that content?

The most significant criticism of Newton's work actually came from Mach, who was possibly the first to create a science education journal, which also surprisingly advocated teaching with a historical approach. In his work, Mach retraces some of Newton's definitions, as he did not accept his concepts of absolute space and time, for example. To prove the strong influence of Mach in teaching, these Machian reformulations are still in the textbooks today, as

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[10] For a more in-depth look at the topic, see Mach (1919, p. 481). In fact, the entire work is marked out by the use of this principle.

shown by Assis and Zylbersztajn (2001), who analysed five important textbooks on physics and realised this influence in the Newtonian mechanics presented. The authors also show that the textbook authors do not realise that they are under Mach's influence.

Thus, Assis and Zylbersztajn believe that the interpretation brought in the books about inertial referential -being adopted as the set of stars in the sky (which is the vision introduced in the manuals by Mach)- as a Newtonian interpretation, owes to a lack of historical knowledge, since this referential was thus determined only with Mach, and not with Newton. This framework was defined by Mach from Newton's bucket experiment. Newton wanted to show that it is the referential of the stars in the sky that causes water to change shape inside the bucket, however, it came to results that made him believe that there was no influence from the stars, but from the bucket with absolute space. Mach criticised Newton on this issue and defined that the set of stars caused water to change shape, thus establishing this standard of inertial reference (Gardelli, 1999). Just like what happens with these definitions, the same can be considered valid for the construction reassembled by Mach (and others) that $F = ma$ is Newton's Second Law, which is presented in a totally decontextualised way and being reduced to the constructions of Newton.

Maltese also believed that Mach was responsible for spreading the idea that Newton built all mechanics, even among physicists. For him, perhaps Newton's great Machian defence comes from the compatibility of his positivist view with Newton's by not caring about the causal explanation of the phenomena. In other words, what happened was a "substantial" evolution of mechanics.

In this work, based on these four main arguments presented, we seek to expose the view defended by Truesdell and his followers - the authors of this work included - that Newton's second law used today is, in fact, the product of a conceptual construction that lasted about sixty years and ended with Euler's elaboration of the fundamental principle of mechanics.

THOMAS KUHN AND NORMAL SCIENCE FROM EMERGENCE OF PARADIGMS

For Thomas Kuhn, the manufacture of scientific knowledge takes place through *paradigms* (Kuhn, 1996) shared among communities of researchers. These researchers employ paradigms to get answers to their research problems.

A successful paradigm develops itself within what Kuhn calls *normal science*: a phase of a scientific discipline in which scientists work based on rules and principles shared by all members of the community. These rules and principles are general directions about how reality must be understood, and they are collectively understood as being a "paradigm." In the history of science, it is possible to find several instances of paradigms. In Lavoisier's chemistry, the paradigm signalled that the phenomena of transformation of matter should be quantitatively managed, instead of through the qualities of matter. In contrast, in Darwin's evolutionism, it was understood that nature was unfriendly. Organisms were involved in a struggle for existence, and therefore there was not any sense working from a harmonious adjustment between organism and nature. In Newtonian mechanics, the paradigm exhibited both Heaven and Earth motions as obeying the same laws and showed that keeping the universe at work did not need God anymore.

It is imperative to notice that those paradigms are both promoting and restricting ways of doing science. A paradigm foster research, since it indicates which methods, instruments, and entities should be considered for solving problems within normal science. On the other hand, although for the same reason, it narrows research, as it does not allow any methods, instruments, and entities to be used (for the resolution of the problems). Besides, even what ought to count as a problem to be solved needs to be legitimised by the paradigm (this point will be resumed afterwards).

So, we can say that a paradigm provides general guidance for research in a field, and such general guidance provides a model for some specific research. That research, in turn, will strengthen the paradigm. Such strengthening is a meaningful element of the scientific activity, since the paradigm is not a finished achievement but a structure that will be strengthened through specific research.

> The success of a paradigm […] is at the start largely a promise of success […]. Normal science consists in the actualisation of that promise, an actualisation achieved by extending the knowledge of those facts that the paradigm displays as particularly revealing, by increasing the extent of the match between those facts and the paradigm's predictions, and by further articulation of the paradigm itself. (Kuhn, 1996, p. 23-24)

The "actualisation of that promise" is the scientific work carried out in normal science. In doing that, scientists seek to:

1) appoint more accurately the facts indicated as meaningful by the paradigms themselves, such as the determination of the place of stars and period of eclipses, accelerations of falling of planets, and resistivity of substances, boiling points and acidity of solutions, the building of synchrotrons and radio telescopes, and so on;

2) contrast the phenomena, if possible, with the predictions of a paradigm;

3) articulate more precisely the paradigm itself and thereby determine, in the same way, for example, the scientific meaning of some physical constants.

Putting some details (about how this work takes place) aside, what Kuhn seeks to call attention to is that all the output from scientific investigations (such as those mentioned above) would not be possible without the direction given by a paradigm. The search, for example, for a gravitational constant, has no meaning outside the framework of the Newtonian paradigm:

> Other examples of the same sort of continuing work would include determinations of the astronomical unit, Avogadro's number, Joule's coefficient, the electronic charge, and so on. Few of these elaborate efforts would have been conceived, and none would have been carried out without a paradigm theory to define the problem and to guarantee the existence of a stable solution. (Kuhn, 1996, p. 28)

A significant point at Kuhn's argument concerns the concept of "scientific novelty". The paradigm advises which part of reality is open for the scientist careful investigation; in doing so successfully, the scientist develops the paradigm and widely improves it, contributing both to the development of the paradigm itself and the development of its specific research.

> By focusing on a small range of relatively esoteric problems, the paradigm forces scientists to investigate some part of nature in a detail and depth that would otherwise be unimaginable. (Kuhn, 1996, p. 24)

It is in that way that we must understand the concept of a scientific novelty. The Newtonian mechanics paradigm introduces itself as a great and amazing scientific novelty (among other reasons, for being a great counterpoint to Aristotle's scientific worldview); however, achievements like Euler's should also be considered new. The problem, however, from Kuhn's theoretical framework, is that the two novelties are conceptually different from each other. While Newton's is a general achievement (because it is a general direction for mechanics), Euler's work (regarding the second law) is specific (and so, Euler's novelty was close to a new formulation of the second law of motion). Thus, according to Kuhn, Euler's performance (and the novelty introduced by him as well) only makes sense if understood as a part of the development of the Newtonian paradigm as a whole.

THE LAW BELONGS TO NEWTON: IT IS A CONSEQUENCE OF NEWTONIAN PARADIGM

Accepting $F = ma$ as proposed by Euler and accepting the four explanatory hypotheses for the obliviousness of Euler's name in this historical episode, it is possible to consider that Newton changed the mechanics' paradigm in the 17th century by suggesting and determining that Earth and Heaven motions would take place because of the same laws, thus, breaking with the Aristotelian paradigm still current at that age.

By employing this Kuhnian perspective about paradigms, we can accept that Newton established a new paradigm. Thus, the *Principia* set the beginning of the autonomy of science: henceforth, science became self-sufficient and no longer subordinated to religion. As Cunningham (1991, p. 380) argues, the *Principia* were designed, due to their author's beliefs, as a transformation of the natural philosophy and not as a transformation from natural philosophy to modern science. For Cunningham, there was still a lack of further developments, which would arrive with Euler, in the 18th century.

Verlet says (1996) that although Newtonian mechanics has been reinterpreted and expanded along many centuries, it was the background for the emergence of theories such as quantum mechanics and general relativity in the 20th century, i.e., the Newtonian paradigm is linked with all further developments on classical mechanics. When we talk about grounds for other studies, we are not talking about conceptual contents but how problems started to be tackled. From the Newtonian work, a change of thought was established, and this means the establishment of a new paradigm.

Many scientists accepted the Newtonian paradigm as a whole, but there was a need for more applications. Thus, today, a physics student needs to know (in dynamics) something that goes far beyond what was developed by Newton. Newton developed his studies focusing on the problem of Heaven mechanics and did not make clear how to use the paradigm for other kinds of motions[11]. Earth problems were then dealt with by other scientists like Bernoulli, among others, in a different way than Newton did (Kuhn, 1996). These approaches were part of a more general theory, which Euler would later unify. These achievements compose what Kuhn calls normal science.

On the theoretical side, Newton had some minor theoretical problems, such as having to consider bodies as material points, ignoring effects such as air resistance, but, even so, this had brought to an approximation, albeit limited, between Newtonian theory and experience. The doubts about his work did not concern experience and observation but theoretical problems. In the 18th century, many scientists, including Euler, were interested in "*improving the match between Newton's paradigm and observation of the heavens*" (Kuhn, 1996, p. 32) from the development of new mathematical manipulation techniques[12], which was far beyond what Newton had thought. Starting from this perspective, one can then consider that Euler worked in what Kuhn calls normal science, in making a more esoteric science, which involved placing the mechanics governed by the paradigm on a fundamental, simple, and general basis. Even if Euler's work represents conceptual advances of what Newton did, the paradigm was shared.

Therefore, we could say that Euler worked within the paradigm at the period of normal science, acting to articulate theory and paradigm, resolving ambiguities, expanding the scope of mechanics, exchanging mathematical and conceptual principles. Regarding the theoretical problems left by Newton, Euler both solved and clarified them and expanded his range of coverage, finally finding the determination of the second law. Once again, as Kuhn accounts, it's hard to separate these two kinds of work: actually, they complement each other.

Kuhn's philosophical analysis of why the law belongs to Newton and why Euler's formulation was ignored leads us to believe that the four hypotheses in this paper support Newton as the author of the law, as too many scientists and the general public noticed the power of the Newtonian paradigm. Euler is considered an articulator of the theory and the Newtonian paradigm: since Newton established the paradigm, what the hypotheses made us wrongly believe is that, after him, nothing else was developed.

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[11] Kuhn argues that the paradigm does not need to explain everything, but a class of problems in particular, and be the best alternative among competitors.

[12] And as Sitko (2020) shows, based on conceptual developments.

Newtonianism was not just a new way of analysing, calculating, and perceiving mechanical problems, but a new culture, a new way of thinking. Newton had so much power in many aspects[13], and this explains why he had so many followers. Those followers, both experts in mechanics, such as Varignon and MacLaurin, and the general public (who were led by the reinterpretation from the dissemination of Newton's work), saw the paradigm as Newton's. For them, since Newton created the structure, the law ought to belong to him. For them, Newton revolutionised the way of thinking mechanics, besides unifying the Heaven and Earth motions. Any later work that was related to his writings would be seen as a translation of formalisms due to this support that Newton received from many people.

As argued in the previous section, the more a science is developed, the more esoteric it becomes, and the less understandable it becomes to those who do not share the paradigm. And that was exactly what happened with *Principia*, which needed to reach the public. The Jesuit Edition of *Principia* was a work that translated the Newtonian writings into more analytical and understandable, and for that, several explanatory footnotes were added. As it was intended to be disseminated, most readers were Newton's followers, i.e., ordinary people. Thus, by reading a more modern formulation in the JE, they may have imagined that Newton would have written all by himself. And even if they did not think it was Newton's, they supposed it could just be a translation of what he had already done. As this formulation in the JE is very similar to Euler's (even though it was conceptually different from the one proposed by Euler), it is possible that the law was understood as an obvious Newton's construction because of the prevalence of the Newtonian paradigm, which may have led the readers to see Newton on a different level from his successors.

After Euler proposed his new principle, Lagrange wrote a treatise summarising and closing the subject in mechanics. The problem is that Lagrange also shared the Newtonian paradigm and understood it as if all post-Newton development were just mathematical and non-conceptual. And that is why Euler was erased, because the Lagrangian idea is that there is a current paradigm, and it is Newtonian.

However, the "Newtonian mechanics" that we know was developed throughout the 18th century due to conflicts in Newton's mechanics that were substantially modified by Euler and others (Pulte, 2001). We also cannot say that one paradigm was replaced by another, since there was not a point in which

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[13] For developments, see Westfall (1995).

Newton was no longer worth and Euler started to be worth; matter of fact, they are complementary. What happened? Euler worked on expanding the paradigm proposed by Newton.

Finally, Mach's defence of Newton as the only creator of mechanics and later normal science with purely mathematical developments reveals a Kuhnian stance, perceiving Newton as the author of the "revolution" in mechanics and as the author of a new paradigm for that area. Indeed, Mach did not see any obstacle between Newton and Mach's own era, thus bringing a generally linear history. Of course, Mach criticised and re-established some Newtonian concepts, but he was still an advocate that all problems could be solved using the general Newtonian laws.

However, from the analysis of the historical materials and letters of the 18th century, we noticed many principles used in different situations until the general and condensed principles were finally found (by Euler), which is completely different from the image of completeness that pervades the textbooks. In hindsight, relying on Machian ideas, it is possible to see one status before Newton, and another after him. But as we approach the episode, what we see is something totally different, blurred, mixed. To keep the episode clean and linear, Mach, Lagrange, the JE, Newtonianism, purposefully or not, put Newton as the only producer of the general principles of motion, and hid everybody else, such as Euler, d'Alembert, Bernoulli, etc.

Thus, Kuhn is brought into this analysis to explain why it is so natural that these hypotheses consider the Newtonian paradigm as the ultimate production in classical mechanics, and consequently, the second law of motion as belonging to Newton. Given his success, it is understandable why the law is considered to still belong to Newton. We must make it clear that we are not referring only to the content of the law, and whether it is the production of Newton, Euler, others, or it is a joint construction: we are analysing and arguing that, from what the history of science brings to us, assigning the law to Newton is plausible, based on the content of the four hypotheses, and from the Kuhnian analysis.

The second law is a construction that involves much more elements and scientists than Newton and his *Principia*, which has been shown in other papers. There were much more developments between 1687 and 1776 so that the second law of motion could emerge. However, the law is "Newtonian," as the textbooks affirm, and, besides that, the history of science was built this way, because the four hypotheses made it that way. It is not a matter of perspective or interpretation: in fact, the history was built like that.

CONCLUDING REMARKS

One of the most relevant philosophical discussions in the field of science education concerns the nature of science, and it is in authors such as Norman Lederman, Douglas Allchin, and Michael Matthews, among others. There is a wide agreement about the idea that an analysis on teaching needs to consider the most relevant aspects of science (the Nature of Science).

One of the issues highlighted by the authors who work to clarify the Nature of Science is the production of scientific knowledge (Matthews, 1994). It is also a consensus that knowledge is not the work of an individual at any given moment but a construction that involves several actors[14]. In the examined case, it is clear that there is a transition between Newton's original statement, its modification by Euler, and the later acceptance of Euler's statement without, however, mentioning him. Thus, $F = ma$ does not belong to Newton alone. From this, we could, at first, understand the question of assignment of authorship of the second law as a question of historical injustice, since Euler's name was erased from its construction. However, things are not so simple.

As we have already seen in the presentation of the four historical hypotheses about Euler's name's erasure, according to the Kuhnian point of view, Newtonian achievement was something unprecedented in the history of science; but, putting any laudatory rating aside, Newton's work asserted itself as a directive structure for research in mechanics, a structure that paved the way for new contributions (such as Euler's) within the same paradigm. This paradigmatic structure was not a solution for all problems, and many of its inaccuracies and imperfections should (as they were, in many ways) be corrected in the future.

One way to understand the constructive nature of scientific achievements (which are therefore not isolated) can be through changes of statements, such as the formulation of Newton's second law for Euler's formulation. If the Kuhnian philosophical conceptual framework is employed, it is possible to understand why Euler was ignored, despite his remarkable scientific contribution: this omission would be the result of a conception of the nature of science that indicates that paradigmatic structures such as the

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[14] Sitko (2020) presents an example of the characteristics of the work of building science in the episode in question.

mechanics of Newton are units that carry a meaning that differs from the meaning of achievements like Euler's.

Then, we can understand why a physics teacher, in his teaching of Newton's laws of motion, *either* omits *or* (which would be more reasonable, especially if the teaching of the laws of motion is historiographically oriented) reference Euler as a scientist who operated within a paradigm already working. Euler did not propose a paradigmatic novelty, but rather a notational novelty as well as an improvement (especially pedagogical) of the Newtonian paradigm (and that is why we stated above that the second law belongs to Newton).

Indeed, we watch out for the science teaching literature, which informs us about, for example, the misconception of the individualistic nature of scientific production (Gil-Pérez, 2001; Allchin, 2013; Bejarano, Aduriz-Bravo & Bonfim, 2019, Sitko, 2020; among many others). Like any other human endeavour, science has, in essence, communal, and collective nature. However, science teaching (historiographically embraced) does not aim to catalogue all members of a scientific construction/discovery. Of course, in Euler's case, the omission, at least, would be inadequate. However, more important than mentioning, in this case, Euler, is to *clarify* the nature of his contribution.

So, worse than omitting Euler in a class about the second law is to overlook that he worked within a paradigm - the Newtonian paradigm. One thing is to say that the formula we currently use is due to Euler's tireless efforts; another, quite different from the former, is to inform the students that Euler's name could have been neglected by some unfairness by the history of science.

Euler cannot be erased from the (historiographically oriented) teaching of the second law. However, what matters is how his name will be inserted. Repairing this *personal* and *historical* injustice cannot be done by creating another one: omitting the fundamental fact that without Newton's pioneering effort – i.e., without Newton's paradigm – Euler may have never developed the second law.

Even more important for science teaching is the image of science that we can retrieve from this episode: science is a collective enterprise, and this very enterprise has, so to speak, formulators of general principles (of paradigms) (like Newton) and developers of these paradigms (like Euler).

In this paper, we restrict ourselves to introduce a plausible explanation (from the historical and philosophical point of view) of why the name is "Newton's second law." However, even understanding why this occurred, Euler's merit should not be omitted, and so, apparently, the most appropriate, from a historical point of view, would have been to call it the "Newton-Euler second law," because we are not dealing with a discreet phenomenon[15], a single scientist doing infallible and complete science (Sitko, 2020), but a continuous one, which endured about sixty years to result in $F = ma$.

In an approach about physics content that would consider the historical process of the construction of the second law of motion, Euler's name should certainly be referenced (along with, of course, his contributions). However, such reference should be made for the students to understand that Euler's work took part of a larger whole (Newtonian paradigm) and that all of this took place within a science that is collectively constructed (Newton, Euler, and so forth).

As pointed out by Matthews (2015, p. 136): "*The pedagogical task is to produce a simplified history that illuminates the subject matter and promotes student interest in it, yet is not a caricature of the historical events*".

It is also worth mentioning that, if it is an overstatement to introduce historically the second law as being exclusively Newton's, it would also be so to call it only Euler's. Furthermore, it would also be a distortion not to qualify their contribution as being, from a philosophical point of view, categorically distinct: Newton's contribution is of a different kind from Euler's. Newton built a paradigm. Euler strengthened it.

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AUTHORSHIP CONTRIBUTION STATEMENT

C. M. S. developed the historical reconstruction of the episode, researched, listed, analysed the four hypotheses mentioned, and developed the Kuhnian analysis. M.R.S. offered theoretical contributions regarding Kuhn, developed Kuhnian analysis, guided and supervised the project. Both authors jointly discussed the issues addressed in the article and contributed to the final version of the manuscript.

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[15] This phenomenon can also be called an idea, scientist, as the reader prefers.

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