Textbook: an Analysis in the Light of the Ontosemiotic Focus on Rational Numbers

Patrícia Pujol Goulart Carpes [a]
Eleni Bisognin [b]

[a]Universidade Federal do Pampa, Campus Itaqui, Itaqui, RS, Brasil.
[b]Universidade Franciscana, Programa de Pós-Graduação em Ensino de Ciências e Matemática, Santa Maria, RS, Brasil.

ABSTRACT

Background: Many teachers consider the textbook the primary guide for the curriculum materialisation. This work analyses the content of the rational numbers in a textbook used in the 7th-grade of elementary school. For this analysis, the theoretical and methodological tools of the Ontosemiotic Approach to Knowledge and Mathematical Instruction (OSA) are used. **Objective:** Our goal was to understand the level of didactic suitability of the instruction process in the textbook. **Design and setting:** Thus, we analysed a section of the textbook about rational numbers using the categories described by OSA. Concepts, procedures, problem situations, definitions and arguments used by the authors were also analysed. **Results**: From the results obtained, we could infer that there are examples of the concepts worked in class, but without the proper definitions and arguments, hindering generalisation. The analysis also allowed us to highlight didactic-mathematical knowledge that can guide the teacher concerning the textbooks' possibilities and limitations, to achieve more didactic suitability in the process of teaching and learning rational numbers. **Conclusions:** the book under study should not be considered as planning for an instructional process to meet the current curriculum guidelines.

Keywords: Textbook, Rational numbers, Onto-semiotic approach.

Livro didático: uma Análise à Luz do Enfoque Ontossemiótico sobre Números Racionais

RESUMO

Contexto: O livro didático é considerado por muitos professores como o principal guia para materialização do currículo. Devido a sua importância, nesse trabalho é feita uma análise de um livro didático do 7º ano do Ensino Fundamental sobre o conteúdo de números racionais. Para essa análise são empregadas as ferramentas teóricas e metodológicas do Enfoque Ontossemiótico do Conhecimento e da Instrução Matemática (EOS). **Objetivo**: o estudo visa verificar o grau de idoneidade didática do processo de instrução utilizado no livro texto. **Metodologia**: Desse modo, foi analisada a sessão do livro sobre números racionais utilizando-se as categorias descritas no EOS. Foram analisados os conceitos, procedimentos, situações-problema, definições e argumentações

Corresponding author: Patrícia Pujol Goulart Carpes. Email: patriciacarpes@unipampa.edu.br

utilizadas pelos autores. **Resultados**: Pode-se inferir dos resultados obtidos, que são apresentados exemplos sobre os conceitos trabalhados, porém sem as devidas definições ou argumentações, prejudicando desse modo a generalização. A análise permite também destacar conhecimentos didático-matemáticos que podem orientar o professor quanto às possibilidades e limitações do livro didático a fim de alcançar uma maior idoneidade didática no processo de ensino e aprendizagem dos números racionais. **Conclusão**: o livro em estudo não deve ser considerado como um planejamento de um processo instrucional para atender as orientações curriculares em vigência.

Palavras-chaves: Livro didático, Números racionais, Enfoque Ontossemiótico.

INTRODUCTION

Textbooks are widely adopted by teachers to plan and develop mathematical knowledge in the classroom environment. Those resources have guided instructional processes, and it is increasingly pertinent that research analyses which and how mathematical objects are mobilised so that teachers can obtain a suitable process. In other words, whether the cognitive (personal) knowledge of the students is in balance with the institutional knowledge intended or used by the teacher, given the context and didactic resources available.

This study proposes the analysis of a textbook approved by the PNLD 2017, used with the 7th grade of elementary school by teachers of the Municipal Education Network of a municipality in the country of the state Rio Grande do Sul, Brazil. The topic is rational numbers, and the analysis is based on the perspective of the Ontosemiotic Approach of Mathematical Knowledge and Instruction, OSA, developed by Godino and collaborators (Godino et al., 2007; Godino, 2017). The OSA is a system of conceptual and methodological tools that subsidise research in the field of Mathematics Education, that can be applied to the analysis of a specific process, in the development of a teaching sequence, or of partial aspects of a study process, such as some didactic material (Godino et al., 2007).

The section of the textbook analysed is taken as a sequence of mathematical and didactic practices of an instructional process developed by the authors. In this sense, we examined the definitions, examples, procedures, and problems proposed for the mobilisation of mathematical knowledge, as well as the previous knowledge necessary for the development of the activities.

Given the intended knowledge, the didactic analysis should guide the teacher as to the possibility of semiotic conflicts or erroneous conceptions that may appear during the instructional process adopted. From the OSA perspective, semiotic conflicts refer to

> [...] any disparity or mismatch between the meanings attributed to the same expression by two subjects (people or institutions) in communicative interaction and can explain the difficulties and limitations in the teaching and learning process implemented. (Godino, 2002, p. 246)

Hence, the textbook can serve as a guide to the teacher to identify potential difficulties and predict solutions.

The theoretical framework that will guide the analysis of the results, the organisation of the didactic configurations of the textbook, and, finally, the final considerations and references adopted are presented below.

THEORETICAL BACKGROUND

The OSA is a modular system that has different tools to describe, analyse and assess an instructional process. In this case, the textbook is considered as a planned instructional process; therefore, we can apply several theoretical notions for its analysis, namely: the onto-semiotic configurations, the didactic configurations, the surrounding norms, and the didactic suitability of the OSA.

The onto-semiotic configurations start from an anthropological formulation of the mathematical object that takes into account Mathematics as a problem-solving activity, a symbolic language and a logically organised system. Thus, the mathematical object is every entity that participates in the *semiosis* process, such as interpretation and language play, concepts, propositions, procedures, and arguments (Godino et al., 2007).

In this sense, the onto-semiotic configurations guide the identification of epistemic conflicts related to the meanings and institutional objects that come from the lesson and cognitive conflicts related to previous knowledge required in the development of the section. They also consider the various meanings of mathematical objects and the identification of objects and processes that emerge from the required mathematical practices (Burgos et al., 2019).

The didactic configurations make a microanalysis of the process, i.e., they provide criteria to decompose the instruction process into units of analysis, which allow for the study of a problem situation or a definition and for the link between the types of knowledge involved, the resources and actions that the teacher and the students perform.

The normative tool is conditioned to the surroundings of the teaching and learning process, i.e., whether the instructional process is as stated in the curriculum, and beliefs and behaviours that can influence the targeted learning.

The didactic suitability, according to Burgos et al. (2019), helps formulate the problem of didactic analysis of books in terms of characterising the suitability of the didactic trajectories proposed and identifying possible changes to improve student learning. The epistemic, cognitive, ecological, affective, mediational, and interactional dimensions, which are part of the didactic suitability, and their coherent and systemic articulations, allow us to evaluate a suitable instructional process, so we can adapt the personal meanings achieved by the students, and the meanings intended by the teacher, according to the context and resources available (Godino, 2017).

The **epistemic** dimension refers to the degree of representativeness of the institutional meanings implemented or intended, regarding a meaning of reference;

the **cognitive** dimension refers to how close the personal meanings achieved are of the meanings intended or implemented; the **ecological** dimension refers to the degree to which the study process adjusts to the educational project of the school and society and the conditioning of the environment in which it develops; the **interactional** dimension refers to the didactic trajectories that allow us to identify the semiotic obstacles and allow us to overcome the difficulties that appear during the instruction process; the **affective** dimension refers to the interest of students in the study process, and the **mediational** dimension refers to the degree of availability and adequacy of the material and temporal resources necessary for the development of the teaching-learning process. All these dimensions can serve as a basis for the organisation of a textbook analysis guide that reflects the many variables that should be considered (Burgos et al., 2019).

The dimensions of the didactic suitability for the topic of rational numbers and the didactic configurations organised to present and analyse the instructional process on the textbook selected are detailed below.

Epistemic and ecological dimensions

The suitability of the epistemic dimension is achieved when we consider the institutional knowledge about the object of study, i.e., the degree of representativeness of the partial and holistic meanings of the rational numbers, as well as the fundamental conceptions of the object and its different records in the instructional process (Carpes, 2019).

The notion of rational number starts from the idea of a theoretical - dynamic and interactive - construct, in which we can build understanding from a simpler notion, called subconstructs, to a more precise or axiomatic form (Kieren, 1988).
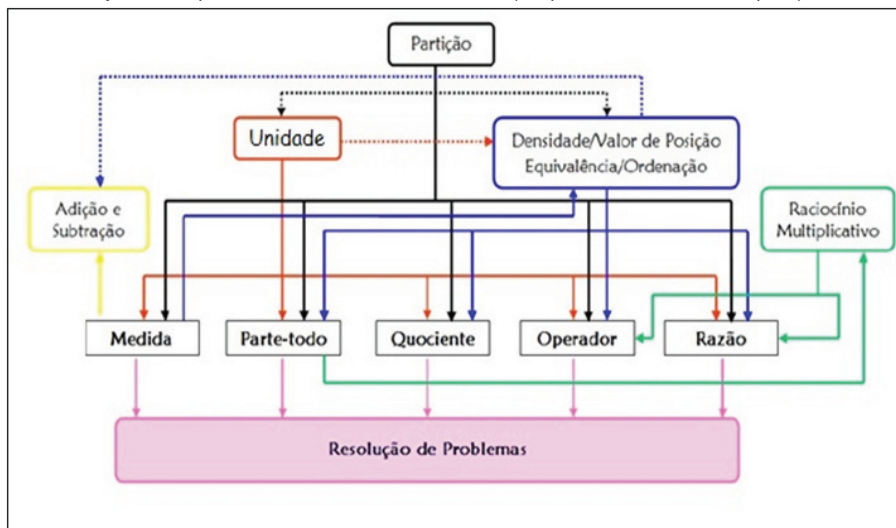
When it comes to referential knowledge, due to the various contextualisations that rational numbers permeate, their meanings are distinct. Kieren (1980) points out that the complete understanding of rationals requires not only the understanding of each of the separate meanings but how they relate. The meanings of rational numbers given by the author are part/whole, quotient, measure, operator, and ratio.

Rational numbers, according to Behr et al. (1992, p.296), "are elements of an infinite field of quotients that consists of equivalence classes and the elements of these equivalence classes are fractions." Also, according to Lamon (2007), there is a distinction between the terms: fractions and rational numbers. Fractions are possible representations of rationals.

Figure 1 illustrates the theoretical model of Behr et al. (1983), that schematises the transversality that involves understanding the rational number. The ability to solve problems involving the different meanings depends on the understanding of the fundamental concepts to help us understand the rational numbers.

The transversality model created for the understanding of the rational number indicates that

> Partition and unity are the basis for developing knowledge of all meanings of the rational numbers, as well as the notion of partition is fundamental for the development of an understanding of unity, equivalence, and ordering. The notion of equivalence/ordering/density of fractions is fundamental for students to sum and subtract fractions, and the meaning of measurement can help them. The meaning of part/whole is the basis for the multiplicative reasoning and fundamental to the understanding of reason and operator meanings. Because it is the basis for others, the meaning of part/whole is the most explored and often the only one worked in the classroom (Magina & Campos, 2008). Understanding the five meanings is the basis for the student to be able to solve problems involving rational numbers. (Carpes, 2019, p.21-22)

The fundamental concepts are explained as follows: partition, unit, equivalence, density, positional value, ordering, addition and subtraction, multiplicative reasoning and the meanings of part/whole, quotient, measure, operator, and ratio.

- **Partition and unit**: the unit that represents the whole $\frac{n}{n}$ $(n \neq 0)$ and the partition that represents the *n* equal parts $\frac{1}{n}$ of the whole.

- **Positional value**: indicates that there is a relationship between numerator and denominator; thus, the fraction must be seen as a single number (quantity).

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- **Density and ordering**: Between two fractional (or rational) numbers, there are infinite rational numbers, and they follow an increasing order.

- **Equivalence**: fractions that represent the same amount (the same quotient).

- **Sum and subtraction**: the denominator determines the size of the parts and therefore it must be the same in both fractions to operate them.

The meaning of **part/whole** presented in the form $\frac{1}{n}$ $(n \neq 0)$ in which this fraction represents one or more parts of the unit that has been divided into equal parts (Lamon, 2006)) is usually the basis, i.e., the first meaning to be explored. In this way, the fraction indicates the comparison between the numerator (number of parts taken from the unit divided) and the denominator (total number of parts in which the unit was divided).

The part/whole meaning is fundamental for the development of other meanings, in which it is possible to develop the notion of partition (in equal parts) and also the idea between parts and whole: the parts together must reconstitute the whole; the more parts the whole has, the smaller each part is; regardless of the shape, size or orientation of the parts, the relationship between the parts and the whole is preserved (Charalambous; Pitta-Pantazzi, 2006).

The meaning of **quotient** refers to the idea of sharing, in which the fraction $\frac{a}{b}$ indicates the quotient $a : b, b \neq 0.$ In this meaning, the understanding of dividend and divisor of the division operation must be clear, because dividing into equal parts is the basis for grasping the rationals as quotients (Lamon, 2006). In this way, a may be greater than, equal to, or less than b. The quotient meaning goes beyond the meaning of part/whole, since there are two variables.

The **operator** meaning is associated with the idea of modifying a continuous - both increase and decrease - quantity, considering the fraction improper or proper, respectively. An equivalent idea when the quantity is discrete (Lamon, 2006). Silva (2005) points out that this meaning enables the student to interpret the fractional multiplier in at least two ways: $\frac{2}{5}$ means 2 of $\frac{1}{5}$ of the unit) or $\frac{2}{5}$ means $\frac{1}{5}$ of (twice the unit).

The meaning of **measure** enables the student to identify the unit of measurement, determine a length, and measure a length by repeating the unit of measurement - iteration (Lamon, 2006). The author adds that from this meaning, unit and partition can also be explored through successive divisions of a unit.

The meaning of **ratio** of the rational number arises from the relationship of two quantities, requiring multiplicative reasoning. Lamon (2006) states that a distinction must be made between the notion of part/part ratio (two parts of a whole "ratio") and the ratio of quantities of different types ("rate"), giving rise to a new magnitude. Silva (2005) argues that the meaning of ratio does not allow associating the idea of partition with the other meanings, but the idea of comparison between two magnitudes.

Given the exposure of the meanings of rational numbers, we perceive the diversity and complexity of the ideas involved and the different contextualisations. Besides the

understanding of each meaning, it is necessary to observe and explore the connections of the meanings. Kieren (1988) considers it necessary to grasp each meaning and its connections for a complete understanding of rational numbers.

Besides the meanings of the rationals, their language (representation) is also necessary to understand the rational numbers, since they are not only fractions or decimals. Still, the interpretation of a situation may be more uncomplicated for an individual in one representation than in another (Carpes, 2019).

For the ecological dimension that includes the surrounding conditions, we took as reference the curriculum proposed by the National Common Curricular Base, BNCC (Brasil, 2017), which is the normative document that defines the organic and progressive set of essential learning that all students must develop throughout the stages and modalities of basic education.

The BNCC proposes five thematic mathematics units: numbers, algebra, geometry, magnitudes and measures, and statistics and probability. Focusing on the rational numbers that are in the thematic unit "numbers," Table 1 presents the objects of knowledge and objectives outlined in the BNCC for the 7th grade of elementary school.

Table 1
Development of objects of knowledge and objectives for rational numbers (Brasil, 2017, p. 304-305)

7th grade - Objects of knowledge	Objectives
• Fraction and its meanings: as part of integers, result of division, ratio, and operator. • Rational numbers in fractional and decimal representation: uses, ordering and association with points of the numerical line and operations.	Solving the same problem using different algorithms. Recognizing that resolutions of a group of problems that have the same structure can be obtained using the same procedures. Representing through a flowchart the steps used to solve a group of problems. Comparing and sorting fractions associated with ideas of parts of integers, result of division, ratio, and operator. Using in problem solving the association between ratio and fraction, such as fraction 2/3 to express the ratio of two parts of a quantity to three parts of the same quantity or three parts of another quantity. Comparing and ordering rational numbers in different contexts, associating them with points of the numerical line. Understanding and using the multiplication and division of rational numbers, the relationship with each other, and their operative properties. Solving and elaborating problems involving operations with rational numbers.

In this study, the BNCC was taken as an ecological contribution, hoping to guide the teacher on the potentials and limitations of the instructional process analysed to promote more excellent didactic suitability of the teaching and learning process. However, it is noteworthy that the textbook precedes the BNCC. Therefore, this study proposes to identify limitations and provide adjustments to the instructional process.

Instructional dimension

The instructional dimension (union of mediational and interactional) refers to the adequacy and relevance of didactic resources that favour students' learning and help them overcome possible misconceptions.

From this perspective, several authors consider that in the study of rational numbers, one meaning should neither prevail over another (Campos & Magina, 2008; Silva, 2005; Lamon, 2007) nor should we need to move between meanings for a complete understanding of this numerical set (Kieren, 1875; 1988).

Also, the early use of rules and algorithms to study fundamental concepts, such as comparison, equivalent fraction, and operations makes the study of fractions a mere double count (Silva, 2005).

There are didactic resources (concrete and digital materials) that can motivate and facilitate the understanding of fundamental meanings and concepts. However, the teacher's referrals and questions are necessary to elucidate and mobilise the knowledge intended (Carpes, 2019).

PROBLEM AND METHOD

The textbook analysed is entitled *Vontade de Saber* (Will to Know), 7th grade, by Souza and Pataro (2015), approved by the PNLD of 2017 and used by elementary school teachers in schools in a municipality in the countryside of Rio Grande do Sul. The analysis focused on the section "Frações" (Fractions) because there was not a chapter entirely dedicated to rational numbers. The sections were presented separately as fractions and decimal numbers.

Considering this textbook was adopted as a resource for the development of the teaching and learning process on rational numbers, and considering the didactic-mathematic knowledge necessary for the process, this study seeks to answer the following guiding question: What is the degree of didactic suitability of the instruction process used in the textbook for the mathematical object rational numbers?

This paper analyses the different problem situations presented in the textbook, the concepts, definitions, representations, and arguments used. This analysis is based on the various dimensions of didactic suitability described in the theoretical framework.

Overview of the Fractions section

The book we are discussing here does not address rational numbers in all their meanings. For this purpose, it presents two sections, one on fractions and the other on decimal numbers, besides a section on integers.

The first section, "Frações", has 29 pages intended for the understanding of the rational number and the representation of fractions. The subsection "Estudando Frações" (Studying Fractions) presents the fraction under its different meanings as part/whole, ratio, and quotient, reading fractions and definition of a proper, improper, and apparent fraction. Subsection "Simplificando Frações" (Simplifying Fractions) addresses equivalent fractions and irreducible fraction. The subsection "Comparação de Frações" (Comparing Fractions) brings the representation of fractions with figural register, the comparison of fractions that have the same denominator and the algorithm of the least common multiple (LCM). It also addresses the operations of addition, subtraction, multiplication, division, potentiation, and square root. Finally, it proposes a review, and brings problems and questions from the OBMEP (Brazilian Mathematics Olympiad of the Public Schools) and ENEM (Brazilian National High School Assessment).

The introductory activity of the section addresses the conscious use of the water available on the planet for consumption. The task proposes the fractional register of the amount of fresh water available on the earth and can be motivating, since it is a recurring theme in the students' daily lives.

Below, we present the units of analysis using the dimensions of the didactic suitability of the OSA.

ANALYSIS OF EPISTEMIC AND COGNITIVE DIMENSIONS

This section discusses the eight didactic configurations organised. Each one describes the intervening mathematical practices and objects.

Configuration 1: the meaning of part/whole of the fraction

The authors begin by pointing out that the unit is complementary to the previous grades, as shown in Figure 2. First, the present three meanings of fractions: part/whole, ratio, and quotient. They represent the meaning of part/whole through a rectangular figure (continuous quantity) and, in the sequence, they indicate the relationship between numerator (number of parts considered) and denominator (number of equal parts of the unit) of a fraction.

Figure 2
Meaning of part/whole of the fraction. (Souza & Pataro, 2015, p.14)

The concepts implicit in this situation are the fundamental concepts of partition and unit (the continuous magnitude divided into equal parts to represent the whole). At this time, a formal definition of the meaning of part/whole is not shown, only numerical examples. The records used are the natural language and the numerical and geometric fraction. The interpretation of numerator and denominator leads to a generalisation of how to represent a fraction.

The definition of the meaning of part/whole was not presented or constructed, nor was it explored through a problem-situation, in a contextualized way. Only one double entry count was described (the area shaded by the total area). The little scrutiny of the meaning of the concept can generate an epistemic conflict.

We observe that there may also be cognitive conflicts if the students do not understand partition, unit, part/whole, continuous, and discrete quantities. The mechanics of double-counting can confuse the student as to the other meanings of the fraction (Silva, 2005; Campos, Magina & Nunes, 2006).

Configuration 2: the meaning of fraction ratio

The meaning of ratio is explored through contextualisation, resuming the idea of the relationship between two quantities (approved people and total people), and the authors present the different readings and interpretations for the ratio given as an example, as shown in Figure 3.

Figure 3
The meaning of fraction ratio. (Souza & Pataro, 2015, p.14)

> • Razão
>
> Em um concurso, 7 de um total de 95 pessoas foram aprovadas. Podemos repre-
> sentar a quantidade de pessoas aprovadas pela fração $\frac{7}{95}$, isto é, 7 das 95 pessoas
> inscritas foram aprovadas ou, ainda, a quantidade de pessoas aprovadas nesse con-
> curso está na razão de 7 para 95.

In this situation, the fraction is presented as the relationship between two distinct quantities rather than a part of a whole. Generalising, ratio 7 to 95 could also be understood as 14 to 190. However, the authors do not present this approach.

Lamon (2006) points out that the interpretation of the ratio as part/part (two parts of a whole "ratio") and the ratio of quantities of different types ("rate"), giving rise to a new magnitude, can generate an epistemic conflict. In the "Proporcionalidade" (Proportionality) section of the book, there is an example of a situation of a ratio of 1 gram of chlorine to 100 litres of water (ratio between two distinct quantities and indicated as 1:100).

Configuration 3: the meaning of fraction quotient

The meaning of the quotient of a fraction is explored numerically, i.e., the numerator divided by the denominator, and uses the distinct division symbols (the bar and the colon), as shown in Figure 4.

Figure 4

The meaning of fraction quotient. (Souza & Pataro, 2015, p.14)

> • Quociente de uma divisão
>
> A fração $\frac{12}{4}$, por exemplo, representa 3 inteiros, ou seja, $\frac{12}{4} = 12 : 4 = 3$.
> Podemos escrever uma fração na forma de divisão e vice-versa. O traço da fração
> representa uma divisão.

In the situation proposed, the authors do not make any remarks about the elements of division being rational numbers, although students had not studied irrational numbers until then. Also, the authors do not highlight the need for two distinct quantities (12 toffees divided equally between 4 people, for example) that differs from the meaning part/whole.

Failure to understand the division operation between whole numbers, the division into equal parts, can generate cognitive conflicts. Examples that only show fractions

whose numerator is greater than the denominator can induce other cognitive conflicts by considering only improper fractions as quotients or only generating integers as a result $(12 : 4 = 3)$.

Configuration 4: Comparison between fractions

The authors propose to compare fractions in context, questioning whether the number of students is higher in the English or Spanish course, according to the activity described in Figure 5. The first procedure adopted to determine the largest fraction is to analyse the meaning of part/whole with the representation of the unit of geometric shape. The largest fraction is determined by identifying the largest shaded area.

Picture 5
The meaning of fraction quotient. (Souza & Pataro, 2015, p.18-19)

Next, the authors present another procedure to compare fractions, through equivalent fractions, determining the least common multiple (LCM) among the denominators of the fractions. The concepts and arguments used in these procedures are: understanding the symbology of greater or lesser ($<$, $>$), why to make the denominators equal, obtaining equivalent fractions, LCM algorithm, and the concept of prime numbers.

The records used to compare fractions were numerical representation (irreducible and equivalent fraction) and geometric representation. The definition or argument of what it is to compare fractional numbers, in a general way, was not explicitly constructed. The procedure of making the denominators equal and comparing the numerators to determine the largest fraction is also not justified. Another comparison procedure could have been adopted, because the authors of the book point out at the beginning of the section the meaning of quotient; therefore, writing the fractions as quotients would also provide the desired comparison and would also determine the positional value of the fractions.

An epistemic conflict is evidenced when the authors did not observe the meaning of an operator of a fraction. The targeted comparison in the example in Figure 4 could also be interpreted as an operator when the total number of language school students was calculated. For example, if you have 36 students, $36 \cdot \frac{5}{12} = 15$ students are studying English and $36 \cdot \frac{4}{9} = 16$ students are studying Spanish. Also, this meaning is highlighted, because, in the other activities, the meaning of operator is explored in other exercises proposed, as shown in Figure 5. However, initially, this concept was not presented.

Comparing fractions and adopting the procedure of only representing fractions geometrically (part/whole) can generate a cognitive conflict for the student.

In Figure 5, if the denominator is a large number, there will be many rectangles. Besides, when comparing fractions, the unit must be the same size (area), so students would need at least one ruler to measure and be able to view the largest fraction correctly. Note that the fractions in this example in Figure 5 represent very close amounts.

Configuration 5: The meaning of an operator of a fraction – exercise

Exercise 14, shown in Figure 6, is among the activities proposed to explore the comparison of fractions. The contextualised activity suggests comparing the fractions in item (a) and operate a fraction on a quantity in item (b). It also describes the social practice of basketball game and the players' score.

Figure 6
The meaning of an operator of a fraction. (Souza & Pataro, 2015, p.19)

The register is proposed only by the numerical fraction. The exercise proposes the recognition of the meaning of part/whole or operator to compare fractions. There is still the possibility of resolution through equivalent fractions. A common, and possibly more efficient, procedure is to operate each fraction on the amount 174. However, the authors do not present this procedure at any time in the section analysed.

According to Kieren (1988), if only one of the meanings is considered, in this case, the meaning of part/whole, it can generate an epistemic conflict. Kieren also points out

that the student must move between meanings so that the concept of rational number becomes understandable.

Configuration 6: Equivalent fractions - exercise

The activity illustrated in Figure 7 presents another procedure to determine whether the fractions are equivalent. The book proposes examples in which the product of the extremes is equal to the product of the means, and a second example in which this does not occur.

Figure 7
Equivalent fractions. (Souza & Pataro, 2015, p.19)

In this activity, the concepts of proportionality ratio and product of the means by the extremes are used to justify that "two fractions are equivalent when the multiplication of the numerator of one fraction by the denominator of the other gives the same result." An epistemic conflict can be generated by affirming a condition, without arguing or defining it. In the book, the concept of ratio and the proportionality constant are presented in the section "Proporcionalidade" (Proportionality), where the procedure of multiplying the numerator of one fraction by the denominator of the other is explored, but without bringing clear arguments.

The student can have a cognitive conflict if he/she cannot understand the comparison and equivalence of fractions. What defines an equivalence - the same amount represented by different fractional numbers - is not explicit in the book. The example presented also provides an interpretation that only two fractions can be equivalent, and not the possibility of having infinite equivalent fractions.

Configuration 7: Mobilising meanings through a problem-situation

At the end of the "Frações" section, there is a proposal to solve a problem-situation, called "Resolvendo Problemas," illustrated in Figure 8. In the book, there is something similar to a script to solve the problem. The question was taken from ENEM assessment and explores the understanding of the whole and its parts without explicitly referring to a fraction.

Figure 8
Problem situation. (Souza & Pataro, 2015, p.40)

The concepts related to this activity are proportion, ratio, part/whole, decimal numbers, volume, and magnitude of measures, and a social practice, to make concrete with cement, sand and gravel. For this activity, the solving process does not involve the simple application of an algorithm; it requires interpretation. In this problem situation, different meanings of fraction are used, since the student can interpret the fraction $\frac{1}{7}$ by the meanings of part/whole, operator, or ratio, as illustrated in Figure 9.

Figure 9
Different procedures of resolution of the activity in Figure 8. (Carpes, 2019, p. 98)

The elaboration of the statement of the question proposed in Figure 8 is more complex than the other questions illustrated in Figures 6 and 7, for example. In this

activity, we highlight the contextualisation, the pertinent adequacy to the students' daily lives and the level of abstraction proposed.

Configuration 8: Comparison and ordering of fractions

Figure 10 illustrates an activity proposed by the OBMEP. The statement asks respondents to identify an integer between two given fractional numbers.

Figure 10
Comparing and ordering fractions. (Souza & Pataro, 2015, p.41)
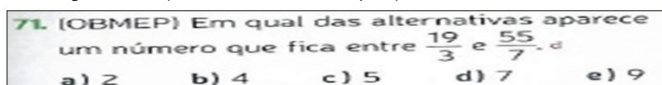
The question is objective, and in the textbook, the comparison of fractions was made by equivalent fractions; however, this procedure alone does not guarantee the solution desired. The exercise thus placed can generate epistemic conflicts, since the authors do not address the concept of the positional value, which is essential for solving the activity.

A typical procedure for solving the question is to apply the meaning of the quotient of a fraction and determine its value in decimal number. This process requires understanding the comparison of decimal numbers

ANALYSIS OF THE INSTRUCTIONAL DIMENSION

The instructional dimension represents the union of mediational and interactional dimensions. Thus, it contemplates the appropriate and pertinent use of didactic resources, the time allocated to the development of the mathematical theme/object, as well as the interactions between teacher and student, between students and teacher, and student and resources used.

The teachers widely adopt textbooks, considered as an instructional process, for the studies of mathematics topics in the classroom. The section of the book analysed initially brings a contextualised situation where rational numbers are mobilised, followed by examples that present three different meanings of fractions and fixation exercises.

In the OSA, according to (Godino et al., 2007), Mathematics is seen as a problem-solving activity, with its own symbology and language. In this sense, the instructional process does not only transmit knowledge, it is a construction process.

The exercises proposed in the section considered are fixation, i.e., repeating the process several times. Of the exercises that explored the meanings of fractions, more than half refers to the meaning of part/whole, and only one exercise on the meaning of quotient

and one of ratio. Therefore, the mobilisation of the partial meanings of the mathematical object may not be enough.

The didactic configurations analysed demonstrate an instructional model characterised by the exposure of the content, with few definitions or procedural arguments and, constantly, the use of rules and algorithms, such as the use of the LCM.

Only one exercise at the end of the section presents the possibility of building an answer and seeking different resolution strategies. The authors of the book reinforce this idea by presenting a kind of script followed by four steps: understanding the problem, elaborating a plan, executing the plan, and performing the retrospect and verification. It is an activity that enables students to dialogue, cooperate and argue, increasing the didactic suitability of the instructional process.

At the beginning of the section, the authors propose, in the teacher's book, an activity using digital technologies. It consists of representing the information (fractions) of an exercise (involving comparison of fractions) in the form of tables and graphs developed in LibreOffice Cal. The activity instigates the student as to the representation of the information through a graph of sectors.

The use of digital technologies meets what Godino et al. (2004) point out when they indicate technological resources to enhance mathematical learning, contributing to the construction of mathematical knowledge.

SYNTHESIS OF THE DIDACTIC CONFIGURATIONS

The textbook employed as an instructional process must mobilise the knowledge of a set of intended mathematical objects. Therefore, through the analysis of the fractions section, eight didactic configurations were described, referring to the development order of the theme.

The first three configurations refer to the exemplification of the concept of rational numbers (part/whole, ratio and quotient), configurations 4 and 6 refer to a procedure (rule), configurations 5 and 7 refer to applications, and configuration 8 to a fixation exercise. Operations with fractional numbers present in this section were not analysed.

According to the configurations analysed, the construction of concepts and their meanings was not observed. Although the book is for elementary school students, the authors only give numerical examples of the situations proposed. Also, they do not address fundamental concepts such as positional value, density, ordering, and multiplicative reasoning. As the authors begin the section by highlighting that it is a resumption of already known concepts, they consider that the students already know those concepts of partition and unit, since they are the basis for the understanding of fractions (Behr et al., 1983).

According to the BNCC (Brasil, 2017), the actions of comparing and ordering fractions should be related to the meanings of part/whole, quotient, ratio, and operator.

However, by describing the configurations, we can observe that not all meanings were explored. The understanding of the fundamental concepts is impaired when we do not move between meanings, because the understanding of partition and unit, for example, is fundamental to the understanding of rational number (Behr et al., 1983; Kieren, 1988).

We highlight the predominance of fixation exercises for the meaning of part/whole with continuous quantities, and exercises involving the meaning of operator of a fraction (Configuration 5). The book does not explicitly address the meaning of operator of a fraction in the concepts and examples developed. We also highlight that the BNCC (Brasil, 2017) indicates for the 7th grade the study of fractions through their meanings as part of integers, result of division, ratio, and operator. Currently, the teacher should be attentive to the new curricular document and adapt the curriculum and planning.

The instructional process analysed, that seeks to exemplify a situation and propose a list of fixation exercises, is considered inefficient and does not include all the competencies and skills required in the current social and academic context. Studies by Godino et al. (Godino et al., 2006; Godino et al., 2014) reinforce the importance of an educational practice focused on the construction of knowledge as a more significant form of students' learning.

Besides, the configurations that illustrate procedural techniques for the application of the mathematical object lack an argument or definition that justifies such steps, and suggests that the teachers need to organise their planning identifying those limitations, and adding their didactic-mathematic knowledge to the object of study to complement their instructional process.

Configuration 7, referred to by the authors of the book as "Resolvendo um Problema" (Solving a Problem), one of the last activities of the section, is a suitable activity to develop knowledge involving rational numbers. It is an enriching activity to create an investigative, argumentative milieu of knowledge exploration.

FINAL CONSIDERATIONS

This study analysed an instructional process based on a textbook of the 7th grade of elementary school focused on the mathematics topic rational number, through theoretical tools of the OSA. The analysis considered didactic configurations in which we identified the dimensions to classify the didactic suitability of the book.

To achieve didactic suitability through the instructional process analysed, in their planning, the teachers must consider the epistemic and cognitive conflicts pointed out for each configuration.

The presentation of the contents neither favours the construction of concepts, nor explores the definitions and arguments of the objects and procedures adopted. It does not directly address rational numbers, just fractions, decimal numbers, and integers, separately.

The actions of ordering and determining the positional value of fractions are not explicitly studied in the fractions section and, according to the BNCC, are concepts that should be addressed through the numerical line for rational numbers with representation in the form of fraction and as a decimal number.

To meet the BNCC guidelines, the textbook can be used as a support material, but not as a planning of an instructional process as, when using it, the teacher can make use of different technological resources and other strategies to overcome the limitations and favour student learning.

AUTHOR CONTRIBUTION STATEMENTS

This article was prepared and organised by the two authors. P.P.G.C. developed the theoretical framework, methodology and collected the data. E.B. analysed the data and worked on the overall construction of the article.

DATA AVAILABILITY STATEMENT

The authors agree to make their data available at the reasonable request of a reader. It is up to the authors to determine whether a request is reasonable or not.

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