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Landscapes of Investigation and Scientific Initiation: Possibilities in Civilizatory Equation

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ABSTRACT

Backgroud: Society today presents a civilizatory equation, which comprises the complex relationship between what is technical and what is human and, therefore, the study of contemporary variables is central to the interpretation of this reality. **Objectives:** Answer the questions: Does Scientific Initiation (SI) approximate in the concerns of critical mathematics education (CME)? Does this approach contribute to critical and reflective mathematics teaching? **Design:** It presents a scenario for research developed from the didactic reflective transposition of a SI project, as well as the purposeful reflections built in the process. The methodological approach was action-research. **Setting and Participants**: The study was carried out at the Federal Institute Catarinense - Brazil and involved 1st-grade students of Technical Course Integrated to High School. The classes were selected because they were students of the researcher herself. **Data collection and analysis:** The data were produced throught the teacher-researcher's notes and the filming of the classes. The analysis was done by comparing the actions/reactions of the students with the theory. **Results**: The SI transposed into research scenarios provides the investigation of contemporary themes that are close to EMC, by encouraging questioning, autonomy, decision-making and critical interpretation of reality. **Conclusions**: With this study, we could perceive a transformation in the mathematics classes to evidence the criticality and understanding of reality. It is evident that landscape of investigation is an excellent strategy for the teaching of mathematics imbricated to the technological and human issues that constitute the complex civilizatory equation.

Keywords: Landscapes of Investigation; Scientific Initiation in High School; Civilizatory Equation; Professional and Technological Education; Critical Mathematics Education.

Cenários para Investigação e Iniciação Científica: Possibilidades na Equação Civilizatória

RESUMO

Contexto: Na sociedade atual se apresenta uma equação civilizatória, que compreende a complexa relação entre o técnico e o humano e, portanto, o estudo das variáveis contemporâneas nas aulas de matemática é fulcral para a interpretação dessa realidade. **Objetivos:** Responder as

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questões: A Iniciação Científica (IC) se aproxima das preocupações da educação matemática crítica (EMC)? Essa aproximação contribui para o ensino de matemática crítico e reflexivo? **Design:** Apresenta-se, um cenário para investigação desenvolvido a partir da transposição didática reflexiva de um projeto de IC, bem como as reflexões propositivas construídas no processo. A abordagem metodológica foi a pesquisa-ação. **Ambiente e participantes:** O estudo foi realizado no Instituto Federal Catarinense - Brasil e envolveu alunos do 1º ano do Ensino Médio integrado ao Ensino Técnico. As turmas foram selecionadas por serem alunos da própria pesquisadora. **Coleta e análise dos dados:** A produção dos dados se deu por meio das anotações da professora-pesquisadora e pelas filmagens das aulas. A análise se deu comparando as ações/reações dos alunos com a teoria. **Resultados da pesquisa:** A IC transposta para cenários para investigação proporciona a investigação de temas contemporâneos que se aproximam da EMC, ao fomentar questionamentos, autonomia, tomada de decisões e interpretação crítica da realidade. **Conclusões:** Com este estudo foi possível perceber uma transformação nas aulas de matemática, de modo a evidenciar a criticidade e a compreensão da realidade. Evidencia-se que cenários para investigação são uma excelente estratégia para o ensino da matemática imbricada às questões tecnológicas e humanas que constituem a complexa equação civilizatória.

Palavras-chave: Cenários para investigação; Iniciação Científica no Ensino Médio; Equação Civilizatória; Educação Profissional e Tecnológica; Educação Matemática Crítica.

INTRODUCTION

During the last decades, technoscientific development has been reaching unthinking levels, which, in turn, boosts a new civilizatory behaviour[1] that reveals that people need to consume and possess, rather than be; an organised society that is susceptible to the commands disseminated by those who dominate the technoscientific apparatuses that, in turn, are treated as instruments of power, and not as a vehicle for human development.

Thus, there is a civilizatory equation – metaphor utilised by Bazzo (2019, p. 21) - that could be “a panacea to bring together the most different variables that arise at all times in a civilisation that is vulnerable to the most accelerated mutations in its daily behaviour” - and more, with the implications that these issues bring to society. In other words, the urge to “provide reflections and changes in our ways of working knowledge in such serious times of human problems” (Bazzo, 2019, p. 20). This equation has in both members some more technical, and others more human, contemporary variables. It is aimed at the overlapping of the variables, so that the result of the equation is, at least, the guarantee of the principles of human dignity.[2] The current social, economic, and political variables are considered essential elements for the analysis and interpretation of reality. Some examples are environmental issues, the immigration process, social inequalities, the hybrid crisis, the atomic bomb, global warming, chemical wars, biological wars, pandemics - such as coronavirus, and so many other variables that compose the civilizatory equation.

[1] Civilizing Behaviour– behaviour according to the constitution of the current civilisation and governed by social transformations, as a social construct. This understanding is in line with Norbert Elias, who, in The Civilizing Process (1994) - original publication in 1939 -, analyses the effects of the formation of the Modern State on the customs and morals of individuals.
[2] According to the Universal Declaration of Human Rights (1948).

Among the various interferences in the contemporaneous civilizatory process, on the one hand, there is research, which has been intertwined with the development of science, technology, education, and on the other, the human skills, such as autonomy, creativity, argumentation, and decision making, with deepening of knowledge. Despite its significant importance in the civilizatory process, many researchers investigate with eminently technical issues. On the other hand, mathematical knowledge is involved as part of the foundation of this society, emerging the need to question its position in this laborious civilizatory equation (Civiero & Bazzo, 2020).

We understand that mathematics is especially relevant in the processing of knowledge, and operates in the process of globalisation,[3] i.e., it interferes in several aspects that integrate society. We admit that globalisation refers to all aspects of life and that, depending on how it is questioned and operationalised, it may or may not be beneficial. Therefore, globalisation "has to do with the construction, codification, and distribution of knowledge which turns into goods for sale" (Skovsmose, 2014, p. 130).

In this environment, we delegate some power to critical mathematical education (CME) by considering it can contribute to the critical formation of subjects by promoting reflections on this process. Bazzo (2019) reflections were also considered when he ponders that we experience a civilizatory equation whose variables need to be discussed in schools, as well as Civiero (2016), whose analysis shows that EMC, nowadays, is the most developed approach to deal with contemporary variables in mathematics classes.

Based on the exposed understandings, we defend the construction of landscapes for investigation[4] in mathematics classes, to provide students and teachers with the opportunity to investigate themes that may provoke reflections on contemporary issues. We identified as a possibility for the development of landscapes for investigation the Scientific Initiation inserted in the curriculum in High School, at the Instituto Federal Catarinense –Rio do Sul Campus, Santa Catarina, Brazil.[5] This campus has Technical Courses Integrated to High School. In 2001, the faculty, understanding that the research must be present in the entire human being's educational trajectory, inserted the Scientific Initiation in the curriculum at this level of education so that all students could access it. It is a space for the development and change of perspective of scientific activity that provides initiation to research in basic and higher education. It is also a path to intellectual independence,

[3] The term globalisation, in this study, is used not only as a mere concept of economic integration but, following the line of Chesneaux (1995), also as a process that involves transformations in the meanings of intensification of communications, time-space, deterritorialization, world integration, technical modernity, and social reflexivity.

[4] For a characterisation of Landscape for Investigation, see Skovsmose (2001a).

[5] Federal The Institutes of Education, Science and Technology (IF) in Brazil are institutions that offer professional and technological education at all levels and modalities, forming and qualifying citizens to act in the different sectors of the economy, with emphasis on socioeconomic local, regional and national development. The IFs are present in all Brazilian states, covering approximately 80% of the country's micro-regions. The Instituto Federal Catarinense (IFC) is part of the federal network and comprises 15 campuses and the rectory distributed in the state of Santa Catarina. They are institutions that guarantee public, free, and quality education.

creativity, curiosity, autonomy and can be a means of sharpening the critical awareness that, according to Freire (1974, p. 15):

> The critical consciousness is characterised by depth in the interpretation of problems; by the substitution of causal principles for magical explanations; by the testing of one's "findings" and by openness to revision; by the attempt to avoid distortion when perceiving problems and to avoid preconceived notions when analysing them; by refusing to transfer responsibility; by rejecting passive positions; by soundness of argumentation; by the practice of dialogue rather than polemics; by receptivity to the new for reasons beyond mere novelty and by the good sense not to reject the old just because it is old - by accepting what is valid in both old and new.

With these understandings, in this research, we aim to answer the questions: Does Scientific Initiation (SI) gets close to the concerns of critical mathematics education (CME)? Does this approach contribute to critical and reflective mathematics teaching?

The research was qualitative, in which some assumptions of action research were used. The character of this investigation requires greater involvement between the researcher and the research subjects, that is, an investigation aimed at producing descriptive data obtained through different observations. Action research was the methodological basis to produce "collective self-reflection" (Kemmis & MC Taggart, 1988). According to Elliot (1997, p.17), action research is a process that continually changes into reflection and action spirals in which each spiral includes: clarifying and diagnosing a practical situation or a practical problem that one wants to improve or solve; formulate action strategies; develop those strategies and evaluate their efficiency; expand the understanding of the new situation and proceed to the same steps for the new practical situation.

In this perspective, the investigation took place during nine meetings, two meetings each week, each meeting with two 50-minute classes with a group from the 1st grade of high school. The researcher was the teacher of the discipline. It took place at IFC, municipality of Rio do Sul, Santa Catarina, Brazil. After the students' acceptance, the classes were filmed and transcribed. The transcription data and the teacher's notes resulted in scripts described in Civiero's tesis (2009) and are depicted throughout this article. The description of the activity is combined with its analysis and reflection to interlink theory and practice. This research explains a landscape for investigation[6] experienced with students. Resulted from RDT,[7] of a Scientific Initiation for the math classes. Therefore, these steps are part of an action research, which essentially has intervention in reality, participation in decisions, and validation.

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[6] The data collection was carried out in 2009, at that time there was no need for prior ethical evaluation by the councils. However, the subjects accepted the invitation and signed a didactic contract with the teacher/researcher.

[7] For RDT, it is not enough to transfer knowledge, it is necessary to instigate reflections on mathematical subjects linked to reality (Civiero, 2009, p. 49).

Finally, we show that the landscape for investigation leads students and teachers to experience actions based on mathematics to understand the imbrications between mathematical knowledge and the contemporary variables of this complex civilizatory equation.

SCIENTIFIC INITIATION IN HIGH SCHOOL

Scientific initiation as well as CME, are potential to discuss contemporary issues and contribute to the student's critical education. Scientific initiation is a primary space for research. Therefore, we defend that research is

> the search, the study, the knowledge, the explanation, and the understanding of the world that surrounds it, motivated by actions of the subject that makes science. This demonstrates that it is not enough to fulfil the requirements of the system, it is also necessary to reduce the gap among areas of knowledge and between the technical and the human. (Oliveira, 2017, p.32)

In line with this conception of research, Scientific Initiation, according to Bazin (1983) and Oliveira; Civiero; Fronza and Mulinari (2013) is a path of intellectual independence, and, as a scientific activity, it does not happen outside a social context.

Given this, we understand Scientific Initiation as a collaborative space of authorship experience as the "search for the understanding in which the human being lives" (Oliveira, 2017, p. 32). Its insertion in basic education is pertinent for providing scientific and technological education that contributes to the formation of the individual by fostering curiosity, creativity, authorship, decision making, and interpretation of reality through research initiation.

Despite its importance, scientific initiation in high school, which comprises students aged between 15 and 18, is recent in Brazil. According to Oliveira (2017), currently, scientific initiation in high school can be classified into three modalities: as an institutional program (since 1986), as public policy (since 2003), and as curricular component (since 2001).

Regardless of the modality, scientific initiation in high school must step away from the technical rationality, that is, it must not focus on universal techniques and methodologies, such as imitation, repetition and the reproductive character, because,

> High school Scientific Initiation can articulate and integrate diverse knowledge, theory and practice and teaching, research, and extension. Dialogicity, problematisation, critical reflection and collaboration are the basis for the development of people's autonomy. Based on these potentials, scientific Initiation in high school is not just a space for methodological learning or research initiation

> focused on training researchers concerned with an object of study that is alien to reality, society, and the civilizatory process. (Oliveira, 2017, p. 147)

Oliveira, Civiero and Bazzo (2019, p. 469) defend scientific initiation as a curriculum component, a space in which the project selected for transposition was developed. For the authors, this type of scientific initiation in high school "is a potential to deal with contemporary issues and bring knowledge from different areas to the student's reality and, therefore, bring reflective and critical discussions."

We defend this potential of scientific initiation, thus, at the basis of this study, we have the modality of scientific initiation as a curriculum component, as it is an environment that drives the permanent reconstruction of knowledge. This modality takes place at the Instituto Federal Catarinense (IFC) - Rio do Sul Campus, [8] locus of this study.

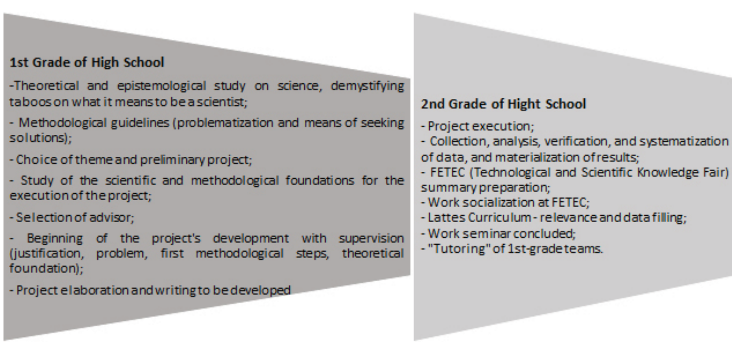
Scientific Initiation as a curriculum component in high school

In 2001, the Scientific Initiation Project started at IFC - Rio do Sul Campus as a constituent project of the diversified part of the high school curriculum matrix with a workload of two hours per week. One or two teachers manage this workload, when students are offered training on epistemological foundations of science, as well as aspects of research methodology, and the writing of projects and reports.

The insertion of the Scientific Initiation Project in the curriculum enables the production of knowledge and the articulation between areas of knowledge, minimising the boundaries between the curriculum components (Scheller, Civiero & Oliveira, 2015). Therefore, there are structural elements that permeate the organisation of scientific initiation from the beginning, as shown in the chart below.

Figure 1

Structuring elements of the curriculum component Scientific Initiation - High School - IFC - Rio do Sul campus - 2001-2019



[8] The IFC - Rio do Sul Campus has existed as a Federal Education Institution since 1994. Currently, it has three technical courses integrated to high school (Agriculture, Agroecology, and Informatics), six higher education courses, two *lato sensu* graduation courses, and one course after high school.

With a view to the planning presented in figure 01, we realise that the teaching plans advocate, at first, the discussion of themes that instigate students' critical reflection on the world, leading them to perceive themselves as subjects in and with the world. From the discussions and deconstructions of myths and taboos about science, technology, and scientists, students start the project that involves choosing a topic under the guidance of a campus professor.

Scientific initiation as a curriculum component allows all students to participate in establishing dialogical teaching and learning relationships in the process of dodiscence. "Teaching, learning and research deal with these two moments of the gnoseological cycle: the one in which the existing knowledge is taught and learned, and the one in which the production of knowledge that does not exist is yet to be worked" (Freire, 1996, p. 28).

Thus, scientific initiation is a space to instigate the student to have pleasure and willingness to learn, to be curious, to be reflective, to argue, to seek answers and to carry out the process of building knowledge in a critical way, which is not a tradition in education and society. In other words, scientific initiation is:

> An educational process capable of equipping the student for critical reading of the social practice in which he lives is the means that will make the school democratic. I understand by the democratic school the one that takes the student to be a transforming subject of his reality; a critical look is not enough, he must be inserted, think and plan necessary changes, believe in them and put them into action. For this process to become effective, it is essential to assume democratic attitudes when restructuring the didactic procedures. (Civiero, 2009, p. 53)

This educational process that constitutes the scientific initiation approximates the landscapes for investigation in problematisation and knowledge production. Besides, scientific initiation intertwines with scientific and technological education; therefore, it is a space for discussion about the variables of the civilising equation, so that the practice of scientific initiation

> [...] needs, in its conduction and supervision process, a dialogical practice that problematises, that questions, that criticises knowledge, that values the other, that integrates, that instigates autonomy, and that takes care of life as the greatest social good, being essential the training of guiding teachers and/or researchers. The understanding and practice of scientific initiation need to go beyond the reproduction only of issues already posed "culturally" for research and teaching, such as, for example, bureaucracy, making it elitist, the selective character, training, focus on the method and reproduction of technical rationality. To impact upon humanising formation, it is necessary to have as a main pact the critical and reflective search to understand the world in which we live, established by a collaborative environment permeated by problematising dialogicity that relates science and technology and the civilizatory process, the advisor and the student. (Oliveira, 2017, p. 275-276)

In this way, we recognise that the scientific initiation works developed in the IFC - Rio do Sul Campus, are scenarios that provoke authorship and critical reflection of knowledge, and can be transposed into the classroom and constitute landscapes for investigation. Therefore, the first author selected scientific initiation projects developed in previous years with other students and leveraged them in proposals for scenarios for research in mathematics classes, whose experience the author reported in her master's dissertation. (Civiero, 2009). She considered this an RDT process, i.e., the knowledge developed in the scientific initiation project was transposed to mathematics classes, according to the CME perspective. We report below the experience lived in one of the scenarios.

EXPERIENCING A LANDSCAPES FOR INVESTIGATION

The landscapes for investigation are milieus of learning built in the classroom that enable investigation, in which students are invited to make discoveries in a process full of questions, curiosities, explanation of perspectives and critical reflection. Therefore,

> The important point is that the landscapes for investigation are not explored based on a previous list of exercises. On the contrary, explorations take place through a "learning guide," in which students can point out directions, ask questions, ask for help, make decisions, etc. (Skovsmose, 2001b, p. 64)

With this understanding, RDT was carried out with high-school students from two classes of 1st grade at the IFC - Rio do Sul Campus. During the activity, the students' reactions and comments, registered throughout the script were observed, as well as the perceptions of the teacher who constituted the landscape. The students are represented with letters of the Roman alphabet to keep anonymity.

The invitation

In a context of uncertainties, in a landscape for investigation, students' acceptance is paramount. According to Skovsmose (2001), a landscape for investigation is constituted from when students accept (and assume themselves as active participants) the process of exploration and explanation

> A landscape for investigation is one that invites students to ask questions and seek explanations. The invitation is symbolised by its "Yes, what happens if ...?" In this way, students get involved in the exploration process. The teachers' question: "Why is this?" represents a challenge, and the students' "Yes, why is this...?" indicates that they are facing the challenge and that they are looking for explanations. (Skovsmose, 2008, p. 21)

Therefore, we sought to encourage the investigation to arouse curiosity, attractively presenting the material, so that the activity was not seen as a command but rather as a different way to learn mathematics, to prioritise the reality in which the students were inserted. Therefore, the teacher in this space is an inquirer and mediator, to avoid defined and unquestionable concepts.

At first, students were invited to accept investigating windrow behaviour through composting based on the work "Composting from various organic wastes," developed in the Scientific Initiation Project (2006/2007). The windrows are made in a structured way with a base of vegetable dry matter and layers interspersed with organic matter. The system works with passive aeration, ensuring the thermophilic composting process. Figures 02 and 03 illustrate the windrows.

The topic was of interest to the students, as they take the Technical Course in Agroecology or Agriculture integrated with High School[9]. Thus, raising concerns about the environment as one of the contemporary variables in this globalised world is part of humanising education as well as professional education.

From the landscape to the investigation

In the first stage of transposition (RDT), from the elements of the work developed in the Scientific Initiation Project, students reflected on the importance of the project, the social relevance inherent to the theme, and raised some hypotheses about its implementation. According to the authors of the Scientific Initiation work,

> The compost is very important for agriculture because it follows the concepts of agroecology, as it is a way of nourishing plants with all the macros and micronutrients that it needs without having to use any type of external inputs that can harm nature. (Battisti, Campos, Souza, 2007)

The students started a series of questions that involved the subject: "What is organic matter? What is compost? What are the benefits provided by the existence of composting in the soil?" Such questions prompted the search for explanations. At the same time, students began to ask new questions: "How to make composting?" (C). "What types of materials can you use to make the compost?" (J). "How do I prepare a compost pile?" (A). "How do I maintain this compost?" (P). "How do you check the maturity of the compost?" (K). "What are the stages of composting?" (B). Initial curiosity, which can be called naive (Freire, 1996), was instated as they accepted the invitation.

However, some students opposed the activity, claiming that they did not perceive a relationship between the activity and mathematics. This resistance to the proposition

[9] An integrated course, it integrates high school with the technical course, which results in a single certificate of completion.

may be due to the exercise paradigm. Student (C) 's question: "What does this have to do with the math class?" is an example of such objection. There was some discomfort as they had to move from a passive to an active position. Resistance strengthened the premise of the ideology of certainty, imbued with a culture that supports the power of containing the ultimate argument attributed to mathematics. For Borba and Skovsmose (1997, p. 133), "students should therefore be persuaded against ideas such as: a mathematical argument is the end of the story; a mathematical argument is superior by its very nature; the numbers say this and this." The only methodological option was to try to detach from the exercise paradigm. We started to discuss how urgent it was to work a different approach with the mathematical contents that were necessary to analyse that reality.

After reading the proposition, the students delved into the theoretical foundations and were free to research more elements, which led them to become collaboratively involved. Classes started to gain momentum; students were not waiting for ready-made answers and investigated them when necessary. Thus, the criticism of the initial curiosity (Freire, 1996, 2006) was manifested. Throughout the activity, the teacher felt she was in the risk zone, because, in a landscape for investigation, uncertainties are part of the process, doubts that students also manifested: "Teacher, when will mathematics appear?" (A,) or: "This class is different, what does the teacher want?" (J).

Given this first referral, students were motivated to recognise the materials and methods used by the Scientific Initiation Project students in the process. According to Chart 02.

Figure 2

Materials and methods of research work - Composting from various organic waste - IFC- Rio do Sul Campus (2006/2007). Adapted from Battisti, Campos and Souza (2007)

Materials used:
Cattle manure; Bird dung with a small percentage of wood shavings (wood dust); Raw kitchen waste; Cooked kitchen waste and fibrous material (chopped elephant grass).
Procedures:
Five windrows were made 90cm long and 50cm wide, each with a different type of organic waste, except for the fibrous material that was present in all windrows. We started the compost with a grass base, 10 cm high, 50 cm wide and 90 cm long. On the same day, we moved to the second layer using the other residues (raw kitchen waste, cooked kitchen waste, bovine manure, poultry manure with wood powder) each row with a type of waste 2cm high. In the third layer, 10 cm of fibrous material was again placed in the fourth 2 cm of waste, and we ended with a fifth layer of grass. Right after finishing each 36cm high windrow, we watered the compost with 6 liters of water. Finally, we cleaned the sides with a hoe.

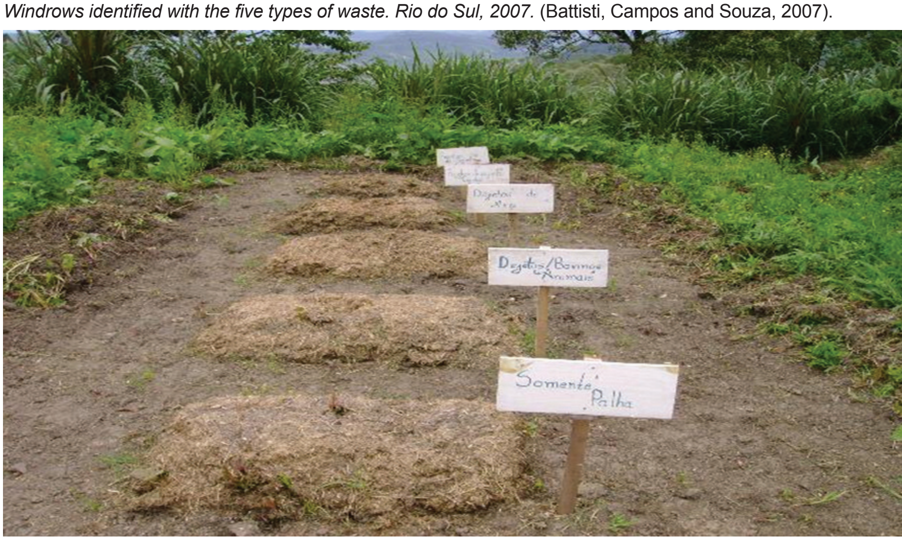
Photos of the experiment were observed to detail each step, to facilitate the understanding of the practice.

Figure 3

Rows with the five types of waste. Rio do Sul, 2007. (Battisti, Campos & Souza, 2007).

Figure 4

Windrows identified with the five types of waste. Rio do Sul, 2007. (Battisti, Campos and Souza, 2007).



After getting to know the composting production process, the next step was to start data analysis. At first, the teacher prompted the students to make estimates: "How do you think the composting occurred? How did the windrows behave? Which decomposed more quickly? How did this happen?". After many conjectures, table 1 was presented with the data.

Table 1
Behaviour of windrows with different residues for composting. Rio do Sul. (Battisti, Campos & Souza, 2007).

Dates	Windrows of other types		Fibrous Widrow	
	Height variation / week (cm)	Final height per week (cm)	Height variation / week (cm)	Final height / week (cm)
09/11/06	0	30	0	30
16/11/06	- 1.2	28.8	-2.4	27.6
23/11/06	-1.2	27.6	-3.6	24.0
30/11/06	-1.2	26.4	-1.2	22.8
07/12/06	-1.2	25.2	-2.4	20.4
…	…	…	…	…

Based on the data, students were encouraged to analyse the results. The teacher acted as a questioner once more, triggering questions such as: "What data are listed in the title of the chart? Does the title tell us what is being presented? Tell us when and where the experiment took place? What can you see in the variation of the heights of the windrows? What was the starting height?"

Through discussions, observations like this appeared: "Look at the windmill with organic material, it always reduces equally. Did this really happen, teacher?" (C).

We emphasise that the way communication develops between students and the teacher can influence the learning (Alrø & Skovsmose, 2002). The teacher plays a vital role in conducting this dialogue, as discussed by Milani (2017) and Milani, Civiero, Soares and Lima (2017).

In this process, the teacher instigated the students to understand the behaviour of the windrows, trying to show the difference between them and the organic material (waste) and the fibrous windrow (straw). For this, they analysed the windrow behaviour:

> The windrow of fibrous material decreased faster due to disruption and varying the decrease, while others gradually decreased by 1.2 cm per week. The windrows were made alternately between fibrous material and organic waste, and at the end, each windrow was 30 cm high. The different windrows gradually lowered around 1.2 cm each week, and in February, their height stabilised at 12 cm. The windrow that had only straw fell faster, varying its decrease due to decomposition and de-structuring of the compost, but ended with the same height as the others. (Civiero, 2009, p. 83).

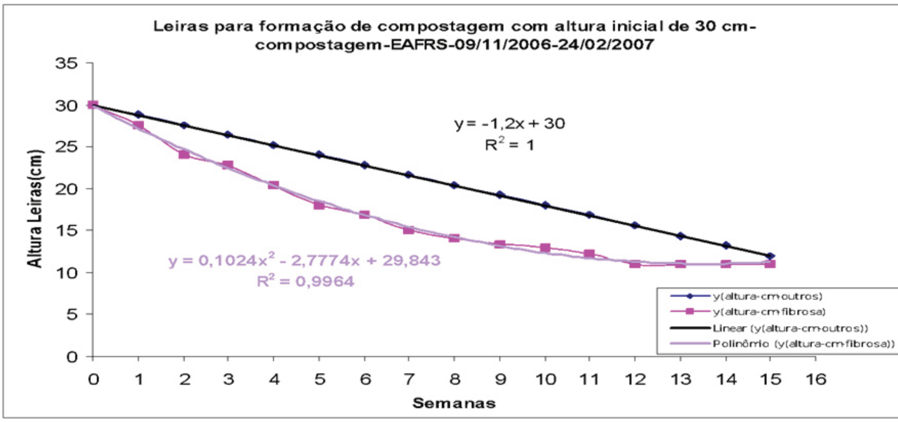
Student (A) observed: "I am already aware of mathematics that appears in these data." Following this speech, other students spoke, willing to establish mathematical relationships and to represent them graphically. The students interacted with the theme and wanted to be involved in the discussions. The commitment and willingness were different from most math classes the teacher experienced.

Math-based actions

To explore other elements, figure 03 was explained. In addition to graphically presenting the behaviour of the data, it also presents the mathematical models that have best adapted to the data.

Figure 5

Evolution of windrow height for composting. Rio do Sul, 2007. (Battisti, Campos, & Souza, 2007)



This stage of RDT is in line with Skovsmose (2001b) regarding the three aspects that a teaching-learning aligned with the social argument of democratisation must present:

> 1) The material has to do with a real mathematical model; 2) The model has to do with important social activities in society; 3) The material develops an understanding of the mathematical content of the model, but this more technical knowledge is not a goal. The goal is to develop an insight into the hypotheses integrated into the model, and thus develop an understanding of the processes (for example, decision processes) in society. (Skovsmose, 2001b, p. 43-44)

The activity developed converges with the three aspects. As for the latter, it was necessary to instigate discussions on the importance of models in a highly technological society and, on the other hand, the verification of estimates and approximations in these models to identify an object of reflective knowledge distinct from the object of technological knowledge when analysing the model and its relations.

The teacher asked questions: "What do they represent? What kind of curves appeared? What do the coefficients mean in the function?" Students were asked to define the mathematical model that best adapted to the curve.

Thereby, students became curious about mathematical models and started a process of mathematical discoveries. In this case, specific mathematical knowledge and its concepts were essential to explain the reality. This speech can express some of the reactions: “Now I understand where the teacher wants to go. The project is full of mathematics” (D).

The students realised that in the windrow formed by various residues, the height varied gradually, exactly 1.2 cm per week. They soon identified this number as the angular coefficient of the linear function, advancing to the concept of rate of change, which is made explicit in the statements: “The windrow is always lowering equally” (G). “Look at the table, each week the windrow dropped 1.2 cm, it’s the same number that appears in the function” (A).

Student (B) reflected: “This number is constant in the organic material windrow.” Student (J) added: “Ah! That is why the graph is a line”. Student (D) immediately asked about the meaning of the correlation index: “Teacher, what does this R^2 represent?”. Students were encouraged to investigate this issue.

Meanwhile, (C) asked: “What about the windrow made of straw? It is different, so how to identify the rate of change if it was not constant?” They realised that their behaviour was different, with the points not adapting to the linear shape, which would need a different mathematical analysis.

The students were curious about the functions that the Excel software presented. Student (E) asked: “Teacher, what do the charts have in common with these functions that Excel listed?” This question encouraged other students to speak out of curiosity. They were ready to start another stage of the investigation.

First, they understood that the Excel program had adjusted the curve with the data that were relating the two quantities, that is, a set of coordinates (x, y). When asked what quantities were being related, A replied: “Of course, we are relating time (weeks) and height (cm).” Therefore, the variables x and y were unveiled, recognising the variable x as the variation of the weeks and the variable y as the variation of the windrow height. The students studied the mathematical concepts with interest and asked questions such as: What is this for? Generally, common in traditional classes, they became obsolete since this premise was stated. This problem disappeared as mathematical concepts were developed from the need generated in a context in which students were inserted.

In this phase of explanation, there was a need to refer to pure mathematics in which students stopped to appropriate the specific mathematical knowledge necessary to understand the project. For the first windrow, which showed a linear decrease, they needed to study the function of 1st degree, recognising its main characteristics and rules. To exercise the content of linear systems, which emerged from the need to adjust curves, they used activities referring to semi-reality. Such activities make up an important part of the list of educational possibilities, however,

> [...] Solving exercises referring to a semi-reality is a very complex competence and based on a well-specified contract between the teacher and the students. Some

> of the principles of this agreement are as follows: the exercise's wording fully describes semi-reality; no other information is essential for solving the exercise; more information is totally irrelevant; the only purpose of presenting the exercise is to solve it. (Skovsmose, 2008, p. 25)

When carrying out the activities proposed, some students showed to be satisfied, which they expressed in some comments: "Now, yes. The class became Mathematics again" (C). This student may still be framed by the exercise paradigm. However, this feeling was not shared by others, who said: "Wow, we did it again without knowing for what" (H). This student showed to feel uncomfortable with decontextualised exercises. And finally: "Of course not. We need to learn to calculate to understand the windrows" (M). We understand that this student realises that specific mathematical knowledge is needed for reality interpretation. The initial curiosity was criticised, but then, as they deepened knowledge, it became an epistemological curiosity (Freire, 1996).

During this traditional class, the students behaved formally, i.e., they reproduced the activities. However, this stage was full of meanings, and the students had a goal. Even so, the teacher needed to create spaces for dialogue, in active listening. According to Milani; Civiero; Soares and Lima (2017, p. 240), "When the teacher tries to perform active listening, he starts the movement of dialogue that seeks to understand what the student says. This movement is not simple and immediate, as it is a change of posture, in the epistemological, methodological, and political sense."

In this process, it is possible to perceive the involvement of students in different milieus of learning[10], which highlights the potential of the landscape developed. Thus, they move between different milieus all the time, according to the necessity. References to pure mathematics, in the context presented, are related to semi-reality and real-life references. The landscapes for investigation with reference to real life emerge naturally from a contemporary variable. Thus, the mixture between the milieus does not happen in a forced way, as grafts, but as a necessity for its development.

At the beginning of the following class, some questions were asked: "What did you see when observing the behaviour of elephant grass windrows and other waste? Why was it represented graphically by a line? What is the relationship between the data and the linear function presented by the Excel program?" After some discussions, the students realised that the height of the windrow varies according to the passage of time (weeks), that is, in the function, it accompanies the x, which is representing the time quantity in weeks. They also concluded that the parameter b of the function was representing the initial windrow height. To conclude, much dialogue was necessary. The discussion went on, and, from time to time, the teacher intervened with some provocation for them to understand the mathematical relationships. Such definitions were evident in the students' statements: "Well, if the table shows that the windrow has always decreased by 1.2 and that each collection was made weekly, then just multiply the week by 1.2, and we will

[10] The milieus were named according to the matrix elaborated by Skovsmose (2001a).

know the height of the windrow" (H). "However, you can't forget that it has to be - 1.2, because the windrow is decreasing and a function y = -1, 2x + 30 that expresses the height of the windrow according to the weeks is obtained" (C). "Interesting, then, that is why the graph is a straight line, week after week the windrow decreases equally" (J). "Yes, the variation is always the same. I also noticed that the line is decreasing, which is logical, because the windrow is lowering" (B). "Teacher, number thirty appeared in the function, is it the initial height of the windrow, or it is pure coincidence?" (G). "It is obvious, you see, now I understand. The graph is showing exactly the behaviour of the windrow. It starts with 30 cm and goes down 1.2 cm, which must be represented by - 1.2 cm, because it is going down, i.e., it is decreasing" (D).

During the dialogue, there was active listening, and the students tried to emphasise the appropriate terminology, for example, when they said that the windrow was lowering, they were taught to use the term decreasing, which was related to the position of the line. Gradually, they adapted language and mathematical symbology.

Next, the students chose their resolution method: either the Simple Linear Regression or the Least Square Method. They found the values of the coefficients a and b, whose meanings and values they had previously recognised, and checked the mathematical algorithm.

At this point, it is possible to infer that this dynamic occurred in line with both Skovsmose (2008, p. 13), who considers "that a new critical mathematical education must seek educational possibilities (and not propagate ready-made answers)," and Postman and Weigartner (1969), who discuss the importance of changing attitudes, moving from a school of answers to a school of questions.

THE MODEL: THE IMPORTANCE OF CRITICAL REFLECTION

When developing RDT in classes, besides what has been exposed so far, we highlight the discussion on the importance of adjusting curves by pointing out that different real situations can present problems that require solutions and decisions. A mathematical formulation can help students to understand the real situation. Therefore, a mathematical model is represented by symbols and mathematical relationships that seek to translate, in some way, a phenomenon of reality. In this perspective, when proposing a model, we must keep in mind that it comes from approximations made to understand a phenomenon better. Thus, these approaches are not always consistent with reality, but they portray aspects of the situation analysed. Therefore, it is necessary to criticise the model in all its dimensions.

We highlighted that several observations could be made regarding mathematics in action and thus justify how mathematics can operate in technologies, production, management schemes, and decision-making. As part of the laborious civilizatory equation, it can change social and cultural behaviours.

We discussed how society is technologised and how much mathematics helps to shape this society. This dialogue passed through the third concern regarding mathematics in action, presented by Skovsmose (2008, p. 112) when considering that mathematics in action "is a paradigmatic space to discuss structures of knowledge and power in today's society."

In this discussion, we emphasise the accelerated change in the civilizatory equation led by the fourth industrial revolution. According to Schwab (2017), this behaviour announces a 4.0 revolution, characterised by the transition towards new systems that overcome the digital revolution. Consequently, the power relations underlying the processes of commercialisation and industrialisation are closely linked to technoscientific development, which, in turn, is conditioned by mathematical algorithms.

We also observed that mathematics provides the possibility of hypothetical reasoning, i.e., it can analyse the consequences of an imaginary landscape. On the other hand, mathematics can also help in the construction of justifications, true or not, in the legitimation of some decisions and actions. The students remembered the electoral season, when we see statistics that, depending on the way they are organised, do not always represent reality, but are used to influence and pressure us to choose a specific candidate. In 2020, we could discuss statistical data from the COVID-19 pandemic. For example, the case fatality rate (CFR%) depends on the number of confirmed cases, and for that, it depends on the number of tests that are carried out. Thereby, it is extremely challenging to estimate the actual death risk accurately.

The students were frightened by the power of mathematisation, which can be seen by the speech: "Wow! Mathematics has the power over everything, it gives a shiver. Knowing that everything is calculated in advance and callously" (I).

We tried to sharpen the discussion by emphasising that mathematical models are not always constructed from a socially just perspective. Encouraging critical mathematics is "integrating students' lives, knowledge, and cultures; having students learn important mathematics and about their world; and supporting them to act on the injustices they perceive and experience" (Gutstein, 2012, p. 65). Therefore, to paraphrase Skovsmose (2008, p. 118), we emphasised that "Mathematics should be a theme for reflection and criticism in all its forms of action."

In this context, we sought to show how critical education can be guided toward emancipation. These discussions were based aligned with Skovsmose (2008, p. 94), when he mentions critical citizenship: "[...] it can "challenge" the constituted authority. It carries with it the opposition to any decision considered unquestionable." We highlighted the relevance of knowing how society is managed and how situations are planned, often mathematically, with its algorithms.

Consequently, this educational possibility was planned, aiming to bring about changes to problematise the need to criticise the social system. During the discussions, the importance of dialogue between the teacher and the students became evident. However, we agree with Skovsmose concerns when he states that

> The important question now is how well mathematics education can prepare for critical citizenship. I do not see that such preparation is related to the school mathematical tradition. I don't even see it linked to the intimate nature of mathematics. It has to do with a possible function of mathematics education. (Skovsmose, 2008, p. 95)

We argue that discussions of this type are those that can help mathematics education set critical citizenship in motion, and that, in this way, students can experience actions based on mathematics to realise the relevance of reflections. One of the students commented in between the discussions in class: "We cannot accept everything as finished, full stop; we need to understand the process to be able to accept it or not" (F).

With this analysis, we finished the study of the function of the 1st degree. However, we ended the class by encouraging students to observe the other windrow's behaviour, made up of fibrous material, whose model refers to a quadratic function. Nevertheless, we will talk about this development at another time, or it can be seen in Civiero (2009).

For Civiero and Sant'Ana (2013, p. 695), the landscapes of investigation from scientific initiation constitute a "critical reflexive approach that can relate teaching to the act of questioning and making decisions, establishing a link with life in society and mathematics".

In this context, we defend the pertinence of the mathematics teacher education regarding scientific and technological literacy to be prepared to promote RDT through landscapes for investigation whose motto is contemporary variables. In this regard, Civiero, Fronza, Oliveira, Schwertl and Bazzo (2017, p. 2673), state that:

> for the teacher to develop mathematical concepts related to reality, he needs to recognise it, read the news critically, expand his list of readings, understand, make decisions, evaluate and criticise social, political, economic, scientific, and technological issues. Recognise the dynamics and complexity of the educational world imbricated with reality outside it, acting cooperatively and collaboratively.

In this way, we emphasise that the teacher must assume a critical epistemological conception to encourage students to have a critical attitude, and enabling them to analyse and make decisions that can interfere in reality and, consequently, in people's quality of life.

ADDITIONAL CONSIDERATIONS

We live in a civilizatory equation different in scale, scope, and complexity of any that has ever occurred before. The challenges for those who aspire to social justice because of the equation are increasingly complex. Therefore, we must urgently enter all possible spaces to show that a better world is possible, where all people have their equal and inalienable rights as the foundation of freedom, justice, peace, and social development.

For this purpose, the description of the RDT of a SI project for mathematics classes provided the development of a landscape for investigation, which, in turn, encouraged reflection and criticism. The development of this approach evidenced the approximation of SI conceptions with the landscapes for investigation. Therefore, when proposing activities from the perspective of the CME, discussions regarding the variables of the civilizatory equation are provided, which tries to find a way of equating the many elements of the interwoven relationship between technical aspects and human issues.

When exploring the behaviour of the composting windrows (the theme of the SI project), we can aim at an environmental study, one of the essential variables whose reflections are fundamental for the existence of the Earth. We invited the students to get involved in the production of the mathematical model, reflect on how the results are related to the criteria used and how they can be used in society. We emphasise that mathematics plays an essential role in social issues and reiterate the importance of looking at the context and appropriating it for reflection and action to foster collective decision-making, given the needs established in the process.

It was also possible to highlight the urgency of the overlapping between the variables, the Scientific Initiation, and the different milieus of learning. When looking for different milieus, one can articulate the dimensions of the methodological specificities of pure mathematics, semi-reality, as well as the very reality. In this context, the landscapes for investigation with real-life references - environmental variable related to the professional course - took shape and its content was essential for the interpretation of life through mathematics, which is contemplated in the words of Skovsmose (2005, p. 96): "In this way, they experienced what actions based on Mathematics can mean and realised the importance of reflection."

Finally, we argue that the development of landscapes for investigation is fundamental to highlight mathematics linked to technological and human issues that constitute the civilizatory equation. To this end, it is essential to provide discussions inherent to reality to appropriate specific mathematical knowledge imbricated with other areas of knowledge and apply them to promote a society where the principles of human dignity are guaranteed, and social justice prevails.

AUTHORS' CONTRIBUTIONS STATEMENTS

PAGC conceived the presented idea. FPZO developed the discussion about Scientific Initiation and PAGC developed the discussion about Landscapes of Investigation and adapted the methodology to this context, created the models, performed the activities, and collected the data. PAGC and FPZO analysed the data. All authors actively participated in the discussion of the results, reviewed and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data that support the results of this study are openly available in the Federal University of Rio Grande do Sul - Brasil, through the link:

https://www.lume.ufrgs.br/bitstream/handle/10183/21588/000737701.pdf?sequence=1

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