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The Interpretation of Statistical Graphs Given in the Press: Advancing Towards Citizen Training

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ABSTRACT

Background: Statistical graphs in communication media validates that which communicates, considering that every media has editorial orientation, and is central to the development of citizen capacity for the critical interpretation of graph information. So it is necessary to understand the processes of interpreting graphs in communication media to improve statistical learning. **Objective**: Systematise elements that concur to the action of critical interpretation of the statistical graphs used in communication media. Design: From the socioepistemology and two studies, we focus on the analysis of textualities of the interpretations of graphs published in the press. We characterise operational, perceptual, and experiential elements that concur to the construction of meanings regarding the graphic phenomenon. **Setting and Participants**: This research was done with two groups: students from the 11th grade of schooling in Chile, who carry out an interpreting activity; and university statistics professors from Peru and Chile, who answered a semi-structured interview. **Data analysis**: Through the content analysis of textualities, units of meaning are identified from open encodings, grouped according to operational, perceptual or experiential aspects. **Results**: The act of interpreting graphs constitutes a space of epistemic activity that allows us to know the graph. In particular, the use of qualitative and perceptual comparisons that make the interpretation permeable to tendentious modifications of graphic elements is evident. **Conclusions**: It is necessary to incorporate press graphs into didactic designs, with a socio-critical discussion regarding the use of graphs in communication, to strengthen citizenship.

Keywords: Statistical learning, Graphs in communication media, Critical interpretation

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La interpretación de gráficas estadísticas dadas en la prensa: avanzando hacia la formación ciudadana

RESUMEN

Contexto: Las gráficas estadísticas en medios de comunicación dan validez a aquello que se comunica y, considerando que todo medio tiene orientación editorial, es central desarrollar la capacidad ciudadana para la interpretación crítica de la información gráfica. Así es necesario comprender los procesos de interpretación de gráficas en medios de comunicación para mejorar el aprendizaje estadístico. **Objetivo**: Sistematizar elementos que concurren a la acción de interpretación crítica de las gráficas estadísticas usadas en medios de comunicación. **Diseño:** Desde la Socioepistemología, y a partir de dos investigaciones, se focaliza en el análisis de textualidades de quienes interpretan gráficas publicadas en prensa. Se caracteriza elementos operacionales, perceptivos y experienciales que concurren a la construcción de significados respecto del fenómeno graficado. **Escenario y Participantes**: Se consideran dos grupos: estudiantes de 11° año de escolaridad de Chile, quienes desarrollan una actividad de interpretación; y profesores universitarios de estadística de Perú y Chile, quienes responden una entrevista semiestructurada. **Análisis de datos:** Mediante el análisis de contenido de las textualidades se identifican unidades de significado a partir de codificaciones abiertas, posteriormente se agrupan según aspectos operacionales, perceptivos o experienciales. **Resultados**: El acto de interpretación de gráficas constituye un espacio de actividad epistémica que permite conocer lo graficado. En particular, se evidencia el uso de comparaciones cualitativas y perceptivas que hace a la interpretación permeable a modificaciones tendenciosas de elementos de las gráficas. **Conclusiones**: Es necesario incorporar a los diseños didácticos las gráficas en prensa, con discusión socio-crítica respecto al uso de las gráficas en la comunicación, para fortalecer la formación ciudadana.

Palabras clave: Aprendizaje estadístico, gráficas en medios de comunicación, interpretación crítica

A interpretação dos gráficos estatísticos apresentados na imprensa: rumo à formação do cidadão

RESUMO

Contexto: O gráfico estatístico nos meios de comunicação valida aquilo que comunica, considerando que todo meio tem orientação editorial e é fundamental para o desenvolvimento da capacidade cidadã para a interpretação crítica da informação gráfica. Portanto, é necessário compreender os processos de interpretação de gráficos nos meios de comunicação para melhorar o aprendizado estatístico. **Objetivo**: Sistematizar elementos que contribuam para a ação de interpretação crítica dos gráficos estatísticos utilizados nos meios de comunicação. **Design**: A partir da socioepistemologia e de dois estudos, privilegiamos a análise das textualidades das

interpretações dos gráficos publicados na imprensa. Caracterizamos elementos operacionais, perceptuais e vivenciais que contribuem para a construção de significados sobre o fenômeno gráfico. **Cenário e participantes**: Esta pesquisa foi realizada com dois grupos: alunos do 11º ano de escolaridade do Chile, que realizam uma atividade de interpretação; e professores universitários de estatística do Peru e do Chile, que responderam a uma entrevista semiestruturada. **Análise de dados**: Por meio da análise de conteúdo das textualidades, unidades de significado são identificadas a partir de codificações abertas, agrupadas de acordo com aspectos operacionais, perceptuais ou experienciais. **Resultados**: O ato de interpretar gráficos constitui um espaço de atividade epistêmica que nos permite conhecer o gráfico. Em particular, é evidente o uso de comparações qualitativas e perceptivas que tornam a interpretação permeável a modificações tendenciosas dos elementos gráficos. **Conclusões**: É necessário incorporar gráficos da imprensa aos desenhos didáticos, com discussão sociocrítica quanto ao uso de gráficos na comunicação, para fortalecer a formação cidadã.

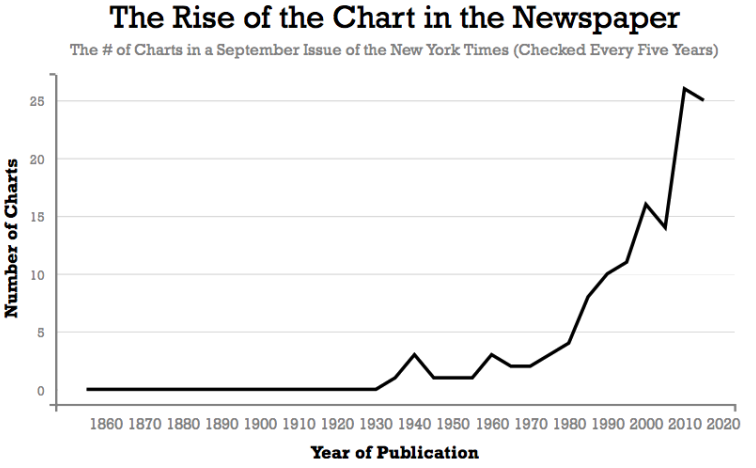
Palavras-chave: Aprendizagem de estatística, gráficos nos meios de comunicação, interpretação crítica

INTRODUCTION

The graphs are widely used by the media and social actors to present succinctly the data that support their claims and political itineraries. They are used both to communicate analyses or results generated by some social actors and to highlight the problems that, according to the perspectives of the person who generates them, must be addressed. This places graphs as one of the main communication tools for informed. The statistically literate citizens can interpret chart information to form an opinion on the policies authorities implemented around a given social problem and exercise their citizenship when choosing any political or public action. In other words, citizens must have the ability to interpret and critically evaluate statistical information, data-based arguments, or stochastic phenomena that can be found in various contexts and based on their analysis, where relevant, act accordingly (Gal, 2002).

Image 1

Growth in the use of graphs in American newspapers (New York Times, 2016)



For example, the study reported by the New York Times (Figure 1) shows the increase of the graphs with statistical information present in the press. Then, the graphs, and therefore the information they deliver, constitute an important input in the citizenship action in the political life of their community. This shows how relevant it is to incorporate into school statistics teaching activities that strengthen the ability to interpret statistical information used in the media, mainly when using graphs. Therefore, to strengthen citizenship education, teaching must address the interpretation of graphs and their articulation, both with the social phenomena that it addresses and the meanings that emerge when reading the graph in various contexts, overcoming, in this way, a teaching focused on "school math" work with graphics.

Although there are several proposals for the analysis of statistical graphs, these, in general, are oriented towards work with accurate graphs and in contexts of statistical analysis (Arteaga et al. 2017; Cavalcanti & Guimarães, 2019; De Carvalho et al., 2011). However, the population has contact in several dissemination spaces with graphs, mainly pictograms, that do not always respond to the mathematical requirements of a statistical graph. In addition, the analysis the citizens make of a chart in the media does not necessarily advance in a reading that identifies implicit or explicit values but constitutes a reading that projects and infers elements associated with the journalist's proposal. It is a reading in which elements of context concur to the situation, as well as those

elements that emerge in the evocation of previous experiences triggered both by what is expressed by the news and by the elements of the statistical graphs, namely: numbers, shapes, colours. Then the semiotic character of the graph becomes central when interpreting the journalistic message. An interpretation that does not necessarily require a detailed data reading. For example, Image 2 shows the debate of the electoral campaign of the year 2017 for the election of the president of Chile.

Image 2

Graph Deliquency three governments. (Electronic Journal El Mostrador, 2017)

The candidate, a former president seeking reelection, uses a graph to reinforce his message, referring to his better performance against delinquency in his last government time compared to the current government. The graph presents the data of the government's closing year (2013) of the three previous presidential periods. In blue, the candidate's previous period, and the other two bars in red represent delinquency during coalition government opposed to the candidate. Looking at the data, we can say that he only achieved a drop of less than 5% compared to what is achieved by the current government and the 7% in 2010; however, the image connotes a different message. We can see that for the results of the candidate's government, the height of the bar is less than half the height of the bars of the governments of the other coalition. Thus, the image connotes a different message from that denoted in the numbers. It is a well-known message in which it seems that the candidate's government is much more effective (than the real thing) in fighting delinquency. This constitutes two central pictorial elements. First: the heights of the bars are generally interpreted as proportional to the value of the variable represented. Thus, the red bars are

more than twice the height of the blue bar; the connoting message is that in the candidate's previous government, delinquency was reduced by more than half than in the coalition governments of the other candidate. A second element is in red. Unlike blue, red refers in our culture to danger, therefore it induces us to say that the results of the coalition that supports the other candidate are dangerous. Then, without assessing the numbers in the graph, a message that does not correspond to the message denoted by the numbers is constituted. In this way, the graph, like any other image, becomes a sociocultural story that presents a narrative in which the various elements that compose it, and their syntax, allow communicating an idea beyond numbers.

This imposes an ethical element to work with the citizen's education in statistics. Considering the text by Huff et al. (2011) "How to lie with statistics", which shows several ways to give a wrong message based on statistical tools, it is necessary to discuss those aspects of the graph image that build meaning in those who observe it. Rather, it is a discussion that is part of an ethical citizen formation that develops skills to understand the use of strategies, such as those mentioned in the text of Huff et al. (2011), regarding the justification of the political message, especially when statistics, and therefore graphs, are used to support and strengthen a message. Assume that the processes of interpretation of a graph refer to the management of mathematical or statistical concepts, as observed in various approaches in mathematical education, makes invisible, or rather excludes from the classroom, the development of a critical reading of the graphic information that strengthens them as citizens.

In this article, we present elements for the analysis of graph interpretation given in the press that incorporates the complexity of a mathematical tool, that does not lose its quality of drawing, and therefore the denoted and connoted messages that the graph presents, and that makes it into a complex semiotic element.

Difficulties in interpreting statistical graphs.

Several studies have shown the difficulties of students and teachers in interpreting statistical graphs. Arteaga et al. (2017) show basic education students' difficulties when reading line graphs. Among the most relevant are: identifying the variable represented in the graph; for example, in a chart, they confuse the values of the annual averages of a variable with a point value of that variable; and the difficulty of the students to compare two distributions present in the same graph, making it difficult to identify the greatest trend.

Several studies (Curcio, 1987; Cavalcanti & Guimarães, 2019; Espinel, 2007, Fernandes & Morais, 2011; De Carvalho, et al. 2011; Inzunsa, 2015; Arteaga, et al. 2018) show how most students have difficulty reading the data, i.e., the explicit data.

In particular, Arteaga, et al. (2017) present a research synthesis on difficulties in reading statistical graphs. In particular, the research shows students' difficulty reading data that are not explicit in the graphs and the difficulties in tasks where they must infer information beyond the data. Most students only manage to read explicit data from the graph. Specifically about these studies, we highlight: the difficulty in the variational understanding of the graphs, referring to recognising the trend of change in the graph (Cavalcanti and Guimarães, 2019); the importance of the statement in the task and the context of the information of the graph in the production of inferences (Fernandes & Morais, 2011); and the answers focused on visual aspects (De Carvalho, et al. 2011).

On their side, Tal and Wansink (2016) show how reading the graph generates a greater belief in the effectiveness of a treatment, unlike when only written information is read. The mere presentation of claims, associated by those who observe them with scientific tools on the effectiveness of a product, increases the perception of product quality. The effects of graphs are maintained even when no additional information, not even implied in the graphs, is provided, and is not moderated by increased understanding or retention of information. However, the study does not make clear as to whether this effect is on the visual nature of the graph or its mathematical character, since incorporating algebraic expressions in a text also increases the reader's credibility. The prestige of science is at the base of an increase in credibility, i.e., the conditions would be: if the information contains graphs and this information contains graphical signs and scientific bases, then, the information has a scientific base. On the other hand, the graphs are easily understandable and do not necessarily give evidence of the scientific bases that legitimate the data. Consequently, the effects of including graphics in persuasive arguments cannot be due to the inferred content resulting from their opacity.

Pandey et al. (2015) carried out a study of information perception from misleading graphs. The study shows the effects of misinterpretation by people with different degrees of schooling. According to the research, several factors can modify a graph, such as: truncate the axis, starting the axis of the ordinates from a number other than zero; use pictograms without adequate proportionality in the area or the ratio of the axis; and change the area in which

it is plotted, using the fourth quadrant of the coordinate axis instead, even when the values of the ordinate are positive. Therefore, the authors found significant differences in the interpretation of the comparison between the values shown in the distorted graphs. In other words, the participants, who saw the misleading graphs, perceived an exaggerated difference between consecutive values.

In summary, several studies have shown the importance of addressing teaching statistical graphs in schools. On the one hand, even when the graphs at school are well constructed, research show that the students find them hard to understand, especially when trying to infer implicit information from them. First, because they are difficult to read and calculate, and second, because of the interpretation of the data in the context that signifies the graphs. On the other hand, the graphs have a role of drawing (which allows inferring the information from the pictorial elements that make up the image), the reading of the axes, the areas of the drawings or the ratio in which the data are plotted induce us to errors in reading and interpreting the data. This is important since graphs, especially when referred to as scientific, give truth value to the information they represent. In this way, the information is not questioned.

THEORETICAL ELEMENTS

In the complexity of education, in this section, we briefly present the theoretical references used in the research we are reviewing. Within the framework of socioepistemology, several articulated theoretical contributions allow us to describe relevant aspects in the processes of interpreting graphs released by the press. First, the enactivist perspective understands the cognitive act as a constant coupling of the world that is built with what we live, with the others with whom we live, and with own what is put into action. Second, the didactic characterisations around understanding statistical graphs and the image that allows us to perceive the graphic as a sociocultural narrative. These aspects will enable us to build a gaze that seeks to recognise in those who act that specific element that concurs to such actions.

Socioepistemological Approach.

The approach to the phenomenon under study is part of a socioepistemological perspective. This theory assumes that learning emerges from the intentional activity from social practices in which the student is involved. We understand that all construction of knowledge is social. This is

materialised in four principles: epistemological relativism, i.e., the validity of knowledge is given by the frameworks of work of each community that creates or uses them; contextual rationality, in terms of the relationship of the subject with the knowledge that is co-defined by the context lived; progressive resignification of the knowledge, which refers to the constant reconstruction of meanings of what is known from the activity in which we are involved; and social practices, in which all human activity is regulated (Cantoral et al. 2018).

Therefore, the classroom, rather than the physical space that groups the apprentices and the teacher, becomes an experiential space in which each student engages in intentional activities for learning. In this way, the noticing moves from the graphic object, and how this should be correctly interpreted by the person who acts and who uses the graph as a tool, i.e., the focus is on the practices of those who use mathematical ideas. Then the activity of interpreting graphs is a practice socially shared by a sociocultural collective, thus we must focus the analysis on recognising those common ways of acting of those who interpret and those who build the statistical graphs. Specifically, by focusing on the press graphs, we have at least two different and diverse social groups that are in work with the graphs: on the one hand, those who build the graphic (politicians, journalists, statisticians and graphic designers) from an editorial line on which they want to communicate something; and on the other, those who read the news and observe the graph aiming to interpret it in the context of the news. This last group, corresponding to citizenship that does not necessarily share the practices of those who built the graph, perceives it more like a drawing than a mathematical tool.

This work focuses on the graph interpreter, it notices the construction of meaning, therefore, focuses on the cognitive. Then it becomes necessary to explain what concurs to the execution of the cognitive act. For this, a look into the cognitive act is articulated from the enactivist perspective (Varela, 2000). In this perspective, unlike representational perspectives, in which learning is to build an adequate representation of reality, learning is a constant adaptation and construction of ideas agreed with those with whom we share the various contexts. Ideas that are socially shared from emerging in community life.

Maturana and Varela's contributions (2004) in the cognitive sciences show that knowledge is given in a mind embedded in the body, an embodied mind. Starting from recognising it from its biological constitution, as a network of neurons that cooperate with each other, knowing has its first action in the specific states of cooperation of the neural network, leaving the banking metaphor (proposed by Sfard 1998) in which the mind is a bank that stores

knowledge to then manage it. Knowing, then, is a particular configuration of our neural network and, therefore, our corporality that emerges before the need to be properly in the world where we must live. For example, when a person appears in our field of vision, the information that reaches our vision centre changes the states of a set of neurons, which motivates a network of neural interactions until a new stable state of cooperation emerges, we recognise who that person is or whether he/she was introduced to us. Thus, knowing refers to our neural network and its states of dynamic configuration. This dynamic does not necessarily depend on what happens in our environment; rather, the environment disturbs our mind. For example, there is the dream, which we live as real, even if it has no direct relationship with our present environment. An external element, for example, an alarm clock, impacts the sleep, ending it suddenly or, when we are very tired, bringing out an image consistent with the dream (a siren or bell that gives meaning to the sound), which makes it hard to recognise the dream. In other words, our cognition generates a coupling of its structure of sense with reality, generating an idea that protects the resting space. However, as the need to work gradually emerges, we return to the vigil. Thus, knowing is a constant coupling what we have known and that relates to what is being lived. This putting in articulation what we know is understood as enacting what is known. Then, learning constitutes the coupling of our world, modifying it so that it incorporates the meanings of what allows us to adequately exercise the activities in which we are involved (Varela, 2000). Therefore, knowing is a living process that always overlaps the corporeal, the interactions with the other and mainly with others, thus being inseparable from the subject's sociocultural environment.

Therefore, learning a mathematical idea emerges from the constant adjustment of what is enacted in the learner's mind, which is modified when developing the activity. This activity involves the subject who learns and those who live with him/her in this process. This implies putting into action what we know, and that has been useful to us in the lived experience of similar experiences, i.e., we put into action the experiential. For example, when a student performs a known math strategy or states, before an exercise presented by the teacher, that he/she is unable to solve the problem and expects the teacher to assign it to another student. This is a successful solution as it frees the first student from the problem. But it also implies putting into action the perceptual aspects, which not only refer to what the senses generate, but to the meaning of what is perceived. In other words, our action involves the experiential, the perceptual, and the operational (Correa, 2011). These categories allow us a description of what concurs to the experience of those who build knowledge.

In short, we seek to build a noticing on the learning of statistical graphs that assumes a sociocultural dimension, in which social practices condition the action, but do not define it unequivocally. A noticing that includes the knowing subject, who, based on what he/she has built in his/her experiences, can offer his/her action while in constant interweaving with others and with the context. Socioepistemology, as an articulating framework of investigative noticing, has focused on the didactic activity guided by social practices and the learnings that are evidenced. Then, we made progress by incorporating tools that allow us to deepen the experience lived by the students. In this way, we can reveal those elements that concur to the cognitive action of knowledge in the framework of the development of an activity. A knowing that, as already mentioned, is always a knowing in constant imbrication with what is lived.

Graphic interpretation as figuration

A figure is understood as the set of lines that seeks -in two dimensions- to make ostensive to the noticing some visible or invisible aspects of an angle of reality. Therefore, it has communicative intentionality by highlighting some elements of others. In particular, the graphs in the press constitute a narrative constructed by pictorial aspects that whoever constructs it wants to communicate. Then this story becomes meaningful only when it is interpreted, i.e., when a subject makes meaning of its elements in terms of what is represented. This make-meaning process is defined as figuration of the phenomenon. A process that encompasses both the construction of the figure and its interpretation.

The approach to graphs has been extensive and mainly from a Peircean tradition, in which the elements of the graph are conceived as a set of symbols that, when interpreted, carry meanings of things from the external to the internal world, guiding the meaning of things (Presmeg, 2008). For example, Cavalcanti et al. (2010) point out that statistical tables and graphs, as forms of representation, allow transmitting information given that it summarises a large amount of data in a small space and allows sharing information quickly. Then, many investigations focus on how students or people properly use the symbol system to interpret the information presented in the statistical graph. This leads to reports that point out the distance between those meanings that the educational institution considers appropriate and those personal meanings that are considered inappropriate. They also show how those signs present or accurately interpret a reality external to our mind. However, an interpretation of the graph will always be mediated by whoever does it. For example, Noss et

al. (2007) show that only the joint work between a technician working with machines and the engineer who constructs the performance graphs of the machinery allowed them to interpret a possible failure based on the graphs. In other words, you cannot do without the experience of those who work with graphs.

Then, assuming that knowing is a constant coupling between the world built by the subject and what is lived, the interpretation of the elements of the graph are mediated both by the experience of the interpreter (referring to what is built in their school and non-school history regarding the interpretation of figures), and by the actions they perform to interpret (mark specific areas to observe eye follow-ups of the elements) and the perception developed.

Perceptual elements in terms of graphs must be perceived, focused on vision. Here we refer to the decoding of the pictorial elements by the eye and to the perceptual phenomenon in its complexity, which leads to the visualisation of those elements, both in the volitional noticing, which seeks to encompass and order the pictorial elements (Villafañe and Mínguez, 2006), and in the image construction capabilities, which the Gestalt current recognises in the act of visualisation (Costa, 2003). The operational elements allow us to focus the look on the graph, intervene it and/or infer non-explicit information and the experiential elements concur to the action of those who interpret. Thus, the interweaving in the interpretive activity of the perceptual, the operational and the experiential allow us to understand the activity of interpretation of a graph.

Towards the interpretation of statistical graphs

For teaching statistical graphs in schools, several proposals characterise stages of skills or strategies to understand them. One of the most used today is Curcio's (1987). This author suggests that we need reading skills to understand graphs. In this model, he proposes:

- Reading between the data: consists of the literal reading of the graph without interpreting the information contained in it.
- Reading within the data: implies the interpretation and integration of the data of the graph; this ability requires the comparison of data or the performance of operations with them.
- Reading beyond the data: consists of making predictions and inferences from the data on information that is not directly reflected in the graph.

- Reading behind the data: is associated with a critical assessment of the representation of the data, the type of graph, how to obtain them, or the conclusions obtained. For example, to analyse whether the sample considered in the study is adequate.

These levels involve a sequence that would seem logical from item (a) onwards. However, as noted, the graph constitutes a narrative of a portion of reality that the author wants to show. In this way, like any image, its interpretation passes through the willful exploration of the beholder. An exploration that considers the identification of the elements of the figure, their ordering and interrelation, i.e., recognising the syntax of the graph. This syntax is normalised in the mathematical-statistical activity and involves those who have learned it from the elements and meanings shared by said community. Thus, the statistical graph, or rather its parts, are shaping meanings in those who work with the image in an integral process that considers the formation of the retinal image, the nerve impulses generated in the biological process of the eye and transported to the vision centres, where it is transformed into the visualised image. This last process occurs in the concurrence of ocular information with those worlds that are connected in the act of seeing and that have been built in the life history of the subject.

Then, in the statistical graph as a sociocultural image, we can recognise three messages: (a) The linguistic message, supported by texts and letters in the image that help anchor the message we wish to deliver when the figurative elements are considered ambiguous; (b) The connoted message, in which the signs present in the image come from a cultural code and therefore are possible to be interpreted by those who work with the figure; it conforms the rhetoric of the image that is specific because it is subject to the physical requirements of the vision, but it is also general, while the "figures" are formal relationships of elements; and, (c) The denoted message, which is the private message of the image as it is constituted by what remains in the image when the signs of connotation are (mentally) erased; it also constitutes a sufficient message as it identifies the figurative scene.

The latter becomes central in work with the interpretation of press graphs. If we consider that pictograms are mainly used, we have that the colours, axes, figures and proportions, among other elements that compose it allow us to deliver a message that complexifies Curcio's (1987) reading levels since they form a cultural narrative, loaded with intentionality about the interpretation that the editorial line of the publication wants to pass to the population. Thus, each

Curcio's (1987) reading level is mediated by the reader's intentionality, in which perceptual and experiential elements that allow perceiving the denoted and connoted messages of the statistical graph concur to the students' activity.

In this way, we are interested in revealing the practices of those who interpret graphs. Practices that we characterise based on those operational elements that concur to the activity. These are understood as tools that are made available to the interpretative activity. Specifically, the tools will be perceived as elements that concur to achieving a goal assumed by the subject (Vigotsky, 1986). For Freudenthal (2001), concepts, structures and ideas are created as tools to organise phenomena of the physical, social, and mental world. In other words, the tools are understood as entities that rally with the intentionality of use to interpreting graphs. Then, the tools become central to the study of figurative practices since the latter generate tools that, available, favour knowledge and the modification of those practices (Ferrari & Farfán, 2008).

Another central aspect in the description of the practice is the arguments. They are understood as systematisations of the experimentally constructed, allowing us to explain and support reality (Maturana, 2001). They are a key element for understanding experiential aspects put into action. And, finally, it is interesting to reveal those meanings enacted by those who exercise figurative practices. Following Wittgenstein et al. (1988), for whom seeking the meaning of something is seeking its use, we understand in this article that the meaning is mediated by the subjects' use and interpretation, by the sociocultural contexts and their rationalities, and by the situations that concur to what is being studied (Espinoza & Cantoral, 2011). Hence, the meaning is configured by the explicit or implicit connotations of the elements of each figure.

In summary, describing what concurs to the performance of an intentional activity by interpreting graphs implies focusing on the tools, arguments, and meanings that facilitate the interpretation of statistical graphs.

Two studies that allow us to move forward

The complexity in the processes of interpreting statistical graphs and, in particular, the contribution of experiential, operational, and perceptual elements together is evident in the two investigations on the interpretation of press statistical graphs: "The experiential, the operational, and the perceptual in the interpretations of Cartesian graphs" (Carrasco, 2016) and "Perceptual, operational, and experiential aspects present in the activity of interpretation of graphs used in the press" (Carrasco & Oviedo, 2019). In these studies, we

present a summary of these processes, following this structure: summary, methodology, discussion, and conclusions.

We seek to resignify the results of these articles by a retroactive view, which is not intended to be true to the moment of development in the publication of that article but seeks to evidence through these results the relevance of the categories and inquiry strategies that have been presented in the previous sections. It is a reflective moment in which noticing what has been done allows us to establish where we are and offer elements to advance towards didactic designs that strengthen mathematics for critical thinking and, consequently, for citizenship.

Carrasco (2016) analyses data obtained in his doctoral work. In his work, the researcher applies a written test to high-school 3rd graders from a low socioeconomic level school. We want to investigate their skills in interpreting two graphs published in the press. The first, a bar graph, reports college selection test results. The second, a line graph, reports the evolution in acquired human immunodeficiency virus (HIV) in men and women over a given period. In the second research, Carrasco and Oviedo (2019) deal with the elements that concur to interpreting a biased statistical graph used in a political campaign. Four academics (three with a postgraduate degree in statistics) were interviewed. The semi-structured interview had three moments that we can classify from Curcio's levels (1987). The first, reading the data; the second, reading beyond the data, asking for recommendations from the decision-makers about what they saw in the graph; and the third, reading between the data, requesting an interpolation.

METHODOLOGY

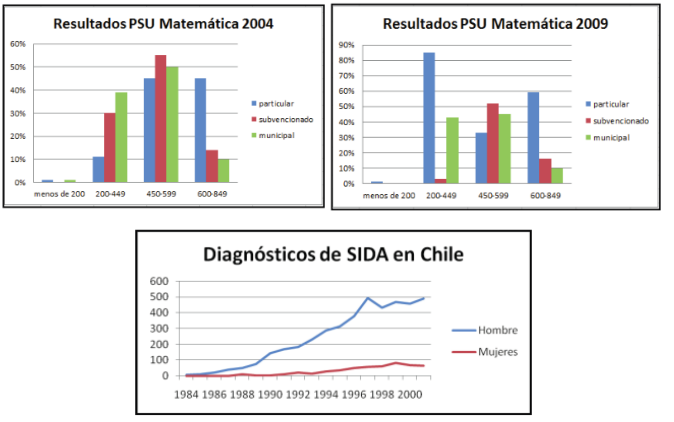
In the first research[1], the data collection was made from a questionnaire applied to high-school 3rd graders (11th year of compulsory schooling in Chile) from a low socioeconomic level school. The test proposes situations of interpretation of two graphs of variation obtained from public websites. The first graphs (Image 3a and 3b) show a histogram with data from the results of

1 University did not require this research to get the approval of the Ethics Committee. The second investigation applied informed consent form. The journal Acta Scientiae is exempt from all liability for any consequence that arises from the damage resulting to any of the participants of each research reviewed according to Resolution No. 510, of April 7, 2016, of the National Health Agency.

the PSU (Prueba de Selección Universitaria [University Selection Test]) in mathematics, which every student who wishes to enter Chilean public universities must take, according to official publications of the years 2004 and 2009, presented according to the school hierarchy (municipal, subsidised, and private) and scoring intervals. The graph (Figure 3c) is a line graph showing the number of diagnoses of HIV-positive cases in Chile by gender between 1984 and 2000.

Image 3

Student questionnaire graphs (Carrasco, 2016)



a) Outcome graphs University Selection Test (PSU)
b) Graphs of HIV-positive infections

The questions in both sections of the questionnaire are open and focus on variational and predictive aspects in the PSU graphs and trends in the HIV-positive graphs. With this, we expected their textuality to reveal aspects of the graph that are considered relevant in predicting situations regarding the trend, the comparison of situations, and generalisations of the information presented.

On their part of the study, Carrasco and Oviedo (2019) interviewed four professors lecturers in statistics. The interview had three moments. In the first, the questions were oriented towards the levels of reading data and beyond the data by requesting the graph description (see Image 4), and projective recommendations about unemployment; in the second, they were oriented towards explaining skills to extract information between the data, requesting

interpolation of values; and, finally, in the third moment, they were asked about the evaluation of the graph based on the respondents' position as professors.

Image 4

Biased graph on unemployment behaviour (Diario El País, 2015).

Both investigations -in their exploratory character- make it possible to obtain a body of textuality built by the research subjects. In the first case, the students' written answers to the questionnaire, and in the second case, the transcripts of the teachers' answers to an interview.

In research with students, the interpretation involved tabulating students' answers. In a first reading, specific phrases of each response were selected and classified and grouped by similarity of meaning. At a second time, the set of sentences was interpreted, and then its textual analysis was carried out, determining categories of analysis from open encodings.

From this, the codifications were grouped into the initial categories regarding whether they reported on operational, perceptual, or experiential aspects. The results, then, are presented around these categories. Since the graphs intervened by the students were not collected, we could not analyse the figures, so we focused on analysing the discourse of the questionnaire answers.

RESULTS

The elements that can be detected after the investigations reviewed are related to what occurs when press graphs are interpreted. Students and academics alike demonstrate similar strategies and tools when interpreting graph information. As an example, Table 1 presents textualities that illustrate the moments the high school 3rd graders analysed the graphs.

Table 1

Student Textualities (Carrasco, 2016)

Category	Textualities
Operational	"In private (schools), in 2004, between the 200-459, 450-599 and 600-849 (PSU test scores) were high, but in 2009, between 200-459 and 450-599, they decreased", "instead, in 600-849 in 2004 they were higher and continued to increase in 2009, in the subsidised in 2004 they were high, instead, in 2009 some remained, and others decreased a lot" (E2). "Municipalities in 2004 in 200-449 did not reach 40%. Individuals did not reach 60% in 2004 in 600-849. The results of 2009 compared to those of 2004 show a decrease in the percentage of students from private schools in scoring 200 to 449 in the PSU" (E13).
Perceptual	"If there is a drop for men, in 1997, it was very abrupt" (E10). "Well, as seen above, we see a drop between a few years that were probably slow, but on the other hand, also there was a strong boost, which made them lose the place" (E10).
Experiential	"That men are more likely to be infected because, socially speaking, men tend to be womanisers and not only that, we have seen a lot that today homosexuality and the disease tend to be brought from abroad" (E24). "It is very noticeable the number of men with AIDS, every year there are more men than women. They have mutual relationships, and so AIDS is spread" (E49).

Especially, the students' interpretation is marked by the first question "what can you say about the data?" To answer, the students make tabulations to help them order the graph elements and information. As shown in Table 1, the descriptions are tabular even if the formats are diverse.

The textualities in table 1 show operational elements that allow the graph components to be sorted. As a first tool are the tabulations of the data that are expressed in the segmentation of the years compared. A second tool is the

constantification, which refers to considering only some variables, leaving other constants aside when analysing a phenomenon (Cantoral, 2016). The students that interpreted graphs in the tabulation process leave one variable constant to compare the other two. This is shown in the qualitative comparisons with respect to the scores, leaving the year as the cases in which it is compared. The year is constantified for comparison. Qualitative comparisons are also used: "more than" or "less than" are phrases to which they resort, without quantifying the differences. It is noteworthy that some comparative phrases such as "it was higher" and "it continued to rise" place a continuity to the phenomenon, which is seen as discrete, but that students understand the temporal continuity of the test that is repeated year after year. It is an experiential element that is embedded in the use of the comparison tool. We can summarise the students' work in table 2.

Table 2

Perceptual, operational, and experiential elements in students

Category	Textuality
Operational	They resort to tabulations. The comparison with the previous state, (not quantified), is used as data for each case. They establish intervals for the variables, mainly the time variables, such as the years of study of AIDS infections. They compare separate curves; no more than two values and, in short, only one, since the other is constantified at intervals given by significant scores. The comparison of the intervals is given in two ways: by outlining maximum or minimum values, or by indicating growth intervals.
Perceptual	It was a perceptual space, in that the graph was not signified from the scores but from the intervals, and the evolution of the variable is traced. Thus, the graph is interpreted on trends, They minimise drops, highlighting only significant drops in a perceptual strategy that ignores minor

	changes in favour of a more simplicity figure, i.e., from cognitive tools that privilege regularity and, therefore, the omission of small variations. It is the globality that marks it. They identify critical points where there is change, or a maximum, or a minimum.
Experiential	It was an experiential space since, in the students' interpretations and projections, the graphs act both as triggers of a reflection whose arguments come from their social being as a resource to justify their assessments.

On the other hand, in the result of the research with the professors, we see that they interpreted a statistical graph from the information that was given in the graph. With this, their interpretations were characterised according to the perceptual, operational, and experiential aspects. Moreover, they were invited to propose possible questions that they would ask their students for the interpretation of statistical graph information. These professors focused on questions to identify formal aspects of the graph, not presenting questions regarding decision-making or interpretation of the information according to the given phenomenon.

In the first part of the questionnaire, two questions guide the interpretation: What did the graph present? And: What would you recommend to decision-makers? The first was intended to promote in the interviewees their willful exploration of the elements in the graph. This question is understood as posed by Villafañe and Minguez (2006), when they describe it as an exploration of the elements of the image that allows identifying its components and the context that the information has.

The following textualities show the result of this volitional exploration, which highlights the significant elements for the interpreter:

> [A-1-CH] These will be the quantities... they are quantities of unemployed people... the scale, the scale here on the axis is by year... and... the axis... well, the press never puts an axis... but down here you could extrapolate that as the number of

> unemployed... then down here I see that or the trend of the unemployed.

Here we see how the professor seeks to identify the different elements that make up the graph until recovering not only its structure but the phenomenon that is being modelled with the graph. It should be noted that, again, experiential elements allow him to understand the graph. He knows that the press does not always respond to the accuracy of a graph. Tabulation and qualitative comparisons are also part of the operational repertoire presented by the professors interviewed, as evidenced in the textuality.

> [A-1-P] According to this graph, years 2007 to 2014, it is notorious that from before 2007 to 2012 unemployment has been growing, has grown from 2,129, 547 to 4,848, 723. Since 2012 unemployment has decreased from 2012 to 2014, it has decreased by approximately 300,000 job posts. Among other things, we observe that from 2007 to 2009 unemployment has been growing linearly, from 2009 to 2012 the behaviour of unemployment growth has been almost parabolic, and from 2012 to 2014 the decrease has been almost linear.

Then, the second question of the first part, "what would you recommend to decision-makers?", is within the reading level given by Curcio (1987), "reading beyond data". Although the respondents were recognised as experts by being invited, the answer was a perceptual reading of the graph. An example:

> [B-3-P] I think whoever created this graph wants to tell me that since 2012 there has been a change and that this change has been positive because look, that trend in unemployment has changed and that things are improving. But to get a better diagnosis of that, we need a bigger picture
>
> [B-1-CH] What I would recommend… would be that they go on doing what they have been doing. Jobs are down.
>
> [A-3-P] The recommendations are fundamentally of a political nature of the governments so that there is more employment in the free-market model. They propose that investments should be encouraged because that would imply less unemployment and, on the other hand, that the state must implement or must invest in the economy to reduce unemployment. The

> government is an important factor to avoid these social problems of an unemployment nature.

As we see, three of the four academics interviewed recommended continuing with the policy based on the graph. This reading is perceptually oriented from the trend that the curve presents. The graph incorrectly amplifies the decrease in unemployment, which was only noticed by one of the interviewees. The other professors focused on the perceptual assessment of the slope and the curve, signifying a connoted message assumed to be true about the behaviour of unemployment. Moreover, when one of the interviewees needed to interpolate two values, he did not answer, preferring to write the formula to get the midpoint of a segment. Being asked why he did not use the values, he says: "that... if I have the ***a*** and the ***b***, only that it is very long to write, but I have them, and then I generate an equation of the y = mx+n type..." [B-1-CH].

In short, it was possible to recognise in several moments of the interviews that Curcio's (1987) reading levels concur to the graph interpretation: perceptual, operational, and experiential aspects, as shown in Table 3.

Table 3

Perceptual, Experiential, and Operational Elements in Professors (Carrasco and Oviedo, 2019)

Curcio's stage	Perceptual aspects	Operational aspects	Experiential aspects
Data reading	Determine the maximum visually, the highest point Verify whether the curve grows or decreases	Determine critical points Tabulation Estimate relevant variation (decrease)	Curve type Linear; Quadratic
Reading between the data	The length of the numbers Black colour The top of the sheet is a higher value	The equation of the line: (a) calculate slope; (b) calculate y-axis intersection; (c) calculate ordered value.	Use linear interpolation

Reading beyond data	The slope of the graph curve	Identify a maximum and describe the inclination of the curve.	Knowledge of economics They assume that the change is the product of governmental measures.

Table 3 shows the possibilities in the description of the graph interpretation processes by the different subjects. Both studies show how the concurrence of the experiential, operational, and perceptual elements shape ways of interpreting graphs at the various levels of reading.

Then, we are surprised that when interpreting -given the need for projective decision-making, which implies reading beyond the data, the professors did not explore in detail the numbers and quantities of the graph. This shows that the connoted message, referring to a significant decrease in unemployment, presented in the unemployment graph (Figure 4), manages to install itself in the interpretation of at least three of the four academics interviewed. This overlapping the message denoted by the numbers, which allow us to interpret that the range of the decrease in unemployment is much lower than that shown. Reality is shaped in an intentional political-social context and not adjusted to the real. Therefore, it is considered essential to strengthen students' capacities to analyse these situations and promote debates on these issues.

DISCUSSION

The two investigations analysed shed light on what concurs to the interpretation of the statistical press graphs. They allow us to understand specifically how the process of interpreting the press graphs always involves the interweaving of the experiential, operational, and perceptual aspects, which shape interpretation practices shared by the various subjects interviewed.

Firstly, when we seek to recognise and interpret the elements that make up the graph, the tabular description of the data graph emerges as an operational element. This data tabulation forms the first step of interpretation, constituting a look at the level of reading in the data that focuses on qualitative comparisons.

Therefore, it is not necessary to determine the values of each point on the axes to establish minimum and maximum moments or changes in trend; but, based on perception, the position "higher" or "lower" of the point on the sheet is sufficient to make a comparison. In this way, the axes are constituted in frames of the curve, and the frame of a figure is not always observed.

This allows us the first interpretation of the message denoted regarding the population behaviour of the variable or variables present in the graph; for example, a professor answers, "Among other things, we observe that from 2007 to 2009, unemployment has been growing linearly, and from 2009 to 2012, the behaviour of unemployment growth has been almost parabolic...", while a student answers, "The results of 2009 compared to those of 2004 show a decrease in the percentage of students from private schools in scoring 200 to 449 in the PSU".

Since the interest in this article is to advance in the reading of press graphs, the above is paramount since both students and professors do not resort to specific values when interpreting the graph information. Still, it is the appearance of the curve that prevails when interpreting. Therefore, the reading of values or quantification of comparisons emerges in the face of specific tasks that request it or in the specific work or school framework of the respondents. Thus, it is only when facing a direct request to calculate or when asked to evaluate the graph as they do with their students that the professors pay attention to the values and their relationship with the axes, identifying then that the graph presented in Image 4 is wrong and induces them to oversize the unemployment rate. Regarding the students, who had not been asked specific questions about quantifications, qualitative comparisons were sufficient for them to conclude and explain the phenomenon.

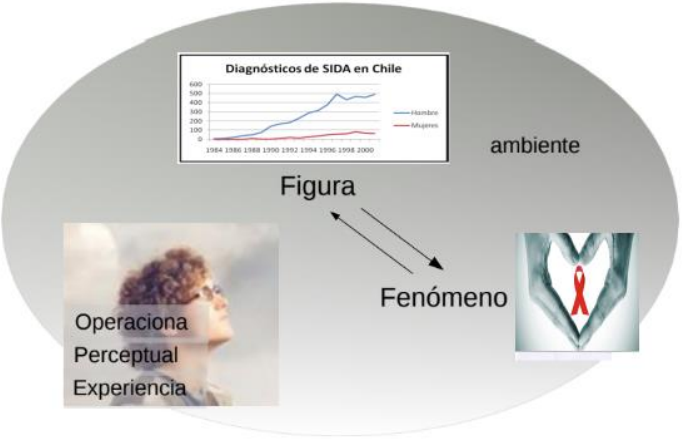
Then, in the face of reading the press graphs in a non-school or professional context, the practice shared by the interviewees refers to a mainly qualitative reading of what is shown, in which the perceptual recognition of the trends -and from there the projection of the behaviour of the graphical variables- configures a reading at the "beyond the data" level. In this sense, professors and students raise explanations and/or projections of the phenomenon from the connoted meanings that emerge by their reading the graph. A reading that starts from the perception of the extreme points and trends that are offered to the eye, as well as from the already known information that appears with respect to the graphical phenomenon. For example, students refer to prejudices about male sexuality to understand and describe the fact that there were more female infections; while professors are not surprised by the

statistical deficiencies of the graph, since their experience validates these types of errors in the press. We strengthen this idea in the fact that, when asked to evaluate the graph in their position as professors, they changed the practice of graph interpretation for an evaluative practice. In the assessment, tools concur to verify that the elements that compose it respond to the proper syntax of the statistical graph. In other words, the experiential is always present and plays a central role in the process of interpreting the graph.

This makes up an epistemic space of work with graphs, in which an articulation between the phenomenon and the statistical graph is lived in the several moments when the practice of interpretation imbues the operational, the perceptual, and the experiential.

Image 5

Schematic epistemic space of figuration (Carrasco, Díaz & Buendía, 2014)



Some tools concur to this epistemic space: tabulations, qualitative comparisons, proportional comparisons, graph intervention highlighting relevant points with another colour or stroke thickness, among others; in which arguments emerge, combining the experiential with the interpretation of the graph. For example, for the students, the infection is debauchery in the association with sexuality as the main source of contagion. Therefore, men, who are culturally more libertine, are more infected (Carrasco, 2016)

And finally, the interpretation and, therefore, the formation of the activity in the epistemic space is mediated by the intentionality of the activity

with the statistical graph. It is the intentionality of the analysis that makes the various levels of reading necessary or not. Therefore, when the intentionality was the simple interpretation (in the students) or the recommendation to the authorities (in the professors), the message that prevailed was the one connoted by the figure, therefore, it is a message that does not necessarily conform to the explicit data but responds to the arguments and meanings that imbricate the previous experiential knowledge -statistical or of the associated contexts- of the one who interprets them.

To the extent that every interpretation includes the construction of the experiential, it presents us with the possibility of strengthening its teaching in statistics as a tool for the construction and transformation of society. If we assume that the graphs are a summary of the most used information and therefore, they allow us to interpret and critically evaluate the statistical information visually (Contreras et al. 2017), their management is of great importance for the citizen culture. Therefore, statistics teaching cannot be limited to just reading the numbers in the graphs. The central part must focus on the critical analysis of the denoted and connoted messages a graph shows, especially the connoted message, which, as Barthes (1986) recognises, is a message that carries a semiotic load of the one who constructs it. Therefore, it responds to the communicative intentionality of the one who creates it. With this, we can say that the idea of teaching a critical statistical education owes to the fact that students are expected to be immersed in political and social issues relevant to their reality as citizens living in a democratic society and have the power to criticise every situation that arises in their lives (Campos, 2016).

CONCLUSIONS

Statistical knowledge, which is present today at all educational levels, has the potential to concur to the future life of citizens, and the personal development and understanding of other areas of the curriculum (Batanero et al. 2002). In this work, we focused on understanding graph interpretations that citizens must make for their lives. From the results obtained in this research, although the study samples of both students and professors were small, we consider that the citizens' cognitive demand when interpreting statistical press graphs implies perceptual, operational, and experiential elements. And this shows a valuable input to be considered in the teaching and learning process of statistical graphs.

Although there are perceptual, operational, and experiential elements at each reading level, we can highlight that they are presented with different emphasis. Thus, at the first level, the greatest emphasis is on the perceptual, since key elements in the graph and its structure must be identified. At the second level, the emphasis on the operational allows us to rebuild the message denoted by the image, i.e., those meanings that the elements of the image make explicit. Finally, at level three, emphasis is placed on the experiential elements, typical of the subject's life history, that allow him/her to develop inferences about the phenomenon forming the connoted message (Carrasco and Oviedo, 2019).

Therefore, in terms of advancing in a didactics that addresses the learning of statistical graphs in the press, the epistemic space of figuration offers a framework to focus the noticing and action on the activity of the student that seeks to articulate the statistical graph with the phenomenon that it presents. It is a graph, which, as a figural account, offers a message with editorial intention. Thus, the process of interpretation exceeds the decoding of the message denoted in the mathematical syntax of a statistical graph but constitutes the discursive intention of the person who constructs it. A message that is interpreted in the message known from the sociocultural condition shared by the journalist and the reader. Therefore, to advance in the achievement of statistical learning that empowers students as critical readers of statistical information in the press, we must develop didactics that demonstrate the concurrence and articulation of the experiential, operational, and perceptual elements that emerge in the activity of interpretation. In this sense, the work with statistics professors reveals that circulating between reading levels contrasting interpretations allows us to tension the connoted and denoted messages of the statistical graph, strengthening a critical reading of the information.

Thus, we must consider and deepen research on how the perceptual, operational, and experiential aspects intervene in the interpretation of press statistical graphs. The two studies reviewed, starting from a socioepistemological perspective that places as a framework for the complex epistemological noticing an imbrication of the cognitive, didactic, and social dimensions, made it possible to integrate into the noticing the contributions from Curcio's (1987) reading levels, and enactive contributions to the enactive proposal of knowing. This moves forward in characterising and interweaving in the epistemic space of figuration the aspects that concur to interpreting statistical graphs showing in the class sessions the inseparability of the statistical graphs and the students' experiential contexts. It is, therefore,

necessary to advance in the design of learning situations that incorporate the sociocritical, and thus ethical, discussion constant, associated with the function of the graph as a tool of social communication: "teaching as the learning of statistics must not be exclusively focused on knowledge, but the development of thought dispositions and the social dimension of beings must be taken into account" (Eichler & Zapata-Cardona, 2016, p. 77).

AUTHORSHIP CONTRIBUTION STATEMENT:

All authors made substantial contributions to the work and take full responsibility for its content. Similarly, all those who have made a significant contribution to the article have been considered authors.

DATA AVAILABILITY STATEMENT

The data are available upon request to the first author of this work, who undertakes to share them with those who require it.

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