ACTA SCIENTIAE

ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.6237

Prospective Teachers' Difficulties in Integrating Technology Into Problem Solving and Teaching on Rational Numbers

Antonio González [a]
Alberto Arnal-Bailera [b]

[a] Universidad de Sevilla, Departamento de Didáctica de las Matemáticas, Sevilla, Spain
[b] Universidad de Zaragoza, Departamento de Matemáticas, Zaragoza, Spain

Received for publication 18 Nov. 2020. Accepted after review 25 Mar. 2021
Designated editor: Claudia Lisete Oliveira Groenwald

ABSTRACT

Background: The use of technology in mathematics teaching is fundamental because it enables students to activate basic mathematical processes. This makes it necessary to carry out studies to identify the prospective teachers' knowledge, so technology can be integrated into their teaching. Models such as TPACK have been developed precisely to analyse the results of this type of study. **Objectives:** Describe the prospective teachers' challenges to integrating technology into their explanations. **Design:** The study carried out is exploratory, with a descriptive purpose. **Settings and participants:** The research was carried out with a sample of 47 pairs of prospective teachers from the University of Zaragoza. **Data collection and analysis:** We used a data collection tool consisting of a task that involves solving a problem of products of fractions and designing the corresponding explanation for some hypothetical students of the early years, with and without technology. These data are analysed under the TPACK framework. **Results:** We identified and analysed the difficulties that our prospective teachers present in the face of some relationships between technology and content, and others of a pedagogical-mathematical nature, to relate different interpretations of the rational number adequately, and their tendency not to include technological tools to design their explanations. **Conclusions:** Our analysis allows us to propose actions to improve our teachers' education to include technology in their classes.

Keywords: Rational numbers; teacher education; ICT; mathematics education; explanations; educational research.

Corresponding author: Alberto Arnal-Bailera. Email: albarnal@unizar.es

Dificultades de maestros en formación integrando tecnología en la resolución y enseñanza de problemas sobre el número racional

RESUMEN

Antecedentes: El uso de tecnología en la enseñanza de las matemáticas tiene una especial importancia debido a la capacidad de esta para promover que los estudiantes activen procesos matemáticos básicos. Esto hace necesario la realización de estudios que permitan identificar los conocimientos que tienen los futuros docentes para poder integrar la tecnología en su enseñanza. Modelos como el TPACK han sido desarrollados precisamente con el propósito de analizar los resultados de este tipo de estudios. **Objetivos:** Describir las dificultades de los futuros maestros a la hora de integrar la tecnología en sus explicaciones. **Diseño:** El estudio realizado es exploratorio, con una finalidad de tipo descriptiva. **Contexto y participantes:** La investigación se realiza con una muestra de 47 parejas de maestros en formación de la Universidad de Zaragoza. **Recogida de datos y análisis:** Utilizamos un instrumento de recogida de datos consistente en una tarea que supone la resolución, y diseño de la correspondiente explicación para unos hipotéticos alumnos de Primaria, de un problema de productos de fracciones con y sin tecnología. Dichos datos son analizados bajo la óptica del marco TPACK. **Resultados:** Identificamos y analizamos las dificultades que nuestros estudiantes para maestro presentan ante ciertas relaciones entre tecnología y contenido y otras de carácter pedagógico-matemático para relacionar adecuadamente distintas interpretaciones del número racional, así como una tendencia a no incluir herramientas tecnológicas en el diseño de sus explicaciones. **Conclusiones:** Nuestro análisis nos permite plantear acciones para mejorar la formación de nuestros maestros en la inclusión de la tecnología en sus clases.

Palabras clave: Números racionales; formación de profesorado; TIC; educación matemática; explicaciones; investigación educativa.

Dificuldades dos professores em formação para integrar a tecnologia na resolução e no ensino de problemas sobre o número racional

RESUMO

Contexto: O uso da tecnologia no ensino da matemática tem uma importância especial devido à sua capacidade de promover que os alunos ativem processos matemáticos básicos. Isso torna necessária a realização de estudos que identifiquem os conhecimentos que os futuros professores possuem para integrar a tecnologia ao seu ensino. Modelos como o TPACK foram desenvolvidos justamente com o propósito de analisar os resultados desse tipo de estudo. **Objetivos:** Descrever as dificuldades dos futuros professores ao integrar a tecnologia em suas explicações. **Design:** O estudo realizado é exploratório, com finalidade descritiva. **Ambiente e participantes:** A pesquisa é realizada com uma amostra de 47 pares de professores em formação da

Universidade de Zaragoza. **Coleta e análise de dados:** Usamos um instrumento de coleta de dados que consiste em uma tarefa que envolve a resolução, e design da explicação correspondente para alguns alunos hipotéticos do ensino fundamental I, de um problema de produtos de fração com e sem tecnologia. Os referidos dados são analisados sob as lentes da estrutura TPACK. **Resultados:** Identificamos e analisamos as dificuldades que os nossos professores em formação apresentam perante certas relações entre tecnologia e conteúdo e outras de natureza pedagógico-matemática para relacionar adequadamente diferentes interpretações do número racional, bem como uma tendência de não incluir ferramentas tecnológicas na concepção das suas explicações**. Conclusões:** Nossa análise nos permite propor ações para melhorar a formação de nossos professores na inclusão da tecnologia em suas aulas.

Palavras-chave: Números racionais; formação de professores; TIC; educação matemática; explicações; pesquisa educacional.

INTRODUCTION

Information and communication technologies have produced numerous changes in learning and teaching, not only because they offer new opportunities to students, but because they have affected teaching methods and teachers' beliefs (Erdogan & Sahin, 2010). Specifically, in mathematics class, the use of technology is a research topic of particular relevance given the need to provide students with experiences that activate fundamental processes, such as conjecture and argumentation (Ljajko, 2016; Morales-López, 2019). However, numerous studies show that teachers - in mathematics and other disciplines - do not usually take advantage of all the benefits technology could bring to their teaching (Bate et al., 2013). Along the same lines, other research studies conclude that prospective and newly licenced teachers generally use technology to a very limited extent, and have limited knowledge on how to integrate and use it in the classroom (Durdu & Dag, 2017), which corroborates the idea that the inclusion of technology in teacher education is a challenge still to be overcome (Cabero, 2014). This reality makes it necessary to create theoretical frameworks to facilitate the analysis of different teaching situations and contexts to find ways to foster the inclusion of technology in them.

The TPACK model (Mishra & Koehler, 2006) was designed precisely to identify the types of knowledge teachers must master to integrate technology into their teaching. In recent years, numerous researchers have adopted this framework perspective to analyse the results of their studies (Bate, 2010; Beltrán-Sánchez et al., 2019; Castellanos et al., 2017; Cózar et al., 2015; Kushner y Ward, 2013; Mouza et al., 2014; Tsai & Chai, 2012) specifically on the teaching of mathematics, which is the area in which our work is framed

(Arnal-Bailera & Oller-Marcén, 2017; Dockendorff & Solar, 2018; Durdu & Dag, 2017; Morales-López, 2019; Özgün-Koca et al., 2010). In fact, the technological component is concretised in these latest works in the use of Geogebra dynamic geometry software, which is also the one we have chosen in our research for its didactic potentialities (promotion of understanding of mathematical content, development of problem solving skills, etc.), as the authors of those works affirm. However, instead of being of a geometric type, the content underlying our study is of an arithmetic nature, specifically on the rational number. We chose this mathematical object because it is important for the curriculum (Real Decreto 126/2014), and to know whether the teaching of this object to our preservice teachers when in elementary and high school might have limited them when making explanations from its different interpretations, as previous studies have already detected (Clarke et al., 2008; Escolano & Gairín, 2005; Freudenthal, 1983; Gairín, 2001; Martínez-Juste et al., 2017; Olive & Vomvoridi, 2006; Simon et al., 2018; Shield & Dole, 2013).

The papers we have cited do not jointly address the use of technology in teaching-learning processes and content on the rational number, which has led us to ask the following research question: *Can our preservice teachers address the explanation of problems of fraction multiplication in which several interpretations of the rational number are connected, by using tecnological support?* To answer this question, we intend to address the following research objectives:

1. Describe the difficulties shown in solving tasks related to the multiplication of fractions with technology.
2. Study if they are prepared to connect different interpretations of the rational number necessary in their teaching practice.
3. Investigate students' inclination to include technology in their future teaching activities.

THEORETICAL FRAMEWORK

In this section, we first introduce the basics of the TPACK (Technological Pedagogical Content Knowledge) model, which will be our framework to study the knowledge our students may need to integrate technology into their future teaching sequences. Below, we present the specific concretions of this model in our work.

TPACK model

This theoretical model, proposed by Mishra and Koehler (2006), was designed from the inclusion of technology in Shulman's construct (1986) on pedagogical content knowledge (PCK), which emerges from the interaction between the components of content knowledge (CK) and pedagogical knowledge (PK). From this third component (TK) stems new sub-components (see Figure 1) on teacher knowledge, so that the components of the TPACK model are configured as follows:

- CK (*Content Knowledge*). Mastery of the knowledge of the content, that is, of the topic to be learned or taught, which will depend on our context and academic level. This is the type of knowledge that teachers should know and understand of the subjects they teach, including knowledge of facts, concepts, and ideas, and the connections between them, as well as theories and procedures that are used according to the corresponding field.
- PK (*Pedagogical Knowledge*). Mastery of pedagogical knowledge, covering teaching and learning processes and general educational values and objectives. This type of knowledge refers to issues such as classroom management, the development and implementation of lesson plans according to the curriculum, the choice of appropriate evaluation methods, etc. Pedagogical knowledge is what enables teachers to understand how their students develop their skills and attitudes towards learning, which requires some management of cognitive, social, and learning development theory.
- TK (*Technological Knowledge*). Mastery of technological knowledge to apply it to perform different tasks, ranging from the most primitive technologies, such as books and whiteboards, to the most advanced, such as digital technologies. Regarding digital technologies, mention that they include both hardware (such as peripheral devices) and software (such as word processors, spreadsheets) handling and installation.
- PCK (*Pedagogical Content Knowledge*). The sub-component of pedagogical content knowledge, which arises from the combination of CK and PK components, refers to that knowledge of pedagogy applicable to teaching specific content, including the management of approaches suitable for a given content and the ability to organise the elements of that content during the teaching process. In this sub-component, the teacher's ability to choose appropriate representations

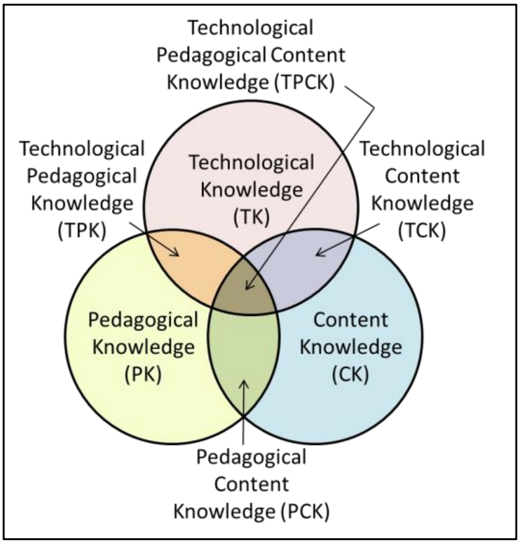
and formulations of concepts is particularly relevant, depending on whether or not they facilitate their understanding.

- TCK (*Technological Content Knowledge*). The sub-component that emerges from the interaction between the TK and CK components, which comprises the interactions and mutual limitations between technology and content, i.e, how the content can be changed because of the use of technology. Indeed, although technology may restrict the types of representation of a particular concept, in many cases new technologies provide greater flexibility in handling such representations. An example of this is the GeoGebra software mentioned above, which allows the user to manipulate geometric objects faster than if he/she did so by drawing them statically, thus contributing to learning processes of abilities such as conjecture and proof. Therefore, this is a case in which the subject to be taught can be changed depending on the application of the technology, since it provides forms of representation that were not available before its appearance.
- TPK (*Technological Pedagogical Knowledge*). The sub-component that relates the TK and PK components, considering the changes that technology produces in teaching-learning and, reciprocally, how teaching could change because of the use of specific technologies. The first path includes knowledge of several technologies as used in educational environments; the second includes knowledge of the pedagogical strategies needed for the use of different technologies.
- TPCK (*Technological Pedagogical Content Knowledge*). This is the sub-component that represents the combination of technology (TK), pedagogy (PK) and content (CK), which deals with sustaining efficient teaching with technology that requires the management of the different representations of concepts within specific content. Likewise, this form of knowledge concerns, among other aspects, teaching methods that use technology to teach specific content, as well as factors that facilitate or hinder the understanding of concepts and how technology can intervene.

The TPACK model places focus on all these components and sub-components when addressing teacher education (Mouza et al., 2014). Numerous studies propose various approaches on which components and sub-components are most important in integrating technology into their educational practice (Kaplon-Schilis & Lyublinskaya, 2019), such as that of Mishra and Koehler (2006), which propose a balance between component and sub-component, in contrast to that of Kushner and Ward's (2013), which emphasises the development of the TPK sub-component.

Figure 1

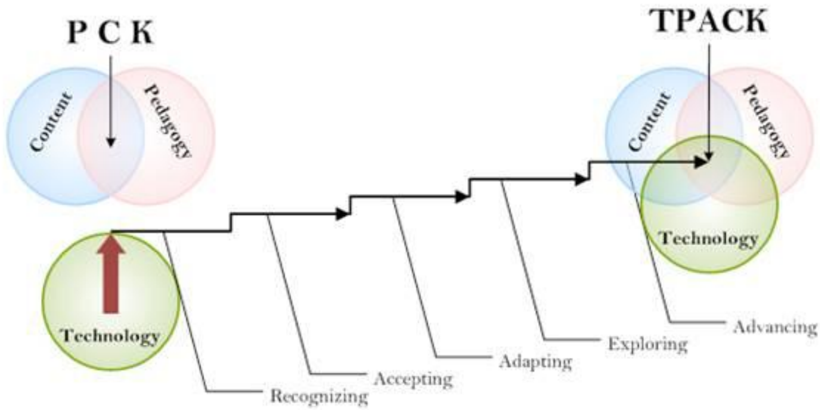
TPACK model components and sub-components (Mishra & Koehler, 2006)



Besides analysing the teacher's knowledge at a given time of the different components and sub-components of the TPACK model, it is also necessary to study the process of acquiring this knowledge and the barriers to overcome. For example, the integration of technology in mathematics teaching and learning is described by Niess et al. (2009), who develop a scheme (see Figure 2) with detailed qualitative descriptors for five levels of TPACK development:

Figure 2

Visual description of the 5 levels of integration of a particular technology into mathematics teaching and learning processes (Niess et al. 2009).



1. *Recognition*, when teachers are able to use technology and recognise the alignment of technology with mathematics content, although without integrating technology in mathematics teaching and learning.
2. *Acceptance*, when teachers form a favourable or unfavourable attitude towards teaching and learning mathematics with appropriate technology.
3. *Adaptation*, when teachers embark on activities that lead them to chose to adopt or reject teaching and learning mathematics with appropriate technology.
4. *Exploration*, when teachers actively integrate teaching and learning mathematics with appropriate technology.
5. *Advancement*, when teachers evaluate the results of the decision to integrate teaching and learning mathematics with appropriate technology.

Regarding the advancement of a teacher from one level to the next, Ertmer (1999) suggested two barriers to integrating technology into educational practice. The first, of external nature, refers to the lack of means or qualification for using technology in teaching; the second, of a personal nature, refers to teachers' beliefs about the interactions between technology and teaching. Tsai and Chai (2012) introduce a third barrier, also of a personal nature, which they call *design thinking*. This barrier refers to teachers' (lack of) ability to create and adapt their teaching practice to technological changes.

Some TPACK model concretions in our work

Regarding the PCK sub-component, in our work, we analyse our students' understanding of the rational number related to the different ways of interpreting it that give rise to different models of teaching it. Behr et al. (1993) and Kieren (1980) propose five constructs or interpretations that, following Clarke et al. (2008), we can summarise, in the representation of the rational number in the form of a fraction, such as:

- Part-whole, consisting of dividing a continuous quantity into equal parts (usually area or length) or constructing subsets of equal size given a discrete set of objects. Thus, the denominator of the fraction represents the parts into which the continuous quantity or discrete set are divided and the numerator the number of parts that are considered.
- Measure, in which the rational number compares a quantity of magnitude with a unit of that magnitude. Thus, the fraction denominator will be the equal parts into which the unit has been divided to measure the quantity of magnitude and the numerator the number of those parts that have been needed to measure.
- Division or quotient, in which the rational number represents the result of equally distributing a given number of objects (numerator) by a given number of people (denominator).
- Operator, which is the one in which the rational number modifies a quantity of magnitude by multiplying it and obtaining another - greater or lesser - quantity of magnitude, expressed in the same unit as the initial one.
- Ratio, which uses the rational number to compare the sizes of two sets or two measures, expressing the measure of one of the quantities of magnitude with respect to the unit of measure of the other magnitude.

Of all the previous interpretations, the most frequent in Spanish textbooks is the interpretation of the part-whole (Gairín & Muñoz, 2005; Olive & Vomvoridi, 2006; Simon et al., 2018), despite the disadvantages this poses for students. Indeed, Freudenthal (1983) analyses, both phenomenologically and mathematically, the limitations posed by the exclusive adoption of part-whole interpretation in teaching, which implies, among other disadvantages, a mechanical learning of algorithms and difficulties in understanding the improper fraction, since, in part-whole interpretation, the quantity of magnitude is both the total and the unit. There are even studies that reveal the scarcity of interpretations in many textbooks, treating the fraction in a purely formal way, without contextualising it (Gairín & Muñoz, 2005; Martínez-Juste et al., 2017; Shield & Dole, 2013).

Regarding the TCK sub-component, we analysed the use of GeoGebra and its interactions with the mathematical content in question. Dockendorf and Solar (2018) report on the influence that the use of this software has on the promotion of learning in high school and on its impact on teachers' conceptions of mathematics teaching and learning. However, it is important to address this sub-component of the model without separating the components that make it up, as can be deduced from a study conducted in Australia by Bate (2010). The study found, on the one hand, that newly graduated teachers were in favour of promoting meaningful learning of the content; on the other hand, it also found that these teachers were competent in the use of basic ICT tools. However, the sum of those two elements did not reverse a significant use of technology in teaching specific content, which means that this sub-component must be especially enhanced in teacher education, as Durdu and Dag (2017) suggest.

The TPK sub-component discusses questions about how technology can influence teaching approaches used in the classroom. This sub-component, like the PK component, transcends the limits of our area of knowledge, the didactics of mathematics, always linked to specific content. So, in this work, we directly analyse the interaction content-pedagogy-technology, through the TPCK sub-component. For this, we studied levels of development (Niess et al., 2009) and barriers of a different order (Ertmer, 1999; Tsai & Chai, 2012) that affect the process of inclusion of technology in our prospective teachers' explanations, categorised from the perspective of Charalambous et al. (2011). Carried out with prospective teachers who designed explanations on the rational number, these authors obtained four factors associated with the quality of such explanations in their study: knowledge of the subject (for example, they cite the understanding of the concept of unit), active reflection on the practice, development of alternative images of teaching (for example, the use of adequate graphics), and development of a productive disposition to give explanations and self-confidence to participate autonomously in this practice.

METHOD AND SAMPLE

The experiment was carried out with 47 pairs of third-year students from the Primary Education Degree (Grado en Magisterio de Educación Primaria) from the University of Zaragoza in the 2017/2018 academic year, at the end of the course called Didactic of Arithmetic II, which covers contents on the teaching of rational numbers.

The task that we show below, designed as a data collection tool for this research, begins with the raising of a problem (word-problem in the sense of Borasi (1986)) on the comparison of different quantities of magnitude. Next, the task continues with questions about resolving the problem through different methods, and about the design of a possible explanation of it to students of the first years. Notice that our students had previously studied the contents needed to address the task, which is a link between the interpretations of measure and quotient studied throughout the course. Likewise, our students had computers to carry it out.

TASK – Given the following problem:

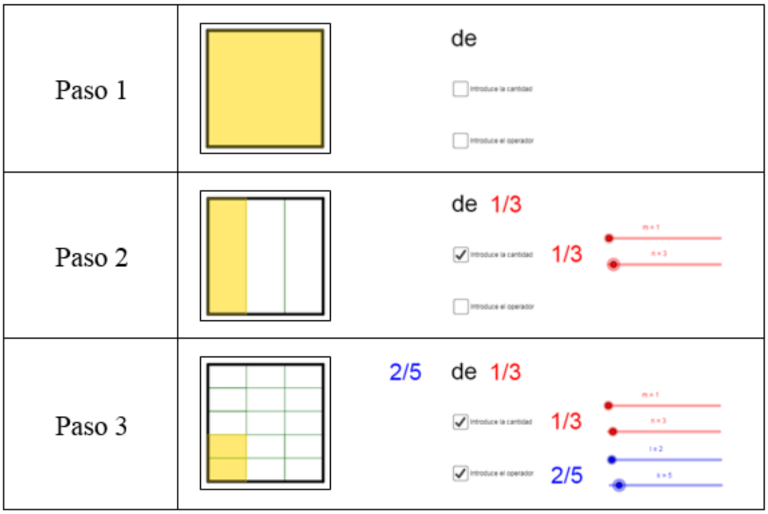
Antonio participates in two distributions of tortillas, one on Monday and one on Thursday. On Monday, there are 3 tortillas for 5 people, and on Thursday there are 5 tortillas for 8 people. From what he gets on Monday, his sister Sara eats the fourth part and he eats the rest. On Thursday, he decides not to share his tortilla, but he drops 1/5 of what he got that day to the ground and does not eat it. What day did Antonio eat the most tortillas? (Note: all tortillas that appear in this problem are equal).

a) Solve it without using arithmetic operations, using the "Multiplicación de Fracciones 1" applet as graphical support. Present your argument. (You can take as many screenshots as you like to make the solution clear.)
b) In the reasoning used in the previous item, what conditions do the graphs you have inserted have to meet?
c) Solve it now without using any graphical strategy, only through arithmetic operations.
d) Imagine that you are going to give a mathematics class to students of the first years of elementary school, in which you have to explain how to solve problems comparing quantities of magnitude from the application of operators. Describe step by step the mathematical directions you would give your students to learn how to solve the problem given at the beginning of this task.

The "Multiplicación de fracciones 1" applet (https://www.geogebra.org/m/b3XaeVVV) allows you to graphically represent the product of fractions in the interpretation of measure: one of the fractions corresponds to a quantity of magnitude (area, in this case) and the other is an operator that modifies it (see Figure 3).

Figure 3

Three steps to represent 2/5 of 1/3 area unit with the help of the applet: unit (Step 1), quantity input (Step 2) and figure operator input (Step 3)



This task allows us to analyse different components and sub-components of the TPACK model that our students demonstrate to carry it out:

- In item a, students are asked to solve the initial problem with the help of the applet and explain their answer. Performing this item involves the management of technology (TK), its relationship with mathematical content (TCK) for the solution of a mathematical problem (CK) and the subsequent argumentation via different interpretations of the rational number that generate different teaching models (PCK).
- In item b, we induced students to reflect on how they should have inserted the graphs in the previous item, as we assumed that students would not make these considerations explicit on their own. This implies, in addition to a reflection on mathematical content (CK), an

appropriate use of specific technology for that content (TCK), in this case, the creation of images (representing the unit) of equal size.

- Item c requires students to use only formal operations, in contrast to item a. Thus, this item focuses solely on aspects of our students' mathematical content (CK).
- Item d assumes the integration of the problem solution in a hypothetical explanation (PCK) to a group of students, which implies, as they have been able to verify in item a, the possibility of including technology in said explanation (TPCK).

In items a, b, and c, the variables are of an emerging nature (except for the interpretation of the rational number) and arise after repeated readings of the students' answers. We present below the variables that appear in each of them together with the corresponding categories in parentheses:

- In item a, we study the presence of conceptual errors in the solution (confusion of the part with the whole), interpretation of the rational number present in the arguments (measure with graphic support, measure with verbal support, quotient, or absence of interpretation) and the correctness of the result (correct or incorrect).
- In item b, we study the relevance of the size of the units and subunits (relevant or irrelevant in both cases).
- In item c, we study the presence of conceptual errors in the solution (confusion of the part with the whole or incorrect application of the operator) and the correctness of the result (correct or incorrect).

In item d, our variables are based on some of those described by Charalombous et al. (2011) on the quality of the explanations. On the one hand, knowledge of the subject is analysed through the mathematical contents explained (references to the operator and the comparison of quantities); on the other hand, the development of alternative images of teaching is studied through two variables: the interpretation of the rational number (measure, quotient, or absence of interpretation) and heuristic reflections (on the result of the task, on the number of different forms of solution, on the presence of technology in the solution and on mathematical aspects).

Our study is exploratory and has been conducted for descriptive purposes (Edmonds & Kennedy, 2017). The procedure we have adopted to collect the information we discussed in the following items has been the analysis of the written productions of the pairs of student mentioned above.

RESULTS

Item a

To solve this item, the students had to use the applet on two occasions, one per distribution. On each occasion, the result obtained from each distribution (Monday and Thursday) had to be entered first, and then the corresponding operator (obtaining 9/20 and 10/20 of tortilla, respectively). On both occasions, the students had to take screenshots with the results obtained, which they would later insert in the answer file to compare them visually (to conclude that Antonio ate more tortillas on Thursday). We want to point out the adequacy of the applet proposed for this item because, although the problem is located in the interpretation of quotient, its solution is the quantity received in Antonio's distribution (and not the other participants'), which the applet expresses directly.

Regarding the correctness in the use of the applet, only 13% of the total made errors when entering the data. Regarding mathematical correctness, we obtained that 55% of the pairs solved the task satisfactorily. The solutions classified as incorrect were those that either lacked the comparison between the quantities of tortilla, or the quantities they proposed for each day were incorrect. The latter case sometimes occurs due to an error that we will call "confusion of the part with the whole" and that was made by 12 pairs (57% of those who did not solve the task correctly). In this error, the complementary fraction with respect to the total represented (1 tortilla) is confused with the complementary fraction with respect to the part corresponding to Antonio (3/5 tortilla). In fact, after calculating the 3/20 of tortilla that Antonio's sister eats, the students claim that the latter ate the complementary of the total represented, that is, 17/20 of tortilla, instead of doing so with respect to what Antonio had earned in the distribution, that is, 3/4 of 3/5 of tortilla.

About the argumentation all but 5 pairs gave some kind of argument based on one of the following interpretations (10 pairs argued in more than one):

- Measure interpretation (with graphic support): it is based on the fact that the amount of coloured area of the square generated by the applet is visually greater for Thursday than for Monday distribution. For example, pair 1 writes "As can be seen in the graphs below, the coloured amount on the unit is less on Monday than on Thursday...".
- Measure interpretation (with verbal support): it is based on the number of subunits or parts into which the unit is divided and its size. For

example, pair number 29 writes "…two subunits of the second drawing are equivalent to one of the first…".

- Quotient interpretation: it is based on the number of tortillas to be distributed and the number of diners for each distribution. Pair 42 writes "...matching the number of people in the two deliveries...".
- Absence of interpretation: the comparison is made through formal arithmetic operations. Pair 2 writes "...as they have the same denominator...".

Recounting the major arguments, we obtained that 15 pairs reasoned only by interpreting the fraction as a measure (with graphic support) and 14 did so without giving an interpretation. The least used argument was that of quotient, used only by 8 pairs.

We observed that there is no significant statistical relationship between the correctness of the response with the presence or absence of interpretations of the fraction in the arguments due to the proximity that exists between the values in Table 1.

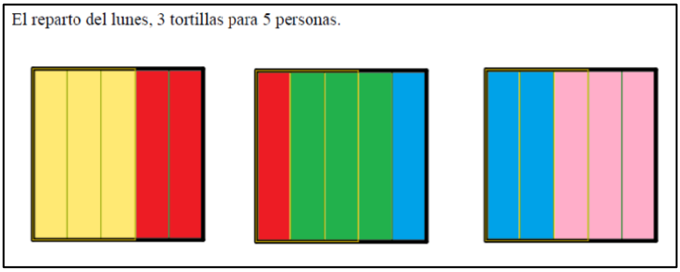
Table 1

Relationship between task correctness and the presence of interpretations of the fraction

	Presence of interpretations of fraction	No interpretations of fraction
Correct solution	18 (38%)	8 (17%)
Incorrect solution	11 (24%)	10 (21%)

Figure 4

Example of adaptation of the applet to the interpretation as quotient



We want to note that four pairs made different use of the applet than the one proposed, although only one solved the task correctly. These pairs represented the distribution as shown in Figure 4 (extracted from pair 9), drawing the three units to be distributed and marking with different colours what corresponded to each participant in the distribution.

Item b

The condition that the graphs of the screenshots taken must meet is that the units, in this case, represented by squares, have the same size so that when visually comparing their shaded areas, it is possible to determine correctly which of the two represents the most quantity of tortillas. However, it is not necessary for the size of the subunits to be the same in both screenshots, as this does not influence the amount of coloured area. Therefore, a correct answer to this task considers the equality of the size of the units relevant and does not consider the equality of the size of the subunits relevant.

Only 23% of students (see Table 2) solved the task correctly. On the one hand, 51% of the pairs did not mention that the size of the units was relevant to make the comparison; on the other hand, 47% of the pairs gave importance to the size of the subunits, with 21% of the pairs making the same error.

Table 2

Relationship between the perception of the relevance of the size of the units and subunits

	Relevant subunit size	Irrelevant subunit size
Relevant unit size	12 (26%)	11 (23%)
Irrelevant unit size	10 (21%)	14 (30%)

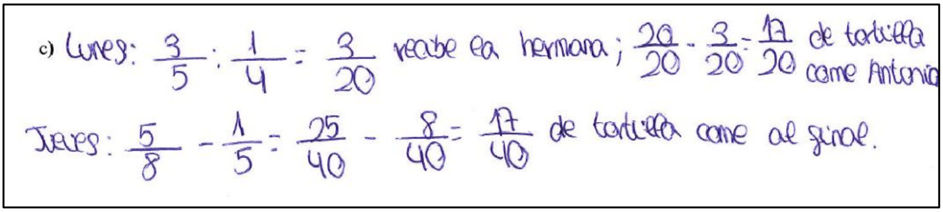
Item c

In this task, the solution is correct when the appropriate operators are properly applied to the quantities of magnitude associated with each day, obtaining the corresponding fractions, and a comparison is made between them.

Thirty-one of the 47 pairs participating (66%) had a correct solution. The most frequent errors were called “incorrect application of the operator” (5 pairs) and “confusion of the part with the whole” (9 pairs). We understand as incorrect application of the operator not to interpret the function of the operator as a product. In Figure 5, the pair 18 raises a quocient of fractions (Monday) and a difference (Thursday) instead of two products. Likewise, in the line of item a, we understand that the students mixed up part and whole when they considered that the total quantity available was the unit. In the same example, the pair performs the complementary with respect to a unit (tortilla), instead of doing so with respect to the quantity available.

Figure 5

Example of incorrect application of the operator in the pair 18



Relating now the correctness in the solutions of items c and a, we provided Table 3, from which a statistically significant relationship of 95% is obtained.

Table 3

Relationship between the correctness of tasks a and c

	Item c correct	Item c incorrect
Item a correct	25 (53%)	1 (2%)
Item a incorrect	5 (11%)	16 (34%)

Finally, note that 14 pairs of students (30%) have obtained different results in both items without having commented on this fact (8 solved the two items incorrectly and 6 solved one or the other item incorrectly).

Item d

This item which was answered by 42 of the 47 pairs, is more didactic than the previous ones, so we cannot talk about correct or incorrect answers in absolute terms. To address their study we first address two variables: what mathematical contents appear when our students design explanations and which interpretations of the fraction are most frequent in their explanations.

Firstly, about the mathematical contents explained, we have studied the presence of references to the operator and the comparison of quantities, which are the contents that appear in the problem posed. These contents are related to the expected mathematical difficulties in students of the first years of elementary school and should therefore be the basis for a quality explanation (Charalambous et al., 2011). We highlight the balance that exists between the number of pairs (20) who considered the need to include in their explanations references to the two outstanding mathematical aspects (operator and comparison) and those who did not. Of the 22 pairs who did not consider this need, 15 do not talk about the comparison and 10 do not talk about the operator.

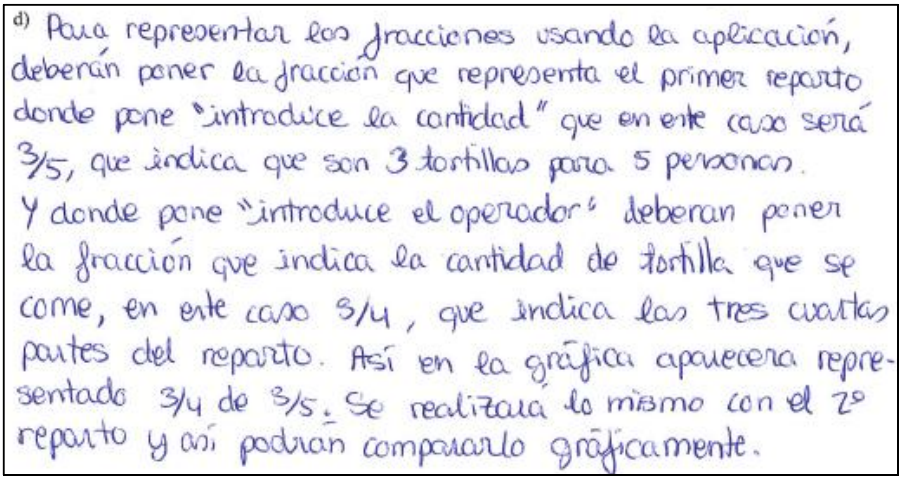
We considered it of interest to relate the selection of mathematical content required by this item with items a and c. We observed that pairs who make partial explanations without referring to the two outstanding mathematical contents (operator and comparison) gave better mathematical solutions from the start; specifically, 68% of these pairs solved the task correctly in the previous items. Conversely, among the 20 pairs who consider it necessary to refer to the ideas of operator and comparison, only 45% solved the task correctly in the previous items.

Regarding the interpretations of the fraction that appear proposed, we emphasise that in 10 cases, none of the fractions relied on any meaning, being considered in a purely formal way. Among those who use some interpretation to give meaning to the rational number, there are 23 pairs who use the measure in - all or part of - their explanation; likewise with 15 pairs who use the quotient in this sense.

Similarly, we can observe general characteristics relating to problem solving and the tools used for explanations: only two of the 42 pairs who solved the task claim that the problem can be solved in more than one way to relate several forms of solution or verify that the result must be the same.

Figure 6

Example of explanation that includes the use of the applet proposed by pair 31


d) Para representar las fracciones usando la aplicación, deberán poner la fracción que representa el primer reparto donde pone "introduce la cantidad" que en este caso será 3/5, que indica que son 3 tortillas para 5 personas.
Y donde pone "introduce el operador" deberan poner la fracción que indica la cantidad de tortilla que se come, en este caso 3/4, que indica las tres cuartas partes del reparto. Así en la gráfica aparecera representado 3/4 de 3/5. Se realizara lo mismo con el 2º reparto y así podrían compararlo gráficamente.

Overall, reviewing the explanations proposed by all pairs, it is remarkable that, although the first items of the task involve intensive use of technology, only three couples used the applet in their indications (see Figure 6). In them, the use intended for the technology, in this case, the GeoGebra applet used in item a, is limited exclusively to its function as a graphic calculator that resolves the need to represent the fraction resulting from applying an operator to a quantity of magnitude. Nor there is a reflection on mathematical issues related to its use, such as the size of the unit or the subunit.

Particularly in pairs who had previously adapted the applet to use it with a quotient interpretation, we can say that two of them are consistent and maintain this adaptation when designing their explanations, but not the other two. None of the four pairs uses technology in their explanations despite having shown a good understanding of it before.

DISCUSSION AND IMPLICATIONS

Next, we discuss the results obtained in the previous section from the perspective of the TPACK model, analysing them from each of the components and sub-components present in our tasks.

TK

The development of the TK component, which includes our students' abilities in the general use of technologies, was not an impairment to the solution of item a. In fact, only a few pairs showed difficulties in using the applet or in placing the data. Moreover, practically all students gave an answer to the problem, correct or not, which goes in the line that when we give students a technological task, they tend to answer regardless of their understanding of the mathematical content they were presented (Ljajko, 2016). These facts could be indications of an acceptable development of the TK component, although to assess it more accurately we would need to have information on the knowledge of our students regarding other technological tools (Kushner & Ward, 2013).

CK

The development of the CK component of our prospective teachers is related to the mathematical errors they showed in the solution of the different items. The most common error is what we call "confusion of the part with the whole" (items a and c), possibly caused by one of the disadvantages of traditional teaching, generally limited to part-whole interpretation, in which the student does not need to recognise the unit because it coincides with the total quantity of magnitude (Gairín, 2001; Gairín & Muñoz, 2005). Simon et al. (2018) also highlight this and other difficulties in some advanced concepts of fractions when teaching is limited exclusively to part-whole interpretation, and how they can be mitigated via sequences of tasks in interpretation as a measure, as Olive and Vomvoridi (2006) also assure. On the other hand, the error "incorrect application of the operator" (item c) could be influenced by the false conception that multiplication increases the quantity, as stated by Clarke et al. (2008).

The inconsistencies observed between the answers to items a and c are remarkable, since many pairs obtained different numerical results without noticing it. This reflects a difficulty in the execution of the verification phase of the solution of a problem (Piñeiro et al., 2019), which is striking given that the Spanish curriculum explicitly addresses the treatment of the coherence of the solutions of a problem (Real Decreto 126/2014). One possible explanation for this, in the particular case of the rational number, is the predominance of part-whole interpretation that we have just discussed, which makes it difficult to find solutions through two different interpretations.

The error made by the pairs that gave importance to the size of the subunits (item b) maybe because they were thinking about the conditions necessary to make formal comparisons between rational numbers. As a matter of fact, to compare fractions formally they usually match their denominators, so they would tend to think that it is always necessary to match the size of the subunits. This error may be related to the previous use of textbooks that tend to confer much importance to formal exercises and in which arithmetic techniques are exercised above all (Gairín & Muñoz, 2005; Martínez-Juste et al., 2017; Shield & Dole, 2013).

The errors and difficulties discussed in this sub-section are clear indicators of weaknesses in the development of our prospective teachers' CK mastery (Alguacil et al., 2016), which may condition the development of the sub-components derived from it.

TCK

The skills the prospective teachers', our students, showed in the TCK sub-component were reflected in their performance in items a and b. In the first, almost half of the pairs were unable to take advantage of the functionalities of the applet that facilitated the understanding of the underlying mathematical content. Moreover, we detected only a few pairs who used the applet to adapt it to the interpretation as quotient, among which only one pair solved the task correctly. In item b, with regard to the error concerning not giving importance to the size of the units, it should be noted that the problem statement already explained that the two were the same. What the students had to consider was that their representations had to be the same size for a visual comparison. This error is possibly caused because when capturing screens twice in the same applet, it was not necessary to change their sizes. All these interactions between technology and content reflect some limitations of our students in their development of the TCK sub-component, which reinforces the validity of

Durdu and Dag's (2017) proposal to focus on future teachers' instruction especially on this sub-component.

PCK

The aspects dealt with in our study that condition the PCK sub-component (items a and d) of our students are: the ability to argue, the completeness of the explanations, and the use of the different interpretations of the rational number. Regarding the first, even some of those who solve the problem incorrectly make the effort to argue within a single interpretation, which would give them an advantage in elaborating future explanations. In contrast, other students who solve correctly reason, either by mixing ideas from various interpretations of the rational number or without using any interpretation, so the numbers lose their meaning, which could result in a didactic difficulty when giving their future students an explanation (Gairín, 2001). These considerations would go along the lines expressed by some authors (Ruíz de Gauna et al., 2013) who show the existence of a prospective teacher's profile of a mostly mathematical nature and another profile of a fundamentally pedagogical-didactic type. These results indicate poor development in the PCK sub-component (Mishra & Koehler, 2006), which is accentuated when there is an imbalance between skills corresponding to CK and PK components, especially when the former is much more developed than the latter (Kushner & Ward, 2013).

Regarding the completeness of the explanations, in the task, our students address the comparison less frequently than the operator. To construct a good explanation, it is necessary to attend to all the characteristics of the task that can be a difficulty of or a misconception from the students (Charalambous et al., 2011). A high percentage of our students did not attend to both difficulties in their explanations, which could affect their ability to build quality explanations, and therefore the development of the PCK sub-component. We noticed that one group provides incomplete explanations along with good mathematical performance, which may be yet another example of imbalance between the PK and CK components, impoverishing the development of the PCK sub-component already indicated previously by Kushner and Ward (2013).

Finally, concerning the use of the different interpretations of the rational number, we observed pedagogical-mathematical weaknesses in some couples that are restricted in their explanations to formal operations between fractions, neglecting the context offered by the problem. Gairín (2001) highlights that prospective teachers who show a lesser understanding of the different interpretations of the rational number design purely formal explanations, which may be related to the instruction received during their pre-university mathematical education, guided by textbooks in which fundamentally procedural exercises predominate (Gairín & Muñoz, 2005; Martínez-Juste et al., 2017; Shield & Dole, 2013). All of the above shows a low development of our students' PCK component.

TPCK

Item d allows us to evaluate our students' TPCK sub-component, which includes the interactions between the three fundamental components of the TPACK model. During the last item, few couples integrated the use of technology in their indications. This fact suggests some scepticism among our prospective teachers about the capacity of technology to transform teaching, even with sufficient technological preparation, as shown by the study by Özgün-Koca et al. (2010). In the line of these authors, this could be related to the fact that our students feel even more identified with the role of mathematics students than with that of mathematics teacher. From another point of view (Ertmer, 1999; Tsai & Chai, 2012), the above could also be interpreted as our preservice teachers have not yet overcome second-order barriers (scepticism) or third-order barriers (adaptation of tasks for inclusion of technology). In terms of Niess et al. (2009), our students would not have reached level 3 (adaptation) of the five levels proposed for the integration of ICT in education. These reflections allow us to conclude this section by noting a low development of our students' TPCK sub-component, which is consistent with the skills shown by them in other sub-components.

CONCLUSIONS AND IMPLICATIONS FOR TEACHER EDUCATION

Regarding the first objective, to describe the difficulties our students found in solving tasks related to the multiplication of fractions with technology, we have observed that they have not taken advantage of the functionalities that the applet offered to solve the mathematical task, nor have they been able to explain the mathematical conditions necessary to solve the problem in a technological environment. Although in our teaching we used technology at different times of the course as a tool to address mathematical content, we see a need to encourage more thoughtful use of it in teacher education. To this end, we propose focusing attention on the relationships between technological actions and their mathematical meaning, thus hoping to overcome the limitations that we have pointed out in the prospective teachers' development of the TCK sub-component.

Regarding the second objective, to study whether our students are prepared to connect different interpretations of the rational number necessary in their teaching practice, the results of this work show some difficulties. Specifically, we find that they are either unable to keep their explanations within the same interpretation of the rational number, or they dispense with any interpretation. Although in our teaching we deal successively with all the interpretations of the rational number mentioned in this work, the results warn us of the importance of placing special emphasis on the connections that exist between them. This idea, which would encourage the development of the PCK sub-component, could be carried out in different ways: the study of textbooks from other countries that show a greater wealth of interpretations of the rational number, or carrying out activities that involve rethinking a problem in terms of a rational interpretation different from the given one.

Regarding the third objective, to investigate students' tendency to include technology in their future teaching activities, we observed a low level of development that could be conditioned by the existence of personal barriers. Our course requires, at a technological level, the use of GeoGebra applets that allow students to create graphics with comfort and precision that cannot be obtained by hand. To expand this use, which does not cover pedagogical aspects, we educators must promote situations where the prospective teachers are forced to integrate technology into their explanations, analysing possible difficulties (both mathematical and technological) that their future students might encounter. We believe that this proposal and those we have made with respect to the two previous objectives will contribute to developing prospective students' TPCK sub-component.

ACKNOWLEDGEMENTS

We would especially like to thank Professor Dr. Rafael Escolano Vizcarra for everything he has conveyed to us about teaching and learning the rational number, and for his contributions and comments to the first versions of this work.

This work was partially financed by the Ministry of Science and Innovation of Spain MICINN (project PID2019-104964GB-I00) and by the "VI Plan Propio de Investigación y Transferencia" of the University of Seville (Spain), as well as by the research groups "Investigación en Educación

Matemática” (S60_20R) officially recognised by the Government of Aragon and “Grupo de Investigación en Educación Matemática” (FQM226) officially recognised by the Junta de Andalucía.

AUTHORSHIP CONTRIBUTION STATEMENT

The authors AG and AA-B discussed the methodology, theoretical basis, data collection and analysis, and the results, and contributed equally to the final version of the article.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be provided by the corresponding author, AA-B, upon reasonable request.

REFERENCES

Alguacil, M., Boqué, M. C., & Pañellas, M. (2016). Dificultades en conceptos matemáticos básicos de los estudiantes para maestro. *International Journal of Developmental and Educational Psychology INFAD, 1*, 419-429. http://dx.doi.org/10.17060/ijodaep.2016.n1.v1.162

Arnal-Bailera, A., & Oller-Marcén, A. M. (2017). Formación del Profesorado y Demostración Matemática. Estudio Exploratorio e Implicaciones. *Bolema: Boletim de Educação Matemática*, *31*(57), 135-157. https://doi.org/10.1590/1980-4415v31n57a07

Bate, F. (2010). A bridge too far? Explaining beginning teachers' use of ICT in Australian schools. *Australasian Journal of Educational Technology*, *26*(7), 1042-1061. https://doi.org/10.14742/ajet.1033

Bate, F. G., Day, L., & Macnish, J. (2013). Conceptualising Changes to Pre-Service Teachers’ Knowledge of how to Best Facilitate Learning in Mathematics: A TPACK Inspired Initiative. *Australian Journal of Teacher Education*, 38(5), 14-30. http://ro.ecu.edu.au/ajte/vol38/iss5/2

Behr, M. J., Harel, G., Post, T., & Lesh, R. (1993). Rational numbers: Toward a semantic analysis-emphasis on the operator construct. In T.P. Carpenter, E. Fennema y T. A. Romberg (Eds.), *Rational Numbers: An Integration of Research* (pp. 13-47). Lawrence Erlbaum.

Beltrán-Sánchez, J. A., García, R. I., Ramírez-Montoya, M. S., & Tánori, J. (2019). Factores que influyen en la integración del Programa de Inclusión y Alfabetización Digital en la docencia en escuelas primarias. *Revista Electrónica de Investigación Educativa, 21*(1), 1-11. https://doi.org/10.24320/redie.2019.21.e31.2088

Borasi, R. (1986). On the nature of problems. *Educational studies in mathematics*, *17*(2), 125-141. https://doi.org/10.1007/BF00311517

Cabero, J. (2014). Formación del profesorado universitario en TIC. Aplicación del método Delphi para la selección de los contenidos formativos. *Educación XX1, 17*(1), 111-132. http://hdl.handle.net/11441/16394

Castellanos, A., Sánchez, C., & Calderero, J. F. (2017). Nuevos modelos tecnopedagógicos. Competencia digital de los alumnos universitarios. *Revista electrónica de investigación educativa, 19*(1), 1-9. http://dx.doi.org/10.24320/redie.2017.19.1.1148

Charalambous, C. Y., Hill, H. C., & Ball, D. L. (2011). Prospective teachers' learning to provide instructional explanations: how does it look and what might it take? *Journal of Mathematics Teacher Education, 14*(6), 441-463. https://doi.org/10.1007/s10857-011-9182-z

Clarke, M. D., Roche, A. & Mitchell, A. (2008). Ten practical tips for making fractions come alive and make sense. *Mathematics Teaching in the Middle School, 13*(7), 372-380. https://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/2008/Vol13/Issue7/Ten-Practical-Tips-for-Making-Fractions-Come-Alive-and-Make-Sense/

Cózar, R., Zagalaz, J., & Sáez, J. (2015). Creando contenidos curriculares digitales de ciencias sociales para educación primaria. Una experiencia TPACK para futuros docentes. *Educatio Siglo XXI, 33*(3), 147-168. https://doi.org/10.6018/j/240921

Dockendorff, M. & Solar, H. (2018). ICT integration in mathematics initial teacher training and its impact on visualization: the case of GeoGebra. *International Journal of Mathematical Education in Science and Technology*, *49*(1), 66-84. https://doi.org/10.1080/0020739X.2017.1341060

Durdu, L. & Dag, F. (2017). Pre-Service Teachers' TPACK Development and Conceptions through a TPACK-Based Course. *Australian Journal of*

Teacher Education, 42(11), 150-171. http://dx.doi.org/10.14221/ajte.2017v42n11.10

Edmonds, W. A. & Kennedy, T. D. (2017). *An applied reference guide to research designs: Qualitative, quantitative and mixed methods*. Sage.

Erdogan, A. & Sahin, I. (2010). Relationship between math teacher candidates' technological pedagogical and content knowledge (TPACK) and achievement levels. *Procedia-Social and Behavioral Sciences, 2*(2), 2707-2711. https://doi.org/10.1016/j.sbspro.2010.03.400

Ertmer, P. A. (1999). Addressing first- and second-order barriers to change: Strategies for technology integration. *Educational Technology Research and Development, 47*(4), 47-61. https://doi.org/10.1007/BF02299597

Escolano, R. & Gairín, J. M. (2005). Modelos de medida para la enseñanza del número racional en educación primaria. *Unión, 1*, 17-35. http://www.fisem.org/www/union/revistas/2005/1/Union_001_006.pdf

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures.* Reidel.

Gairín, J. M. (2001). Sistemas de representación de números racionales positivos: un estudio con maestros en formación. *Contextos Educativos, 4*, 137-159. http://dx.doi.org/10.18172/con.490

Gairín, J. M. & Muñoz, J. M. (2005). El número racional positivo en la práctica educativa: Estudio de una propuesta editorial. In *IX Simposio SEIEM*. http://www.seiem.es/docs/comunicaciones/GruposIX/pna/gairinmunoz.pdf

Kaplon-Schilis, A. & Lyublinskaya, I. (2019). Analysis of relationship between five domains of TPACK framework: TK, PK, CK Math, CK Science, and TPACK of pre-service special education teachers. *Technology, Knowledge and Learning, 25,* 25-43. https://doi.org/10.1007/s10758-019-09404-x

Kieren, T. (1980). The rational number construct — its elements and mechanisms. En T. E. Kieren (Ed.), *Recent research on number learning* (pp. 125-149). ERIC/SMEAC. https://eric.ed.gov/?id=ED212463

Kushner, S. N. & Ward, C. L. (2013). Teaching with technology: Using TPACK to understand teaching expertise in online higher education. *Journal of Educational Computing Research, 48*(2), 153-172. https://doi.org/10.2190/EC.48.2.c

Ljajko, E. (2016). Does the problem complexity impact students' achievements in a computer aided mathematics instruction? *Teaching of Mathematics, 19*(1), 41-55. http://www.teaching.math.rs/cap/browse.php?p=TM191

Martínez-Juste, S., Muñoz-Escolano, J. M., Oller-Marcén, A. M., & Ortega, T. (2017). Análisis de problemas de proporcionalidad compuesta en libros de texto de 2º de ESO. *Revista latinoamericana de investigación en matemática educativa, 20*(1), 95-122. http://dx.doi.org/10.12802/relime.17.2014

Mishra, P. & Koehler, M. (2006). Technological pedagogical content knowledge: a framework for teacher knowledge. *Teachers college record, 108*(6), 1017-1054. https://www.tcrecord.org/content.asp?contentid=12516

Morales-López, Y. (2019). Conocimientos que evidencian los futuros profesores cuando realizan una tarea que involucre geometría, enseñanza y uso de tecnologías. *Acta Scientiae, 21*(2), 75-92. https://doi.org/10.17648/acta.scientiae.v21iss2id5081

Mouza, C., Karchmer-Klein, R., Nandakumar, R., Ozden, S. Y. , & Hu, L. (2014). Investigating the impact of an integrated approach to the development of preservice teachers' technological pedagogical content knowledge (TPACK). *Computers & Education, 71*, 206-221. https://doi.org/10.1016/j.compedu.2013.09.020

Niess, M. L., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper S. R., Johnston, C., Browning, C., Özgün-Koca, S. A., & Kersaint, G. (2009). Mathematics teacher TPACK standards and development model. *Contemporary Issues in Technology and Teacher Education, 9*(1), 4-24. https://www.citejournal.org/volume-9/issue-1-09/mathematics/mathematics-teacher-tpack-standards-and-development-model

Olive, J. & Vomvoridi, E. (2006). Making sense of instruction on fractions when a student lacks necessary fractional schemes: The case of Tim.

Journal of Mathematical Behavior, 25, 18-45. https://doi.org/10.1016/j.jmathb.2005.11.003

Özgün-Koca, S. A., Meagher, M., & Edwards, M. T. (2010). Preservice teachers' emerging TPACK in a technology-rich methods class. *The Mathematics Educator, 19*(2), 10-20. http://tme.journals.libs.uga.edu/index.php/tme/article/view/210

Piñeiro, J. L., Castro-Rodríguez, E., & Castro, E. (2019). Concepciones de profesores de primaria sobre problemas matemáticos, su resolución y enseñanza. *Avances de Investigación en Educación Matemática, 16*, 57-72. https://doi.org/10.35763/aiem.v0i16.253

Real Decreto 126/2014, de 28 de febrero, por el que se establece el currículo básico de la Educación Primaria. *Boletín Oficial del Estado*. Madrid, 1 de marzo de 2014, núm. 52, pp. 19349-19420. https://www.boe.es/buscar/act.php?id=BOE-A-2014-2222

Ruíz de Gauna, J., García, J., & Sarasua, J. (2013). Perspectiva de los alumnos de grado de educación primaria sobre las matemáticas y su enseñanza. *Números. Revista de Didáctica de las Matemáticas, 82*, 5-15. http://www.sinewton.org/numeros/numeros/82/Volumen_82.pdf

Shield, M. & Dole, S. (2013). Assessing the potential of mathematics textbooks to promote deep learning. *Educational Studies in Mathematics, 82*(2), 183-199. https://doi.org/10.1007/s10649-012-9415-9

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14. https://doi.org/10.3102/0013189X015002004

Simon, M. A., Placa, N., Avitzur, A. & Kara, M. (2018). Promoting a concept of fraction-as-Measure: a study of learning through activity. *Journal of Mathematical Behavior, 52*, 122-133. https://doi.org/10.1016/j.jmathb.2018.03.004

Tsai, C. C. & Chai C. S. (2012). The "third"-order barrier for technology-integration instruction: Implications for teacher education. *Australasian Journal of Educational Technology, 28*(6), 1057-1060. https://doi.org/10.14742/ajet.810