ACTA SCIENTIAE

ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.6307

Professional Tasks in Early Childhood Education Teacher Education: Promoting Reasoning at 0-3

Yuly Vanegas[a]
Joaquín Giménez[b]
Montserrat Prat[c]
Mequè Edo[d]

[a]Universidad de Lleida, Facultad de Educación, Psicología y Trabajo Social, Departamento de Matemáticas, Lleida, Cataluña, Spain
[b]Universidad de Barcelona, Facultad de Educación, Departamento de Educación Lingüística y Literaria, y Didáctica de las Ciencias Experimentales y de la Matemática, Barcelona, Cataluña, Spain
[c]Universidad Ramon Llull, Facultad de Psicología, Ciencias de la Educación y el Deporte - Blanquerna, Departamento de Educación, Barcelona, Cataluña, Spain
[d]Universidad Autónoma de Barcelona, Facultad de Ciencias de la Educación, Departamento de Didáctica de la Matemática y las Ciencias Experimentales, Bellaterra, Cataluña, Spain

Received for publication on 1 Dec. 2020. Accepted after review on 7 Sep. 2021.
Designated editor: M. Mar Liñán García

ABSTRACT

Background: Teacher training at all levels has been the subject of continuous study in recent decades, however, in the case of early childhood education, little research has been carried out to analyse the mathematical knowledge of prospective teachers. To characterise this knowledge, it is essential to organise professional tasks focused on recognising how future teachers learn to look at different teaching situations in a structured way. **Objective**: We intend to describe the design and validation of a professional task aimed at prospective teachers to observe, analyse, and reflect on real experiences of initiation to logical-mathematical reasoning in cycle 0-3 cycle. **Design**: The structuring of this professional task covered five stages: review of studies on mathematical construction in the cycle of 0-3 years of age; review of studies on the beginning of logical-mathematical reasoning; choice of narratives/videos to observe and analyse; implementation in a pilot group and initial analysis of the teaching competence, and redesign and construction of the final version of the task. **Settings and Participants**: The initial version is implemented with 37 prospective teachers of the early childhood education degree of a Catalan university. **Data and analysis**: Qualitative-interpretative research is carried out, analysing the written productions prepared by the prospective teachers, through content analysis. **Results**: The observations of the pilot study show that prospective teachers recognise how children identify and relate, but there are difficulties in identifying when children are

Corresponding author: Yuly Vanegas Muñoz. Email: yuly.vanegas@udl.cat

operating. **Conclusions**: We found that the task designed allowed us to improve the knowledge of the prospective teachers on the reasoning in the 0-3 cycle and that the use of narratives is pertinent and enriching in the structuring of professional tasks.

Keywords: Teacher education, Task design, Reasoning, Early childhood education

Tareas profesionales en la formación de maestros de educación infantil: promoviendo el razonamiento en 0-3

RESUMEN

Antecedentes: La formación de docentes en todos los niveles ha sido objeto de estudio permanente en las últimas décadas, sin embargo, en el caso de la Educación Infantil, se han realizado pocas investigaciones para analizar el conocimiento matemático de futuros maestros. Para caracterizar dicho conocimiento es fundamental organizar tareas profesionales enfocadas en reconocer cómo los futuros docentes aprenden a mirar de manera estructurada diversas situaciones de enseñanza. **Objetivo**: Nos proponemos describir el diseño y validación de una tarea profesional orientada a que los futuros maestros observen, analicen y reflexionen sobre experiencias reales de iniciación al razonamiento lógico-matemático en el ciclo 0-3. **Diseño**: La estructuración de dicha tarea profesional contempló cinco etapas: revisión de estudios sobre la construcción matemática en el ciclo de 0-3 años; revisión de estudios sobre el inicio del razonamiento lógico-matemático; elección de narrativas/videos a observar y analizar; implementación en un grupo piloto y análisis inicial de la competencia docente, y, rediseño y construcción de la versión final de la tarea. **Escenario y participantes**: La versión inicial se implementa con 37 futuras maestras del grado de Educación Infantil de una universidad catalana. **Recoleción y análisis de datos**: Se realiza una investigación cualitativa-interpretativa, analizando las producciones escritas elaboradas por las futuras maestras, mediante análisis del contenido. **Resultados**: Las observaciones del estudio piloto permiten constatar que las futuras maestras, reconocen como los niños identifican y relacionan, pero, se observan dificultades en identificar cuando los niños están operando. **Conclusiones**: Se constata que la tarea diseñada ha permitido mejorar el conocimiento de las futuras maestras sobre el razonamiento en el ciclo 0-3 y que el uso de narrativas resulta pertinente y enriquecedor en la estructuración de tareas profesionales.

Palabras clave: formación de docentes; diseño de tareas; razonamiento; educación infantil.

Tarefas profissionais na formação dos professores da educação infantil: desenvolvendo o raciocínio em 0-3

RESUMO

Antecedentes: A formação de professores em todos os níveis é objeto de estudo permanente, porém, no caso da Educação Infantil, poucas pesquisas foram realizadas para analisar o conhecimento matemático dos futuros professores. A fim de caracterizar tais conhecimentos, é essencial organizar tarefas profissionais centradas no reconhecimento de como os futuros professores aprendem a olhar para várias situações de ensino de forma estruturada. **Objetivo**: Propomos conceber e validar uma tarefa profissional que visa aos futuros professores observar, analisar e refletir sobre experiências reais de iniciação ao raciocínio lógico-matemático no ciclo 0-3. **Desenho**: A estruturação desta tarefa profissional incluiu cinco etapas: revisão dos estudos sobre construção matemática no ciclo de 0-3 anos; revisão de estudos sobre o início do raciocínio lógico-matemático; escolha de narrativas/vídeos para observar e analisar; implementação em grupo piloto e análise inicial da competência docente, e redesenho e construção da versão final da tarefa. **Participantes**: A versão inicial é implementada com 37 futuros professores, alunos de graduação do curso de Educação Infantil (pedagogia) de uma universidade catalã. **Dados e análise**: Realizamos uma pesquisa qualitativo-interpretativa, analisando as produções escritas dos futuros professores, por meio da análise de conteúdo. **Resultados**: As observações do estudo piloto permitem verificar que os futuros professores reconhecem como as crianças ss identificam e relacionam, mas observam-se dificuldades em identificar quando as crianças estão operando. **Conclusões**: Verifica-se que a tarefa desenhada tem permitido aos futuros professores aprimorar seus conhecimentos de raciocínio no ciclo 0-3 e que o uso de narrativas é relevante na estruturação de tarefas profissionais.

Palavras-chave: formação dos professores; desenho de tarefas; raciocínio; educação infantil.

INTRODUCTION

Alsina (2006) states that the erroneous claim that it is not appropriate to talk about acquisition of mathematical knowledge in early childhood has prevailed for a long time. However, authors such as Fuson et al. (2009) and the approaches of Associations such as the *National Association for the Education of Young Children* - NAEYC and the *National Council of Teachers of Mathematics* – NCTM (2013); and the *National Research Council* - NRC (2014) highlight the importance of considering mathematical activity in the 0 - 6 years stage and, therefore, the need to incorporate it into current curricula.

Children develop mathematical ideas naturally from their daily experiences (NCTM, 2000), so the teacher must be aware of their potential and incorporate them into the classroom to support the construction of children's mathematical ideas. Teachers can design rich school activities focused on competencies if they are able to identify the power of developing mathematical processes in their classrooms (NCTM & NAEYC, 2013). Authors such as Clements and Sarama (2020) and Alsina (2015) point out that to achieve a quality mathematical education in the first year, it is important to implement curricula focused on mathematical processes systematically.

Teachers' mathematical knowledge at any level plays an important role in mathematics teaching. Evidently, prospective teachers have few (or no) memories of their early years at school. The first teaching knowledge with which prospective early childhood education teachers set off is limited and based on personal theories and preconceptions (Jaworski & Gellert, 2003). This makes it necessary for prospective teachers to observe what the practice is like in mathematics teaching at this stage, since they know well that mathematics is fundamental (Ball et al., 2005).

Ivars et al. (2016) point out that in initial training, it is important to encourage prospective teachers to experience different contexts that allow them to notice at teaching situations in a structured way. One of those possible contexts is the analysis of school experiences. Such experiences described as a narrative can be a powerful instrument for the construction of professional tasks. According to Ponte (2001), the use of narratives is an ideal tool for the teachers' professional development and a useful research methodology for those interested in the study of teacher education.

Our interest in the initial training of early childhood education teachers has led us, among other aspects, to consider different approaches and professional tasks to face future teachers with situations that are closest to their professional context. Hence, in this article, the objective is to describe the process of designing and structuring a professional task focused on the analysis of experiences that promote reasoning in the cycle of 0-3 years of age. This type of task seeks to include prospective teachers in processes of interpretation and decision-making (of teaching situations) so that they begin to "notice" the mathematical activity of the classroom (Sabater et al., 2012).

THEORETICAL BACKGROUND

Below, we highlight aspects that we consider key in this study on reasoning in early childhood education, teacher education, the development of the teaching competence "noticing," and the professional tasks.

The Importance of Analysing Reasoning in Early Childhood Education

Working with logical-mathematical reasoning in the early ages does not aim to learn mathematical concepts but to enhance capacities (Canals, 2009), for example, the ability to relate, to deduce cause and effect, to generalise in order to group or classify, etc. In early childhood education, to justify their answers, children follow a combination of methods: "perception, empirical evidence, and short chains of deductive reasoning based on previously accepted facts" (NCTM, 2000, p. 126). Perception and empirical evidence allow the child to face sensory stimuli for the construction of mathematical relationships (Baroody et al., 2016).

Table 1

Main logical-mathematical structures from 0 to 6 years of age (Alsina, 2006)

Identify, define, and/or recognise sensory qualities	Relate sensory qualities	Operate sensory qualities
Recognise the different sensory qualities: colour, measurement, thickness, texture, etc.	Equivalence relations: classifications	Direct logic operators
Group elements by a common quality	Order relations: ordinations	Reverse logic operators
Group elements by two common qualities	Qualitative correspondences: pairing and associations	Neutral logic operators
	Seriations	

The logical-mathematical reasoning of children from 0 to 6 years of age can be organised according to their complexity (from lowest to highest) in three structures: (i) identifying, defining and/or recognising qualities, (ii) relating qualities, and (iii) operating qualities, in short, observing changes, the

so-called logical operators (Canals, 1992; Alsina, 2006). This organisation can help the teacher not only in designing activities but analysing children's understandings and difficulties when they face a task that requires logical-mathematical reasoning. Table 1 shows the description proposed by Alsina (2006) for the three structures mentioned before.

We agree with Alsina (2006) that the perceptual relationships in which children establish similarities and differences are the basis for learning to classify and order objects. For this reason, we recognise the importance of discoveries in the early ages. It is worth mentioning that Alsina (2019) discriminates the terms "logical-mathematical reasoning" and "early algebra" where the latter is more typical of the 3-6-year cycle and mainly oriented to the activities of pattern discovery and identification. Other authors such as Vukatana (2013) refer to reasoning at 0-3 as "early inductive reasoning."

Activities with non-specific material, such as heuristic play, help to recognise relationships of belonging, and through appropriate codes (English, 2004), allow children to attribute meanings and help identify common characteristics. The heuristic play from this perspective is about curiosity and exploration. As children select, manipulate, explore, and compare items, they discover what the object can and cannot do. Indeed, when young children make a discovery, for example, when one element fits into another or an interesting sound is produced, they often repeat the action several times to test the result, which strengthens their cognitive development and the fine control of the muscles and the coordination between hands and eyes (Goldschmied & Jackson, 2004).

According to brain research, the game provides a scaffold for cognitive development, a vehicle for augmenting neural structures, and a way by which all children practice the skills they will need in the future (Isenberg & Quisenberry, 2002). Ruesga et al. (2005) observe that 3-year-olds are able to solve situations based on attributions of common characteristics (e.g. sizes and colours) using double-entry logical inference charts. On the other hand, Mora's (2016) studies allow us to understand how neural synapses are generated in the early ages and the role of the game as a childhood activity that allows combining curiosity and pleasure, two powerful weapons to favour learning.

One of the difficulties of future early childhood education teachers lies in recognising the knowledge that underlies the children's actions, even more so when these are not verbalised (De Castro et al., 2015). It is essential that prospective teachers know different activities in which reasoning is promoted in the 0-3 cycle. And be able to design and manage tasks that address processes

such as attribution, designation, and characterisation that lead to the idea of classification (Digneau, 1981; Peres, 1984, Brousseau, 1986; Byington, et al., 2016).

Teacher education and teaching competence

The interest in investigating the practice and the teacher professional knowledge has promoted the development of models for the analysis of the interaction and educational practice in the classroom (Coll & Sánchez, 2008). Some of these models focus on the promotion of the teacher's reflection on the action, thus seeking to identify key factors that influence the teaching processes, which can help support decision-making about various professional situations.

This, together with the current curricular approaches (both at school and university level) focused on the development of skills, leads us to consider new challenges in initial teacher education: How do children and young people learn mathematical knowledge? What aspects of classroom situations are relevant for interpreting such learning? What mathematical activity should be promoted in the classroom? What kind of professional situations must the teacher who teaches math face at different levels? Answering these questions implies that teachers must be able to build professional knowledge and develop teaching skills that allow them to identify and interpret relevant aspects of the teaching and learning situations. In the words of Mason (2001) and Sherin et al. (2011), learn to notice the different teaching situations in a structured way.

The professional knowledge of the mathematics teacher in early childhood must contemplate the knowledge of the curricular framework, the knowledge of the subject, the knowledge of the nature of the mathematics teaching and learning processes, and the knowledge of the complexity of teaching (Alsina, 2009). It is therefore necessary to provide prospective teachers with sufficient tools so that they can develop specific skills that are necessary for educational practice (Llinares, 2013). One of these competencies is the so-called *noticing*, which involves the development of the prospective teachers' cognitive skills to identify and interpret the students' mathematical thinking and enable informed decision-making about their teaching proposals. As stated by Ivars et al. (2016) in recent years understanding how this teaching competence is developed has become a research objective in the didactics of mathematics.

Criswell and Krall (2017) argue that the noticing competence would allow teachers to go beyond issues that are easily observed, such as students' behaviours and actions. And address issues that need to be meaningfully inferred, such as students' thinking about a given mathematical notion. Ginsburg (2016) states that teachers will not be able to teach well if they do not understand children, mathematics, and the pedagogy associated with them. And, therefore, it is essential that teachers and prospective teachers learn to recognise and understand the children's mathematical thinking.

We believe it essential to have elements that indicate what the early childhood prospective teachers' initial perceptions are when analysing different experiences. For this, the design and implementation of professional tasks are required. Placing these initial positions will promote a reflective posture as a permanent driving for the transformation of their teaching practice and professional development (Samuel et al., 2018).

Design of professional tasks

The relationship between task design and teacher education has been discussed by many researchers (Sousa et al., 2020). Professional tasks are situations teachers pose to students that can be of different nature: implementing material prepared by another student in class, investigating, developing an exercise in class, etc. The format of the professional tasks can also vary, it may be a report, an investigation, a video, etc.

Research on task design focuses on different aspects. For example, Swan (2007) studied the nature and type of tasks; Stein et al. (2009), the characteristics that a task must meet to stimulate or challenge the student; Charalambous (2010), the teacher's role in implementing the task to have students achieve a relevant cognitive process; on the other hand, Giménez et al. (2013) analysed the task design and their suitability in the education of prospective middle and high school mathematics teachers.

One of the situations that can be raised in the pre-service teacher education as a professional task is the analysis of narratives. According to Ponte (2001), the narratives can be understood as a way of representing a school experience for oneself or for others. Narratives involve people, scenarios, and events that take place at a given time, and present the following characteristics: a) situations that involve some conflict or difficulty; b) one or more agents acting in the situation with intentions of their own; and c) a temporal sequence of related events, in which the conflict is resolved in a specific way.

In pre-service teacher training, the narratives provide relevant setting off points to discuss problems the teachers face when making decisions about several teaching and learning situations such as how to translate some curriculum requirements into school activities, what type of management to develop in class, and/or how to assess and track students' learning. In the context of research on teacher education, authors such as Chapman (2008) and Ponte (2001) use narratives to reflect on the thinking and actions of prospective mathematics teachers towards mathematics content and mathematics teaching and learning.

As prospective teachers' educators, we must select, plan, and design professional tasks that allow them to identify the children's behaviour to support the development of their thinking and consequently make justified professional decisions. We know that in the case of early childhood education, prospective teachers are not specialists in mathematics teaching, and this means that such selection and design of professional tasks should be more careful so as to foster a good reflection on mathematical notions and processes and the didactics associated with them.

METHODOLOGY

In early childhood teacher education, it is essential to study aspects such as the design and implementation of activities and the analysis of professional tasks for prospective teachers to notice situations that are specific of their professional context and, consequently, develop teaching competence. Approaching these professional perspectives will allow us to characterise how prospective teachers move from observing descriptively the actions/problems as innovative practices to knowing how to argue about the didactic value of the tasks by associating types of reasoning.

The research has a qualitative approach of a descriptive nature (Cohen et al., 2018). It is based on intervention research that focuses on studies on action, seeking to understand and explain its effects (Chizotti, 2006). Those studies are characterised by the design and development of interventions, which includes determining the extent to which an intervention is defined by practical principles, objectives, and explicit activities (Fraser & Galinsky, 2010).

The intervention is carried out with students of the early childhood education degree of two Catalan universities during the academic years 2018-2019 and 2019-2020. The prospective teachers attend the school subjects Matemáticas, Ciencias Experimentales y Educación and Conocimiento y

Actividad Matemática en Educación Infantil [Mathematics, Experimental Sciences and Education and Knowledge and Mathematical Activity in Early Childhood Education], which are offered in the third semester of both degrees.

We design a professional task to allow prospective teachers to observe, analyse, and reflect on real experiences that promote logical-mathematical reasoning in 0-3 cycle. The structuring of this professional task covered different stages: 1) reviewing studies about the teaching and learning of logical-mathematical reasoning in 0-3 cycle; 2) reviewing studies, materials, and activities to work the logical-mathematical reasoning in early childhood education; 3) choosing narratives for prospective teachers to observe and analyse; 4) implementing the professional task designed in a pilot group; and, 5) defining levels to characterise the teaching competence in advance.

Previous professional task: design of reasoning activities from nonspecific materials

The professional task presented in this study has as its beginning a previous professional task (PPT) posed to prospective early childhood teachers. This task focused on the design and implementation of activities based on the use of non-specific materials[1]. Non-specific material was chosen as it is a material present in usual activities in this cycle. Among these activities are the treasure basket, the heuristic game, and the experimentation trays (Goldschmied, 1986, 2002; Edo, 2012; Majem & Òdena, 2001). Non-specific materials are those that do not have an educational purpose at first but that allow children to carry out explorations in which they begin to identify the objects and their characteristics. These materials can be natural: pineapples, leaves, sticks, pumpkins, sea sponges, etc., or everyday materials: brushes, zippers,

[1]In the case of this PPT (which is not the one described in this article), the families of all children involved in the research received an informed consent form stating that they could withdraw from the study at any time. About the design and structuring of the professional task shown in this article, it is a task elaborated with the prospective teachers, without the intervention of minors. All participants collaborated on a completely voluntary basis. A previous ethical evaluation of this research has not been requested due to its nature, since it is carried out from the productions of the prospective teachers, whose participation has been voluntary and in a training process. In no case do names, images, or any reference to the participants appear. At the same time, the authors of this work assume and exempt Acta Scientiae from the possible consequences that may arise from the data collected in this research including full assistance and eventual compensation for any damage that results to any of the research participants, in accordance with Resolution No. 510, of April 7, 2016, of the National Health Council of Brazil.

pushbuttos kitchen roll paper or toilet paper rolls, pieces of rubber tube of different diameter, etc.

These types of activities (with non-specific materials) resides not only in its potential for exploration but also in the opportunity it provides for children to characterise objects, establish relationships to create an order of what they perceive. There are fundamental processes in the construction of children's initial mental structures, which, as Edo (2012) argues, will be present throughout the mathematical enculturation process.

The PPT was developed in the 2018-19 academic year by 80 students of the early childhood education degree of two Catalan universities, organised in teams of 4-5 people. Each team designed and implemented an activity (with non-specific materials) with the aim of enhancing logical-mathematical reasoning in children aged 0-3 years. The goal was to provide opportunities for children to discover and establish characteristics (common or not) through manipulating and experimenting with different objects. This will be the perceptual basis of processes such as attribution, designation, and classification.

The PPT included, besides the design and implementation of the activity, a reflection on the material used and its potential and relevance to promote reasoning in children in this cycle. As a product, the prospective teachers presented a written document and a visual record of the implementation (Boukafri et al., 2016). The PPT was developed in different phases, which can be seen below (Figure 1).

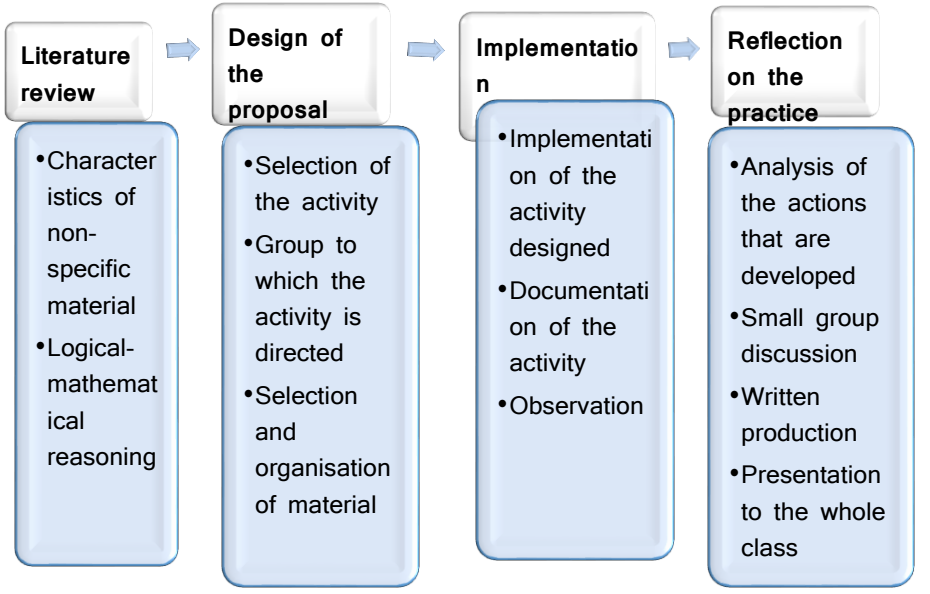
The productions of prospective teachers in the PPT have allowed us to have a diverse set of activities structured from non-specific material. Among these activities, we find proposals such as treasure basket, sensory wall, experimentation trays, lights and shadows, heuristic game, sensory carpet, among others. Each of them has incorporated different types of materials that have allowed different attributes to be explored and lead to the establishment of different relationships. Likewise, we have a variety of implementation examples with a single child and small groups, which is essential to analyse the suitability of the proposals and enrich the reflection on the practice in teacher education.

The development of the reflection phase on the implementation of the activity allowed students of the early childhood education degree to observe the childrens' reactions and actions, identify the potential of the activities with non-

specific material to promote reasoning and recognise key aspects in the management of this type of activity.

Figure 1

Phases in the development of the previous professional task (PPT)



For the next course (2019-2020), some of the proposals of the prospective teachers who carried out the PPT were used as input for the design of another professional task to be proposed to a new group of prospective teachers. This new task, which is described in detail in the following section, focused on the study of narratives (proposals derived from the PPT). The aim of the new task is to strengthen in this new group of prospective teachers other aspects related to their teaching competence, specifically the analysis of real classroom experiences.

Initial design of the professional task and first implementation

In this professional task, we propose a new training objective. We are interested that prospective teachers not only learn to plan and design activities but also consider it appropriate to influence other aspects of their professional competence, those related to the assessment, analysis, and proposals for improvement on experiences carried out by others.

A key element in the design of this professional task is the choice of narratives to be analysed. In this case, we chose two different experiences focused on the use of non-specific materials for two different ages from the set of proposals developed by the prospective teachers in the PPT (in the previous year, 2018-19). As a documentary register of each of the experiences, there is a video and a written document. The video shows how the proposal planned was implemented with children aged between 0 and 3 years old. The written document corresponds to the work that was requested from the prospective teachers in the PPT. This work was structured in four sections: a) review of literature on the type of activity and first exploration of the materials; b) design of the proposal and organisation of the material; c) implementation and documentation of the evidence of the children's main actions; and d) reflection on the practice and what was learned with the development of the task globally.

The first of the selected experiences was implemented with a 9-month-old girl and consisted of an exploratory activity on a *sensory wall*. The second experience was a *heuristic game* activity implemented with a group of seven 24-month-old children. By observing and analysing these experiences, the prospective teachers are expected to identify how logical-mathematical reasoning can be promoted in early childhood education. At the same time, they can observe the importance of working the logical-mathematical reasoning in the cycle from 0-3 years of age from real situations.

The objects presented in the first experience (sensory wall) includes natural objects, wood, metal, fabrics of different textures and colours, mirrors, pushbuttos, brushes, among others. These objects are organised as shown in Figure 2. The sensory wall is designed to provide children with a variety of objects and materials to experiment with different senses.

Figure 2

'Sensory wall' Experience (proposal made by the prospective teachers in the PPT)

In the second experience (heuristic play), the objects are arranged into five bags containing about 10-15 items in each bag. The objects include: jars of different diameters without lids, wooden sticks of different lengths, rubber tubes of different lengths and diameters, wooden rings of different diameters and, chains of different lengths and thicknesses. Some of these can be seen in Figure 3.

Figure 3

'Heuristic Play' Experience (proposal made by the prospective teachers in the PPT)

The professional task is organised in three parts. In the first one, the prospective teachers are requested to ask about the main characteristics of the type of activities described in the two experiences (sensory wall and heuristic play). Identifying the type of objects that are used in each experience and indicating the attributes corresponding to each object or groups of objects, and describing the actions that could be developed with each object. In the second part, the prospective teachers were requested to associate each of the children's actions with the logical capacities (identifying, relating, and operating and evaluating the design and management of each of the activities studied. Finally, in the third part, the prospective teachers are asked to explain whether or not they would use (and for what reasons) objects similar or equal to those presented in the two activities.

A first implementation of the task was carried out with 37 students of the second year of the early childhood education degree at a Catalan university.

The group is taking the first subject of didactics of mathematics in the degree. The prospective teachers have studied content related to logical-mathematical reasoning and the problem solving process. We proposed that they develop the task in small groups (3-4 people), for a joint reflection that could promote the didactic analysis competence of learning situations. As a result of their analysis, each group submits a written work, which constitute our data. In total we have 10 groups and therefore 10 written productions.

RESULTS

The results presented below are organised into two sections. Firstly, we show them the observations of a first approach to the development of teaching competence and secondly, the final proposal of the professional task.

A first approach to the development of the teaching competence

Once the task was implemented, the answers of the different groups of prospective teachers were organised to refine the task for subsequent implementations, but also to characterise their teaching competence: noticing. We built levels associated with the different capacities of the teaching competence: identifying, interpreting, and making decisions (Jacobs et al., 2010). We used the methodology of successive observations and analysis to elaborate the descriptors for each level (Strauss & Corbin, 1994). We also used open code in which the data can be explored, and analysis units can be identified to codify aspects of each of the capacities of the teaching competence (Cohen et al., 2018).

Table 2

Examples of responses and levels associated with the identifying skill

Group	Level	Examples of answers about the objects and their attributes	Examples of answers on the actions

7	N1	" *Metal* jars – *sound.* Chains – *movement, touch, colour, sound"*	" *Jars* allow them to *put* in and *take out* objects. They spinned them and made noise with the sticks. They can also *stack* them."
3	N2	"Bag of *rattles* - develop the *sense of hearing*. *Zipper* – develops the curiosity of children for the so pleasant *texture*. *Comb and brush* - *develops touch*"	"Relating the *action of playing to* the *sound* (if I touch and move it, it sounds). Perform the *action of the clamp* and identify that it must be *slipped.* Explore the different *textures*, from *soft to rough.*"
9	N3	"According to Piaget, the child uses *inductive reasoning to extract general characteristics from the observation* of the *similarities* and *differences* of objects, facts, or actions...Some of the arguments we see are hint.ed at when the girl identifies that the bells *ring* by *moving* the bag. Or when she discovers that *by pressing* the switch, *the light turns on*"	"The knowledge of objects and their *functions*/*actions* appear by *experimenting* with them"

The answers and comments obtained in the implementation related to the ability to identify attributes of the objects and their mathematical potential in the construction of future mathematical notions (see examples in Table 2) are associated with the following level descriptors:

N1 – Recognise only the attributes listed in the written document associated with experiences 1 and 2 of the PPT.

N2 – Recognise both the attributes described in the written document of experiences 1 and 2 of the PPT; add other attributes and suggest that some of the attributes described in the experiences are not adequate.

N3 – Recognise the attributes in the two experiences of the PPT and distinguish relevant differences between them.

As for the identifying skill, we find that prospective teachers recognise measurable attributes, spatial relationships associated with movements, and attributes associated with comparisons and order (large-small; thick-thin; very-poor, among others). They also identify various relationships, for example, as children give unequivocal samples that recognise similar elements. Because of the type of function they perform, they allude to causal relationships.

Regarding the interpreting skill, three levels are previously established, and the following criteria are associated:

> N1 – Identify comparison actions and some cause-effect relationships by pointing at them descriptively
>
> N2 – Allude to types of reasoning metaphorically, indicating terms such as visual, deductive, but without specifying what it is. Describe actions associated with the different logical capacities (identify, relate, operate) without theoretical justification.
>
> N3 – Evaluate children individually, associating types of abilities and possible types of mathematical reasoning appropriately. Discriminate key aspects on the adequacy of activities according to age.

Table 3 below shows, by way of example, extracts of answers from different groups and the allocation at the different levels.

Table 3

Examples of responses and levels associated with the interpreting skill

Group	Level	Answer Extract	Analysis
10	N1	"…They are able to perceive that when they take a *metal tube* and *hit* it, they associate that it makes *noise*, and that the *louder they hit*, the *louder the sound*'	The group of prospective teachers describes actions that allude to cause-effect relationships, without justification, they only verify them.
2	N2	"On the sensory wall, we see that the girl can *identify different elements* thanks to *touch*. It is a process of identification by touch."	This group interprets different actions and associates them

		"...They observe that when *they move some objects* in a *specific way* or when they *collide with each other*, they make *noise*. They establish a *relationship* between the *action (hitting) and the sound*'	with different capacities (identifying – relating)
4	N3	"…In other words, with this experience, children begin to *discover and know the different objects* they have in the classroom, *then* they begin to *do actions* on them, for example, make noise, and finally, they *apply the symbolic game*, making those objects stop being what they are to be other things such as necklaces, snails, telescope, etc.' "Perhaps if we suggested a treasure basket instead of this activity, *it would be more appropriate*. As Eva *does not crawl*, the educator had *to help her to reach* the objects'	The group of prospective teachers recognises key moments in the development of experiences where children move from observation to discovery. They also interpret the value of the heuristic play to introduce the symbolic game. They appreciate the relevance of the activities according to the ages and characteristics of the children (for example, their mobility possibilities and how this conditions the exploration)

Regarding the interpreting skill, we observe that the prospective teachers allude to types of reasoning (inductive, abductive, etc.) metaphorically, erroneously referring to the visual, and deductive, without specifying how to verify it. They explain descriptive traits of actions associated with logical capacities, without theoretical discussion.

Table 4

Examples of answers and levels associated with the making decisions skill

Group	Level	Excerpts from answers	Analysis
2	N1	"If we had to prepare an activity for early childhood education, we could perfectly use the materials from the experiences, although it would be better to *use and enhance other objects*, such as *natural objects*, choosing objects that we can find at home or in our daily environment. It is important *to get as much variety* as possible regarding *size, weight, colour, and texture.* Natural objects: pineapples, chestnuts, leaves, stones..."	The group makes a slight reflection, but does not give a specific reason, nor does it provide what could be worked with the proposed material.
5	N2	"Get informed, i.e., ask the children why they are doing things, since otherwise, the educator cannot clearly see how their reasoning changes"	The group recognises the value of a metacognitive action (asking questions)
4	N3	"On the sensory wall, we would add balls of different textures, *spring a ling* (circuits with wooden pieces of different colours, shapes, and sizes); bubble plastics will give them a feeling of change, it will make it easier for them to interact with sound and touch. In the heuristic game, we would add natural objects such as cork stoppers for different sizes and shapes, forest pineapples, chestnuts, stones, shells, coconut shells, because of the specific shapes they have." "When they are very small, they develop this reasoning through repetitions, therefore, we would pay a lot of attention to the children's repetitions with the objects, if they make them touch each other or if they hit them against the floor, or against something, or everything in general"	This group of prospective teachers proposes more distinctive comparative elements for the attributes texture, form, and measure. They recognise the role of repetition in identifying children's reasoning.

Regarding the making decisions skill related to the assessment of activities that promote reasoning and their possible improvement, Table 4 presents examples of answers and the levels that have been assigned. The criteria defined for these levels are:

N1– Consider small changes with limited justification.

N2 – Propose incorporating new materials and more relationships and operations. justify, but without alluding to any theoretical reference.

N3 – Make proposals for improvement appropriately considering the children' age, materials, timing, and management of activities. Justify incorporating theoretical references.

As for making decisions skill, the prospective teachers propose some changes in the format of experiences, making some justifications. Fundamentally, they consider, in the case of heuristic play, that it would be appropriate to include and ask some questions to help children verbalise their discoveries. No new contributions are shown in terms of mathematical structures to be developed.

After the first implementation and an initial analysis of the productions of the group of the prospective teachers involved in this research, we decided to made some adjustments in the final structuring of the professional task. Four sections are underscored in the task. We suggest that a table be included for the organisation of the answers in section 2. This is intended for the prospective teachers to give more precise answers on the relationships between objects and the actions carried out with them. We also added a question (in section 4) about the possible improvements in the experiences analysed. We intend prospective teachers to justify more widely the aspects they assume should be considered in an activity (with non-specific material) that promotes reasoning in 0-3 cycle. In this way, we could have more evidence about the making decisions skills of the prospective teachers in specific professional situations.

The details on how the professional task is organised are presented in the following section.

Proposing a professional task

One of the products generated by this study was a professional task that aims to mobilise the professional knowledge of prospective early childhood teachers about a type of experiences and activities that help promote logical-

mathematical reasoning in the 0-3 cycle. The final version of this task considers the skills that characterise the professional noticing: identifying, interpreting, and making decisions (Jacobs et al., 2010; Ivars et al., 2016); the notion of narrative and its potentiality to encourage reflection on situations of mathematics teaching and learning (Ponte, 2001; Ponte & Quaresma, 2017); two types of activities with non-specific material, which we consider suitable to promote reasoning with children at this stage (Goldschmied, 2002; Edo, 2012; Boukafri et al., 2016) and specific aspects of reasoning in early childhood education such as logical capacities: identifying, relating, and operating (Canals, 2009).

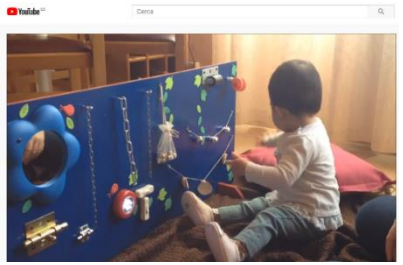
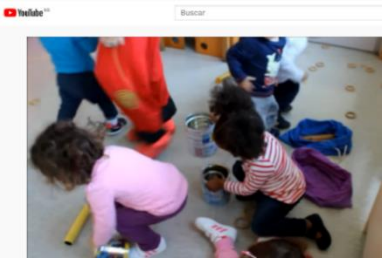
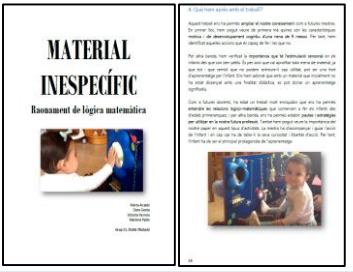
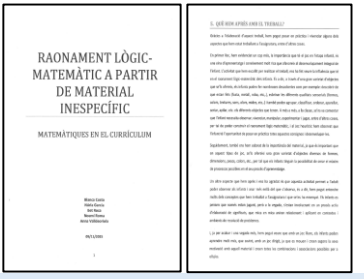
The professional task is structured in four parts:

1. In the first, the prospective teachers are requested to ask about the main characteristics of the type of activities described in the two experiences (sensory wall and heuristic play).
2. In the second, they are requested to identify the type of objects that are used in each experience and indicate the attributes corresponding to each object or groups of objects, and describe the actions that could be developed with each object.
3. In the third part, they are requested to associate each of the children's actions with the logical capacities (identify, relate, and operate); and, assess the design and management of each he activities studied.
4. Finally, in the fourth part, the prospective teachers are asked to decide whether or not to use (and why) objects similar or equal to those presented in the two activities, and reflect on the type of improvements they would make in the two activity proposals to better recognise the children's reasoning.

Figure 4 below presents the final version of the professional task, which includes the documentary supports of the two experiences (video and written document), the questions raised in each of the sections, which serve to guide the analysis of the experiences, and the corresponding objectives for each section.

Figure 4

Professional task of analysis of experiences that promote reasoning in 0-3 cycle

DOCUMENTARY SUPPORT	
Narrative 1: 'Sensory Wall' Experience	Narrative 2: 'Heuristic Play Experience
	
	
QUESTIONS TO GUIDE THE ANALYSIS	**OBJECTIVES**
1. Let us look for information Probe and describe the main characteristics of the activities described in both experiences: Sensory Wall and Heuristic Game).	Recognise the main characteristics of activities with non-specific material for children aged 0-3.

2. Let us describe the experience

2.1 Identify the types of objects that are used in each experience.
2.2 Indicate the attributes corresponding to each object.
2.3 Describe the actions that could be developed with each object (For example, put, remove, stack, etc.).

Recognise the importance of object exploration in helping children identify attributes and establish relationships.

We suggest you answer points 2.1 to 2.3 by making two tables such as the following:

'Sensory Wall' Activity		
Object	*Attribute*	*Actions*

'Heuristic Play" Activity		
Object	*Attribute*	*Actions*

3. Let us interpret

3.1 Interpret what type of children's reasoning is evident in each of the experiences.

Highlight relevant aspects in the interpretation of children's reasoning in different experiences.

3.2 Associate with each of the capacities: ***identifying, relating, and operating*** with children's concrete actions.

Appreciate the importance of suitability in the design of activities to be carried out with children

3.3 Knowing that both the sensory wall and the heuristic game can be carried out in different ways, assess what you think of the design and management carried out in each of the activities.

Recognise the impact of the teacher's role in the proper development of activities with non-specific material

4. Let us make decisions

Identify key aspects in the selection/planning of activities with non-specific materials.

4.1 If you plan to do these activities with children, would you use other objects? Why?

Learn to evaluate the scope and limitations of different proposals.

4.2 What would you do to improve the proposals for activities, so that you recognise better the children's reasoning?

Recognise aspects susceptible to improvement in the design, planning, and implementation of proposals

FINAL CONSIDERATIONS

We consider, as Pochulu et al. (2013), that it is in the process of designing and redesigning tasks that the possibility of increasing the knowledge of teachers arise. In our case, the professional tasks designed give the opportunity to involve future teachers in what Serrazina (2010) calls the process

of *planning – action – reflection*, which is fundamental in the configuration of the mathematical and didactic knowledge of the prospective teacher.

Learning to evaluate the adequacy of the planning and design of activities based on the analysis of the proposals made by others is also a key competence to be developed by the prospective teachers. Being able to see and study the implementation of these proposals and observe how the activity with children works in different contexts allows integrating both the knowledge developed in training, as well as the observations on how certain processes are actually developed in early childhood education.

The chosen narratives allowed us to involve the prospective teachers in real professional situations and were suitable as an example of activities for the promotion of logical-mathematical reasoning in 0-3 cycle. It is not enough to show a type of activity; it is necessary to be able to discuss with the prospective teachers what happens during its implementation, what the key aspects in its design are, what makes an activity be a successful experience, among other aspects. Specifically, the study of both experiences (sensory wall and heuristic play) allowed the prospective teachers to recognise different types of attributes (sensory and measurable characteristics); interpret children's actions associated with different types of relationships (equivalence, order, causal, spatial, etc.). As well as observing actions that indicate changes (shape, position, measurement, among others). As Alsina & Berciano (2016) affirm, these aspects are key in the promotion of intuitive and informal mathematics in children under 3 years of age.

The professional tasks described have made it possible for prospective teachers to be more aware of what happens in different experiences carried out with children aged 0-3. One of the tasks focused on the processes of design, planning, and implementation of activities to promote reasoning in the 0-3 cycle, while the other professional task placed the prospective teachers before a situation of evaluating said activities to make decisions on their possibilities of improvement. All this implies analysing the mathematical content involved, reflecting on how to organise this content to teach it, analysing and interpreting children's productions, and considering how this type of activity can be managed. In short, as it was proposed by Fernández et al. (2020), this aims that prospective teachers learn to notice and have more and more elements that allow them to make effective teaching decisions.

We know that the process of designing professional tasks must be permanent and adjusted to the particular needs of the educational contexts. We expect to continue advancing in this perspective, designing and redesigning

proposals to improve the professional knowledge of prospective teachers and be able to contribute with elements to the discussion on the type of education in mathematics and didactics of mathematics that we consider should have the prospective teachers of early childhood education.

ACKNOWLEDGEMENTS

This study has been carried out in the context of the projects: PID2019-104964GB-I00 (MICINN) / PGC2018-098603-B-I00/ 2017-SGR-101 and 2017-SGR-1353.

AUTHORSHIP CONTRIBUTION STATEMENT

Y. V., J. G., and M. P. conceived the idea presented and developed the theory. Y.V., J. G., and M. P. designed the professional tasks, collected the data, and wrote the initial draft. All authors actively participated in the construction of the methodology, discussion of the results of the implementation, and jointly produced the final version of the article.

DATA AVAILABILITY STATEMENT

Data used and analysed during the present study are in the authors' archives, the first author Y.V. being in charge of their custody and consultation by the interested parties.

REFERENCES

Alsina, À. (2006). *Como desarrollar el pensamiento matemático de 0 a 6 años*. Octaedro-Eumo.

Alsina, À. (2009). Un análisis optimista de la educación matemática en la formación de maestros de educación infantil. *UNO-Revista de Didáctica de las Matemáticas, 51*, 30-43.

Alsina, À. (2015). *Matemáticas intuitivas e informales de 0 a 3 años*. Narcea.

Alsina, À. & Berciano, A. (2016). Una aproximación a las acciones matemáticas de niños de 1 a 3 años. En J. A. Macías, A. Jiménez, J. L. González, M. T. Sánchez, P. Hernández, C. Fernández, F. J. Ruiz, T. Fernández y A. Berciano (Eds.), *Investigación en Educación Matemática XX* (pp. 137-146). SEIEM.

Alsina, À. (2019). Del razonamiento lógico-matemático al álgebra temprana en Educación Infantil. *Edma 0-6: Educación Matemática en la Infancia*, *8*(1), 1-19.

Ball, D., Hill, H. & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, Fall*, 14-22.

Baroody, A., Purpura, D. J., Eiland, M. D., Reid, E.E. & Paliwal, V. (2016). Does fostering reasoning strategies for relatively difficult basic combinations promote transfer by K-3 students? *Journal of Educational Psychology*, *108*(4), 576-591. https://doi.org/10.1037/edu0000067

Boukafri, K., Vanegas, Y. & Prat, M. (2016). Comencem amb la formació inicial. *Actes del Congrés Català d'Educació Matemàtica (C2EM)*. Federació d'Entitats per a l'Ensenyament de les Matemàtiques a Catalunya. Barcelona. https://c2em.feemcat.org/wp-content/uploads/actes/3C182.pdf

Brousseau, G. (1986). Fondements et méthodes de la didactique des mathématiques. *Recherches en didactique des mathématiques*, *7*(2), 33-115.

Byington, T., Kim, Y., Nazarechuk, C., & Weigel, D. (2016). *Math in the Preschool Classroom: Classification, Matching, Seriation and Patterning*. University of Nevada Cooperative Extension.

Canals, M. A. (1992). *Per una didàctica de la matemàtica a l'escola: 1. Parvulari*. Eumo.

Canals, M. A. (2009). *Lógica a todas las edades*. Associació de Mestres Rosa Sensat.

Chapman, O. (2008). Narratives in mathematics teacher education. In: *The Handbook of Mathematics Teacher Education: Volume 2* (pp. 13-38). Brill Sense. https://doi.org/10.1163/9789087905460_003

Charalambous, C. Y. (2010). Mathematical knowledge for teaching and task unfolding: An exploratory study. *The Elementary School Journal*, *110*(3), 247-278. https://doi.org/10.1086/648978

Chizotti, A. (2006). *Pesquisa qualitativa em ciências humanas e sociais*. Vozes.

Clements, D. H. & Sarama, J. (2020). *Learning and teaching early math: The learning trajectories approach*. Routledge. https://doi.org/10.4324/9781003083528

Cohen, L., Manion, L. & Morrison, K. (2018). *Research Methods in Education*. (eight edition). Routledge.

Coll, C. & Sánchez, E. (2008). Presentación. El análisis de la interacción alumno-profesor: líneas de investigación. *Revista de Educación*, *346*, 15-32.

Criswell B. & Krall R.M. (2017). Teacher Noticing in Various Grade Bands and Contexts: Commentary. In: Schack E., Fisher M., Wilhelm J. (Eds.) *Teacher Noticing: Bridging and Broadening Perspectives, Contexts, and Frameworks*. *Research in Mathematics Education*. Springer. https://doi.org/10.1007/978-3-319-46753-5_2

De Castro, C., Flecha, G. & Ramírez, M. (2015). Matemáticas con dos años: buscando teorías para interpretar la actividad infantil y las prácticas docentes. *Tendencias pedagógicas*, *26*, 89-108.

Digneau, J. M. (1980). *Création d'un code À l'école maternelle: étude d'un saut informationnel*. Mémoire de DEA, IREM de Bordeaux.

Edo, M. (2012). Ahí empieza todo. Las matemáticas de cero a tres años. *Números*, *80*, 71-84.

English, L. D. (2004). Mathematical and analogical reasoning in early childhood. In L. D. English, (Ed.), *Mathematical and Analogical Reasoning of Young Learners* (pp. 1-22). Lawrence Erlbaum. https://doi.org/10.4324/9781410610706

Fernández, C., Bernabeu, M., Callejo, M. L., González, J. M., Ivars, P., Llinares, S., Zorrilla, C. & Torregrosa, B. (2020). Desarrollo de competencias docentes para la enseñanza de las matemáticas en la formación de maestros de Educación Infantil y Primaria. In *Memòries del Programa de Xarxes-I3CE de qualitat, innovació i investigació en docència universitària: Convocatòria 2019-20*. (pp. 141-144). Instituto de Ciencias de la Educación. Universidad de Alicante.

Fraser, M. V. & Galinsky, M. J. (2010). Steps in Intervention Research: Designing and Developing Social Programs. *Research on Social Work Practice*, *20*(5), 459-466.

Fuson, K. C., Clements, D. H. & Beckman, S. (2009). *Focus in prekindergarten: Teaching with curriculum focal points.* NCTM & NAEYC.

Giménez, J., Font, V. & Vanegas, Y. (2013). Designing Professional Tasks for Didactical Analysis as a research process. In C. Margolinas (Ed.) *Task Design in Mathematics Education. Proceedings of ICMI Study 22*. (pp. 581-590). ICME.

Ginsburg, H.P. (2016). Helping early childhood educators to understand and assess young children's mathematical minds. *ZDM Mathematics Education, 48*, 941–946. https://doi.org/10.1007/s11858-016-0807-739

Goldschmied, E. (1986). El joc heurístic. Una activitat del segon any de vida. *Infància*, *33*, 11-15.

Goldschmied, E. (2002). *Educar en la escuela infantil.* Octaedro.

Goldschmied, E. & Jackson, S. (2004). *People under three: Young children in day care*. Psychology Press.

Isenberg, J. & Quinsenberry, N. (2002). A Position Paper of the Association for Childhood Education International PLAY: Essential for all Children. *ChildhoodEducation, 79*(1), 33-39. https://doi.org/10.1080/00094056.2002.10522763

Ivars, P., Fernández, C. & Llinares, S. (2016). Cómo estudiantes para maestro miran de manera estructurada la enseñanza de las matemáticas al escribir narrativas. *La matemática e la sua didáttica*, *24*, 79-96.

Jacobs, V. R., Lamb, L. L. & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for research in mathematics education*, *41*(2), 169-202.

Jaworski, B. & Gellert, U. (2003). Educating new mathematics teachers: integrating theory and practice, and the roles of practising teachers. In A. J. Bishop (Ed.), *Second international handbook of mathematics education* (pp. 823-876). Kluwer. https://doi.org/10.1007/978-94-010-0273-8

Llinares, S. (2013). Professional noticing: A component of the mathematics teacher's professional practices. SISYPHUS. *Journal of Education*, *1*(3), 76-93. https://doi.org/10.25749/sis.3707

Majem, T. & Òdena, P. (2001). *Descubrir jugando. Temas de infancia, 1*. Octaedro.

Mason, J. (2001). *Researching your own practice: The discipline of noticing.* Routledge. https://doi.org/10.4324/9780203471876

Mora, F. (2016). *Neuroeducación, solo se puede aprender aquello que se ama*. Alianza.

NAEYC & NCTM (2013). Matemáticas en la Educación Infantil: Facilitando un buen inicio. Declaración conjunta de posición. *Edma 0-6: Educación Matemática en la Infancia, 2*(1), 1-23.

NCTM (2000). *Principles and standards for school mathematics*. NCTM.

NRC (2014). Fundamentos cognitivos para la iniciación en el aprendizaje de las matemáticas. *Edma 0-6: Educación Matemática en la Infancia, 3*(1), 21 - 48.

Peres, J. (1984). *Utilisation d'une théorie des situations en vue de l'identification des phénomènes didactiques au cours d'une activité d'apprentissage scolaire: construction d'un code de désignation d'objets à l'école maternelle*. Thèse de 3ième cycle Université Bordeaux II, IREM de Bordeaux.

Pochulu, M., Font, V. & Rodríguez, M. (2013). Criterios de diseño de tareas para favorecer el análisis didáctico en la formación de profesores. En SEMUR, Sociedad de Educación Matemática Uruguaya (Ed.), *Anales del VII Congreso Iberoamericano de Educación Matemática*. (pp. 4999-5009). SEMUR.

Ponte, J. P. (2001). Professional narratives in mathematics teacher education. *Proceedings of the Canadian Mathematics Education Study Group* (pp. 61-65). University of Alberta.

Ponte, J. P. & Quaresma, M. (2017). O papel do contexto nas tarefas matemáticas. *Revista Interações*, *22*, 96-216. https://doi.org/10.25755/int.1542

Ruesga, P., Giménez, J. & Orozco, M. (2005). Diagramas de relaciones lógicas en tareas de transformación para preescolares. *Enseñanza de las ciencias: Revista de investigación y experiencias didácticas*, *23*(3), 403-418.

Sabater, L., Penalva, M. C. & Callejo, M. L. (2012). Competencias docentes en la formación de maestros de educación infantil desde la educación matemática. En J. Álvarez, M. T., Tortosa, N. Pellín (Coords.). *Actas X Jornades de Xarxes d'Investigació en Docència Universitària*. (pp. 2982-2994). ICE-Universidad de Alicante.

Sámuel, M., Vanegas, Y. & Giménez, J. (2018). Caracterización del conocimiento matemático de futuras maestras de educación infantil. *Bordón, 70*(3), 61-75. https://doi.org/10.13042/Bordon.2018.62907

Sherin, M. G., Jacobs, V. R. & Philipp, R. A. (Eds.) (2011). *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 35-50). Routledge. https://doi.org/10.4324/9780203832714

Serrazina, M. de L. (2010). A Formação Contínua de Professores em Matemática: o conhecimento e a supervisão em sala de aula e a sua influência na alteração das práticas. *International Journal for Studies in Mathematics Education of the Educational Researches and Publications Associations*, *2*, 2-24. https://doi.org/10.17921/2176-5634.2010v2n1p%25p

Sousa, J. R., Gusmão, T. C. R. S., Font, V. & Lando, J. C. (2020). Task (re) design to enhance the didactic-mathematical knowledge of teachers. *Acta Scientiae, 22*(4), 98-120. https://doi.org/10.17648/acta.scientiae.5711

Stein, M. K., Smith, M. S., Henningsen, M. A. & Silver, E. A. (2009). *Implementing standards-based mathematics instruction* (2nd ed.). Teachers College Press. NCTM.

Strauss, A. & Corbin, J. (1994). Grounded Theory Methodology: An Overview. In N. K. Denzin, & Y. S. Lincoln (Eds.), *Handbook of Qualitative Research* (pp. 273-285). Sage.

Swan, M. (2007). The impact of task-based professional development on teachers' practices and beliefs: A design research study. *Journal of Mathematics Teacher Education*, *10*, 217-237. https://doi.org/10.1007/s10857-007-9038-8

Vukatana, E. (2013). *Early Inductive Reasoning: Examining 11-Month-Olds' Abilities* (Unpublished master's thesis). University of Calgary, https://doi.org/10.11575/PRISM/25768