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The High School Student Protagonism through a Research with the Projects Methodology

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ABSTRACT

Background: Developing autonomy, the ability to solve problem situations, make decisions and act for the benefit of your social environment are modern life skills and can be developed in the school environment, along with mathematical content, and can be viable through the methodology of project projects, using active methodologies and the resources of digital technologies. **Objectives**: Discuss the Mathematics Curriculum or the work projects as a pedagogical proposition based on the development of three projects with the thematic Cryptography, Music, and Project launching applicable to the High School. **Design**: Qualitative research that sought to investigate work with projects in High School was used. **Setting and Participants**: Experiments developed with two classes of high school students in the Rio Grande do Sul state. **Data collection and analysis**: Data collection took place during the development of the project stages through students' written records and questionnaires. **Results**: It is considered that the Work Projects developed constituted a possibility to modify the role of the student and the teacher, allowing students to become active, participative, and committed to the development of their knowledge. **Conclusions:** It is understood that students, their learning and development must be the focus of the educational process. Therefore, the school curriculum must enable students to assume the role and responsibility for their learning.

Keywords: Mathematics Curriculum; Active Methodologies; Work Projects; Digital Technologies; Skills of modern life.

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O protagonismo do estudante no Ensino Médio por meio da investigação com a metodologia de Projetos

RESUMO

Contexto: Desenvolver a autonomia, a capacidade de resolver situações problemas, tomar decisões e atuar em benefício do seu meio social são competências da vida moderna e podem ser desenvolvidas no ambiente escolar, juntamente com os conteúdos matemáticos, podendo ser viável por meio da metodologia de projetos de trabalho, utilizando-se metodologias ativas e os recursos das tecnologias digitais. **Objetivo:** Discutir no Currículo de Matemática os projetos de trabalho como proposta pedagógica a partir do desenvolvimento de três projetos com as temáticas Criptografia, Música e Lançamento de Projéteis aplicáveis no Ensino Médio. **Design:** Foi utilizada a pesquisa qualitativa que buscou investigar o trabalho com projetos no Ensino Médio. **Ambiente e participantes**: Experimentos desenvolvidos com duas turmas de estudantes do Ensino Médio, no Rio Grande do Sul. **Coleta e análise de dados:** A coleta de dados ocorreu ao longo do desenvolvimento das etapas dos projetos, por meio de registros escritos dos estudantes e questionários. **Resultados:** Considera-se que os Projetos de Trabalho desenvolvidos se constituíram em uma possibilidade de modificar o papel do estudante e do professor, possibilitando aos estudantes tornarem-se sujeitos atuantes, participativos e comprometidos com a elaboração de seu conhecimento. **Conclusões:** Entende-se que os estudantes, suas aprendizagens e desenvolvimento devem ser o centro processo educativo. Para tanto, é preciso que o currículo escolar possibilite aos alunos assumir o protagonismo e a responsabilidade sobre suas aprendizagens.

Palavras-chave: Currículo de Matemática; Metodologias Ativas; Projetos de Trabalho; Tecnologias Digitais; Competências da vida moderna.

INTRODUCTION

The conditions imposed by modern life, when we are called to act in a constantly changing world, increasingly dependent on technologies and which, at all times, presents us with new challenges, both individual as collective, require individuals to develop autonomy, the capacity of solving problem situations, making decisions, acting for the benefit of their social environment. In this context, Education, and particularly Mathematics Education, has the responsibility to develop work that enables students, since early, to act in environments that contribute to their formation as active citizens in this increasingly demanding world.

However, an education centred on the protagonism of students, where they have the opportunity to participate actively and assume responsibility for their learning and development, requires curriculum planning that allows this action and protagonism. In this context, we understand that active methodologies, including work with projects, allied to digital technology resources, have a lot to contribute.

Projects are considered an active methodology based on the theoretical principle, which we consider significant: the formation of autonomy, critical students, with conditions to act in the modern world. Freire's (2014) ideas subsidise this theory, breaking with the idea that students are only passive recipients of the information. Far beyond being passive students, they are restless people who can reflect on different themes and develop solution strategies to face problem-situations of a certain complexity (Mora & Rivera, 2004). The project methodology allows the use of various topics of interest that are important for the student's education, providing opportunities not only for work with school contents but for training that relates them to social, cultural and political aspects.

Thus, in this article, we present three work projects developed with high school students. The first, with the theme Cryptography and Functions, the second with the theme Music, involving Trigonometry and the third, on Projectile Launch and the study of Functions, Uniform and Uniformly Varied Rectilinear Movement and Oblique Launch.

CURRICULUM OF MATHEMATICS AND CONTEMPORARY THEMES AND OF INTEREST FOR THE FORMATION OF A CITIZEN

According to Pacheco (2005), the School Curriculum needs to be organised according to educational objectives and purposes, considering the beliefs and knowledge of the school community, seen from a political perspective of education, needs to establish the relationships between school and society, regarding individual, collective, political, and ideological interests.

Researches such as Olgin's (2015) highlight the need to approach the knowledge defined in the Mathematics Curriculum of High School, using themes of interest, which will approach contemporary and relevant issues for the education of students, aimed at enhancing the curricular tracks of to provide the necessary knowledge for the development of the contents expressed in the Curriculum, integrated to the people' professional, personal and social life situations.

We understand that working with themes of interest, where students investigate situations in different contexts, can provide them, in addition to knowledge from the discipline itself, other knowledge related to social, cultural, political and economic contexts, being a path to the formation of a future active and committed citizen.

According to Olgin (2015), mathematical knowledge can be developed interconnected to eight thematics: Contemporary, Social-Political, Culture, Environment, Technological Knowledge, Health, Local Themes and Intramathematics. In the Mathematics Curriculum, this set of themes aims to contemplate a critical, transformative, reflective education, rich in contexts, going beyond a fragmented curriculum organisation and not connected to reality (Skovsmose, 2008, 2013, 2014). Olgin (2015) considers that, when working with themes of interest, it is necessary to reflect on: what to teach, how to teach and why to teach, following the principles of Coll et al. (1998). What to teach refers to the development of didactic activities integrating the themes with mathematical knowledge; how to teach is related to the process of developing interdisciplinary work through research projects; why teaching using themes and the methodology of interdisciplinary projects is associated with the enhancement of the teaching and learning process, through essential themes for the education of students.

We found in Brazilian curriculum documents the need to approach, throughout the Curriculum, issues in the current world that impact life in society (Brasil, 2017, 2019a). In this context, the Common National Curriculum Base - BNCC (Brazil, 2017), points to the importance of relating school and extracurricular knowledge for the formation of students able to deal with different problem situations, indicating the need to develop in the School Curriculum the pedagogical proposals that contemplate the approach of other themes that permeate human life.

Such themes are presented in the BNCC in the skills of each curricular component, indicating work in a contextualised, transversal and integrating way of the Curriculum, through topics such as environmental education, food and nutrition education, human rights education, education for consumption, financial and fiscal education, work, Science and technology, among others (Brasil, 2017).

According to the document that presents the Transversal Contemporary Themes (TCT) (Brasil, 2019a), they seek to relate the different curricular components, establishing connections between situations already experienced by students in their contexts, as well as contributing to the contextualisation of the objects of knowledge of different areas listed in the BNCC (Brasil, 2019a).

The development of TCT in the School Curriculum aims to ensure that, in Basic Education, themes are approached that enable students to have a broad education, focused on work, citizenship and democracy, taking into account regional and local peculiarities. These themes are presented in six thematic macro-areas: the Environment, Economy, Health, Citizenship and Civility, multiculturalism, and Science and Technology (Brazil, 2019a). The work methodology proposed for the development of TCT is based on four pillars: problematising the reality and learning situations; overcome the fragmented conception of knowledge towards a systemic view; integrate curriculum skills and competencies to problem-solving; promote a continued educational process and knowledge as a collective construction (Brasil, 2019b). Thus, these pillars underlie the development of strategies that connect the different curricular components to the TCT to provide opportunities for the "student to reframe the information coming from different disciplinary and transversal knowledge, integrating them into a social context" (Brazil, 2019b, p. 9).

In this sense, we defend that the methodology of work projects, approaching themes of interest and interdisciplinary work, constitutes one of the methodologies suitable for an Education that leads to the student's necessary training, with the development of the required skills to live in modern society.

WORK PROJECTS: INVESTIGATING STUDENT PROTAGONISM

Teaching methodologies are changing, always intending to motivate, stimulate, and establish conditions for students to take responsibility for their learning, act as active agents, and be committed to their learning. These reflections lead to the possibility of didactic planning for the development of competencies, supported by the so-called Active Methodologies, which value the effective participation of students in the construction of knowledge and the development of competencies. Thus, the possibility opens up for students to learn at their own pace, time and style, through different forms of experimentation and sharing, inside and outside the classroom, with the mediation of teachers, incorporating digital resources (Bacich & Moran, 2020).

For Bastos (2006, p. 10), Active Methodologies are "interactive processes of knowledge, analysis, studies, research and individual or collective decisions, to find solutions to a problem." The authors Miter et al. (2008) point out that Active Methodologies use problematization as a strategy for the development of the teaching and learning process, intending to reach and motivate the student, because, in the face of the problem, he stops, examines, reflects, relates to the knowledge that it already has and starts to reframe its discoveries, understanding that problematization can lead the student to contact with information and to produce knowledge.

We agree with Berbel (2011) when he states that learning through problematization and/or problem-solving, in this case, problems involving mathematical knowledge, enables the active involvement of students in their formation process. If it is crucial for students to be proactive, it is necessary to adopt methodologies in which they engage in increasingly complex activities, in which they have to make decisions and evaluate the results, with the support of relevant activities, materials and resources.

Ramos, Teodoro and Ferreira (2011) highlight that one of the characteristics of contemporary society is a new relationship between people and Science, where scientific, technological and socio-historical knowledge, with emphasis on forms of communication and management of social processes and productivity, they would be presuppositions of life in society, in which social relations are increasingly mediated by technology and information. We understand that the Work Projects methodology can favour the development of the mentioned aspects, as it provides interaction between students, teamwork, and the relationship between knowledge from different areas, as it is an eminently knowledge-building activity (Barbosa & Moura, 2013) where it is up to the students, during their performance, to solve problems, have new experiences and build new knowledge.

For Groenwald, Silva and Mora (2004), this method can be defined as an organised search for answers to a set of questions around a problem or theme that is relevant from a social, individual or collective point of view, which can be worked on in the or outside the classroom with collaborative and cooperative work involving the entire school community.

Hernandez and Ventura (2017) point out that, during the development of a work project, it can be seen that it contributes to the formation of students, aiding in learning, as they start to organise a plan; search, select and interpret the researched information; carry out a synthesis dossier; and evaluate the whole process in group and with the teacher. In this movement, "they form a continuous ring of meanings in the learning process" through the project developed (Hernandez & Ventura, 2017, p. 73).

Barbosa and Moura (2013) add that participating in the development of a project can enrich students' experiences, knowledge and skills, as a project, being unique, depends on the application of specific knowledge and appropriate methodologies to organise, interpret and analyse the data obtained to answer the problem.

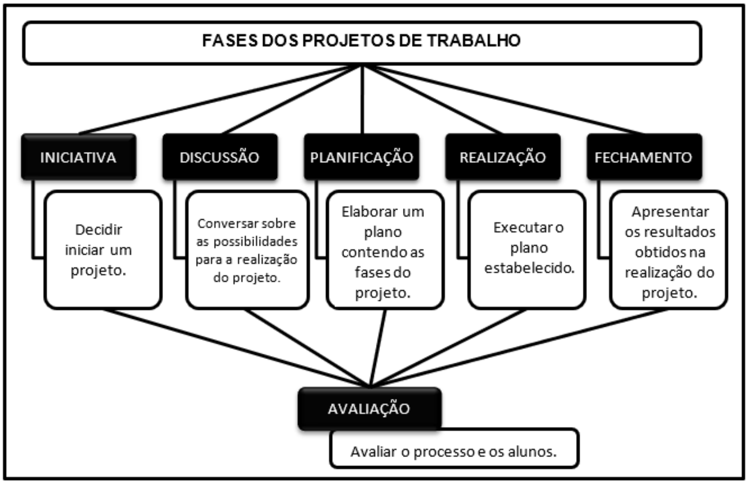
For Groenwald, Silva and Mora (2004), the project methodology is a teaching concept that considers students restless people who can reflect on different themes and develop solution strategies to face problem-situations of a certain complexity so that they develop skills and knowledge as they work in the search for solutions to the proposed problems, integrating theory and practice, seeking the development of skills that lead to the formation of flexible and adaptable students to contemporary reality, without detriment to a solid base of systematised school knowledge.

In a work project, four pillars can be identified: "freedom to choose the topic" by the students, based on negotiation with the teacher; "the formation of groups of students" for the development of the team project; "the vision of an open laboratory, without borders", in which students use different resources and spaces to carry out the project; and the "socialisation of results", where students present the data obtained and the answers found in the development of the project (Barbosa & Moura, 2013).

According to Mora (2003), there are different ways of organising the phases of a work project, the most common being shown in Figure 1.

Figure 1

Phases of a Work Project (Adapted from Mora, 2003).



The “Initiative” phase is related to the elaboration of the project, and different ideas may arise for determining the theme. Quite often, the ideas and initiatives that precede the pedagogical work through the project method come from the teachers. Still, the initiative can come from students or parents and other members of the school and extra-school community as a booster of thematics that must be developed.

The "Discussion" phase refers to the interaction between teachers and students regarding the chosen theme. The project members decide what to do and how to do it. Each participant must be able to express their opinion or point of view around the characteristics of the project, and each student must be aware of their role in the project, pointing out their ideas, knowledge and experiences.

The “Planning” phase is the development of an action plan, in which the following will be decided: deadlines; the responsibilities of each group member, the resources human and technical resources needed to carry out the project. It is essential that all participants take an active role and be aware of their role in each project's activities. It is important to note that this phase can be revisited throughout the entire project and can be reformulated at any time.

The "Realisation" phase is characterised by the involvement of students in the investigation, being the richest and most active phase, as the students show their creativity and skills in the face of the problems that arise in the execution of the project. Participants go through the detailed execution of each aspect of the project, and teachers, in addition to helping students, need to adequately and systematically prepare that specific knowledge of the subjects integrated into the project and consolidate such contents, taking care not to neglect the contents of the areas specific.

The "Closing" phase corresponds to the presentation of results, which can have two orientations: projects centred on the process and others whose fundamental objective is to obtain a product. In each case, it must be considered whether the participants were able to achieve the objectives in carrying out the project.

The "Assessment" phase involves the analysis of the construction and implementation phases of the project, considering both the written and presented work, as well as the involvement of students throughout each stage. The project method requires a permanent formative evaluation, complemented with the final presentation of the results. The evaluation must take place in a group, collective and with self-assessment.

We insist that students must be the centre of the teaching and learning process and that teachers and other participants act as moderators and facilitators of the process. This allows the active participation of students through research, discussion and decisions on the themes investigated, which can lead to the development of creativity and independence of students, enabling greater motivation and interest in carrying out the project.

We understand that the project methodology aims to promote teaching supported by the active, reflective and critical participation of the student in autonomous and collective work, in which he is firmly committed to the construction of his knowledge by working with problems or themes of interest for their comprehensive formation and the development of skills so that active and committed citizens are formed with a strong presence in society.

WORK PROJECTS AND THE PROTAGONISM OF HIGH SCHOOL STUDENTS

Three work projects will be presented, developed with high school students, involving the themes Music, Cryptography and Projectile Launching. Concerning the organisation of work with projects, in the three projects, the option was to follow the guidelines and phases pointed out by Mora (2003): Initiative, Discussion, Planning, Realisation and Closing.

1st Project – *Mathematics, Physics and Music: an interdisciplinary research proposal*

This project is under development within the scope of a Master's thesis, in the Graduate Program in Science and Mathematics Teaching (PPGECIM), at the Lutheran University of Brazil (ULBRA), authored by Lucas Teixeira[1], supervised by Professor Claudia Lisete Oliveira Groenwald.

The objective is to integrate the themes of Mathematics, Physics and Music, with contextualised activities, integrating Brazilian musical culture into the Mathematics curriculum in High School. A proposal is presented for a 2nd-year high school class from a state school in the city of Canoas, in the state of Rio Grande do Sul.

We justify the choice of this theme because we understand that Mathematics is a discipline that develops abstract knowledge and that, in many classroom situations, students do not find a use for what is being proposed, being essential for learning that students can relate theory and practice.

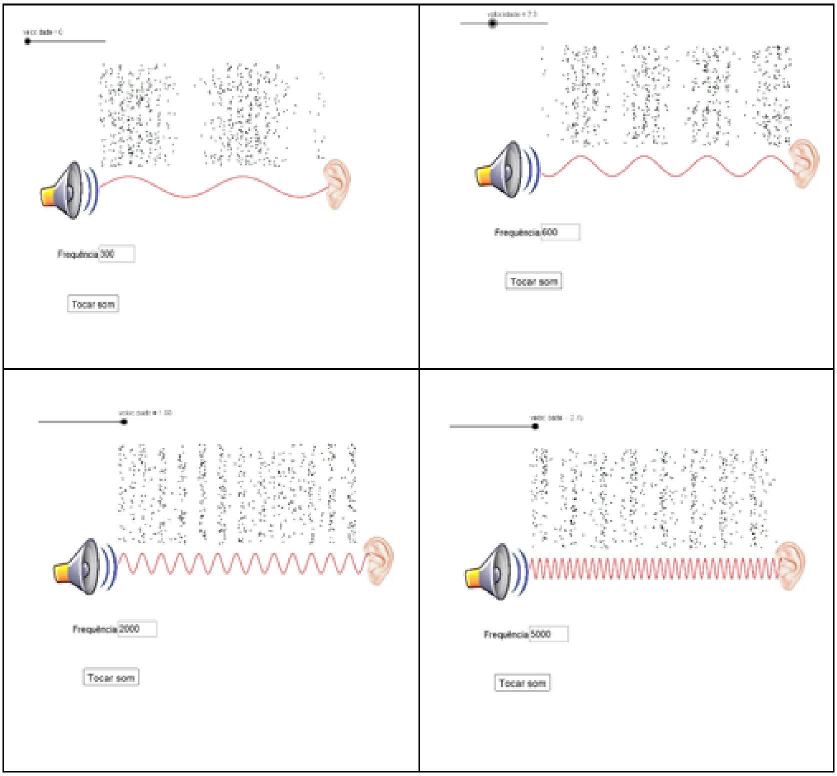
[1] Master's student from the Graduate Program in Science and Mathematics Teaching (PPGECIM) of the Lutheran University of Brazil, scholarship by the Coordination for the Improvement of Higher Education Personnel (CAPES).

According to Nunes (2018), knowledge, to be valid and have meaning for students, must go through contextualisation and globalisation, without which we run the risk of working only the contents, without using them as instruments to think about space in which we are inserted.

We seek to relate the study of trigonometric functions with the physical characteristics of sound in the execution and elaboration of musical sounds. In addition to the trigonometric functions associated with the sound phenomenon, fractions are also related to the musical tempo, and the concept of multiples can be used for different musical scales. In Figure 2, through a learning object developed in the GeoGebra software, we observe the relationship between the sine function, the frequency, and the sound produced.

Figure 2

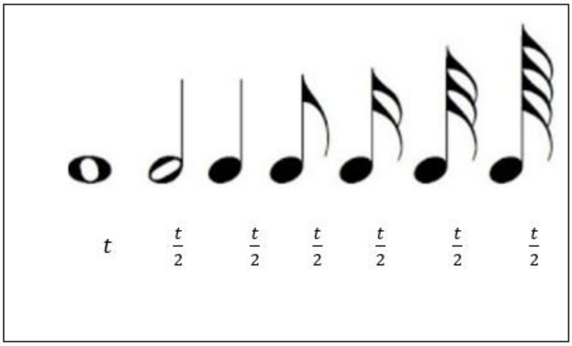
Learning object in GeoGebra software (http://ppgecim.ulbra.br/laboratorio)



In Figure 3, we observe the relationship between fractions and musical notes.

Figure 3

Musical notes and fractions



We perceive interdisciplinarity (Mathematics, Physics and Music) composing interdisciplinary aspects of the knowledge to be developed. We also highlight the cultural factor that we aim to address in this project. Sarti (2001) discusses how culture affects forms of expression, as feelings are governed by cultural codes, constituted by the collectivity, which sanctions forms of manifestation, revealing culture as a powerful influencer of social expression, either sensations or feelings. This expression takes place in different ways, not only in art but also in the beliefs and values of a civilisation.

Music is a form of cultural expression that is very popular globally, undoubtedly part of the young world. However, in addition to bringing Science and Art closer together, we aim to assess the impacts of an approach in the cultural perspective of Mathematics, combining aspects of interdisciplinarity and digital technologies.

Brazil has a great musical variety, representing the weight of culture and the main characteristics of Brazilian regions. For an example of this diversity, it is enough to analyse the characteristic musical styles of each region. In the southern region, the main cultural highlight is traditional Music from the Rio Grande do Sul, or Nativist Music. Even for an experienced musician, not knowing the gaucho culture would bring difficulties for him to interpret the musical compositions of this region, since nativist music is built in a slower tempo, with arrangements and rhymed lyrics, in general, connotative and metaphorical, mainly referring to the countryside, the love for the State of Rio Grande do Sul, the horse (one of the symbols of Rio Grande do Sul) and the woman, the gift.

In the initiative and discussion stages, the importance of introducing the project methodology for developing competencies considered necessary for high school students was discussed in the research group and the project development school. It was also discussed which year of high school and which students would be interested in participating. It was decided to organise a didactic sequence with activities that explore the relationships between the themes, with a constant reconstruction of meanings, involving knowledge, practical activities and technological resources, seeking to relate theory and practice. Due to the adversities experienced in 2020, the Sars-Cov-2 virus pandemic, an adaptation of the activities to the Google Classroom platform was proposed. The choice of this platform is in line with the strategies adopted by the State of Rio Grande do Sul in continuing high school classes with distance learning. In the planning stage, the sequence of activities and resources that would be possible to be worked with the students were organised. We present the activities in Table 1.

Table 1

Table with project activities

Activities	Project Actions
Activity 1	Revisit the concepts of Trigonometric Functions since, when working with the organisation of sounds, it is essential that this modelling tool is easy to understand so that it is possible to relate it to Physical concepts. Then, the relationship between Mathematical and Physical concepts within the sound phenomenon is proposed. The concepts of frequency are associated with the perception of high and low sounds, for example.
Activity 2	Introduce the musical universe, treating Music as the Science and Art of combining sounds. At this point, the entire history of the development of Western Music is explored, from the "monochord" experiment carried out by the Pythagoreans to the current digital musical, also passing through the main musical scales. The focus of

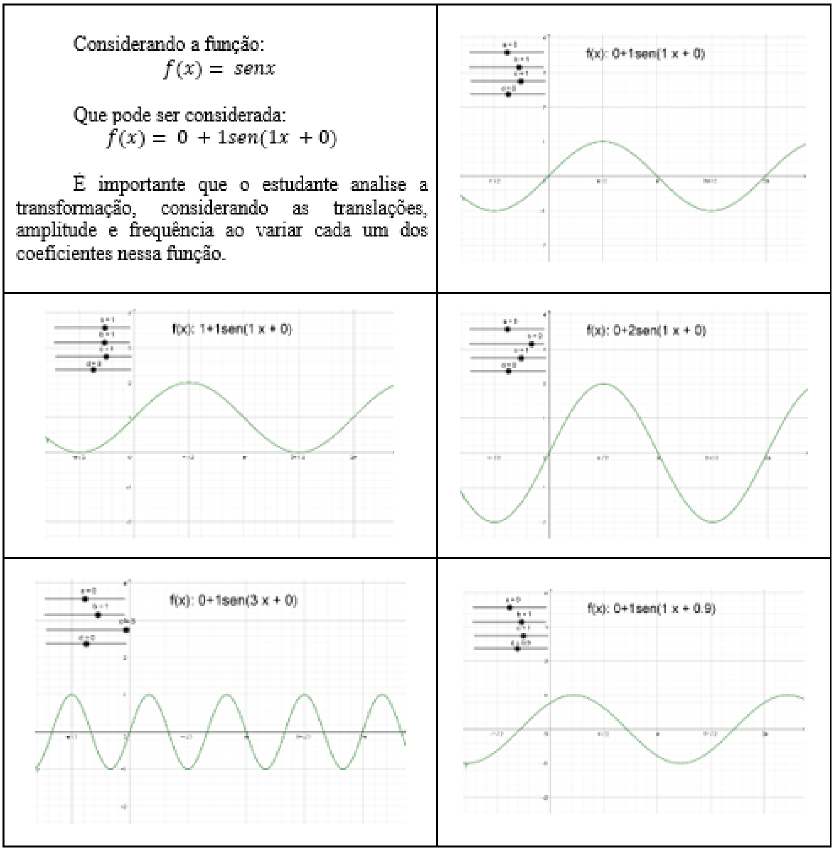
	this moment is to analyse, mathematically, how the musical organisation takes place.
Activity 3	Study of musical movements, where the speed of execution of certain Songs and how this organisation occurs are analysed. The concepts of fractions and decimals are explored, as the marks and pulses of a song can be written musically using these concepts.
Activity 4	Analysis of Brazilian musical styles, these segmented by region. Investigation into the Brazilian cultural variety in the "Music" item. In addition, the concepts noted above in the sequence of activities can be used to “analyse” the typical musical compositions of each region.
Activity 5	Space for creating, without criteria or recommendations, a song following the knowledge acquired in the previous activities. The Chrome Music Lab page is used so that students can create their own compositions.

As an example of activity, in the initial part of the sequence, which deals with the review of trigonometric functions and sound characteristics, the GeoGebra software is used to study the variation of the function parameters.

For the execution of sounds from the graph of periodic functions, it is necessary to construct a sine function to construct the object of study (Silva, Groenwald & Homa, 2017). According to the authors, in the Input field at the bottom of GeoGebra, we insert the function $(x) = a + b * \sin(c * x + d)$, where a, b, c and d are sliders representing the parameters of the created function. Using this tool, the activity consists of creating opportunities for students to relate the function parameters to their influence on their graph and how these parameters influence the sound coming from it (Figure 5). (Figure 4).

Figure 4

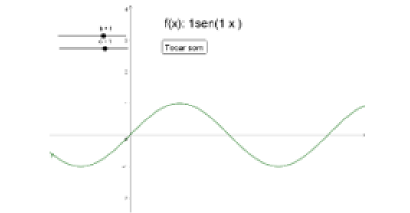
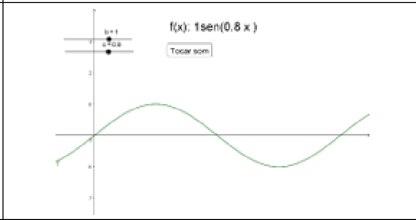
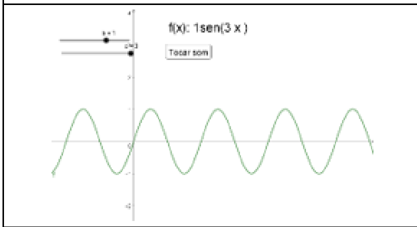
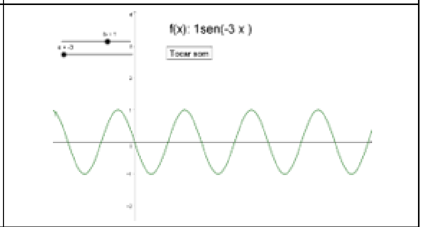
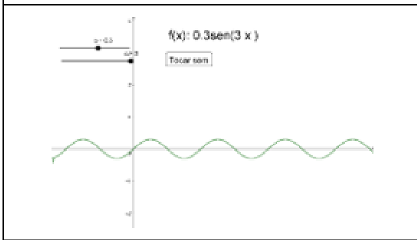
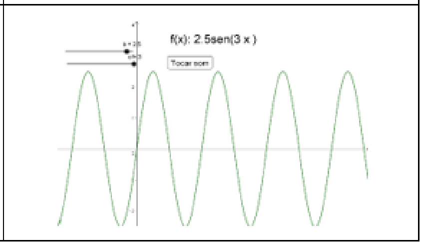
Sine function and coefficient variation with GeoGebra software GeoGebra (Silva, Groenwald & Homa, 2017)



Next, we present the learning object, where it is possible to identify the sound according to the variation of the sine function (Figure 5).

Figure 5

The sound according to the sine function

Som de referência, com frequência padrão.	Som com 0.8 da frequência, mais grave que a referência.
	
Som com o triplo da frequência, mais agudo que os anteriores.	O coeficiente da frequência é negativo, mas o que importa é o valor absoluto que continua o triplo do som de referência.
	
Amplitude menor que a referência, som com baixa intensidade.	Amplitude maior que a referência, som com alta intensidade.
	

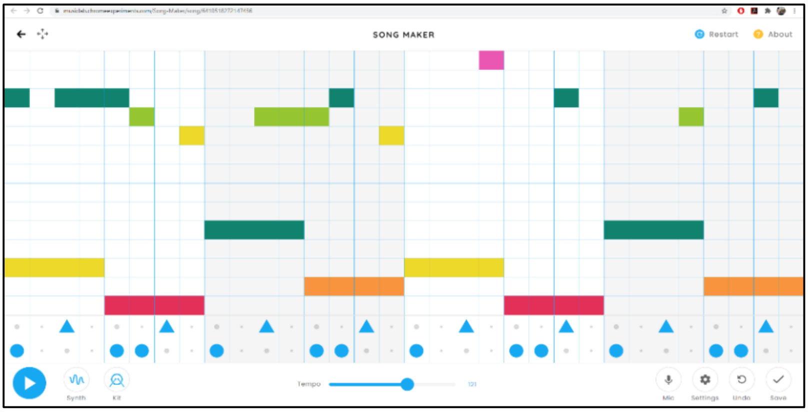
We emphasise that the mathematical concepts of amplitude and frequency are easily contextualised by analysing the physical properties of sound. The amplitude of a trigonometric function is equivalent to the concept of sound intensity (known as "volume"); the greater the intensity of a sound, the greater the number of decibels emitted by the sound source. In the case of the frequency of a trigonometric function, we have the high-pitched versus low-pitched relationship characterised by pitch. The higher the frequency, the resulting sound is increasingly higher and the lower the frequency, the lower is this sound. In everyday language, we use the terms "high" and "low" to characterise high and low sounds, but this description should be avoided, as high and low are commonly used to refer to sound intensity.

In the "musical" part of the sequence, the proposed activities consist of analysing the musical movements of current Music, asking students to give a brief description of the sensations felt when listening to certain Music and how the musical tempo adopted by the artist influences this issue.

In the end, the groups were asked to create songs freely. The Chrome Music Lab page allows you to create, intuitively and playfully, songs from pre-defined instruments (Figure 6).

Figure 6

Music on Song Maker do Chrome Music Lab (https://musiclab.chromeexperiments.com/Song-Maker/song/6410518272147456)



In the Song Maker module, users can create songs with different tempos and combinations of musical notes just by filling in coloured squares. There is also the option of other melodic and percussive instruments to increase the variety of possibilities. What is expected of students at this point is to analyse whether the musical notes chosen in the compositions belong to the same scale (object of study of the didactic sequence that portrays the harmonious combination between different notes) as well as verifying whether the 4/4 division is respected in the musical organisation.

In the evaluation, it was decided to constantly follow the entire process during the work in the groups. The observation and analysis of students' productions is the project's primary data extraction tool.

The expected results follow two main questions:

1. Are students able to link the existing relationships between Mathematics and Music?

2. What are the educational aspects considered positive for the development of projects using a cultural perspective?

We hope that students can understand the direct relationships between Mathematics and Music and how Science is present in art. In addition, the student's protagonism will be evaluated when we request the musical analysis of different musical styles and their compositions since the description of feelings/feelings is a factor with high subjectivity and a very striking cultural characteristic. We emphasise that we seek to enable moments of reflection in workgroups about the impact that culture has on their environment and that musical phenomenon are related to knowledge of Mathematics and Physics.

2nd Project – *Cryptography in teaching polynomial functions in the 1st degree*

The project "Cryptography in the teaching of polynomial functions in the 1st degree" aimed to address, in high school, a current theme that is part of life in society and that allows the development of didactic situations with mathematical contents. It was developed with 18 students from a 3rd-year high school class, from a private school in Farroupilha, in the Rio Grande do Sul.

Cryptography is a contemporary theme, present in various situations of everyday life, such as payment authentication, internet communication, bar code verification, internet browsers, and online financial transactions, among others. According to Groenwald and Franke (2007), this theme can help the teaching and learning process of Mathematics through coding and decoding didactic activities involving the mathematical contents.

Olgin (2011) complements that when this theme is developed in an organised and planned way, it can enable: the improvement of students' knowledge; the enrichment of Mathematics classes, with coding and decoding activities and games with the contents that are worked on in High School; contextualisation of the content covered; the work with problem-solving, where students carry out the survey of relevant information, elaboration of hypotheses, verification and validation of them; the resignification of mathematical contents.

The project introduced the theme with a historical approach, seeking to identify the emergence of the need to use methods to keep information safe. It aimed to present activities for reviewing and deepening the content of functions.

In the "Initiative" and "Discussion" stages, the following actions were carried out presentation of the Cryptography theme, determination of student groups (4 groups, called Group A, Group B, Group C and Group D); research on the theme and discussion among the members of each group about the researched theme, and this moment was mediated by the teacher.

The "Planning" stage was the elaboration and organisation of teaching activities to develop the project (Table 2).

Table 2

Model of activity planning and development

N°	Activities	Objectives
1	Presentation of the research carried out by students, on the Internet, on the concept of Cryptography.	Investigate the concept and applications of the Cryptography theme, aiming to understand its importance for life in the modern world.
2	Presentation of Cryptography history and discussion of examples of encrypted situations.	Understand the evolution of the Cryptography theme throughout history. Know and apply the different methods used to encode and decode messages.

3	Decrypt encrypted messages using 1st-degree functions.	Understand the relationship between the Cryptography theme and the content of the 1st-degree function, deciphering messages.
4	Challenge aimed at groups for decoding encrypted messages.	Explore the contents of the 1st-degree function image and solving two-variable linear equation systems.
5	Development of an encrypted message addressed to groups.	Apply the knowledge developed during the project to prepare a message to be unveiled by another group.

As an example of activity 2, we present in Figure 7 Caesar's cypher, the Pigpen cypher, the Playfair cypher and the ADFGVX cypher.

Figure 7

Examples of activities involving applications from the history of Cryptography (Adapted from Olgin, 2011)

CAESAR's CYPHER

The cypher used by Julius Caesar consisted in replacing each letter of the original message with one that was three places ahead in the same alphabet, as shown in the table below. Caesar used the regular alphabet to write the message, and the cypher alphabet was used to encode the message that would be sent.

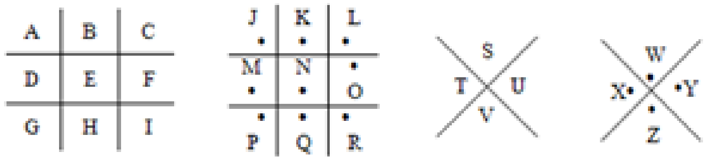
ALFABETO NORMAL	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
ALFABETO CIFRADO	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

Encoding example using Caesar's Cipher.

NORMAL TEXT	ALEGRE
CODED TEXT	DOHJUH

PIGPEN CIPHER

A monoalphabetic substitution cypher is the Pigpen cypher, used by the Freemasons to keep their secrets (Singh, 2003). The cypher consists of replacing a letter with a symbol, following the pattern shown in the Figure below.



Standard used by Pigpen cypher.

The coding of the Pigpen cypher is carried out by finding the letter's position in one of the four grids in Figure and drawing the portion of the grid that represents the letter to be encoded.

Encoding example, using the cypher.

NORMAL TEXT	CASAL
CODED TEXT	

PLAYFAIR CYPHER

Another type of cypher is the Playfair cypher, created by Charles Babbage and his friends Sir Wheatstone and Baron Lyon Playfair. This cypher replaces each pair of letters in the message to be encoded with another pair of letters. To encode, first choose a keyword, for example, ULBRA. Before encryption, the letters of the alphabet are written in a 5X5 square, starting with the keyword and combining the letters I and J into a single element, as shown in the following Figure.

U	L	B	R	A
C	D	E	F	G
H	I/J	K	M	N
O	P	Q	S	T
V	W	X	Y	Z

Playfair Cipher Table.

The original message is written in pairs of letters or digraphs. The two letters, in any digraph, must be different, which is achieved by inserting, for example, a letter x if the same letters appear or if the number of letters is odd. Encryption starts as follows: if the two letters are on the same line, they are replaced by the letter immediately to the right of each one; if one of them is at the end of the line, it is replaced by the letter at the beginning of the line. If both letters are in the same column, they will be replaced by the letter immediately below each one, in which case, if one of the letters is the last letter in the column, it will be replaced by the one at the top of the column.

If the letters in the digraph are neither in the same line nor in the same column, the following rule is used: to encrypt the first letter, look along its line until you reach the column where the second letter is; the letter at that intersection will replace the first letter. To encrypt the second letter, use the same reasoning.

The following is an example of encoding using the cypher mentioned above.

NORMAL TEXT	VIDA
ORIGINAL TEXT IN PAIRS	VI – DA
CODED TEXT	WH – GL

CIFRA ADFGVX

The ADFGVX cypher uses a 6x6 grid to encode filled with 36 squares, where the 26 letters of the alphabet and 10 digits are placed. In the first row and column, place the letters A, D, F, G, V and X, as shown in the following Figure.

	A	D	F	G	V	X
A	8	P	3	D	1	N

D	L	T	4	O	A	H
F	7	K	B	C	5	Z
G	J	U	6	W	G	M
V	X	S	V	I	R	2
X	9	E	Y	0	F	Q

ADFGVX Cipher Table.

Coding begins, taking each letter of the message to be sent, locating its position in the grid and replacing it with the letters in the row and column. For example, d will be replaced by AG. A message encoded by this cypher will look like the following Figure.

NORMAL TEXT	LAGO
CODED TEXT	DADVGVDG

Coding of the ADFGVX Cipher.

To encrypt the letter L, its position in the grid is located and replaced by the letters in its row and column, as shown in Figure.

	A	D	F	G	V	X
A	8	P	3	D	1	N
D	L					
F	7	K	B	C	5	Z
G	J	U	6	W	G	M
V	X	S	V	I	R	2
X	9	E	Y	0	F	Q

L=DA

An example of code with a 1st-grade function is shown in Figure 8. The activities aim to review and deepen the concept of function, function image, inverse function calculation to allow the student to expand their knowledge regarding these contents.

Figure 8

Example of activity with 1st-degree function (Adapted from Olgin, 2011)

Code with 1st Degree Function

Consider the following Figure that, for each letter of the alphabet, associates an integer from 1 to 26 and encodes the message “A vida é bela”

("Life is beautiful."), using Code with Linear Function, knowing that the encoding function is f(x) = 5x + 1.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	2

Table of the numerical value of each letter.

It is known that: A = 1, B = 2, C = 3, ... e $f(x) = 5x + 1$

The normal text is A vida é bela.

The numerical sequence of normal text is 1 – 22 – 9 – 4 – 1 – 5 – 2 – 5 – 12 – 1.

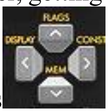
To encode, the image of the function is calculated for each digit of the numerical sequence of the normal text:

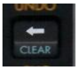
f(1) = 5.1 + 1 = 6 f(22) = 5.22 + 1 = 111 f(9) = 5.9 + 1 = 46

f(4) = 5.4 + 1 = 21 f(5) = 5.5 + 1 = 26 f(2) = 5.2 + 1= 11

f(12) = 5.12 + 1 = 61

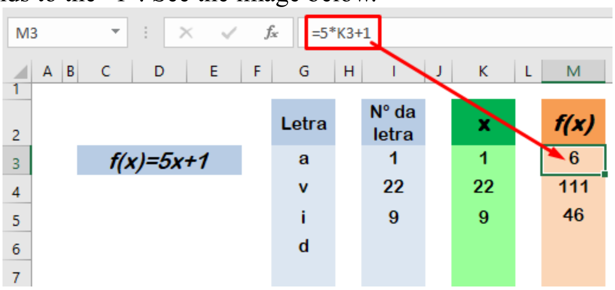
In this activity, you can guide students to use the resources of a calculator or electronic spreadsheets for calculations.

The student is expected to use the calculator as follows: type 5 x 1 + 1 and press enter, getting 6. For the remaining digits, they are expected to use the cursor keys 

to move around the display and the "clear" 

key to erase the digit you wish to modify, thus modifying only the variable x. This example uses the HP 35S calculator.

In Excel software, it is expected to type in the cell corresponding to f(x) the formula for determining the image of the function, being "=5*K3+1". The representation "K3" means the position of the element of x that corresponds to the "1". See the image below.



The text being encoded, the image of each digit found in the function: 6 – 111 – 46 – 21 – 6 – 26 – 11 – 26 – 61 – 6

To decode the message: the inverse function is calculated to check if the results found are correct.

The inverse function of f(x) = 5x + 1 is:

f(x) = 5x + 1
f(x) -1 = 5x

$$\frac{f(x)-1}{5} = x$$

Therefore, the inverse function corresponds to $f^{-1}(x) = \dfrac{x-1}{5}$.

In the "development" stage, educational activities with the theme of Cryptography were carried out. In Figure 9, we present an example of a situation involving decoding a message using the 1st-degree function content.

Figure 9

Example of activity developed in the Project

Code with 1st Degree Function

Activity 1 - Decode the message: "8/63/28/38/93/48/8/148/153/18/78/68/83/ 8/93/103/48/63/ 43/28/153/18/78/68/153/103/78/78/23/78/98/143".

Using the following alphabet and knowing that the encoding function is *f(x) = ax+b* and that the message starts with a synonym for the word contentment, for example, "euphoria, joy, happiness, satisfaction".

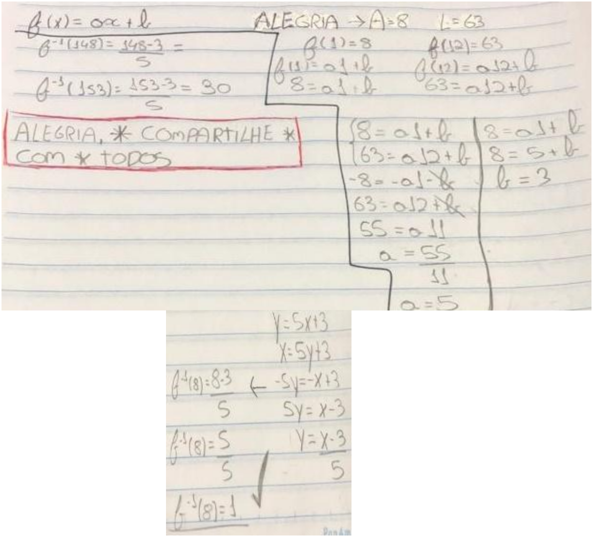
A	**B**	**C/Ç**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
R	**S**	**T**	**U**	**V**	**W**	**X**	**Y**	**Z**	**!**	**.**	**,**	*****	**-**	**:**	**?**	
18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	

Note: The symbol "*" represents space between words.

Group A started the resolution of the activity, trying to find out what was the keyword and the ciphering function to decode the message. For this, they took as keyword "joy", and from the function $f(x) = ax + b$, they determined two equations and found the values for "a" and "b", solving the system, as can be seen in Figure 10.

Figure 10

Resolution of group A (students of group A)

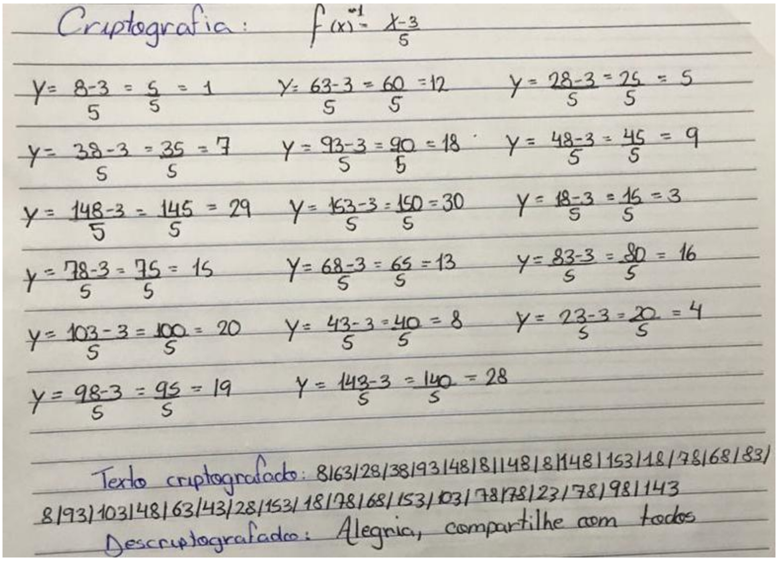


The students in this group verified if the function found was correct, calculating the function image.

Group B, in Figure 11, presents the inverse of the function referring to the activity illustrated in Figure C and the image calculations of the inverse function performed to decode the message. In this way, the group members can decipher the message in the numerical text, using the encoding and decoding alphabet.

Figure 11

Resolution of group B (students of group B)



In the "Finalisation" and "Assessment" stages, students created coded messages involving the content of 1st-grade functions to communicate between groups. Each group presented the message they had encoded and the message they received. Regarding the message he received, it was necessary to show the strategies used for decoding. The evaluation was carried out throughout the development of each stage that makes up the project.

3rd – Project *Projectile Launching*

Here, we present the project "Projectile Launching", carried out within the scope of a Master's thesis, in the Graduate Program in Science and Mathematics Teaching (PPGECIM), at the Lutheran University of

Brazil (ULBRA), authored by Valmir Ninow [2] (Ninow, 2014), , supervised by teacher Carmen Teresa Kaiber.

The project was developed by a group of third-year high school students from a public school called Group-A, whose objective was to investigate the development of Work Projects, considering different strategies, procedures and resources, aiming to construct mathematical knowledge and related skills and competencies. The project was part of a set of projects from which we sought to provide opportunities for students to develop actions such as group work, organisation, planning, research, decision-making and the confrontation and solution of both practical and theoretical situations, having been developed throughout of eight weeks, with tasks performed both in the classroom and in other environments, outside school hours.

The Project Launching project aimed to work with knowledge related to the areas of Mathematics and Physics, both theoretical and practical, focusing on aspects related to Affine and Quadratic Functions, Uniform and Uniformly Varied Rectilinear Movement and Oblique Launch. Elements from Mathematical Modelling were used, as well as digital technologies (SketchUp Excel software), which were used to build, model and analyse the presented situations. In addition to the objects of knowledge dealt with, we aimed to provide students with a job where they had the opportunity to take responsibility for their learning.

In the context of the first phase of the work, when the groups were formed and started to discuss and decide what to work with, Group-A defined the theme of its project considering converging interests and curiosity to learn more about themes already studied in Physics and Mathematics, but who were interested in deepening and "knowing in practice". Based on this decision, they started to prepare a proposal for the development of the work, with a definition of objectives and actions necessary to reach the proposed objective, schedule of actions and attributions of the group members. We believe that this phase was very productive because the students, in addition to organising themselves, distributing tasks, researching the topic, discussed a lot about the responsibility of each group member for the success of the work.

[2] Doutor em Ensino de Ciências e Matemática pelo Programa de Pós-Graduação em Ensino de Ciências e Matemática (PPGECIM) da Universidade Luterana do Brasil.

The group, in its planning, presented the first organisation of its actions in stages, which is highlighted in Table 3.

Table 3

Group-A Planning

Phases	Description
1	Study about projectile launching.
2	Build a catapult.
3	Make releases.
4	Perform mathematical and physical calculations.
5	Study and use a program.
6	Do the final work and present.

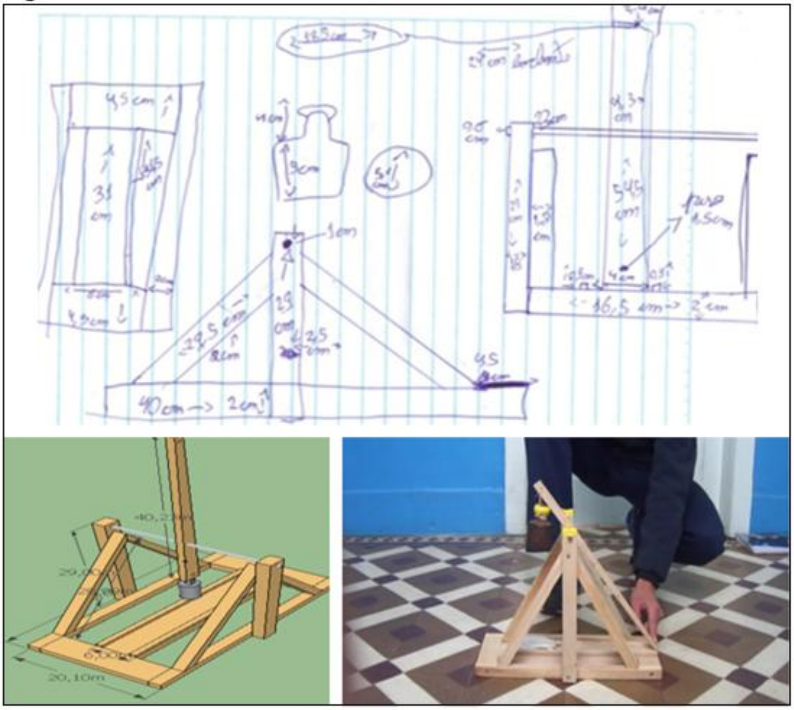
The steps were taken up and expanded, as each one required studies, research, new experiments, more studies. From this initial planning, the group established a schedule with the actions that would be developed and the deadlines for each stage. This work of establishing a topic of interest, discussion and planning of actions, and the first research and readings refer to the first three phases, as Mora (2003) pointed out. During this period, the teacher assumed the role of an advisor, encouraging the search for study materials, motivating them from questions and discussions.

In the project development phase, the students constructed the sketch of the design of a catapult of the type Trebuchet[3] as a projectile launcher. From the first sketch formulated, they became a representation using the SketchUp software and, finally, the construction of the wooden catapult, as illustrated in Figure 12.

[3] The Trebuchet is a catapult used as an instrument of war in Europe during the Medieval period. It uses a massive counterweight to hurl projectiles attached to the opposite end (Wikipédia, 2014)

Figure 12

Designing and Building a Catapult



Once the catapult was built, the group used the Uniform Rectilinear Movement (MRU) and Uniformly Varied Rectilinear Movement (MRUV) equations, already known and studied, seeking to adapt them to the oblique launch involved in the activity. The studies continued with the search for models regarding the maximum reach of the mobile after launch, the maximum height reached, and the time for the object to reach this maximum height.

Along with this study, the experiments were carried out using launches with two weights, one of 10 grams and the other of 20 grams, duly registered. The experiment demanded creating the conditions to carry it out, such as the location, conditions and, mainly, how the data resulting from the experiment would be collected and registered. To make the measurements related to these launches, the students used a brick wall as a base, and they made the marking and measurements for each of the weights after each launch, with 15 launches for each weight and values duly registered. This organisation demanded study, group discussions with the professor, and the prediction of what could happen. At this stage, they sought help from the Physics teacher, who was available to help them whenever they needed it.

With the data obtained in the experiment, the group started to analyse them against the studied equations. However, the experiment showed, in practice, what the students had already learned regarding the catapult: one of the basic principles is to vary the launch angle so that a particular target, a distance or a height is reached. However, the physical model created by the group did not allow for the necessary variations, as the launch angle was fixed (by construction). Thus, what would have to be changed was the weight to be launched and the counterweight for variations in the mobile trajectory to occur, a fact perceived by the students in the first tests, who sought to make adaptations but were unsuccessful. This entire process proved to be a lot of learning for students who studied a lot, counting on the help of physics and mathematics teachers.

Seeking an alternative to the question, the idea arose to develop another projectile launcher, from a prototype of a pet bottle rocket, water and compressed air[4]. The construction carried out by the students can be seen in Figure 13.

Figure 13

Pet bottle rocket launcher

[4] developed by a group of Engineering students from Unisal in Lorena, São Paulo.

The experiment with the new launcher (always accompanied by the teacher) allowed the desired angle variations, always keeping the amount of water and the same pressure.

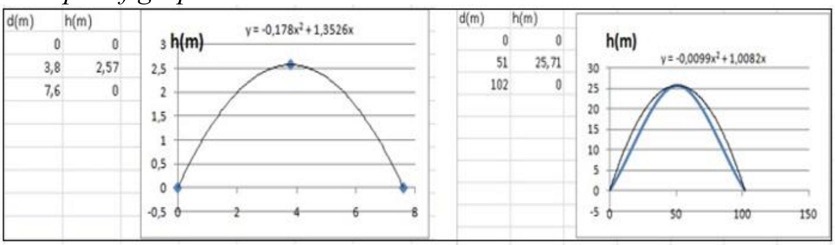
During the launches, the students noticed that, with the launch angle of 45º, the projectile reached the greatest distance. Already with an angle greater than 45º, the projectile reached a greater height, but it reduced its distance and, with a launch angle smaller than 45º, it decreased the height and distance in relation to the launch with an angle of 45º, showing that the group I understood what was going on in each variation of the launch angle. They also calculated the time for the pet bottle projectile to reach its maximum height and the total time of its trajectory.

All calculations so far had been produced using pencil, paper and calculator, so the group decided, as provided in the schedule, to perform the calculations using Excel software to model the data obtained and explore the possibility of making comparisons between the model created in the program and the one made with pencil and paper. We emphasise, however, that Excel was not the first option for the job.

Thus, graphs referring to the two launches (catapult and pet bottle) were constructed (Figure 14).

Figure 14

Example of graphs built in Excel

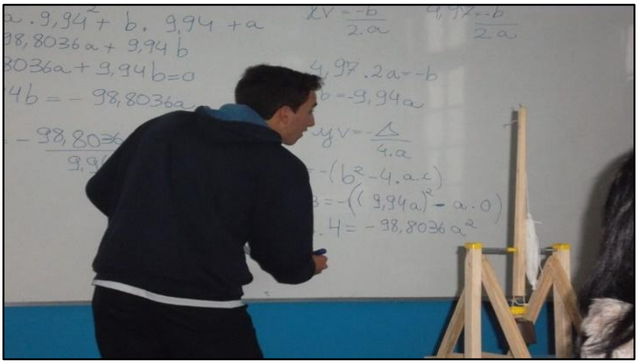


The first graphic represents one of the models (distance by height) that worked with the catapult, and the second, the launch of the PET bottle projectile at an angle of 45º, also with the variation of distance by height. The "trend line" was used for the construction and the finding of the mathematical model that represented the movement and construction by “scatter”, which approximates the points on the graph.

This was the last activity carried out by the group that started to finalise the project report and organise the presentation in the classroom, with the project reaching its final stage. In Figure 15, one of the students is highlighted, explaining the functioning of the catapult, representing and explaining the work developed, including presenting all the Mathematics parts.

Figure 15

Student demonstrating one of the mathematical models



We believe that this socialisation of knowledge is of fundamental importance in working with Projects, with each group being responsible for presenting and promoting discussions and reflections on the knowledge developed.

Throughout the development of the project, we noticed that students used prior mathematical and physical knowledge in a good way and when faced with new questions outside their domain, they sought to research, discuss in the group and with the teachers. Furthermore, the group met all the situations that arose that required the search for solutions very seriously. However, the work had moments of tension and discouragement, especially when it realised that the built catapult was fixed, not allowing the change of angle, when they concluded that the Modellus was "very difficult" and when they intended to shoot a film that ended not happening.

We consider that regardless of the knowledge of Mathematics and Physics produced, the Work Projects developed in the class, and mainly the Project presented here, constituted a possibility to change the role of the student and the teacher, enabling students to become active subjects, participatory, committed to the development of knowledge that is not the only individual but of an entire group.

CONCLUSION

The theoretical notes and projects that we present here highlight a view we have on the teaching and learning of Mathematics: the understanding that students, their learning and development must be the centre of the educational process.

For this vision and understanding to be put into practice and to give space in the school for the student to take the lead and responsibility for their learning, we have to work with methodologies, strategies and make use of resources that create environments with potential for students to act, act and protagonise.

AUTHORS' CONTRIBUTIONS STATEMENTS

C. de A. O., C. L. O. G., and C. T. K. discussed the theoretical framework to contribute to the production of this article and collectively participated in its construction.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author upon reasonable request.

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