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The Role of the Ostensives in Understanding Quantitative Statistical Variables

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ABSTRACT

Background: One of the main challenges of teaching statistics in basic education is the treatment of quantitative statistical variables because, besides the calculations involved in the abstract measures, understanding the behaviour of these variables and the meaning of their statistics is complex by their very nature. In this sense, using manipulable concrete materials and the student's action as ostensive mathematical objects in the management and representation of this type of variable can help understand them. **Objectives**: this research investigates how the concrete manipulable materials and the student's action contribute to understanding and representing quantitative statistical variables in different registers. **Design**: we make theoretical reflections on the active use of the ostensive objects in the representation and transformation of data into statistics – tables, graphs, and summary measures – in basic education based on the anthropological theory of the didactic and the theory of registers of semiotic representation. **Data collection and analysis**: this is a theoretical study that uses results already published in related research. **Results**: the analyses point out that the use of the ostensive objects helps students' understand the statistical concepts and that from this point on, they represent and transform the representations of statistical information in different registers more fluidly. **Conclusions**: the study reveals that using the ostensives and the students' actions in managing statistical concepts is a fundamental aspect of learning.

Keywords: Quantitative statistical variables; Statistical data; Concrete manipulable material; Student actions; *Ostensives*.

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O papel dos ostensivos na compreensão das variáveis estatísticas quantitativas

RESUMO

Contexto: um dos principais desafios do ensino de Estatística na Educação Básica é o tratamento das variáveis estatísticas quantitativas, pois além dos cálculos envolvidos nas medidas resumo, a compreensão do comportamento dessas variáveis e o significado de suas estatísticas são, pela sua própria natureza, complexas. Nesse sentido, o uso de materiais concretos manipuláveis e o papel da ação do estudante como ostensivos na gestão e na representação desse tipo de variável podem auxiliar nessa compreensão. **Objetivos**: esta pesquisa investiga como os materiais concretos manipuláveis e a ação do estudante contribuem para a compreensão e representação das variáveis estatísticas quantitativas em diferentes registros. **Design**: tecemos reflexões teóricas sobre o uso ativo dos ostensivos na representação e transformação dos dados em estatísticas – tabelas, gráficos e medidas resumo – na Educação Básica a partir da Teoria Antropológica do Didático e da Teoria de Registros de Representação Semiótica. **Ambiente e participantes**: trata-se de um estudo teórico que utiliza resultados já publicados em pesquisas correlatas. **Coleta e análise de dados**: esses dados foram coletados a partir de artigos publicados em periódicos científicos bem como em dissertações disponíveis em meio digital. **Resultados**: as análises apontam que a utilização dos ostensivos ajuda na compreensão dos conceitos estatísticos pelos estudantes e que a partir disso esses estudantes tem maior fluidez na representação e transformação das representações das informações estatísticas em diferentes registros. **Conclusões**: o estudo revela que a utilização dos ostensivos e da ação do estudante na gestão dos conceitos estatísticos é um aspecto fundamental na aprendizagem.

Palavras-chave: variáveis estatísticas quantitativas; dados estatísticos; material concreto manipulável; ações dos estudantes; *ostensivos.*

INTRODUCTION

Statistics contents in basic education in Brazil were officially included in 1997 with the publication of the National Curriculum Parameters by the Ministry of Education (MEC, 1997) and later ratified in the BNCC Common Curriculum National Base (MEC, 2018).

This inclusion is due to the recognition of the importance of thought and statistical literacy for reading and understanding the world, which became much more evident in the context of the Covid-19 pandemic, when terms that were usually within academic scope became part of the population’s jargon, assisting in public managers’ and citizens’ decision making, as shown in the works of Alsina, Vásquez, Muñiz-Rodríguez, and Rodríguez-Muñiz (2020), Rodríguez-Muñiz, Muñiz-Rodríguez, Vásquez, and Alsina (2020), Correia and Cazorla (2020), and Samá, Cazorla, Velasque, Diniz, and Nascimento (2020).

In this sense, the scientific production in the field of statistical education has been very profitable, as can be seen in the various review works (Santos, 2015; Oliveira Junior & Vieira, 2015; Schreiber & Porciúncula, 2019; Tavares & Lopes, 2019; Oliveira & Paim, 2019). These mappings indicate an increase in the studies in statistics teaching, mainly in basic education.

There has also been some progress in the production of material for statistics teaching. Lima, Bezerra, and Valverde (2016) used colourful little balls with bags and, by drawing lots, developed basic concepts of probability and statistics with elementary school students. Silva (2019) built the "statistical box" using wood and styrofoam pellets to develop population concepts, sample, and sampling methods with high school students. Porciúncula and Schreiber (2019) reported the positive perception of prospective mathematics teachers when using manipulable concrete materials didactically.

Despite all this progress, we still find there is a lack of understanding of basic statistical concepts. When investigating the specific knowledge of mathematics teaching degree students, Oliveira (2020) found that the freshmen and senior students still had conceptual gaps and misunderstood basic concepts.

One factor that explains that gap is that the students do not understand how data can and should be collected, or how raw data becomes statistics - which are thought, here, as the set of concepts and procedures that involve the process of organising and summarising data, i.e., tables, graphs, and summary measures, a procedure that gets more complex when we consider the quantitative variables.

According to Pfannkuch and Rubick (2002), one of the key points in statistical research is to define measures that "capture" qualities or characteristics of the elements that compose the population of the phenomenon to be investigated. For this reason, we should pay particular attention to the concepts of variables and the statistical data generated by their operationalisation. In this context, the practical material that can be manipulated can play a fundamental role, especially when those concepts are presented for the first time.

Manipulable concrete material in mathematics teaching is very much used, especially in early childhood education and in the early years of elementary school. This trend is also seen in statistics teaching, as shown by the works of Selva (2003), Caetano (2004), Alsina (2017), Cazorla, Henriques, and Santana (2020), among others. This seems to be partly explained because, in this stage of basic education, more emphasis is placed on the treatment of qualitative variables, according to the BNCC (MEC, 2018).

However, the use of manipulable concrete material in the final years of elementary school and high school is a little more restricted, which, in our view, owes to the fact that in this level of education, we must work with the quantitative variables, which requires greater investment in the design of these materials to make them accessible to teachers. Studies by Nascimento (2007), Silva, Magina, and Silva (2010), Silva (2019), Cazorla and Santana (2020), and Santana (2020) show that those materials bring benefits in the apprehension of the meaning of the quantitative variables, allowing the representation and transformation of statistical data between different registers.

Thus, this article aims to reflect theoretically on the role of the manipulable concrete material and the students' actions in the representation of quantitative statistical variables in different registers. To this end, we find a basis in the theory of registers of semiotic representation (TRSR) and the anthropological theory of the didactic (ATD).

With this article, we extended the theoretical framework about the use of ostensive objects in the teaching of qualitative statistical variables (Cazorla, Henriques, & Santana, 2020) for quantitative variables. However, this study is limited to statistical concepts; therefore, we will not deal with probabilistic concepts.

THEORETICAL BACKGROUND

In this section, we present the two theoretical frameworks, namely: the anthropological theory of the didactic (ATD), proposed by Chevallard (1992), restricting us to *ostensive* and *non-ostensive* objects that allow us to analyse the role of the students' actions regarding the use of concrete manipulable materials, and the theory of the registers of semiotic representation (TRSR), developed by Duval (1999), in which we focused on the conversion of the raw data of quantitative variables into corresponding representations from the numerical register into the graphic register.

The anthropological theory of the didactic: the role of the ostensive objects

Similar to what Cazorla, Henriques, and Santana (2020, p. 1249) proposed, given the use of *ostensives* in the representation of qualitative variables, we based this study on the anthropological theory of the didactic (ATD) regarding the concepts of *ostensives* and *non-ostensives*, which allow us to explore "the potential of manipulable concrete materials as *ostensives* suitable to the representation of theoretical statistical data based on the mental mobilisation of the inherent *non-ostensive* objects, as well as the observation and analysis of the students' quantitative actions," aiming to expand that theoretical framework for the representation of the quantitative variables.

According to Henriques (2019, p. 65), the *ostensives* are all objects that have a sensitive nature and certain materiality in a perceptible reality; the *non-ostensives* are objects that, like ideas, intentions, or concepts, exist institutionally "without being seen, said, understood, perceived, or shown by themselves: they can only be evoked or summoned up from the proper manipulation of *ostensive* objects associated." For example, writing $f(x) = 2^x$ can be understood as a simple manipulation of *ostensive,* objects, but this representation could not be made intentionally without the intervention of specific *non-ostensive* objects, such as the exponential function concept. Chevallard (1999) uses the term manipulation to designate the various possible uses of *ostensive* objects by man. In fact, several concrete materials can be part of this manipulation in the students' actions.

To Henriques (2019, p. 33), the manipulable concrete material or manipulable didactic resource is "every ergonomic and cognitive, hand-free tangible instrument, capable of enabling knowledge management, and is therefore useful in the teaching-learning process of objects of institutional knowledge."

We, therefore, believe that access to or management of knowledge through the mediation of those materials is an essential condition in learning since the students' manipulation of appropriate materials can enhance their action in the face of the activities proposed in various ways, particularly in visualisation, design, and transformation of raw data into statistics.

Thus, counting on this theoretical reference, we analysed the role of the concrete material free-hand manipulable and the students' action in the representation and conversion of the statistical data, because we understand that this process pervades the mobilisation of *ostensive* and *non-ostentative objects* in each register of representation considered.

The Theory of Registers of Semiotic Representation: the Role of the Conversion of Registers

This theory was proposed by Raymond Duval, for whom a representation register is a *semiotic* system endowed with signs that allow the identification of a representation of an object of knowledge. Three cognitive activities are involved in this process. The first activity refers to the construction of the representation of the mathematical object by means of traits that allow identifying the object represented within a system of its own, with clear rules (Duval, 2009).

Such rules, called compliance rules, must be respected when building a representation, not only so that the object represented can be identifiable but also because they allow the occurrence of the second cognitive activity, the treatment of the representation, within the semiotic system used. Thus, a new representation is obtained, showing new characteristics for the same object that was first represented and maintaining the same original register system (Duval, 2009).

The third cognitive activity consists of representing this same object in another register or semiotic system by means of conversion. This activity will allow us to elucidate other characteristics of the object represented, but in a new system, emphasising that this conversion process involves cognitive processes distinct from those used to treat a representation. In the treatment, the system presents their own internal rules for transformations to occur; however, when one wishes to start from one semiotic system to another through conversion, there are no rules defined to perform it, as would occur in the translation of a language, for example, or in a decoding process, which makes conversions more complex (Duval, 2009). The author states that:

> Conversion and treatment are totally independent sources of problems in mathematics learning, and it seems that conversion is a more complex cognitive process than treatment. The problem most students find is so deep that conversion can be considered as the **limit** of understanding. The conversion of semiotic representation usually appears as a trick that cannot be well learned and is not taught! (Duval, 2006, p. 149)

As in mathematics, those activities are extremely important in the teaching and learning process of statistics and are at the heart of the process of understanding the objects under study, such as the statistical variables, especially the quantitative ones. Therefore, we argue that those three cognitive processes should be mobilised in statistical education. We also stress that in the absence of mobilisation of those processes, the students' learning about the object can be compromised. For this very reason, Duval says:

> Learning mathematics is learning to discriminate and to coordinate semiotic systems of representation in order to become able of any transforming of representation. That can summed up in one sentence. Mathematics learning does not consist first in a construction of concepts by students but in the construction of the cognitive architecture of the epistemic subject. What is at stake in mathematics education through particular content acquisition is the construction of this architecture, because it creates future abilities of students for further learning and for more comprehensive understanding. (Duval, 2000, p. 14)

According to Duval (2012), the process of understanding mathematical objects always involves the mobilisation of at least two different types of semiotic registers, although only one register is chosen to represent this object in a given situation, according to the characteristic that is to be evidenced by the object represented. In view of this, it is clear that the study of mathematics will always require coordination between the various possible registers to represent a single object. In this way, "The coordination of the registers is not a consequence of the understanding of mathematics; on the contrary, it is an essential condition for it" (Duval, 1999, p. 11, our translation).

Although we have included the core principles of the TRSR, our focus is on the role of the *ostensives* and the hand-free manipulable concrete material in the representation or formation of the quantitative variable, in the numerical register and its corresponding conversion into the graphic register. Based on these two references, we present the results obtained in our research and its discussions below.

RESULTS AND ANALYSES

According to Cazorla and Oliveira (2010), a quantitative statistical variable is the one whose results provide quantities. They can be classified as discrete variables, if likely to be counted, generating whole numbers, or as continuous numbers, when they are the result of a measurement, generating any real value in a range. The statistical variables can also be classified as empirical, which are those that have a reference in the sensitive world, and conceptual, which "cannot be observed directly, but inferred by the behaviour of the subjects involved in the research" (ibidem, p. 121), which requires the creation of instruments for their operationalisation.

In this article, we work only with quantitative empirical variables since those can be counted (discrete) or measured by measuring instruments (continuous), making it possible to use concrete manipulable materials, which becomes complex in the case of conceptual variables. We will also explain better the operationalisation of the variables because, according to Pfannkuch and Rubick (2002), it is essential to define measures that "capture" qualities or characteristics of the elements that make up the population; also, there are several ways to apprehend the same variable (Cazorla, Utsumi, & Monteiro, 2021a).

As we want to analyse the role of students' actions in the face of the use of concrete manipulable materials and the conversion of representations of statistical data into different registers, we address the situations in which the students collect the data, since this step has already been performed with data collected from secondary sources.

According to Cazorla and Oliveira (2010), in the variables collected by the students, we have three sources that generate statistical data: observation, experimentation, and simulation. In an observation, the data would be collected directly from the source, without any interference in the phenomenon, as in the problem of the eutrophication of a lake by the aquatic plant water hyacinth (Correia & Cazorla, 2020). The observation step would consist of visualising the natural phenomenon as it occurs, without any intervention, only data collection. In an experiment, this situation would be replicated under controlled conditions, and the doubling of the plant biomass would be observed in a water tank, for example. As the authors propose, the simulation can be developed with bottle caps (as if they were the water hyacinths) in a paper drawing simulating the lake, as we will see later.

In a school situation, the data to be observed can be from the students' themselves, such as age and anthropometric measurements (height, body mass, etc.), as done by Santana (2020); or through the students' action, for example, by checking the number of people inside cars passing by (Watson, 1996), or the number of vehicles per colour (Alsina, 2017), among other things.

The data collected in the experiment could be shared with science, biology, and physical education, such as checking the amount of canary seed that germinated (Vendramini & Magina, 2010) or the growth of sunflowers (Santos, 2018). Similarly, we could generate data by simulating the phenomenon, such as the eutrophication of the lake (Correia & Cazorla, 2020).

We observe that many genuinely continuous variables, as a rule, are discretised in a school situation, i.e., they are represented by whole numbers, such as student height (H) in centimetres without millimetres, or in decimal numbers limited to a fixed number of decimals (in metres, with two decimal places); body mass in kilogrammes without grammes, or age in complete years, allowing the use of manipulable concrete material. However, in some cases, this strategy does not work, as in the case of body mass index – BMI (BM/H^2), because to work with BMI ranges, one of the limits is 18.5, as reported by Santana (2020).

Another issue that must be taken into account is the "degree of ostensibility." In the case of empirical variables, some can be seen by the naked eye, such as the height of students, and others need some material to assist in the representation, such as the number of letters of the first name, where we would need posters/paper cutouts/statistical cube to note down the value. Conceptual variables are not noticeable to the naked eye and need concrete material, such as posters with scores given by judges in olympic competitions or instruments such as Likert scales.

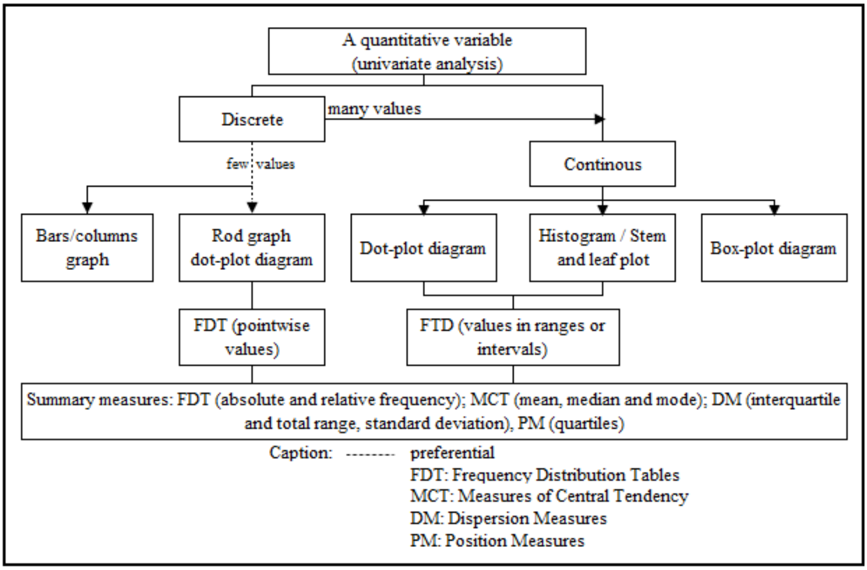
Before analysing the prototypes of the use of the *ostensives* and the students' action in the collection, representation of the data and their possible transformations, we should score the concepts and procedures involved in the analysis of quantitative variables in a univariate and bivariate way.

According to the BNCC (MEC, 2018), the statistics knowledge objects that must be taught throughout basic education are: sample and census survey; qualitative and quantitative variables, population, and sample; statistical tables and frequency distribution tables (FDT), single and double-input; bar/column, line, and pie charts, pictogram, stem and leaf diagram, box-plot, and histogram; summary measures: absolute and relative frequency; measures of central tendency – MCT (mean, median, and mode); dispersion measures – DM (range and standard deviation). However, as Cazorla, Utsumi, and Monteiro observed (2021a), the quartiles and interquartile range were not listed, but they are implicit in the construction of the box-plot. Likewise, we advocate the teaching of the dot-plot and the rod graphs, which were not covered.

Cazorla, Utsumi, and Monteiro (2021b) present a scheme of the possible transformations of the raw data in charts, tables, and summary measures for quantitative variables, which we adapted, as shown in Figure 1. From this, we analysed the potential of the use of the *ostensives*. We observe that in the bivariate context, this use becomes complex, requiring some attention, as we will see later.

Figure 1

Transformations of data of quantitative variables in the univariate context



With this theoretical framework, we present three prototypes of the representation and transformation of raw data of quantitative variables, using concrete manipulable material and the action of students, namely: (a) in the univariate analysis of discrete variables that take few values; (b) in the univariate analysis of continuous or discrete variables that take many values; and (c) in the bivariate context in the analysis of two quantitative variables.

We also focus on two processes in the use of *ostensives*. The first concerns the representation or formation of quantitative variables, and the second concerns the conversion of data representations between registers, as we will see in the three prototypes announced.

(A) the use of ostensives in the univariate analysis of discrete variables that take few values

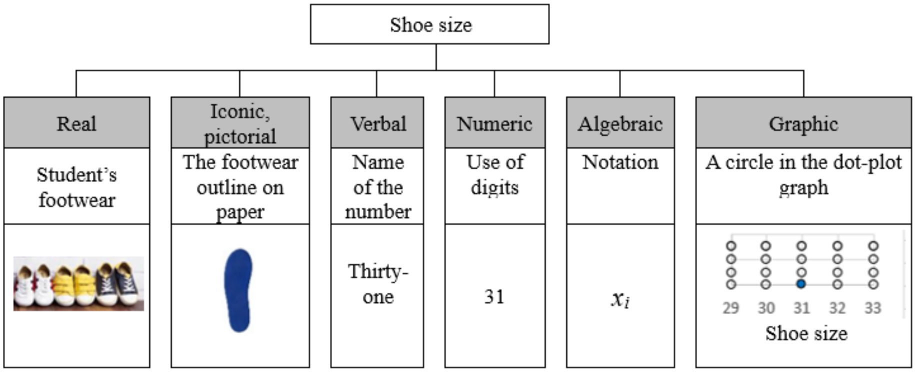
In this prototype, we include discrete variables that take few values generated from the students' actions, that can be observed by the naked eye, such as the "shoe size," or that students can use *ostensives* (posters) for viewing, such as the "number of letters of the first name."

For the formation or construction of an idea in a register, we mobilised some signs and compliance rules. In this sense, we referred to the various representations (real, iconic, verbal, numerical, and graphic) for a qualitative variable proposed by Cazorla, Henriques, and Santana (2020) and expanded into the quantitative variables. We included algebraic representation because we use the concept of ordered pairs and variables to form a mathematical function in the final years of elementary school and high school.

In Figure 2, we present a proposal considering as an example a discrete variable that assumes few values, referring to the students' "shoe size." The first representation, which we call "real," occurs when the student shows his shoes. The second, called "iconic" or "pictorial," appears when we use figures or *ostensives* to represent the size of the footwear. In this case, we may ask students to draw the orthogonal projection of the footwear on paper, as performed by Fielding-Wells (2018, p. 1). The third is the "verbal" representation, when we represent the shoe size orally or in written form in the mother-tongue register, "thirty-one". The fourth, is the "numeric" representation, when we write the numeral "31". The fifth, algebraic, is when we represent the letter x with a subscript i (x_i) to denote the value of the variable X "shoe size" of the i-th student. Finally, the graph, when we use a circle in the dot-plot diagram in a proper frame or a "unit" trace/line segment in the rod graph or a "unit" portion of a bar in the bar chart.

Figure 2

Types of representations of a discrete variable that takes few values



Once the data have been collected, the next step is to organise and summarise them to briefly describe the variable "shoe size" using the *ostensives*, so that students can follow the data transformations in the "statistics" from direct, hand-free manipulation.

In Figure 3a, we observe the students formed rows according to the shoe size. Here, the students themselves acted. In Figure 3b, the student represented the size of his/her shoes on one side of the statistical cube and stacked them. Finally, in Figure 3c, each student drew the orthogonal projection of his/her right foot footwear (it could be the left footwear, the important thing is to be always just one of them) on a paper or cardboard, cut, and glued it on the wall. In these three situations, we observed that the students acted actively, either by forming ordered rows representing their data in the statistical cube or by stacking or drawing and sticking the drawings of their shoes to the wall.

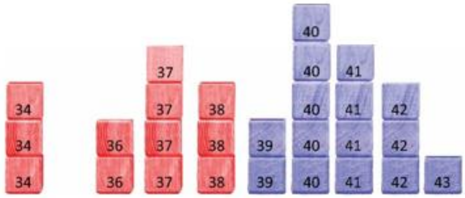
Figure 3

Representation and organisation of a discrete variable using ostensives

(a) forming rows according to shoe size

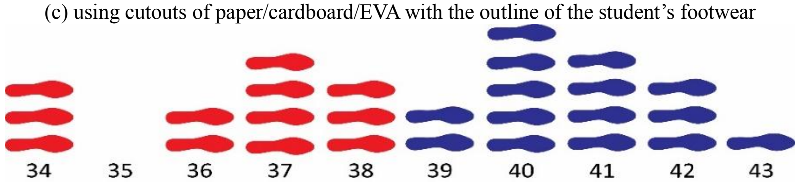
Source: Silva, Cazorla, and Kataoka (2011, p. 41)

(b) stacking statistical cubes with student data



Shoe size

(c) using cutouts of paper/cardboard/EVA with the outline of the student's footwear



Shoe size

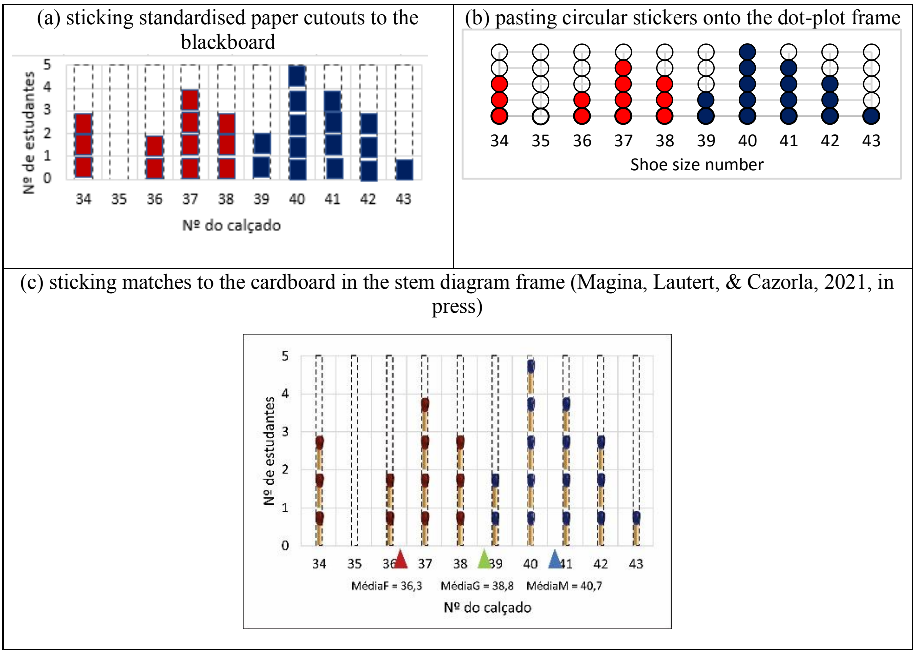
Caption: red – girls, blue – boys.

In all these cases, students recognise their data and can visualise the distribution of the variable under study. In other words, there is a two-way relationship between the student and his/her data, which can be emphasised by placing the student's name on one side of the cube or the drawing of his/her footwear, and different colours can be used to distinguish a qualitative variable. For example, red for girls and blue for boys; or one colour for each age, which will allow students to visualise possible characteristics of the variable.

We can also use other materials, not as *ostensives*, as those in Figure 3, but that the students can still manipulate. In Figure 4a, we show a "bar graph," widely used in basic education, with standardised paper cutouts stuck to the squared paper and glued to/drawn on the blackboard; circular stickers to the frame of the dot-plot diagram (Figure 4b) or matches/straws to the rod graph frame drawn in the cardboard (Figure 4c). As we can see, students can still recognise and track their data.

Figure 4

Organisation and representation of the variable "shoe size" using ostensives



(a) sticking standardised paper cutouts to the blackboard

(b) pasting circular stickers onto the dot-plot frame

(c) sticking matches to the cardboard in the stem diagram frame (Magina, Lautert, & Cazorla, 2021, in press)

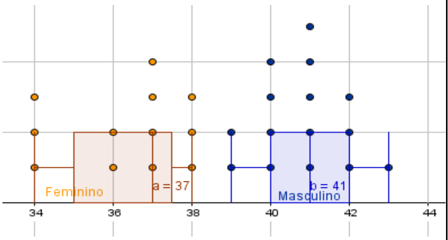
We used Figure 4c to represent the mean shoe size numbers of girls, boys, and in general by using triangles at the base of the numerical scale (abscissa). In this figure, we can observe that girls' feet are smaller than the boys', and the distribution of this variable presents a bimodal behaviour influenced by gender.

Figure 5a presents the dot-box-plot, coloured by gender; the general dot-box-plot (Figure 5b); and the box-plot for each gender and global (Figure 5c), the three built with the software GeoGebra. We also present the diagram of stem and leaves by gender and general (Figure 5d). In these charts, we are mobilising the *non-ostensive* elements, referring to the Cartesian plane, the dots, the straight line segments that make up the box-plot, besides having calculated the median, the quartiles, and examined the presence of discrepant values.

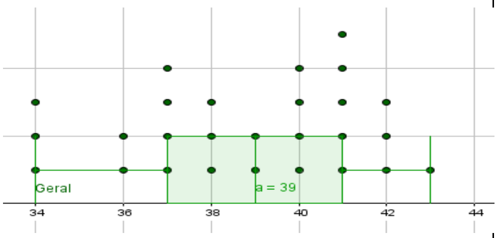
Figure 5

Organisation and representation of the variable "shoe size" using non-ostensives

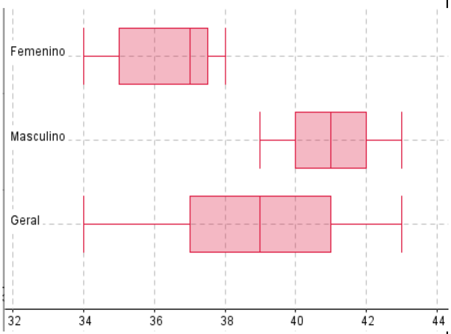
(A) *Dot-box-plot* of the shoe size number per gender



(b) *Dot-box-plot* of the shoe size number



(c) Box-plot of the shoe size number per gender



(d) Stem and leaves diagram

Opposite (bivariate) per gender

Feminine	Stem	Masculine
444	3	
	3	
66	3	
7777	3	
888	3	
	3	99
	4	0
	4	1111
	4	222
	4	3

Univariate

Stem	Leaves
3	444
3	
3	66
3	7777
3	888
3	99
4	0
4	1111
4	222
4	3

Stem: tens
Leaf: units

As we have already mentioned, the dot-plot was not listed in the BNCC. However, we can observe its contribution to constructing and understanding the box-plot because the students can still "see" the dot that can represent their data and track their data in the stem and leaf diagram. The box-plot detaches from the data and becomes an abstract object so that the students no longer recognise theirs. We, therefore, believe that the dot-plot must be taught before the box-plot.

When variables are not noticeable to the naked eye, such as the number of letters that make up the first name, we need to use concrete material, such as standardised paper cutouts, posters or statistical cubes, in which the student represents their data (Figure 6a). This strategy can also be used for continuous variables. The treatment of this variable can follow all the steps described above, and the students will form rows according to the number of letters in their name, stacking the cubes. They can also stick their paper cutouts or write their names in full on the board (Figure 6b).

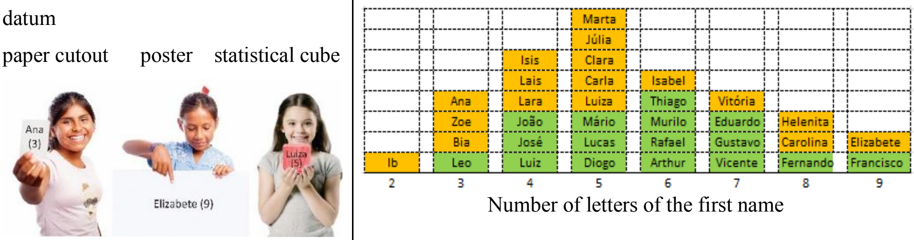
Figure 6

Ostensives to represent the variable "number of letters of the first name"

(a) using *ostensives* to represent the datum

paper cutout poster statistical cube

(b) sticking standardised paper cutouts to the blackboard



Source: Images from freepick.com

Caption: orange – girls, green – boys

As part of the experiment, Vendramini and Magina (2010) presented the "seed germination" task, in which the students planted ten seeds of birdseed in small disposable coffee cups, guessed and registered the number of seeds that would germinate (conceptual variable) and, after some time, counted the seeds that sprouted (empirical variable). Santos (2018) conducted a similar experiment with pre-school children using sunflower seeds.

This prototype of analysis and use of the *ostensives* applies to any discrete variable that takes few values that results from the observation, such as the number of siblings or letters that form the first name, or values produced by them in an experiment, such as the quantity of seeds that germinated, or via simulation, by flipping coins, instead of planting seeds.

(b) The use of ostensives in the univariate analysis of continuous or discrete variables that take many values

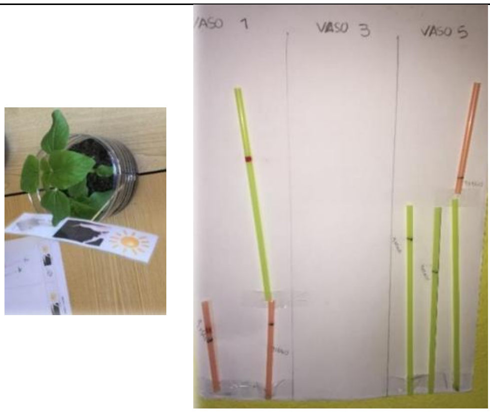
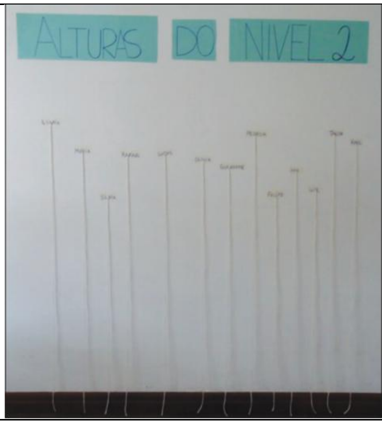
Just as we use the *ostensives* to represent and organise the discrete variables that take few values, we can use the same strategies for the representation and organisation of discrete variables that take many values, such as the number of passengers travelling a day on an urban bus or continuous variables, such as the height of a plant or a student.

We observed that there are strategies that allow us to collect data from continuous variables on a continuous scale, such as that performed by Santos

(2018), in which children track the growth of sunflowers using 10 and 20 cm-long straws, making marks to observe how much the plant grew (Figure 7a) or in the constant suggestion in the Pacto Nacional pela Alfabetização na Idade Certa [National pact for literacy in the right age] (MEC, 2014), for students to measure the height of their colleagues with a string, cutting and sticking it to the blackboard (Figure 7b).

Figure 7

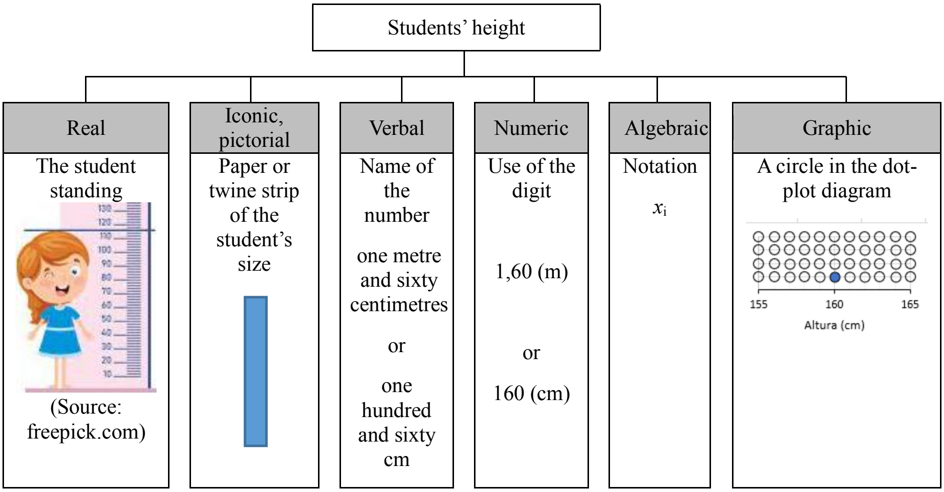
Use of ostensives to measure height (continuous variable, continuous scale)

(a) Height of sunflowers (Santos, 2018, pp. 51-52)	(b) Students' height (MEC, 2014, p. 63)
	

To illustrate how we can represent the continuous variables, we consider as an example the “students’ height,” in which we have the student himself (real) and we can cut paper/twine strips with a mark representing the student’s height and stick them to the wall, represent them orally or in writing fully, and so on, as shown in Figure 8.

Figure 8

Types of representations of a continuous variable (discretised)



Silva, Magina, and Silva (2010) worked with anthropometric variables with 25 students from the 9th grade. To work on the variable "students' height", the authors mobilised students to experience data representation using their own bodies, asking them to form a row in ascending order (lower to higher) (Figure 9a). Students at the same height were asked to stand behind each other ("stacked"), forming rows perpendicular to the main one, forming a human "dot-plot". In this figure, we see the researcher asking the student that occupied the 13th position to take a step forward to imply that her height represented the median height.

In Figure 9b, researchers used a giant tape measure (as an *ostensive*) to show students that being side by side does not mean that the distance is the same, therefore the representation in the numerical line is important. In Figure 9c, we observed the representation of the *human dot-plot* data in the numerical register. It plays an important role in finding the position measures, the median (M_d), the first quartile (Q_1) and the third quartile (Q_3). However, the corresponding conversion in the graphic register is necessary, aiming at the real quantification of the variable "height", as shown in Figure 9d.

Figure 9

The role of the students' action in the representation and of ostensives of the variable height

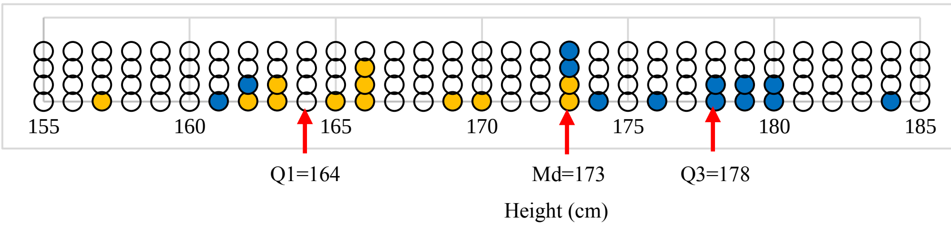
(a) The representation of the median

(b) the use of the giant tape measure

(c) the numerical register and the determination of the median and quartiles

Gender	F	M	F	M	F	F	F	F	F	F	F	F	F	F	M	M	M	M	M	M	M	M	M	M	M
Height	157	161	162	162	163	163	165	166	166	166	169	170	173	173	173	173	174	176	178	178	179	179	180	180	184
Position	1st	2n	3rd	4th	5th	6th	7th	8th	9th	10t	11t	12t	13t	14t	15t	16t	17t	18t	19t	20t	21s	22	23r	24t	25t
Statistics						Q1 = 164							Md						Q3 = 178						

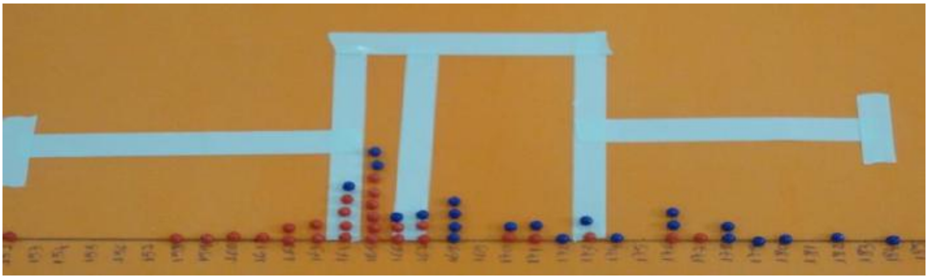
(d) the graphic register and the determination of the median and quartiles



Regarding the use of concrete material, Silva, Kataoka, and Cazorla (2014) worked with 24 mathematics teachers of high school, who built the dot-box-plot (Figure 10) using the Styrofoam base, coloured pins/thumbtacks to represent the dots and the crepe tape outline the box-plot.

Figure 10

Dot-box-plot built with concrete material (Silva, Kataoka, & Cazorla, 2014, p. 4)



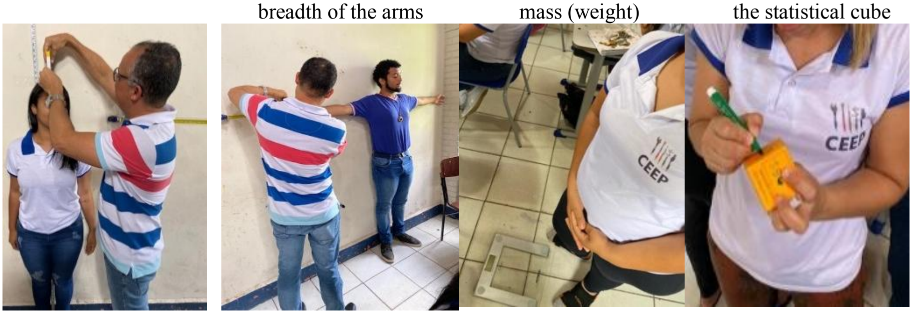
Santana (2020) advanced with the use of *ostensives* and students' actions in the management of quantitative variables, in the study of the role of the statistical variables in the contextualisation of the affine function. The author used the statistical cube (Cazorla & Santana, 2020) as an *ostensive* to represent seven variables, namely: gender (qualitative with two categories), using the colour of the cube, being orange for girls and blue for boys, and on the faces of the cube, the age, height, and arm span and the body mass (continuous discretised). BMI (with one decimal) and BMI ranges (ordinal qualitative). The author used a tape measure, a digital scale, and a statistical cube in the process of representation, formation, and representation of the variables (Figure 11).

For the students to see the distribution of the variables, the author built four banners, two for univariate analysis and the other two for bivariate analysis. The first banner was three meters long and half a meter wide (Figure 12), with a squared sheet to represent the height/arm span (150 to 200 cm) and body mass (50 to 100 kg). Each square measured 5.5 cm on the side to accommodate the statistical cube, formed by squares of 5 cm on the side.

Figure 11

The role of the students' action in the representation and the ostensives in the representation of variables (Santana, 2020, Figure 55, p. 111)

(a) measuring height | (b) by measuring the breadth of the arms | (c) measuring body mass (weight) | (d) writing the data in the statistical cube



To analyse the data distribution, Santana (2020) asked each student to put their cubes in the banner (Figure 12a). After all the students placed their cubes in the banner, the author asked them to describe the data distribution (Figure 12b). To indicate where the mean was likely to be found, the students used a black pencil case (Figure 12c).

Figure 12

The role of the students' action in the management of the height variable using the statistical cube (Santana, 2020, Figure 56, p. 113)

(a) Student placing the cube in the banner	(c) students analysing the likely location of the mean	(c) students analysing the likely location of the mean
		

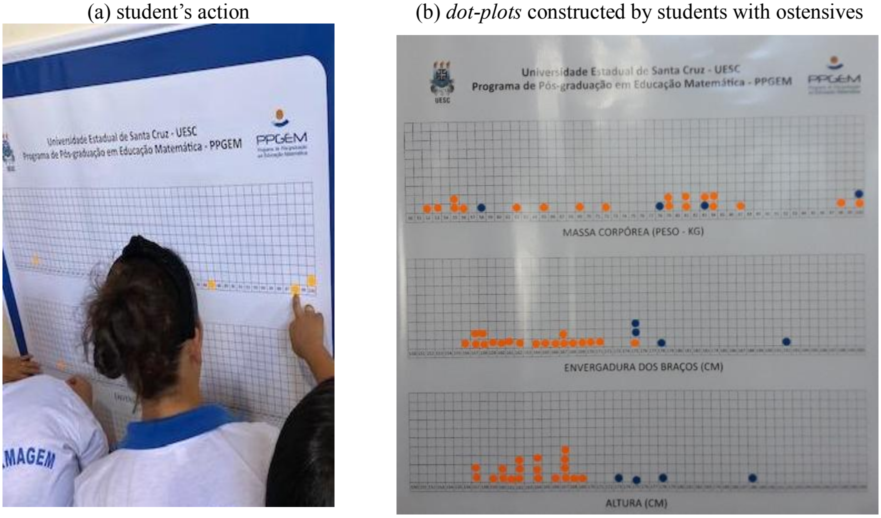
In this case, the statistical cube acts as an *ostensive* that still maintains a biounivocal relationship with the students' data because each student recognises its cube. Therefore, the representation still adheres to the students' data.

The second banner was built to accommodate the 2 cm diameter circular stickers (Figure 13). Thus, each square had 2.1 x 2.1 cm^2. Next, the researcher delivered circular stickers (orange for women and blue for men) and asked them to stick on the places related to the measures in the cube (Figure 13a), building the dot-plot of height, arm span, and body mass, coloured by gender (Figure 13b).

Figure 13

The role of the ostensives in the management of representation and conversion of statistical data of the numerical register into graphic register using circular coloured stickers (Santana, 2020, Figure 57, p. 113)

(a) student's action (b) *dot-plots* constructed by students with ostensives



Since the banner was large and glued to the wall, once the students finished pasting their stickers, the author asked them to look at the data distribution by asking them guiding questions: Who are the highest ones? The shortest? Where should the mean be? Comparing the height and arm span distribution, which of the two varies the most? When comparing the body height and body mass distribution, which of the two varies the most? The students registered their guesses of the mean values in their notebooks, which were then compared with the mean calculated using the data. The author observed how close the guesses of the mean values were. This strategy of collective analysis was very promising because one student's reasoning complemented the reasoning of the other, forming a collective knowledge.

(c) the use of ostensives in the bivariate analysis of quantitative variables

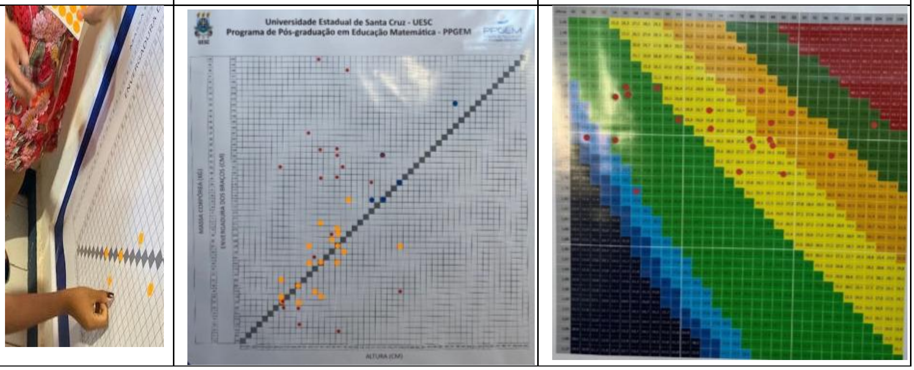
For the bivariate analysis, the author built a third banner simulating the Cartesian plane (Figure 14b), with 1.5m in the side, the height was in the abscissa, the measurements ranging from 150 to 200 cm, in units. In the ordinate, there were two scales, one for the arm span, taking the same height values, and a second scale for the body mass, whose measurements ranged from 50 to 100 kilograms. The difference for the traditional Cartesian plane is that the lines of the mesh intersect in the whole numbers. In this construction, the numbers are found between the squared lines to include the circular sticker in this squared sheet. This adaptation was a didactic strategy to represent the continuous variables, which had to be discretised for the concrete material to be used. In this graph, the measurements of short and thin people are located in the lower-left corner, those of thin and tall people in the lower-right corner, the short and obese people in the upper-left corner, and those of tall and obese people in the upper-right corner.

The fourth banner (Figure 14c) is a dispersion diagram suitable to track the BMI. It had body mass in the abscissa, ranging from 46 to 110 kg, and the height in the ordinate, ranging from 1.46 to 2.10 meters (both discretised, varying in one unit). The reading starts at the dot (1.46 m; 46 kg) in the upper-left corner, the reading for the abscissa from left to right, and the reading of the ordinate from top to bottom. At the intersection of each measurement is the BMI calculated value, which is coloured according to the BMI ranges. In this graph, the measurements of people within the normal range are situated diagonally, slightly to the left (green band), the measurements of people with low weight problems are at the bottom of the diagonal (blue band) and the measurements of people with signs of obesity are above the diagonal (yellow, orange, dark green and red bands).

Figure 14

The role of the ostensives in the management of representation and the conversion of statistical data of the numerical register into graphic register using circular coloured stickers (Santana, 2020, Figure 57, p. 113)

(a) student's action	(b) "traditional" dispersion diagram	(c) BMI dispersion diagram



Legenda: Figura 14ª e 14b: círculo laranja (feminino), azul (masculino) para o par ordenado (A, MC) e círculo vermelho para o pa ordenado (A, MC) nos dois *banners* (Figura 17b e Figura 17c).
Figura 14c: azul "Abaixo do peso" (menos do que 18,5); verde claro "Peso normal" (de 18,5 a 24,9); amarelo "Sobrepeso" (d 25 a 29,9); laranja "Obesidade grau 1" (de 30 a 34,9); verde escuro "Obesidade grau 2" (entre 35 e 39,9) e, vermelho "Obesidad grau 3" (40 ou mais).

Caption: Figure 14a and 14b: Orange circle (feminine), blue (masculine) for the ordered pair (H, BM) and red circle for the ordered pair (H, BM) in both banners (Figure 14b and Figure 14c.
Figure 14c: blue "Underweight" (less than 18.5); light green - "Normal weight" (between 18.5 and 24.9); yellow - "Overweight" (between 25 and 29.9); orange - "Class 1 Obesity" (between 30 and 34.9); dark green - "Class 2 Obesity" (entre 35 and 39.9) and red - "Class 3 Obesity" (over 40).

To perform the bivariate analysis, Santana (2020) asked the students to check their heights (H) and arms spans (AS), represented in the statistical cube, to identify the ordered pair (H, AS) and to stick the circular sticker in the square (blue for men and orange for women) From the third banner (Figure 14a), forming a cloud of dots quite adherent to the line of equality (AS = H) and asked them to place themselves in relation to Vitruvius's hypothesis that the arm span is equal to the height (AS = H).

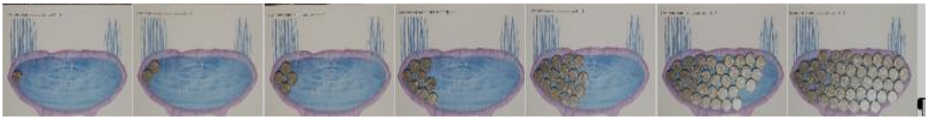
Next, the author asked the students to look at their heights (H) and body masses (BM) and stick a red sticker (without distinguishing gender) in the corresponding location, thus building the dispersion diagram of BM and H (Figure 14b), whose dot cloud was randomly dispersed, no pattern.

Finally, the author asked the students to look at their heights and body masses and glue their stickers on the fourth banner (Figure 14c), when the students were impacted by the tendency to obesity of most of them.

By way of illustration, we present the example of simulation for modelling the exponential function performed by Correia and Cazorla (2020) – in the context of deterministic covariation – inspired by the video by Maurício Féo[1], which explains the exponential growth in the eutrophication process of a lake by water lilies, by asking the following question: "Until when can I clean the lake, before I lose control and the water lilies take all the place?" In this case, the authors proposed that the students simulated the eutrophication process by the water plant *E. crassipes*, known as water hyacinth, which, in ideal conditions, manages to double its biomass in approximately 15 days. For this, they suggested that they used bottle caps/coins/beans or any concrete material of an equal shape and size to simulate the water hyacinths and to draw a lake in cardboard (Figure 15).

Figure 15

The simulation of lake eutrophication by the water hyacinth, exponential growth (Correia & Cazorla, 2020, Figure 3, p. 12)



This student knows that the process baseline is zero (x=0), with one water hyacinth, and, at each period, it doubles its biomass in a process similar to meiosis (cellular division). The authors suggest that the linear growth with a rate equal to two be worked in parallel and that a table and a squared sheet are at hand to have them build the graphs in the paper/pencil environment.

In this process, the student has the bottle caps as *ostensives* to represent the number of water hyacinths in the lake at each period. The student's action by adding caps each time may enhance his understanding of the covariation present in the simulation because, strategically, the student may count the total number of caps he must have on the lake for each period or identify that given

[1] https://g1.globo.com/bemestar/coronavirus/noticia/2020/04/10/enigma-da-vitoria-regia-vira-exemplo-em-video-que-explica-o-que-e-o-crescimento-exponencial-da-pandemia.ghtml

the quantity existing in the period X_n, he must add only one quantity $X_{n+1} - X_n$ to complete the quantity required in X_{n+1}. In other words, by working with this reasoning, completing the amount that is lacking, the student may, through his action, develop the notion of covariation, identifying that in the linear situation, the increment is equal in all periods. Still, in the exponential situation, it increases more in each period.

For the statistical covariation, the authors propose the follow-up of the accumulated cases of infected individuals and deaths by Covid-19, which at the moment of expansion can be modelled by the exponential function. However, those are secondary data, so that students do not have direct action unless by searching for official sources. In addition, we will hardly be able to use *ostensives* to represent them, not least because the order of magnitude exceeds thousands, and it is necessary to use free electronic spreadsheets or mathematical/statistical software, such as *GeoGebra*, for example, but that we will not address in this work.

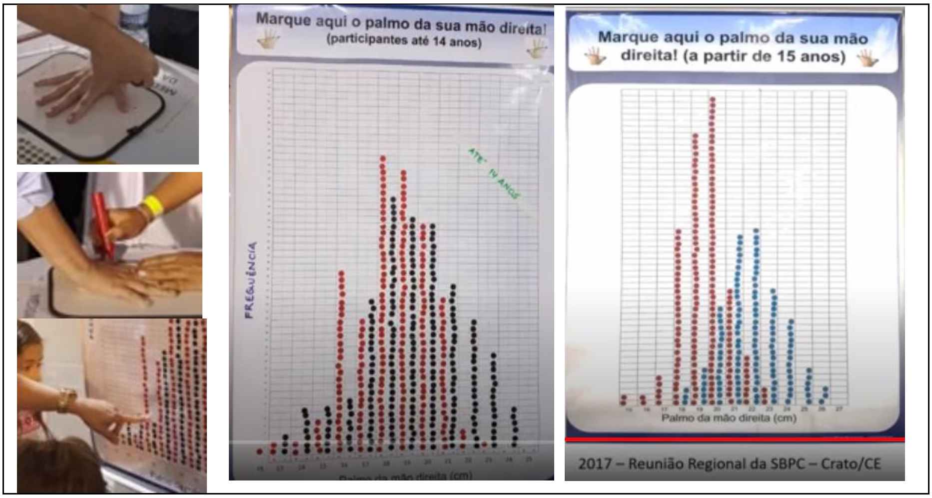
As a suggestion of additional reading, Cazorla, Samá, Velasque, Diniz, and Nascimento (2021) bring several possibilities of work with the statistical concepts inherent to the data generated in the Covid-19 pandemic context. They introduce two applets to simulate exponential growth and contagion rates, sensitising students about the importance of adherence to prevention measures, such as mask use, avoiding closed sites and human agglomeration, and how it impacts the graph paths - which sometimes drastically change.

There are other studies in which researchers have used the *ostensives* in an encouraging way, such as, “Let us measure the palm of our hand?”, carried out by the “Statistics Tent”, led by the professors. Lisbeth Cordani and Doris Fontes, from the Brazilian Statistical Association, who have travelled the country in the meetings of the Brazilian Society for the Progress of Science – SBPC, popularising the teaching of statistics, as we can see in the video on YouTube (https://www.youtube.com/watch?v=cqIQAXFO8Aw).

During the visit to the Statistics Tent at SBPC meetings, professors Lisbeth and Doris encouraged visitors to measure their right-hand palm and glue a sticker (red for girls and blue for boys) on one of the banners, one for children and adolescents (up to 14 years old) and the other for adults (15 years or more), as shown in Figure 16.

Figure 16

The use of ostensives in the popularisation of statistics (Statistics Tent)



In the above-mentioned video, professor Cordani contextualises and analyses the possibilities for understanding the fundamental ideas of statistics, such as the difference by gender, by age group, the more symmetrical and concentrated distribution of the older and more widespread of the younger ones, which is explained by the presence of children and adolescents in full growth, among other aspects. We encourage readers to watch the video and participate in the online experience.

CONCLUSIONS

The fundamental ideas of statistics present a paradox. They are intuitive and complex because although they do not have mathematical complexity in their concepts, they are not easy to interpret. The most emblematic case is that of the mean, since most students know how to calculate it but cannot attribute meaning, as Santana's work (2020) shows, when the students were asked to give their guesses about the average height value by examining the distribution of the cubes in the banner. However, based on a key question-driven analysis, students understand the real meaning of the mean, so much so that their guesses were very close to the value calculated with the data.

Also, one of the key points is the definition and operationalisation of variables, the generation of data, and how they are transformed into statistics – tables, graphs and summary measures. In this sense, the use of the *ostensives* and the students' actions in managing statistical concepts are fundamental aspects of learning.

When students participate in the whole process, from collecting the data to tracking how data turn into statistics, they can understand and give meaning to the measures, as Cardoso, Nagamine, and Cruz (2010, p. 8) report about one student's speech, whose height was the median value in the human dot-plot. She was so amazed that when she saw the median in the box-plot diagram, she said: "That little dot in the median is me." Or Santana's (2020, p. 116) students, who exclaimed: "We must watch out about our food!" and "about physical activities." Those statements reveal that involving the student in the whole learning process seems to be a promising didactic strategy.

Throughout this article, we have seen numerous possibilities for using the *ostensives*. We hope that, with this, teachers feel encouraged to use free-hand concrete manipulable materials that actively involve students in the whole process of data collection and transformation.

However, as can be seen in the construction of the banners, it is necessary to be flexible and pay attention to the modifications that we must make so that the *ostensives* can fulfil their role, assisting the student to capture data and transform them into different representation registers.

With this article, we hope to have built a theoretical reflection on the role of the free-hand concrete manipulable material and the student's action in producing statistical knowledge, empowering them to read the world.

AUTHORS' CONTRIBUTIONS STATEMENTS

I. M. C. is responsible for the conception of the work, A. H. for the theoretical framework of the TRSR and the ATD, C. V. S. provided the data of his master's dissertation regarding the contribution of the statistical variables in the contextualisation of the affine function and G. S. C. is developing his dissertation with the modelling of the exponential function and the context of deterministic and statistical covariation.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the first author of this article, I. M. C., upon reasonable request.

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