ACTA SCIENTIAE

ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.6543

Mathematical Modelling and Didactic Moments

Gleison de Jesus Marinho Sodré[a]

[a] Universidade Federal do Pará (UFPA), Escola de Aplicação da UFPA, Belém, PA, Brasil

Received for publication 30 Mar. 2021. Accepted after review 23 Apr. 2021
Designated author: Claudia Lisete Oliveira Groenwald

ABSTRACT

Background: from the perspective of mathematics education, an important issue addressed vehemently in international conferences on mathematical modelling teaching, regardless of the theoretical current adopted on the concept, is how we can teach modelling? **Objective**: to highlight the didactic moments announced by the anthropological theory of the didactic in the process of studying problems in concrete contexts in mathematical modelling. **Design**: a study and research path was conducted based on theoretical and methodological tools of the anthropological theory of the didactic. **Settings and participants**: preservice teachers of a degree course of a public institution, who had to solve a problem of application in savings. **Data collection and analysis**: From an empirical excerpt of a study developed by Sodré (2019) with preservice teachers. **Results**: the elements found in the empirical research confirm the hypothesis that regardless of the path taken in the modelling process, one or more didactic moments is/are performed. **Conclusions:** ultimately, the study of problems in concrete contexts, besides highlighting the encounter of teachers with different didactic moments, revealed the remarkable dependence between mathematical and non-mathematical know-how that stimulates us to further investigations.

Keywords: Mathematical modelling; Didactic moments; Anthropological theory of the didactic.

Modelagem matemática e os momentos didáticos

RESUMO

Contexto: na perspectiva da educação matemática, uma questão importante abordada com veemência em conferências internacionais sobre ensino de modelagem matemática, independente da corrente teórica adotada sobre o conceito, é como podemos ensinar modelagem? **Objetivo**: evidenciar os momentos didáticos anunciados pela teoria antropológica do didático no processo de estudo de problemas em contextos concretos em modelagem matemática. **Design**: para isso, foi realizado um percurso de estudo e pesquisa orientado a partir de ferramentas teórico-metodológicas da teoria

Corresponding author: Gleison De Jesus Marinho Sodré. Email: profgleisoneaufpa@gmail.com

antropológica do didático. **Ambiente e participantes**: professores em formação de um curso de licenciatura de uma instituição pública a partir do enfrentamento de um problema de aplicação em poupança. **Coleta e análise de dados**: a partir de um recorte empírico desenvolvido por Sodré (2019) com professores em formação. **Resultados**: os elementos encontrados na empiria confirmam a hipótese que independente do percurso realizado no processo de modelagem um ou mais momentos didáticos é/são realizado(s). **Conclusões:** em última análise, o estudo de problemas em contextos concretos além de evidenciar o encontro dos professores com diferentes momentos didáticos, revelou a notável dependência entre os saberes matemáticos e não matemáticos que nos estimulam a futuras investigações.

Palavras-chave: Modelagem matemática; Momentos didáticos; Teoria Antropológica do didático.

Modelización matemática y momentos didácticos

RESUMEN

Contexto: desde la perspectiva de la educación matemática, un tema importante abordado con vehemencia en los congresos internacionales sobre la enseñanza del modelado matemático, independientemente de la corriente teórica adoptada sobre el concepto, es ¿cómo podemos enseñar el modelado? **Objetivo:** destacar los momentos didácticos anunciados por la teoría antropológica de la didáctica en el proceso de estudio de problemas en contextos concretos en la modelización matemática. **Diseño:** Para ello, se llevó a cabo un curso de estudio e investigación basado en herramientas teóricas y metodológicas de la teoría antropológica de la didáctica. **Escenario y participantes:** docentes en formación de una carrera de grado en una institución pública ante un problema de aplicación en el ahorro. **Data collection and analysis:** desde un abordaje desarrollado por Sodré (2019) con docentes en formación. **Resultados:** los elementos encontrados en la experiencia (empiria) confirman la hipótesis de que independientemente del camino recurrido en el proceso de modelado, se realizan uno o más momentos didácticos. **Conclusiones:** en el análisis final, el estudio de problemas en contextos concretos, además de mostrar el encuentro de profesores con diferentes momentos didácticos, reveló la notable dependencia entre conocimientos matemáticos y no matemáticos, lo que nos estimula a futuro investigaciones.

Palabras clave: Modelización matemática; Momentos didácticos; Teoría antropológica de lo didáctico.

INTRODUCTION: A BRIEF OVERVIEW ON MATHEMATICAL MODELLING

Regardless of the theoretical current adopted on the notion of mathematical modelling, MM from now on, from the perspective of mathematics education, an important issue addressed vehemently in the International Conferences on the Teaching of Mathematical Modelling and Applications, hereinafter ICTMA, is "How can we teach modelling?" [1] (Schukajlow, Kaiser, & Stillman, 2018, p. 11, our translation).

In this sense,

> A quite usual starting point in research lines is what Gascón (2011) calls a teaching problem of mathematical modelling. According to García, Gascón, Ruíz-Higueras, and Bosch (2006), the two most frequent formulations of the problem are: how to teach mathematical modelling? and how to teach mathematics through modelling?[2] (Florensa, Garcia, & Sala, 2020, p. 22).

Although research on the MM teaching by Borromeo Ferri (2006), Blum and Borromeo Ferri (2009), Perrenet and Zwaneveld (2012), Blum (2015), Greefrath and Vorhölter (2016), Vorhölter, Greefrath, Borromeo Ferri, Leiß and Schukajlow (2019), and Barquero and Jessen (2020), among others, highlight almost dominantly the didactic technique of MM cycles to minimise the complexity existing in the MM process, it is necessary to consider that this technique of cycles from the perspective of the anthropological theory of didactics, from now on ATD, is likely to be questioned, as highlighted by García, Gascón, Ruiz Higueras, and Bosch (2006), Bosch, García, Gascón, and Ruiz Higueras (2006), Sodré and Guerra (2018), Sodré (2019), and Sodré and Oliveira (2021), but instead of criticising it, providing it with solid theoretical frameworks that allow better understanding and, if possible, making it more accessible to the teaching and learning of MM.

In this line of thought, Sodré and Guerra (2018) and Sodré (2019) proposed the *Investigative Cycle of Mathematical Modelling*, hereinafter ICMM, "as a methodology for the development and analysis of mathematical models of situations in concrete contexts" (Sodré & Guerra, 2018, p. 253), which should "always be understood as relative and provisional, open to

---

[1] Text fragment: Como podemos ensinar modelagem?

[2] Text fragment: Un punto de partida bastante común en las líneas de investigación es lo que Gascón (2011) denomina el problema docente de la modelización matemática. Según García, Gascón, Ruíz-Higueras y Bosch (2006), las dos formulaciones más frecuentes del problema son: ¿cómo enseñar modelización matemática? y ¿cómo enseñar matemáticas a través de la modelización?

questioning and review, and pertinent to the extent that it is rich for the identification of didactic phenomena and formulation of didactic problems"[3] (García, Barquero, Florensa, & Bosch, 2019, p. 78).

The ICMM (Sodré & Guerra, 2018; Sodré, 2019; Sodré & Oliveira, 2021) paraphrased from the three genres of genuine tasks of mathematical activity announced by Chevallard, Bosch, and Gáscon (2001): *using known mathematics*, *learning (and teaching) mathematics*, and *creating new mathematics*, should be forwarded sequentially by the study director or the class teacher, as guided by the following genres of tasks reconstructed from Sodré (2019):

> **G_1**: Use socially legitimate mathematical models for situations in social contexts to answer questions about them, highlighting the associative relationship between situations in contexts and mathematical models;
>
> **G_2**: Study a mathematical model in the face of different situations and contexts, and study a situation in a concrete context in the face of different mathematical models;
>
> **G_3**: Create a mathematical model associated with a new situation from the study of situations and their associated mathematical models, considering the analogies or homologies between these situations and the new situation.

Under this bias, we assume these three types of tasks here "understood as a reference epistemological model oriented to the teaching of school MM from the study of situations in concrete contexts that unfolded in the study of other situations" (Sodré, 2019, p. 121), considering what Gascón (2014) highlights about the need for the researcher or the teacher to question the *reference epistemological models* explicit or not about the institutional activity. Epistemological models, in general, are products of theoretical or practical knowledge that guide the teacher's or researcher's practice in an institutional environment.

> A characteristic and differentiating fact of the ATD's investigations on mathematical modelling processes is the role of the reference epistemological model of the scope to teach.

[3] Text fragments: Ser entendidos siempre como relativos y provisionales, abiertos a cuestionamiento y revisión, y pertinentes en la medida en que sean fértiles para la identificación de fenómenos didácticos y la formulación de problemas didácticos.

.

> This model is an explicit and alternative reconstruction (to the organisation of school knowledge) of the knowledge to teach.[4] (Florensa, Garcia, & Sala, 2020, p. 23)

It is necessary to ratify, in any case, that the REM should be "understood as a hypothesis of researchers that should be enriched and validated with the design, experimentation, and analysis of the study processes" (Ibidem).

In any case, it is noteworthy that the notion of MM from the perspective of the ATD is sustained dominantly:

> Linked to the notion of mathematical activity since the first developments of this research framework, when it is assumed that doing mathematics consists essentially of producing, transforming, interpreting, and developing models to be able to provide answers to given problematic questions.[5] (Florensa, Garcia, & Sala, 2020, p. 23)

Also according to Florensa, Garcia, and Sala (2020), other investigations in the line of the ATD on the teaching of MM expand the initial descriptions proposed by Chevallard (1989) and Gascón (1994), in particular, the investigations by Bolea (2002), García, Gascón, Ruiz Higueras, and Bosch (2006), Barquero (2009), and Fonseca, Gascón, and Oliveira (2014) that begin to interpret the MM processes from the structuring and articulated development of "increasingly complex mathematical praxeologies" (Florensa, Garcia, & Sala, 2020, p. 23, our translation), with emphasis on the role of mathematical activity.

However, our assumptions based on Sodré and Guerra (2018), Sodré (2019) and Sodré, and Oliveira (2021), although following elements of the ATD, depart from the perspective of MM as a praxeological organisation of increasing complexity, considering that "school MM activities are not limited

---

[4] Text fragment: Un hecho característico, y diferenciador de las investigaciones desde la TAD sobre los procesos de modelización matemática, es el papel del modelo epistemológico de referencia del ámbito a enseñar. Este modelo es una reconstrucción explícita y alternativa (a la organización del saber escolar) del saber a enseñar.

[5] Fragments of the text: vinculada a la noción de actividad matemática desde los primeros desarrollos de este marco de investigación, cuando se asume que hacer matemáticas consiste esencialmente en la actividad de producir, transformar, interpretar y hacer evolucionar modelos matemáticos para poder aportar respuestas a ciertas cuestiones problemáticas.

to the application of mathematical knowledge, since these activities are conditioned by other non-mathematical knowledge, practical and theoretical, which condition and are conditioned by mathematical knowledge" (Sodré, 2019, p. 90).

Otherwise, "non-mathematical knowledge articulated and integrated with mathematical knowledge is what allows the recognition of the situation with mathematics related to the type of problem in context (Sodré & Guerra, 2018, p. 259). From this perspective, the development of MM practices that allows the articulation and integration of mathematical and non-mathematical knowledge can even highlight important aspects of the MM process, especially the notion of situation, the mathematical model, the choice of method or technique to face the model and, no less important, in close relation to the use of computers or calculators to make it possible to perform a practice as Guerra e Silva (2009) warn:

> In this thinking, modelling is not restricted to the formulation of the M model and the interpretation of a solution to the S situation, but also to the adequacy or creation of a P method, since a model without a method is not useful because it does not provide a solution to the situation and, on the other hand, the method may not be useful in the face of a model already validated for a type of S situation because it does not produce coherent solutions for the specific context of the situation. (Guerra & Silva, 2009, p. 110)

In this sense, the process of studies in MM, including the situational aspects, the mathematical model, the choice of the technique to face this model and the use or not of calculators and/or computers with the purpose of *desmagificar* ("demagicfy") (Bosch, Chevallard, & Gascón , 2006) the situation in context can, in any case, show interdependence between the quadruple - {situation/mathematical model/method/machine}, not always visible in the study of problems in contexts, making it seem that the desirable application of mathematical know-how, for example, may be sufficient to understand a problem in context, without, however, considering that "the processing of a real problem with mathematical methods is limited because the complexity of reality cannot be translated completely into a mathematical model"[6] (Vorhölter, Greefrath, Borromeo Ferri, Leiß, & Schukajlow, 2019, p. 101)

We assume that the realisation of the MM process, regardless of the path in a context of the institutional practices "gestures of studies" (Chevallard, 1999), are manifested, i.e., "*the time necessarily comes when* this or that "gesture of study" must be fulfilled" (Chevallard, 1999, p. 241, author's emphasis, our translation), considering that every gesture of study "aims, directly or indirectly, or through conditions that are intended to create or modify, the existence and functioning of a didactic system (or a category of didactic systems)"[7] (Chevallard, 2009, p. 16).

With this look, Fonseca, Gascón, and Oliveira (2014) highlight that different *didactic moments* can emerge during the MM process, in particular, the *exploratory moment, the work of the technique, the technology-theoretical, and the evaluation* (Fonseca, Gascón, & Oliveira, 2014, p. 296), announced "as a tool to perform the concrete analysis of observable didactic organisations"[8] (Bosch & Gascón, 2010, p. 76) that can be experienced by a class, for example, during the study of a problem in a concrete context.

Chevallard (1999) adds:

> Whatever the course of study, some *types of situations* are necessarily present, albeit quite variable, both qualitatively and quantitatively. These types of situations will be called *moments of study or didactic moments* because it can be said that, regardless of the path taken, *there necessarily comes a time when* this or that "gesture of study" must be fulfilled: where, for example, the student must "correct" the elements elaborated (moment of institutionalisation); where "what is worth" of what was built up until then must be asked (moment of assessment); etc. (Chevallard, 1999, p. 241, author's emphasis)

---

[6] Text fragment: the processing of a real problem with mathematical methods is limited, as the complexity of reality cannot be translated completely into a mathematical model.
[7] Text fragment: vise, directement ou indirectement, à travers la ou les conditions qu'il s'efforce de créer ou de modifier, l'existence et le fonctionnement d'un système didactique (ou d'une catégorie de systèmes didactiques).
[8] Text fragment: como herramienta para llevar a cabo el análisis concreto de las organizaciones didácticas observables.
.

According to Chevallard (1999), the six didactic moments do not follow the character of the temporal structure of the study process and can be experienced several times, depending on the type of situation faced. Therefore, before being a chronological reality, the six moments are first, a *functional* reality of the study (Chevallard, 1999, p. 242, author's emphasis), which we interpret here in the following terms:

- **Moment of the first meeting** – **MD_1:** refers to the meeting or reunions with the praxeological organisation or any of its components or to a problematic question Q that this praxeology can contribute to answer;
- **The second moment – MD_2:** the exploratory of the types of tasks T_i and the elaboration of techniques related to this type of tasks, because, according to Chevallard (1999), what is at the heart of the mathematical activity is more the development of techniques than the resolution of isolated problems. In other words, studying some types of problems is a means of creating and putting into play a technique related to the problem of the same type;
- **The third moment – MD_3:** the *technology-theoretical questioning* related to the technique used to answer the types of tasks. Otherwise, it is the "discourse" undertaken on the technique, but which is closely related to each of the other moments **MD_1** and **MD_2**;
- **The fourth moment – MD_4**: the moment *of the technique work*, in which the use of one or more routine techniques may be limited to respond to certain problems and, thus, require the construction of a new technique with greater reach. This technique work should continue until students or teachers achieve a more robust mastery of the set of techniques available. This didactic moment somehow completes the exploratory moment. The choice of a technique, according to Chevallard (1999), the most effective and reliable, requires observing the discourse of the technology developed so far;
- **The fifth moment – MD_5**: the moment of institutionalisation that aims to highlight what is the praxeological organisation previously elaborated, without referring to isolated aspects of this organisation, but that any of its components should make more or less explicit reference to the organisation as a whole;
- **The sixth moment - MD_6:** consists of the *evaluation* that allows "taking stock," according to Chevallard (1999), which includes

evaluating what has been constructed so far about the praxeological organisation and, with that, putting to the test the domain of use of the praxeology, which here can be understood by the proper use or not of mathematical models in a situation facing the problems that may emerge during the study process.

Thus, we assume that the study of problems in concrete contexts and, in a more inclusive manner, guided by the genres of tasks described by $\mathbf{G_1}$, $\mathbf{G_2}$ e $\mathbf{G_3}$, under the direction of a study director or teacher in the classroom, different *didactic moments* as assumed by Fonseca, Gascón, and Oliveira (2014) can be evidenced, including the mathematical activity itself as desired by Florensa, Garcia, and Sala (2020), as an integral and indispensable part of the MM process.

THE INVESTIGATION QUESTION

Based on the assumptions above and considering that the study of problems in concrete contexts as desirable in school education, regardless of the path taken by students or teachers, for example, can lead to the encounter of didactic moments (Chevallard, 1999), not necessarily all, but which can potentially be revealed in the set of conditions in the sense of the ATD, created by the teacher or researcher before a *community of studies*[9].

In this sense, the ratification or not of our hypothesis leads us to the following investigation question:

How does the study of problems in concrete contexts in MM highlight the didactic moments announced by the ATD?

Thus, we aimed to highlight the *didactic moments*, not all perhaps, about the study of problems in concrete contexts, specifically, from the proposition of the *Percurso de Estudos e Pesquisa Orientado* (Guided Study and Research Path) forwarded by Sodré (2019) as a didactic device for the teaching and learning of MM, to be described below.

[9]In this text, the expression community of study refers to the students and the teacher in the classroom.

METHODOLOGY

In this investigation, we assumed the notion of *Percurso de Estudos e Pesquisa Orientado* forwarded by Sodré (2019), based on the ATD theoretical and methodological resources, more precisely, from the notion of the *Study and Research Path* (Chevallard, 2004, 2005, 2013, 2019), henceforth SRP, which, under the paradigm of questioning the world, takes its concrete form, calling for itself disciplinary and non-disciplinary knowledge, which may prove useful, if not indispensable to the use, study, and eventually to learning, and even to the creation of mathematical models about situations in concrete contexts.

The SRP, according to Ladage and Chevallard (2010), is linked to the emergence of the paradigm of questioning the world against the dominant school paradigm, in which in the latter, "the teacher visits several 'monuments' that he introduces and comments with students"[10] (Ladage & Chevallard, 2010, p. 2). On the other hand, in the paradigm of questioning the world, "what matters is not so much what the student will know *in advance*, but what he will be able to learn from *his/her research*, in order even to advance"[11] (Ibidem, p. 2, emphasis added).

In general, a SRP is referred from *indeterminate questions* Q_i that are answered by *specific questions* Q_{ij} during the investigation (Chevallard, 2009), which "can lead a class to rediscover a complex of works that can vary depending on the path taken (which depends on the activity of X, the decisions of Y, but also on the praxeological resources $R_i^\lozenge$ and O_j currently accessible)"[12] (Chevallard, 2009, p. 28,), which can be modelled by the notion of the main didactic system S (X, Y, Q) capable of producing or not auxiliary didactic systems to build strong answers.

In this sense, "a question with a strong sense, an answer with a strong sense: the answer is now not a simple information, *it is an entire praxeological*

[10] Text fragment: le professeur conduit la visite de différents « monuments » qu 'il présente et commente aux étudiants.
[11] Fragments of the text: Ce qui importa n 'est pas tant ce que l 'étudiant saurait à l 'avance que ce qu 'il pourra apprendre par son enquête en vue même de la faire avancer.
[12] Text fragment: Une même question Q peut ainsi conduire une classe à rencontrer un complexe d 'œuvres qui peut varier selon le parcours emprunté (lequel dépend de l 'activité de X, des décisions de y, mais aussi des ressources praxéologiques R ◊ i et Oj actuellement accessibles).

organisation that is yet to be built"[13] (Chevallard, 1999, p. 233, author's emphasis) and, as such, the fulfilment of a "complete" SRP, according to Chevallard (2013, p. 3, author's emphasis):

> It assumes the realisation of *five basic "gestures"*, which are five types of tasks H_i that are consubstantial with the investigative situation and that can be formulated as follows:
> H_1. *Note* the answers $R^\lozenge$ that live in the institutions.
> H_2. *Analyse*, especially in the double experimental and theoretical plane, those answers $R^\lozenge$.
> H_3. *Assess* those answers $R^\lozenge$.
> H_4. *Develop* your own response $R^\heartsuit$.
> H_5. *Spread and defend* the answer $R^\heartsuit$ thus produced.

For Chevallard (2013), the technique that consists of performing those types of tasks in a coordinated manner does not necessarily follow a linear logic, but "as a process of study and investigation from a *functional epistemology of know-how*"[14] (Bosch & Gascón, 2010, p. 86, emphasis added).

Preserving the characteristics of the SRP, Sodré (2019) proposed the *Guided SRP* considering the three genres of tasks $\mathbf{G_1}$, $\mathbf{G_2}$ and $\mathbf{G_3}$, which it is up to the *topos* of the principal or the study advisor teacher and, therefore, is not within the students' reach, as it is assumed in the development of any SRP.

Considering the three genres of tasks $\mathbf{G_1}$, $\mathbf{G_2}$, and $\mathbf{G_3}$, these should more inclusively forward the studies, and, thus, Sodré (2019, p. 96) points out that the guided SRP "is not confused with the SRP when devoid of any a priori guidance, even if restricted to the study director's *topos*."

To understanding better its operation in action, we forward the excerpt of a lived experience (*empiria*) carried out by Sodré (2019), to provide us with answers that can somehow highlight the *didactic moments* lived by a community of studies, consisting of five preservice teachers[15] of a public university from problems in concrete contexts.

---

[13]Text fragment: A cuestión en sentido fuerte, respuesta en sentido fuerte: la respuesta no es ahora una simple información, es toda una organización praxeológica que está por construir.

[14] Fragments of the text: Como un proceso de estudio e investigación basado en una epistemología funcional de los saberes.

[15]This teacher education took place in the context of an institutional project by the Support Programme for Methodological Intervention Projects (Programa de Apoio a

Specifically, we take the following excerpt from the problem described by Sodré (2019, p. 114):

> ***Q_{14}*** *- What is the fixed amount to be deposited monthly in a savings account, considering a 0.5% monthly rate, to obtain, at the end of 12 (twelve) months, the R$ 3,000.00 necessary to purchase an electronic device?*

The analysis of empirical data and the results found below consider the practices manifested by teachers when facing the situations, under the basic postulate of the ATD that considers human action in the situation.

Thus, the teachers were organised into two groups that we represent here symbolically from the theoretical modelling of two auxiliary didactic systems described by: $\boldsymbol{S_1 (x_1, x_3, x_4, Q_{14})}$ and $\boldsymbol{S_2 (x_2, x_5, Q_{14})}$. In this specific case, the auxiliary didactic systems derived from the main didactic system: $\boldsymbol{S_i}$ ($\mathbf{X, Y, Q_{14}}$), in which $\mathbf{X = \{x_1, x_2, x_3, x_4, x_5\}}$ symbolises the set of teachers who participated during studies, **Y** is the study director, here understood as the researcher, and $\mathbf{Q_{14}}$ is the question to be answered by the study community.

ANALYSIS OF EMPIRICAL DATA AND RESULTS FOUND

From task $\mathbf{H_5}$ of the SRP and guided by the genres of tasks $\mathbf{G_1}$, $\mathbf{G_2}$, and $\mathbf{G_3}$ (Sodré, 2019), which includes the other tasks $\boldsymbol{H_i}$ of the SRP, each didactic system $\boldsymbol{S_1}$ and $\boldsymbol{S_2}$ defended their answers following the performance of the other tasks of the SRP, i.e., $\mathbf{H_1}$, $\mathbf{H_2}$, $\mathbf{H_3}$, and $\mathbf{H_4}$, before the study community.

> It is worth mentioning that the empirical data of this investigation were collected from the conceptual photographic records revealed by the didactic systems both in the study

Projetos de Intervenção Metodológica - PAPIM) of the Dean of Undergraduate Education (Pró-Reitoria de Ensino de Graduação - PROEG) of the Federal University of Pará. In this sense, the teachers' records mentioned here do not expose their identities, their images, and voices, thus ensuring the dignity and due protection of the participants in scientific research. For this reason, no prior ethical evaluation was requested by the appropriate councils of the research project from which the work takes place. Thus, we assume and exempt the Acta Scientiae from any consequences arising therefrom, including full assistance and possible compensation for any damage resulting from any of the research participants, as directed by Resolution No. 510, of April 7, 2016, of the National Health Council of Brazil.

process and in the dissemination and defence phase of the answers obtained, of course, from the researchers' logbooks, to better collect and point out the spontaneous manifestations of the didactic systems. (Sodré & Oliveira, 2021, p. 44)

Regarding the delimitation of the study process undertaken by the auxiliary didactic systems, S_1 (x_1, x_3, x_4, Q_{14}) e S_2 (x_2, x_5, Q_{14}), whose path was guided by the study of a problem in a concrete context of mastery of school financial mathematics, it is necessary to take into account that the mathematical models of social practices of the related financial institutions or banks, for example, are defined a priori, according to Chevallard (1989), as:

> Some of the systems that one wishes to study are subject to objective laws, which do not depend on one's will. This is the case of the physical phenomena and, more generally, of phenomena studied by the natural sciences. On the other hand, some systems, the creation of culture, are explicitly regulated, sometimes very precisely, by social conventions. This is the case of financial transactions, capital loans, etc., social practices that are *defined a priori by a mathematical model*.[16] (Chevallard, 1989, p. 28, author's emphasis)

The text extract reveals that mathematical models, including those used by the related financial institutions, are creations of human culture, i.e., they can be endowed with interests and intentions that are not always visible to the users of those models and, thus, show themselves as necessary knowledge for society and, consequently, for school education and teacher education, according to Sodré (2019).

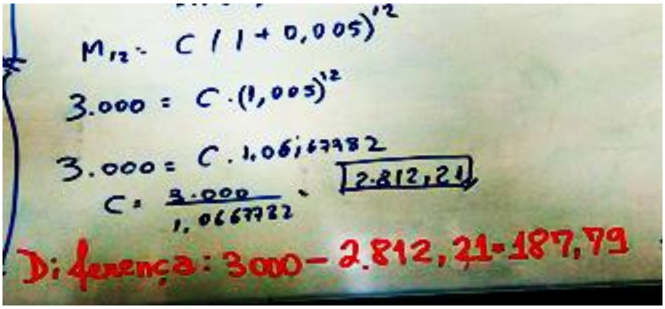
Specifically, the problem Q_{14} faced by the auxiliary didactic systems, more precisely under the guidance of the genres of tasks**G_1** and **G_2**, which included the use of mathematical models, showed some practices of S_2 (x_2, x_5, Q_{14}) before the study community, as shown in Figure 1.

---

[16] Text fragment: Certains des systèmes que l 'on peut vouloir étudier sont soumis à des lois objectives, qui ne dépendent pas de la volonté des hommes. C 'est le cas des phénomènes physiques et, plus généralement, des phénomènes qu 'étudient les sciences de la nature. En revenge, certains systèmes, création de la culture, sont explicitement réglés, de manière parfois fort précise, par convention sociale. C 'est le cas des transactions financières, du prêt à intérêt, etc., pratiques sociales qui sont en fait définies a priori par un modèle mathématique.

Figure 1

Record of the auxiliary didactic system ***S_2 (x_2, x_5, Q_{14})***. (Sodré, 2019)



The situation evidenced by the teachers of ***S_2 (x_2, x_5, Q_{14})*** seems to lead us to meet the notion of praxeological organisation endowed with a type of tasks **t_1:** *calculate the value of a fixed portion to be deposited monthly in a savings account*, which revealed the use of a mathematical technique **τ_1** from the mathematical model of the situation of compound interest, with the clarity of the limitation of this technique **τ_1**precisely by what ***S_2*** said:

- ***S_2 (x_2, x_5, Q_{14})****:* We asked this question with the feeling it was not correct because the solution found represents the difference produced by the amount of R$ 3,000.00 with R$ 2,812.21, it corresponds to the interest on the capital of R$ 2,812.21 if it were applied in 12 months.

The fragment of **S_2** is the clear confession that they did not have, until then, a mathematical technique more suitable for the type of task, because: "in a universe of routine tasks, problematic tasks arise all the time, here and there, which we cannot – yet – perform. New types of tasks, which are then the types of *problems*, are thus established, and new praxeologies will be built around them"[17] (Chevallard, 1999, p. 227, author's emphasis, our translation).

Some intertwined didactic moments seem evident to us, in particular, the first meeting – **MD_1,** evidenced by *the reunions* of the didactic systems***S_1*** and ***S_2*** in the situations and the mathematical model associatedand embedded

[17] Text fragment: en un universo de tareas rutinarias, surgen en todo momento, aquí y allí, las tareas problemáticas que no se sabe -aún- realizar. Nuevos tipos de tareas, que son entonces los tipos de problemas, se asientan así, y nuevas praxeologías vendrán a constituirse a su alrededor.

at this moment, the *work of the technique* – **MD_4**, in view of the limitation of the mathematical τ_1.

Otherwise, and guided by the task genres**G_3**, the expansion of the quality of relations (Chevallard, 2005) of the didactic systems ***S_1*** and ***S_2*** referred them to the work of the mathematical model (Chevallard, 1989) of the situation of compound interest, as we deduce from Figure 2.

Figure 2

Record of the auxiliary didactic system ***S_1 (x_1, x_3, x_4, Q_{14})***. **(**Sodré, 2019)

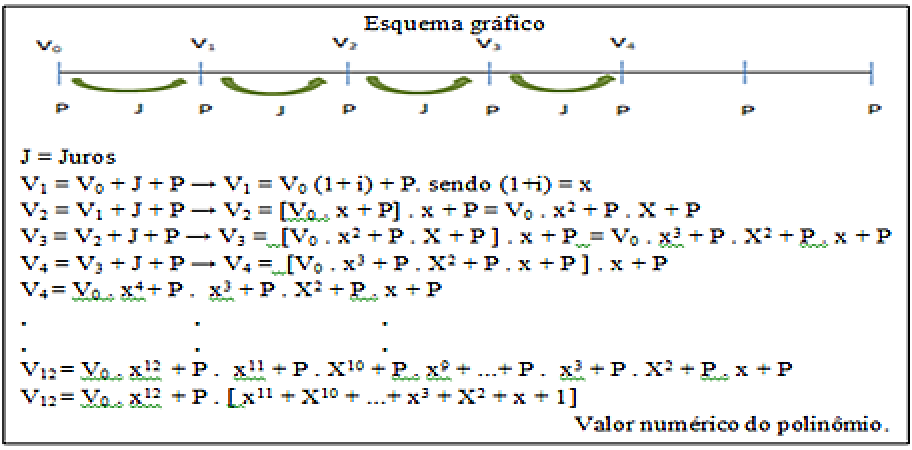
The type of problem faced by the didactic systems **S_1** and **S_2** led them to the study and investigation of works, more precisely, by the rediscovery of situations and mathematical models previously studied on the problem of financing for the calculation of fixed instalments that allowed the use of the graphical scheme of the timeline, according to Figure 2. With this, it was also possible to develop the mathematical model of the problem of financing as a type of model that also accounts for another situation: the accumulation of capital in savings. Therefore, this model, by meeting different types of situations in concrete contexts, seems to meet one of the qualities of a mathematical model, i.e., *multivalence*, according to Revuz (1971).

The relationship of the didactic systems **S_1** and **S_2** with the know-how at stake seem to highlight the following *didactic moments*:

- **MD_1** , in association with **MD_2** – considering that the “type of problem always goes hand in hand with the constitution of at least one embryo of the technique, from which a more developed technique can emerge” [18] (Chevallard, 1999, p. 243, our translation), and, also according to this author, the study of problems is a means and not an end in itself, which allows creating or using a technique;
- **MD_3 -** by the careful introduction of *technology-theoretical resources* through the “cash flow” of the timeline and, no less important,
- **MD** – the work of the “new” mathematical technique $\boldsymbol{\tau}_2$ posed for the type of problem considered;
- **MD_6**– the evaluation, i.e., to be put to the test on the mastery or not of the proper use of the mathematical model of the financing problem in the face of the savings situation, by analogies or homologies of other purchase and sale situations investigated.

Figure 3

Transcript of the record of the auxiliary didactic system ***S_1 (x_1, x_3, x_4, Q_{14})***. (Survey author, 2021)

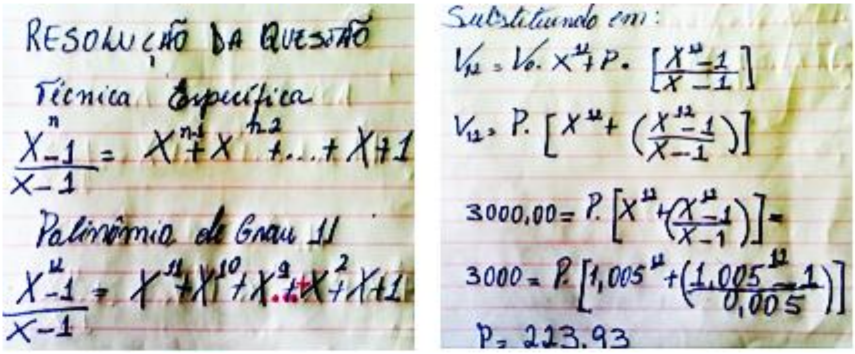


[18] Text fragment: un problema de un tipo determinado va siempre a la par con la constitución de al menos un embrión de técnica, a partir del cual una técnica más desarrollada podrá eventualmente emerger.

In any case, the fulfilment of those moments resulted from the moment of institutionalisation – **MD_5** – of the mathematical models and associated situations, legitimised until then by the didactic systems **S_1** and **S_2**, of the study of vehicle financing.

It is worth noting that the complexity of the mathematical model the teachers faced showed – besides the construction of a more ergonomic mathematical technique $\boldsymbol{\tau_3}$ to calculate the numerical value of the polynomial [$\mathbf{x^{11} + x^{10} + x^9 + ... + x^3 + x^2 + x + 1}$] –, the use of scientific calculators, delimiting with this new auxiliary didactic systems installed: ***S_1 (x_1, x_3, x_4, ₵)*** e ***S_2 (x_2, x_5, ₵)***, because teachers do not know those praxeologies of use of the calculator, more precisely, enter the numerals to calculate exponentials, include or not parentheses or other specific symbols of the calculator to make feasible the practice demanded to construct the answer, but which became necessary before the situation and the mathematical model, as shown in Figure 4.

Figure 4



The work of building a new specific mathematical technique $\boldsymbol{\tau_3}$ highlighted the following didactic moments:

- **MD_4** – in particular, by constructing the specific technique to calculate the numerical value of the most laborious polynomial: $\mathbf{x^{11} + x^{10} + x^9 + ... + x^3 + x^2 + x + 1}$.
- **MD_3 –** from the perspective, even if implicitly, of the technological-theoretical discourse interpreted from the “discourse” of the series of powers, more precisely described here by: $\left[\frac{x^{12}-1}{x-1} = x^{11} + x^{10} + x^9 + \cdots + x^3 + x^2 + x + 1\right]$.

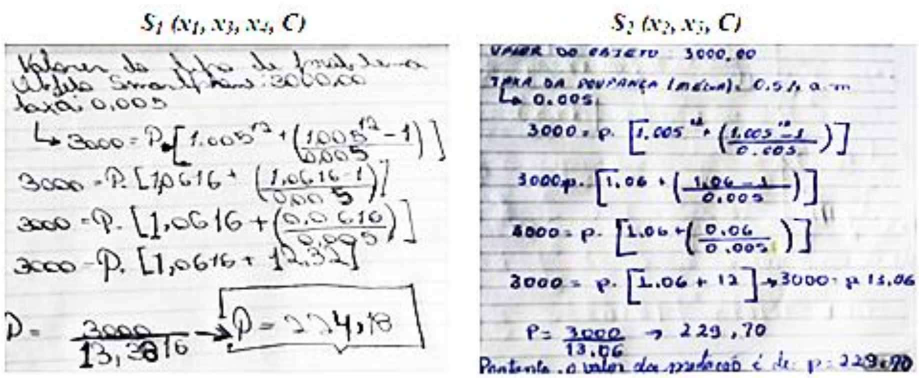
It is worth noting that this quality of relationship with knowing (Chevallard, 2005) a polynomial constructed or reconstructed by teachers allowed them to meet “a situation includes the ‘raison d'être’ or *rationality* that gives meaning to the mathematical activity carried out under institutional restrictions that provide and limit the application of the corresponding mathematical knowledge” [19] (Bosch, Chevallard, & Gascón, 2006, p. 3, emphasis added), in particular, giving meaning to and resignifying other objects of the basic school:

- ***S_1 (x_1, x_3, x_4, ₡)**:* This seems to have to do with geometric progressions, when it talks about the sum of the terms of a PG... in this case, x can fulfil the role of the ratio symbolised by the letter q as the progression rate, I found them similar .

In addition, the didactic system ***S_1 (x_1, x_3, x_4, ₡)*** forwarded a generalisation of the specific technique τ_3 described in Figure 4 by: $\left[\frac{x^{n}-1}{x-1} = x^{n-1} + x^{n-2} + \cdots + x + 1\right]$. However, the use of the scientific calculator showed a confrontation of practices between the didactic systems ***S_1 (x_1, x_3, x_4, ₡)*** and ***S_2 (x_2, x_5, ₡)*** in the face of the answers generated by the didactic systems in the use of the mathematical model, as shown in Figure 5.

Figure 5

Records of the auxiliary didactic systems. (Sodré, 2019)



[19] Text fragment: a situation includes the “raison d’être” or rationale that gives sense to the performed mathematical activity. And it also contains institutional restrictions that provide and limit the application of the corresponding mathematical knowledge.

Teacher x_1 of the auxiliary didactic system S_1 initially found as a numerical answer the value of the portion given by **p = 223.93** (Figure 4) considering all possible digits of the scientific calculator, while teacher x_5, of the same didactic system S_1, considered, in his defence, the use of four decimal places after the comma, finding the value of **p = 224.18,** as the answer to the value of the portion. Those answers found, under the indispensable conditioning of the scientific calculator, generated, not just "conflicts" between auxiliary didactic intra-system practices ***S₁*** S_1 $(x_1, x_3, x_4, ₵)$, but also confrontations with the answer produced by $S_2 (x_2, x_5, ₵)$, with a value of **p = 229.70**.

It is necessary to consider the teachers' highlight:

- $S_1 (x_1, x_3, x_4, ₵)$**:** I think this happened (I'm talking about the difference in the answers found)... because of the number of digits we used or was used in the calculator in relation to the answers of colleagues, which was not really unanimous.

In this sense and analogously to the situation $S_1 (x_1, x_3, x_4, ₵)$**,** the teacher raised the following question:

- $S_1 (x_1, x_3, x_4, ₵)$**:** So... why does the electricity supplier use more than two decimal places in the unit value charged by kWh in the composition of the electricity tariff?

This issue produced the following defences of didactic systems:

- $S_2 (x_2, x_5, ₵)$*:* If the electricity consumption of a residence is 150 kWh in the period of a commercial month with the unit price of kWh of R$ 0.663333, and not exactly R$ 0.66, the amount to be paid disregarding other charges of the final tariff is R$ 99.50 (in the case of the unit price of kWh R$ 0.663333) and not R$ 99.00 (if the unit price of the kWh is R$ 0.66). So, if the energy supplier uses rounding in the unit price of kWh, there is a reduction in the collection of this company, because the difference of R$ 0.50 may seem insignificant if you consider only one residence, but if you consider the universe of residences in the state of Pará, for example, the impact on the collection generates more profitability for this energy supplier; [...] There is a detail, that the state is benefited because, when it charges the ICMS, it also increases the collection;

- ***S_1 (x_1, x_3, x_4, ₡):*** Like gasoline also, there is numerical rounding in the composition of the value charged by the litre of fuel that has implications for greater revenue of these companies.

  (Dialogues of the didactic systems S_1 and S_2).

Those dialogues of the didactic systems seem to clearly reveal, besides the didactic moment – **MD_1** - due to the encounter with tasks of using the calculator, the influence or limitation of the scientific calculator machine to obtain the answers produced by the mathematical model to the situation. We must consider that the initial answer produced by the model is for the same and not for the situation in context. The answer constructed by the mathematical model, which is directly conditioned by the use of the scientific calculator, may not always prove useful to the situation and thus, clearly show the conditioning of the situation in context to the use of the calculator.

This pace of the didactic systems ***S_1 (x_1, x_3, x_4, ₡)*** and ***S_2 (x_2, x_5, ₡)*** revealed "disturbances" (Chevallard, 2005) caused by the use of the scientific calculator, because "the digital tool, however, can only be used after the mathematical terms have been translated into the computer language" [20] (Greefrath & Vorhölter, 2016, p. 22, our translation), i.e., according to the need that the teachers presented to construct qualities of relations (Chevallard, 2005) with the praxeologies of use of the scientific calculator.

Ultimately, our investigation seems to reveal, although partially, possibilities for forwarding answers to the questions raised by Niss, Blum, and Galbraith (2007) (apud Greefrath & Vorhölter, 2016), more precisely:

> How should digital tools be used to varying degrees for modelling processes? What is the effect of the digital tools on the spectrum of modelling problems to be worked on? How is culture teaching influenced by the existence of digital tools? When do digital tools improve or hinder learning opportunities in the modelling process?[21] (Greefrath & Vorhölter, 2016, p. 23).

---

[20] Fragments of the text: The digital tool, however, can only be used after the mathematical terms have been translated into the computer's language.

[21] Text fragments: How are digital tools supposed to be used in different grades to support modelling processes? What is the effect of digital tools on the spectrum of modelling problems to be worked on? How is teaching culture influenced by the

Although these questions need more empirical experimentation, it is necessary to highlight the indispensable role of the calculator or computer in the practice of MM; i.e., here we evidence a dependence on the so-called real problems of the use of those digital tools, which, despite being non-mathematical know-how, work conditioned by mathematical know-how, which distances the gaze of the MM as a strict practice of mathematical know-how, as can make us believe, for example, Niss, Blum, and Galbraith (2007) (apud Stillman, 2019).

RESEARCH ROUTES AND FUTURE PERSPECTIVES

This article, answers, albeit partially, to some questions pointed out by Niss, Blum, and Galbraith (2007) (apud Greefrath & Vorhölter, 2016) about the impacts of the use of digital tools in the study of MM problems and their role as an integral and indispensable part of the MM process by creating conditions and, thus, institutional constraints, revealed evidence of *didactic moments* (Chevallard, 1999), not all of them, surely, in the study of problems in concrete contexts, specifically, from the study of tasks of school financial mathematics.

The specific tasks of financial mathematics based on the empirical fragments forwarded by Sodré (2019) proved challenging to teachers and potentially rich for creating conditions in the sense of the ATD, which allowed the encounter with mathematical and non-mathematical know-how, and thus the encounter of those teachers with different didactical moments. In other words, the empirical results ratified Chevallard's (1999) statement about the didactic moments depending on the *creation of appropriate didactic situations*.

The research evidenced the encounter of teachers in training with different didactic moments, in particular, the **MD_1** moments in association with **MD_2**, exploration of tasks and techniques in the face of the new problems and, thus, the encounter with**MD_4** of the work of the technique for those issues, which referred them to the investigation of new techniques and, in a more inclusive manner, pointed out the moment of institutionalisation **MD_5** and the assessment – **MD_6** – before the proper use or not of the mathematical models in situation.

It should be noted that the work of the technique of the didactic moment, here symbolised by **MD_4**, was found at different stages of the study

existence of digital tools? When do digital tools enhance or hinder learning opportunities in the modelling process?

process, in particular, by the teachers' encounter with praxeologies that involved the use of "new" techniques in the face of the problem of calculating the fixed portion to be deposited in a savings account, in view of the moment when the work of the technique is one of the most *discredited moments*, according to Bosch, Chevallard, and Gascón (2001).

Moreover, changes in the quality of teachers' relations with the know-how were evident (Chevallard, 2005), in this case, the praxeologies related to the proper use of the scientific calculator, introduced in the study process as one of the conditions that made it possible to carry out problematic tasks hitherto faced by teachers.

It seemed valid to us that the study of problems in concrete contexts in MM does not depend only on mathematical know-how, because there is a clear dependence on other non-mathematical know-how, in our case, the use of scientific calculators that work with mathematics as a tool not only to verify results but as an integral and indispensable part of the MM process, which conditioned the answers the teachers produced before the situation in context.

Otherwise, besides the didactic moments verified in the lived experience of teacher education, the study of problems in a concrete context ratified elements of the MM process pointed out by Guerra and Silva (2009), taking into account the dialectic of the interdependence of the quadruple {*situação / modelo matemático / método / máquina*} with awareness of the indispensable role of the scientific calculator that produced different answers to the study community.

We should note that the answers the teachers found may be useful to the mathematical model, but they may not necessarily prove suitable before the situation in context, in addition, of course, to the dependence of choosing the mathematical method to face the mathematical model.

Ultimately, and in view of the results found here, we are motivated to conduct future research that includes problems that may be unveiled from the use of mathematical models in concrete contexts.

DATA AVAILABILITY STATEMENT

The data supporting this study and investigation will be made available by the corresponding author (GJMS), upon prior request.

REFERENCES

Barquero, B. & Jessen, B. E. (2020). Impact of theoretical perspectives on the design of mathematical modelling tasks. *Avances de Investigación em Educación Matemática, (17)*, 98–113.

Barquero, B. (2009). *Ecología de la modelización matemática en la enseñanza universitaria de las matemáticas*. [Trabajo de Tesis Doctoral, Universitat Autònoma de Barcelona].

Blum, W. & Borromeo Ferri, R. (2009). Mathematical modelling: can it be taught and learnt?. *Journal of Mathematical Modelling and Application, 1*(1), 45-58.

Blum, W. (2015). Quality teaching of mathematical modelling: what do we know, what can we do? In: Cho, S. J. (ed.). *The proceedings of the 12th International Congress on Mathematical Education,* 73-96.

Bolea, M. P. (2002). *El proceso de algebrización de organizaciones matemáticas escolares*. [Trabajo de Tesis Doctoral, Universidad de Zaragoza].

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *The International Journal on Mathematics Education, 38*(2), 86-95.

Bosch, M. & Gascón, J. (2010). Fundamentos antropológicos das organizações didáticas: das "oficinas de práticas matemáticas" às "rotas de estudo e pesquisa". In: Bronner, A. Larguier, M. Artaud, M. Bosch, M. Chevallard, Y. Cirade, G. & Ladage, C. (ed.) *Diffuser les mathématiques (et les autres savoirs) comme outils de connaissance et d'action*. (p. 49-85). IUFM de l'Académie de Montpellier.

Bosch, M. Chevallard, Y. & Gascón, J. (2006). Science or magic? the use of models and theories in didactics of mathematics. *Proceedings of the fourth congress of the european society for research in mathematics education*.

Bosch, M. García, F. J. Gascón, J. & Ruiz Higueras, L. (2006, agosto). La modelización matemática y el problema de la articulación de la matemática escolar: Una propuesta desde la teoría antropológica de lo didáctico. *Educación Matemática, 18*(2), 37-74.

Chevallard, Y. (1989). Le passage de l'arithmétique à l'algèbre dans l'enseignement des mathématiques au collège. *Petit X*, *19*(19), 43–72.

Chevallard, Y. (1999). L’analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches En Didactique Des Mathématiques,* 19(2), 221–266. https://revue-rdm.com/1999/l-analyse-des-pratiques/

Chevallard, Y. (2004). Vers une didactique de la codisciplinarité. Notes sur une nouvelle épistémologie scolaire. *Journées de didactique comparée.*

Chevallard, Y. (2005). *La transposición didáctica: del saber sabio al saber enseñado*. Aiquegrupo editor.

Chevallard, Y. (2019). On using the atd: some clarifications and comments. *Educ. Matem. Pesq*., *21*(4), 1-17.

Chevallard, Y. (2013). Éléments de didactique du développement durable – Leçon 1*: Enquête codisciplinaire & EDD*.

Chevallard, Y. (2009). La notion d´ingénierie didactique, un concept à refonder. Questionnement et élémentos de réponses à partir de la TAD. In: Margolinas, C. et al. (org.): En amont et en aval des ingénieries didactiques. *Recherches em Didactique des Mathématiques,* (1), 81-108.

Chevallard, Y. Bosch, M. & Gascón, J. (2001). *Estudar matemáticas: o elo perdido entre o ensino e aprendizagem*. Artmed.

Florensa, I. García, F. J. & Sala, G. (2020). Condiciones para la enseñanza de la modelización matemática: estudios de caso en distintos niveles educativos. *Avances de Investigación en Educación Matemática,* (17), 21–37.

Fonseca, C. Gascón, J. & Oliveira, C. (2014, noviembre). Desarrollo de un modelo epistemológico de referencia en torno a la modelización functional. *Revista Latino americana de Investigación en Matemática Educativa, 17*(3), 289-318.

García, F. J. Baquero, B. Florensa, I. & Bosch, M. (2019). Diseño de tareas en el marco de la teoría antropológica de lo didáctico. *Avances de Investigación en Educación Matemática,* (15), 75-94.

Garcia, F. J. Gascón, J. Higueras, L. & Bosch, M. (2006). Mathematical modelling as a tool for the connection of school mathematics. *ZDM Mathematics Education, 38*(3), 226-246.

Gascón, J. (1994). El papel de la resolución de problemas en la enseñanza de las matemáticas. *Educación Matemática, 6*(3), 37-51

Gascón, J. (2014). Los modelos epistemológicos de referencia como instrumentos de emancipación de la didáctica y la historia de las matemáticas. *Educación Matemática*. (Ed. Especial), 99–123.

Greefrath, G. & Vorhölter, K. (2016). *Teaching and learning mathematical modelling: approaches and developments from german speaking countries.* Springer.

Guerra, R. B. & Silva, F. H. S. (2009). Reflexões sobre modelagem matemática crítica e o fazer matemático da escola. *Perspectivas da Educação Matemática, 2*(3), 95-119.

Ladage, C. & Chevallard, Y. (2010). La pédagogie de l'enquête dans l'éducation au développement durable. In: *Actes du colloque Education au développement durable et à la biodiversité*.

Perrenet, J. & Zwaneveld, B. (2012). The many faces of the mathematical modeling cycle. *Journal of Mathematical Modelling and Application, 1*(6), 3-21.

Revuz, A. (1971). The position of geometry in mathematical education. *Educational Studies In Mathematics,* (4), 48-52.

Schukajlow, S. Kaiser, G. & Stillman, G. (2018). Empirical research on teaching and learning of mathematical modelling: a survey on the current state-of-the-art. *ZDM – Mathematics Education, 50*(1–2), 5-18.

Sodré, G. J. M. & Guerra, R. B. (2018). O ciclo investigativo de modelagem matemática. *Educ. Matem. Pesq*., *20*(3), 239-262.

Sodré, G. J. M. & Oliveira, M. L. S. (2021). O ciclo investigativo de modelagem matemática: uma transposição didática escolar. *Vidya, 41*(1), 35-57.

Sodré, G. J. M. (2019). *Modelagem matemática escolar: uma organização praxeológica complexa*. [Tese de Doutorado, Universidade Federal do Pará, Belém, PA, Brasil].

Stillman, G. A. (2019). State of the art on modelling in mathematics education-lines of inquiry. In. Stillman, G. A. & Brown, J. P. (eds.), Lines of Inquiry in Mathematical Modelling Research in Education.

Proceedings of the *13th International Congress on Mathematics Education*. (p. 1-20).

Vorhölter, K. Greefrath, G. Borromeo Ferri, R. Leiß, D. & Schukajlow, S. (2019). M*athematical modelling*. In: Jahnke, H. N. Hefendehl-Hebeker, L. (eds.). Traditions in German-Speaking Mathematics Education Research. *Proceedings of the 13th International Congress on Mathematics Education*. (p. 91-114).