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Learning opportunities experienced by mathematics teachers: unveiling actions and role of the teacher educator during a formative process[1]

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ABSTRACT

Context: One of the research challenges related at the professional learning of mathematics teachers is to understand the importance of the role and actions of the teacher educator during a formative process. **Objective:** This research seeks to understand what the role was and how the actions of the teacher educator took place, in a process of continuing education with teachers of basic education, about the teaching of patterns and regularities, with a view to providing learning opportunities to the teachers involved. **Design:** It is a qualitative-interpretative study aligned to an intervention research. **Settings and Participants:** We developed the research in a formative process involving 33 mathematics teachers and future teachers, and 3 teacher educators. **Data collection:** We use data from audio and video recordings of planning and enacting of the formative process, and protocols from teachers' tasks and planning from teacher educators. **Results:** We found that teacher educators, since planning the formative process, sought to provide opportunities for participating teachers professional learning, since he/she structured the process through professional learning tasks, using videos of mathematics lessons to highlight the classroom practice, as well as encourage discussions among teachers about patterns and regularities in and for the teaching of algebra. **Conclusions:** We identified that teacher educators played a mediating role in the orchestration of the discussions and an articulator between

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mathematics and didactics, thus providing opportunities for professional development and the learning of teaching algebra.

Keywords: Teacher educator; Teacher professional learning; Teaching of Algebra; Mathematics teacher education; Professional Learning Task.

INTRODUCTION

Research has highlighted the need to investigate the continuing education of teachers who teach mathematics, in particular, with a focus on the teacher educator [2] (Fiorentini et al., 2016) and on the constitution and development of the teacher's professional learning (Webster-Wright, 2009). Studies point out that it is essential to consider the teacher's practice as a significant element of the formative processes (Ball & Cohen, 1999; Lampert, 2010; Smith, 2001) and also emphasize the importance of taking into account that, if we aim to organize formative programs that help teachers in their professional learning, it is essential to compose new investigations on the Role and Actions of the Teacher Educator in this context (Ribeiro & Ponte, 2019).

Directing the discussions to researches that deal with teacher educators, both in the context of initial and continuing education, it is necessary to consider that this is an area that is still little explored in Brazil (Coura & Passos, 2017; Fiorentini, 2018). Coura and Passos (2017) highlight the lack of specific training for educators to exercise their role, who have the same set of professional knowledge as the teacher for teaching in basic education. In this way, teacher educators acquire the particularities of their profession only in practice, when they are already educating teachers.

Given this fact, the important role of teacher professional development for teacher educators is reinforced (Passos et al., 2016; Superfine & Li, 2014), as it is in the elaboration and implementation of formative processes that the teacher educator has his/her own knowledge for the exercise of their profession. Superfine and Li (2014) state that, for the teacher's professional development of teachers, they must work with mathematical content and concepts, propose activities that encourage the establishment of a Community of Practice (Lave & Wenger, 1991), through cooperative work, and seek activities situated in real teaching practice.

[2] In our paper, we are using the term "teacher educator" to both those who educate prospective teachers and those who educate practicing teachers, that is, those who initiate, guide, and support teacher learning across the lifespan.

The work of teacher educators is important, as they inspire teaching, they are the ones that future teachers look up to when they enter the classroom (Romanowski et al., 2017). In addition, teacher educators play a central role when thinking about the Professional Learning Opportunities for Teachers (PLOT) model, as proposed by Ribeiro and Ponte (2020). This model considers the intersection of three domains that, according to Ribeiro and Ponte (2020), compose it. They are the Professional Teachers Learning Tasks (PTLT), the Discursive Interactions Among Participants (DIAP) and the Role and Actions of the Teacher Educator (RATE). This last domain focuses specifically on the teacher educator, which is the central theme of this article.

This model, in addition to guiding the preparation and development of formative processes, can be used as an organizational tool for analyzing the results of the educative process, during or at the end of its implementation. In this way, it is possible to identify and assess whether, and how, a formative process that includes the three domains of the model provides opportunities for teachers' professional learning (Ribeiro & Ponte, 2020).

In view of this and given the importance of the teacher educator in formative processes, whether in initial or continuing education, we aim in this paper *to understand the role and how the teacher educator's actions took place in a process of continuing education with teachers from basic education about teaching patterns and regularities, with a view to providing learning opportunities for the teachers involved.* To operationalize this objective, we seek to answer the following questions: (i) How did the teacher educator favor the articulation of the mathematical and didactic dimensions of the teacher's professional knowledge? (ii) How did the teacher educator, during a formative process, orchestrate the mathematical and didactic discussions among the participants?

THEORETICAL FRAMEWORK

For the organization of the theoretical framework, we discuss the RATE presenting the particularities of his/her work (Jaworski & Huang, 2014), we raise the strategies and five practices to orchestrate discussions in classrooms presented by Stein et al. (2008) at the level of the educators' work (Prediger et al., 2019) and we explore the use of video as a facilitator in the teacher educators' work to manage collective discussions (Borko et al., 2014) in order to provide opportunities for teacher learning. We also seek to focus on the construction of substantive links between academic and school mathematics in

education for school practice discussed during a course (Moreira & David, 2008). Such elements are essential for understanding the RATE domain in the *PLOT* model, which we chose to address at the end of this section, in order to interconnect it with the references discussed here.

About the teacher educator, Prediger et al. (2019) point out that there is a relationship between the work of the teacher educator and the teacher and, therefore, there is also a consensus between the need for a body of knowledge necessary for the work of teacher educators, as well as teachers. Complementarily, the authors indicate that it is necessary to be aware of the particularities that are present in the teacher educator's work, such as the need for knowledge about relevant theories about the professional learning of teachers and the interaction between research results and the practice that supports teaching and learning (Jaworski & Huang, 2014; Prediger et al., 2019). Jaworski and Huang (2014) highlight that teacher educators work to enable the transposition of theoretical ideas present in research results into ways to teach in basic education and, for this, they lack the specific knowledge necessary for teaching. In summary, teacher educators' knowledge can therefore be seen as superimposed on mathematics teachers' knowledge but does not contain it entirely (Beswick & Chapman, 2012 cited by Jaworski & Huang, 2014, p. 176).

Among other aspects considered essential to teacher educators, Jaworski and Huang (2014) direct their work towards reflection. They present a list of six competences, elaborated by Smith (2005 cited by Jaworski & Huang, 2014), with aspects necessary for the reflective teacher educator's work. It highlights self-awareness, reflection on action to apply the implicit knowledge of teaching, the scope of theories and their tests in practice, active participation in curriculum (re)formulation in your country, the potential capacity to teach all the age groups of basic education students, the comprehensive knowledge of the educational system and the achievement of a high level of experience and professional maturity.

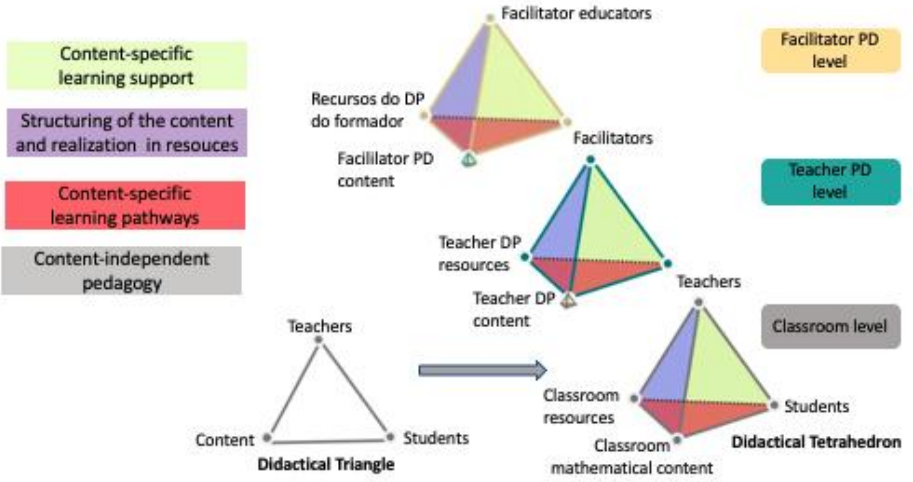
The concern with the teacher educator's work has been recurrent in research on teacher professional development, especially at the international level (Prediger et al., 2019), a fact arising from its importance in teacher education. An example of this is the study by Prediger et al. (2019), in which the authors systematize existing research strategies that take into account the multifaceted structure of investigations into professional development, presenting a model to systematize and explain the approaches that have been used, in addition to listing new research that is needed. The Three Tetrahedrons model (Figure 1), presented by Prediger et al. (2019), captures the complexity

of learning in the classroom and the work of teachers and teacher educators, seeking to establish necessary questions to be investigated on the subject.

From the Didactic Triangle (Jaworski, 2012), Prediger et al. (2019) propose to consider the importance of resources for the teaching and learning process through the Didactic Tetrahedron (Figure 1) and, with this, extend the model to other levels of professional development. We highlight the Central Tetrahedron (Figure 1), the one that most interests our study, in which Prediger et al. (2019) present the Professional Development of the Teacher. In this tetrahedron, we are interested in directing our gaze to the Professional Learning Opportunities (Ribeiro & Ponte, 2020) that teacher educators can provide teachers through their practice.

Figure 1

Sketch of the Model of the Three Tetrahedrons. (Adapted from Prediger et al., 2019)



In the same sense, Borko et al. (2014) make an analogy with the classroom and present some practices to be developed by teacher educators in order to promote high-quality discussions among teachers. The purpose of the discussions is to deeply explore math concepts, students' reasoning, and teachers' behavior when teaching. For the authors, managing a rich discussion

in a teacher education space is linked to the planning and orchestration managed by the teacher educator. Teacher educators need to give teachers a voice, recognize the ideas and proposals that arise, and incorporate them into subsequent questions to promote high-quality conversations that explore mathematical concepts, students' mathematical reasoning, and teaching behavior (Borko et al., 2014).

Therefore, good planning must be accompanied by the determination of clear objectives, the definition of valuable resources, such as the clippings of videos from classes for teacher education, as well as the elaboration of questions that guide the discussion (Borko et al., 2014). In the orchestration of the discussions, the teacher educator must awaken the teachers' thinking to the mathematical task at stake, highlight their speeches and propositions and help the group of teachers in establishing the relationships of their ideas with mathematical and pedagogical ideas (Borko et al., 2014).

Regarding teachers' mathematical knowledge, Moreira and David (2008) point to the possibility of integration between academic mathematical knowledge and knowledge associated with school teaching practices, an integration that does not always happen naturally by the teacher. For Moreira and David, such integration consists in showing teachers how the concepts of academic mathematics contain particularities of the concepts of school mathematics, even though this is not normally presented in a formal or more evident way by the teacher educators. With this, a harmonious relationship between such knowledge is not established; thus, the need for new studies that help to clarify better what this integration would be is demonstrated.

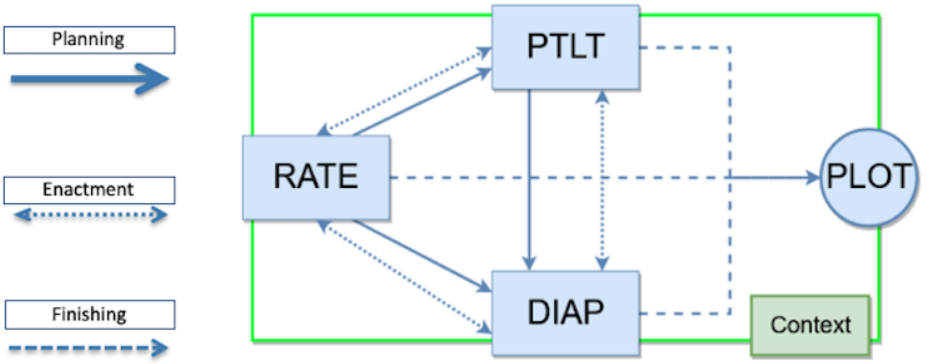
Finally, as previously presented, the PLOT model (Ribeiro & Ponte, 2020), through its different domains - PTLT, DIAP and RATE, the focus of our article -, seeks to support the organization and implementation of formative processes with aimed at providing learning opportunities for the participants. In addition to the three domains, the model consists of three operationalization phases, a fact that allows the articulation between such domains in a single system. The *planning* phase is the moment in which the teacher educator prepares the educative process; *enactment*, in turn, begins when the PTLTs and DIAPs, elaborated in the previous phase, are put into practice, and trigger the formative process; finally, the *finishing* occurs when, from the integration of the three domains, learning opportunities for participating teachers are realized (Figure 2).

Considering the focus of our study, we now go on to explore the RATE domain, which is presented in the *PLOT* model as a trigger for the promotion

of professional learning for teachers in a training process. This domain has as components, in its conceptual dimension, the articulation between mathematical and didactic knowledge and the approximation between academic and school mathematics. On the other hand, there is the management of an exploratory teaching and learning environment and the orchestration of didactic and mathematical discussions, components that form the operational dimension of this domain.

Figure 2

Model of Professional Learning Opportunities for Teachers (PLOT). (Adapted from Ribeiro & Ponte, 2020)



The four components of the *RATE* domain have great relevance for its understanding, as can be confirmed by the works discussed above, since they consider the importance of the role of the teacher educator from the planning stage (Prediger et al., 2019; Jaworski & Huang, 2014) even the orchestration of discussions (Borko et al., 2014) during a formative process, as well as taking into account the teacher educator's actions and the way in which these should be based on knowledge about the professional development of teachers (Prediger et al., 2019). All of this occurs with the purpose of promoting articulations and approximations between academic and school mathematics (Moreira & David, 2008), and favoring teachers' professional learning opportunities (Ribeiro & Ponte, 2020).

STUDY CONTEXT

We propose to analyze part of a formative process entitled "Patterns and Regularities in School Mathematics", carried out during 2018 at a Brazilian public university. The formative process lasted 60 hours, was developed by 3 teacher educators, 2 of them authors of this article, and had 33 participants, 7 teachers in initial education and the other teachers already graduated (21 of them with experience in public and private schools). The formative process was carried out using 5 professional learning tasks, the first 2 for surveys of prior knowledge (Aguiar et al., 2019) and the last 3 to involve participating teachers in a cycle of planning, development, and reflection of mathematics classes, henceforth PDR Cycle (Ribeiro et al., 2020; Trevisan et al., 2020).

After the phase of surveying prior knowledge, with the completion of the first two PTLTs, and a phase consisting of educational workshops, the PDR Cycle began. For this, the teachers were separated, taking as a criterion their professional experience in basic education, which generated six small groups of five or six members. Within these groups, the members worked collectively to plan lessons that would later be developed in different classes: two groups for the sixth and seventh grades, two for the eighth and ninth grades of Middle School and two groups for High Scholl.

During the third PLTT, the one focused on planning within the PDR Cycle, each of the six groups prepared a lesson plan for a group of students, according to what had been agreed, whose theme would be the use of patterns and regularities in the construction of algebraic thinking. At the end of the third PTLT, during a plenary presentation and discussion of the elaborated plans, the participating teachers chose, in a negotiated manner, a lesson plan for each of the groups indicated above. This choice would be preceded by the development of the lesson plan by a member of the group in his/her class of students, with the observation of two teacher educators and some members of the group of teachers who prepared the plan.

Once the three lesson plans were chosen, the development phase of the lesson began, subsidized by what we call the fourth PTLT, that of development within the PDR Cycle. The classes took place in public schools of basic education, in which the three chosen teachers taught regularly. To enable the subsequent monitoring of the rest of the group of teachers in formative process, a video recording of the class was made, audios of the work of students in small groups were collected, as well as the protocols produced by them during the class. The teacher educators, back at the university, to prepare the fifth PTLT, that of reflection within the PDR Cycle, made a careful choice of records and

small excerpts from the classes recorded on video, as well as prepared scripts to support the analysis of these classes, which would occur during the teacher education process, in the reflection phase of the PDR Cycle.

The fifth PTLT was composed of three parts, each one based on a class developed during the fourth one. Thus, in each meeting, a part of the PTLT was held (sixth and seventh grades, eighth and ninth grades, and high school), and in these meetings, first, the teachers received the PTLT and discussed them in small groups, in possession of a notebook with the video episodes selected by the teacher educators. After the resolution of PTLT in the small groups, plenary sessions were held led by the teacher educators. This approach was inspired by the three-phase class model, better known as exploratory teaching (Ponte, 2005). For this article, we selected the second part of PTLT related to the Polka Dot Sequence task (Figure 3).

Figure 3

Math assignment for the 9th grade. (Research data, 2018)

Observe the sequence of figures:

Figure 1 Figure 2 Figure 3 Figure 4 Figure n

a) Describe the regularity you observed in this sequence of figures. How else can you represent regularity?

b) How many balls should figure 5 have? Assemble the sequence with caps.

c) How many balls should figure 120 have?

d) Write an algebraic expression that represents the terms of this sequence.

e) Is it feasible to form figure 120 with bottle caps? Explain.

Thus, to compose the data corpus that we analyze in this article, we will bring the plenary referring to the lesson plan that contains the Polka Dot Sequence task (Figure 3), applied in a ninth-grade class of Elementary School,

the plenary is conducted by the teacher educator Ribeiro. Next, we will explain how the data collection was carried out.

METHODS AND PROCEDURES FOR DATA COLLECTION AND ANALYSIS

Developed from an interpretive perspective (Scheiner, 2019; Creswell, 2014), our study had its data collected through audio recordings during the planning of the formative process, with the participation of the three teacher educators. There is also data from the plenary held during the fifth PTLT of the formative process, collected through audio and video recordings. It is noteworthy that, during the plenary, the three teacher educators were present, but it was teacher educator Ribeiro who assumed the central role in the orchestration of the discussions.

With the material in hand, which had a video recording of the plenary and the audios of the planning moments of this plenary, we carried out the complete transcription of this information. Our intention was to focus our attention on the content related to the RATE domain when managing Professional Learning Opportunities for Teachers (Ribeiro & Ponte, 2019, 2020).

After selecting the plenary of the fifth PTLT as the object of analysis, due to the richness of details in the interactions that took place between the teacher educator and the participating teachers at that time, the authors of this article performed, independently, a floating reading of the transcripts and watched the video of the plenary, to take notes on their perceptions about this material. Returning to the group, they discussed the individual notes and identified possible relationships, in addition to selecting the three excerpts in which the Teacher Educator's Role and Actions in the formative process are analyzed. Back to the individual work, new annotations of the selected excerpts were made, which were important for the writing and refinement during the analysis carried out in group, later. The choice of two forms of work (individual and collective) during the first part of the analysis allowed us to capture details at different moments of observation of the material, as the individual clinical look can unveil what the collective has not seen and, thus, add to study.

Without losing sight of the procedures adopted for the analysis of the plenary video, we also used the audio of the plenary planning by the teacher educators. Listening individually, we chose some excerpts in which there

seemed to be direct relations with the excerpts selected from the plenary for further discussion in the collective. At a certain point in the analysis, we decided that, for the rest of the work, the collective gaze would be of better benefit, due to the multiple paths that individuality could take us. Thus, we started to carry out the rest of the analysis together so that the work would not lose the quality of the collectivity. In this last phase, in addition to the joint analysis, we carried out individual readings of the texts that were produced to find possible gaps and, thus, bring the demands to be discussed to the group work. Finally, after several comings and goings, when we believe that we have exhausted the possibilities for analyzing the chosen excerpts, we carry out the final discussions, presented in this text.

DATA ANALYSIS

For the analyses, we selected three excerpts from the fifth PTLT plenary, related to the ninth-grade class, in which there are interactions between the teacher educator Ribeiro and the participating teachers. We used three excerpts from the transcripts of the dialogues that took place in the plenary; they highlight the role and actions of the teacher educator to promote learning opportunities for teachers.

The first excerpt, "*Orchestrating a Discussion*", presents the teacher educator's actions to encourage discussions among teachers about the generalizations described by students in the protocols of groups 9A and 9D. In the second excerpt, "*Connecting ideas*", we highlight the participants realizing that the teacher educator's actions, throughout the formative process, referred to exploratory teaching, similar to what was encouraged by teachers in carrying out the classes they planned. In the third excerpt, "*Consolidation of ideas*", the teacher educator discusses the importance of planning to develop a class based on exploratory teaching and on relevant points, such as: choosing a good mathematical task, promoting collective discussion, and managing the class (Bridge, 2005). The choice of the third excerpt reinforces the importance of the role and actions of the teacher educator in the previous excerpts and highlights the discussion of educators in the planning of the plenary.

Regarding the analyzes related to the planning of the plenary, we sometimes resorted to audio to understand how the objectives set by the teacher educators were carried out at the time of the plenary. In this way, planning serves us as a reinforcement to consolidate evidence about the role and actions of the educator.

Orchestrating a Discussion

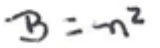
During the plenary session, the teacher educator uses questions from the fifth PTLT to engage teachers in a collective discussion about what was observed during the work in small groups, as well as seeking to generate new reflections that could be made at that time. The first excerpt analyzed begins when the teacher educator calls the teachers' attention to a PTLT issue. As a strategy, the teacher educator compared the resolution of two groups of students, the first group, 9 A (Figure 4) presents the following answer:

Figure 4

Protocol of students from group 9A. (Research data, 2018)

We highlight the group 9A protocol when answering the question:

Write an algebraic expression that represents the terms of this sequence.



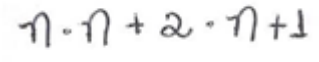
Differently, the 9 D group (Figure 5) presents the resolution of the mathematical task through the following answer:

Figure 5

Protocol of students from group 9D. (Research data, 2018)

We highlight the 9D group protocol when answering the question:

Write an algebraic expression that represents the terms of this sequence.



To trigger such reflection in small groups, PTLT presented, with student protocols, a request for teachers to relate the discussions and solutions presented by groups 9A and 9D in light of the following questions:

(i) *What comparison do you make between the discussions and solutions presented by Group 9A and Group 9D?*

(ii) *Is there a difference in the way of thinking about the mathematical task in the discussions and solutions of Group 9A and Group 9D?*

(iii) *Were the answers found the same? Comment.*

During the plenary, Ribeiro pointed to the resolutions of the student groups (Figures 4 and 5) presented on the blackboard, seeking to take the discussions that the teachers themselves had carried out in small groups to all the participants. At that moment, we noticed the teacher educator's intention to invite the teachers to participate in the plenary, in order to mobilize the mathematical knowledge that had been raised in the small groups, so that the teachers could share it with the others.

With that, we noticed that the teacher educator encouraged the teachers to explain the emerging ideas in small groups regarding the students' mathematical resolutions. Thus, he promoted reflections on the different algebraic expressions presented by the students regarding the generalization of the pattern of the dotted sequences (Figures 4 and 5).

Teacher Joana declared her reflection on this moment in the class, pointing out the students' difficulty in the resolutions:

> *Joana*: (. . .) *I think that what "clicks", for the student, is understanding who "n" is. What is "N"? Is it the figure? "N" is the position, so, this is what we* [teachers] *must put into their heads (. . .). I think that's what we needed to ask them: "this 'n' you're talking about, what is the 'n'? Is the figure, the position? And they have an idea of what a position is, because, even in*

> *the ninth grade, there are students who don't understand what a position is.* (Plenary, 2018)

The teacher pointed to possible questions that could have been made to help students in their reasoning about the generalizations presented. However, the teacher's speech was driven by teacher educator Ribeiro, who had the purpose of "provoking" this reflection in teachers about the generalization made by students. The teacher educator's intention is evident during the planning moment, when the teacher educators discussed and explained the following ideas, reflected in Ribeiro's speech:

> *Teacher Educator Ribeiro: I wanted them to observe what Felipe* [the teacher who taught the class] *is doing, or actually not doing, so that we can enrich with what could have been done.* (Planning of the fifth PTLT, 2018)

The teacher educators, in the planning, discussed what they wanted to highlight during the plenary, in order to show how Professor Felipe's actions had repercussions in the classroom. With this in mind, at the time of the plenary, the teacher educator took advantage of Joana's speech when she said: *"because, even in the ninth grade, there are students who don't understand what position is"*. Thus, he pointed out that the situation involves a recurring doubt among students; therefore, it must be provided for in the teacher's planning and dealt with at some point in the class:

> Ribeiro: *So, if the teacher knows that there is a recurrent difficulty when interpreting the problem, perhaps the teacher, in his actions, is at the moment of presenting the task, because then he could already work with the whole group, or (. . .) at the moment when I go through the groups [separately], because then, when I'm going through the groups, that group that has already realized who the "n" is, I won't intervene (. . .).* (Plenary, 2018)

At that moment, still based on Joana's report, the teacher educator asked two more questions that could be used by the teacher during the class. These questions were intended to direct the teachers' discussion towards what actions Professor Felipe could have taken in his class to help students with mathematical thinking:

> Ribeiro: (. . .) *but, if I already know that this is a recurrent difficulty, when I go through the groups I can observe; so, in the case of 9A, I will ask: "But who are you calling 'n'? Why*

> *are you calling it 'n'?". So, asking some questions that don't give the answer, but make them think, think about it.* (Plenary, 2018)

The teacher educator tried to draw attention to the teacher's action in asking students questions that would help them in their reasoning during the monitoring phase (Stein et al., 2008) of a class that intended to promote collective discussions. This warning was soon noticed by one of the teachers, who continued to exemplify how the "five practices" proposed by Stein et al. (2008) for the case studied:

> Hélia: (. . .) *for example,* [the teacher] *saw the two responses there in the monitoring; then* [if they] *selected the two answers to make the plenary and asking this question "who is 'n'?", they* [students] *would think before the plenary, to answer: "Well, the 'n' is 2 and up".* [Group 9A considers n to be the number of marbles on the side of the figure - see Figure 4]. The *other group* [states:] "The *'n' is 1 and forward*" [the 9D considers the *n*, the position of the figure, see Figure 5]. *Then* [they might] *think why, then, are the expressions different* (. . .). *But then you could do the fifth step, which is connecting* (. . .). (Plenary, 2018)

The teacher educator took advantage of Hélia's speech to emphasize how anticipation and monitoring (Stein et al., 2008) could be applied in this class. When he used the speeches of teachers Hélia and Joana about the mathematical task (Figure 3) to exemplify how the class could have been carried out, an articulation was evidenced (Ribeiro & Ponte, 2020) between the mathematical dimension and the didactic dimension of professional knowledge of the teacher:

> Ribeiro: (. .*) when I* [the educator speaks as if it were Professor Felipe acting during the class] *I decided to go through the groups, remember that there is a phase called* anticipation*, one of the five practices, in anticipation, the knowledge that Joana put here is present there — look, they have a hard time recognizing the 'n', so I'll be on my toes. In* monitoring*, I started to notice "Look, group 9A is interpreting the 'n' as the side, for this to be true, the set from which I will take the value of 'n' has to be different from another interpretation, which is when I think of the 'n' as the figure's position."* (Plenary, 2018)

Then, the teacher educator directed the discussion to the role of the teacher and to the mathematical and didactic knowledge that were mobilized and expected of him, in order to establish again the relationships with the five practices of Stein et al. (2008), which had been studied at the beginning of the training process:

> Ribeiro: (. . .) *in this practice called monitoring, which is to go through the groups and understand what is happening, I'm going to make some notes so that, later, in the next practice, which she* [Hélia] *already said is the practice of* selecting*, I'll think : "I'm going to select these two, and I'm going to sequence them in a way that later, in the last practice, I can make the connections between what one did and what the other did." OK?* (Plenary, 2018)

One of the teacher educator's intentions was to raise issues identified in Professor Felipe's class, to establish relationships with two of the five practices proposed by Stein et al. (2008), which, in this case, were equivalent to anticipation and monitoring. Therefore, we can conclude that the teacher educator himself used the *orchestration* component (Ribeiro & Ponte, 2020) to provide opportunities for reflections with teachers about the importance of anticipating student responses, during the planning of a class, in order to structure the next steps.

The teacher educators' intention to discuss the mathematical and didactical aspects of the task applied to students can be evidenced in the planning of the fomative process, when they discussed how the anticipation and monitoring took place during the class:

> Ribeiro: *You said he [Felipe] didn't ask big questions in small groups.*
>
> Teacher Educator Marcia [who observed Felipe's class]: *No, he wanted everyone to come to* n^2*, for him, he was already happy with* n^2.
>
> Ribeiro: *But it wasn't the* n^2*!*
>
> Marcia: *And it wasn't the* n^2*!*
>
> Ribeiro: *Had they [*the teachers in the group who planned the class] *not done it* [the math task solving]*? In preparation, they didn't make it to the ...*

> Marcia: *That's the question, I think they arrived at* $(n+1)^2$*, but I don't know why he [Felipe] thought it wasn't...* (Planning from the fifth PTLT, 2018)

In this first excerpt, "Orchestrating a discussion", we observe that the teacher educator prioritized discussions about the teacher's work, turning to pedagogical issues, and did not deepen the mathematical discussion that explained the difference between the students' answers, as foreseen in the planning of the educators for the fifth PTLT. However, when observing the protocols produced by the teachers during the fifth PTLT, we found that the discussion about the "possible values of n" had permeated all groups, but none of them presented a direct relationship on how the results of the responses of the students could have helped Professor Felipe during the plenary session with his students.

In this circumstance, the teacher educator could also have taken the opportunity to highlight the *approach* (Ribeiro & Ponte, 2020) between school mathematics content and academic mathematics. Although the mathematical task (Figure 3) deals with school mathematics contents, the mistakes presented by the students could lead to a deeper discussion, from a mathematical point of view, with the teachers. Such an opportunity would have the potential to provide them with better connections between student responses and help them establish a useful relationship for the practice of sequencing student responses (Stein et al., 2008) during the classroom plenary.

We observed that the teacher educator acted intentionally and in accordance with the planning of the fifth PTLT. We also noticed that the strategies provided by the teacher educators in the planning in relation to the teacher's difficulties during the class development were used and necessary for the discussion in the fifth PTLT.

Conneecting ideas

After the participants had watched episode 6, both in small groups and at the time of the plenary, the teacher educator presented one of the questions of the fifth PTLT (Figure 6) which includes the completion of the mathematical task (Figure 3) by Professor Felipe:

Figure 6

One of the questions of the fifth PLT. (Research data, 2018)

> **Episode 6:** Completion of the Math Task by the Teacher
>
> Now answer the questions:
>
> 1) Did the teacher make it possible for students to present different ways of performing the task (including possible incorrect strategies)? Comment.

After the teacher educator encouraged teachers with this question (Figure 6), Julia related the development of the class in the elementary school to the training model they were experiencing. She compared the moment of discussion in small groups and the subsequent discussion in plenary and characterized the exploratory teaching present both in the proposed class that was being analyzed and in the orchestration model that had been used in the training:

> Júlia: (. . .) *because he* [Felipe] *did the same thing you* [educators] *do here with us, he put them* [students] *to discuss in a group, then he asks a representative, or the whole group, to go ahead (*. . .).
>
> Ribeiro: *So, does Julia say it is the same or is it similar to what we do?*
>
> Júlia: *Similar...*
>
> Educator Ribeiro: *Keep this well, similar to what we do.* (Plenary, 2018)

Following this dialogue with teacher Júlia, the teacher educator gave voice to two other teachers, who pointed to time management to connect the ideas presented by the students, as well as the need to resume what was done by the students to forward them for the purpose of the job.

> Maria: ... *I understand that, in the matter of time, we get desperate, but we didn't connect the ideas, I think. Everyone gave a presentation of what they thought* [one student from each group went to the board to explain how their group found

> the generalization]. *And I think what ended up missing in the end was* [the teacher] *connecting these ideas of each one* [student] *to be able to understand what they had done.*
>
> Ribeiro: *So, wait... You see, Maria is saying that a suggestion that would have been important to have been made would be that, during the presentations of the different strategies, there was a moment to connect these strategies between one group and another. Did anyone make any other suggestions, or make any other comments other than those already mentioned?*
>
> Lucas: *Look, I didn't comment on paper, but, in my strategy, at the end of the last presentation, I would, in this case, make a demonstration for each type of reasoning there, where the error was, and what would be the final path that the group should have taken to achieve the goal.* (Plenary, 2018)

After listening to them, the teacher educator connected what was said and continued talking about the need to carry out a moment of systematization and formalization of the ideas presented by the students. The teacher educator also reinforced what was pointed out in Júlia's previous speeches, revealing some similarities between the training design and the class that was planned and applied by Professor Felipe:

> Ribeiro: *Okay, so what Lucas said is important* (. . .) *how can I, in a final moment, in a moment of systematization, take the different strategies and work both with the correct and with the incorrect strategies, so that the correct strategies are formalized, and the incorrect ones are reconstructed, so that the students are able to remedy the difficulties they had. But, still, about the way the professor conducted this plenary, we have already seen that he gave the floor, everyone presented it.* Julia said that the way he did it is similar to the way we have been doing our meetings (. . .). (Plenary, 2018)

We observed that the teacher educator made several directions about the orchestration of a class to the participating teachers to draw attention to the teacher's performance in view of what was planned, based on the five practices of Stein et al. (2008), and what was perceived by them, but was not applied properly during the development of the class:

> Ribeiro: (. . .) Maria said that she missed him making a connection between the different strategies, *but I think maybe*

> *we could think of something else that could have been done there, so that this connection became more necessary than simply doing it for to do. What did you guys observe if you hear a sequencing in the way the strategies were presented, or if the sequencing was something... that was thought by the teacher, you could see if there was 'look, first I'll call this one, then that one, after that" or if they were kind of presenting who's coming now, who's coming now?* (Plenary, 2018)

With these guidelines, the teacher educator allowed participants to reflect on the action of Professor Felipe and proposed other actions during the class based on exploratory teaching.

Consolidation of ideas

The last part of our analysis contemplates the end of the plenary, in which the teachers and the teacher educator talked about the difficulties encountered in carrying out exploratory teaching.

> Júlia: ... *Maybe that's* [about Professor Felipe never having done a plenary in class], *that's not why he didn't want to do it, even if he had anticipated it, as he's never been through this, there are things that will come up later, later that you presented the class, you thought: "Wow, I could have done that and I didn't, so next time I'll do it."So, it's a habit, so if we get used to* anticipating, monitoring, selecting, sequencing, and connecting*, then there will be a time when you do it so calmly that you don't even notice; so, I think you have to practice a little...* (Plenary, 2018)

Júlia's speech shows her understanding of how a class takes place in the exploratory teaching approach, as well as demonstrates her understanding of the difficulties experienced by Professor Felipe, since she had also been one of the teachers who taught a planned class in the same training process (Aguiar et al., 2021). Knowing this, we consider this statement as support and encouragement to Professor Felipe, while she points out the difficulties encountered when trying to incorporate exploratory teaching into her classes.

Taking advantage of this moment of reflection and discussions among teachers about the difficulty related to time management, teacher educator Ribeiro sought to systematize some principles that should be adopted by

teachers in the use of exploratory teaching combined with the use of the five practices:

> Ribeiro: *So, this is a care I have to take, a balance I have to seek, but I have to be careful, because the discussions in small groups can be interesting, sometimes, in one or another group and in the plenary time and time for me to socialize with the whole group. So, it is important for me to have collective discussions in small groups, but it is important to have collective discussions in the large group, because these five practices, for example, are only possible when I think about doing the work until the plenary, because otherwise the that I will be able to do, anticipating and monitoring, at the most, that in monitoring I may have to stay where the discussions are and I can make my interventions, okay, but I can't do the selecting, I can't do the sequencing and not connecting it, it's not because they created it, "I'll do it because it's the recipe", no, it's because the debate between them, the confrontation of ideas is the time to produce their mathematical knowledge...* (Plenary, 2018)

The teacher educator, when describing the importance of time management, reinforced the relevance of moments of discussion among students for the construction of mathematical knowledge. With that, he highlighted the necessary elements for exploratory teaching: classroom management and interaction among students (Ponte, 2005).

DISCUSSION OF RESULTS

In this section, we discuss the results considering the theoretical framework presented above. We therefore seek to answer our research questions.

In the first excerpt, "*Orchestrating a discussion*", we identify the intentions of teacher educators in planning the fifth PTLT by using student protocols to select the different strategies for their resolution (Borko et al., 2014), to highlight the action teacher in small group discussion during class. The teacher educators wanted to present different answers to the teachers, one of which was not in accordance with the pattern of the sequence, thus requiring a mediation by the teacher so that the students could better reflect on that generalization. Based on this idea, during the plenary session in the formative

process, the role of the teacher educator was fundamental in orchestrating (Ribeiro & Ponte, 2020) a discussion that would lead the participating teachers to realize the attitude of Professor Felipe and, on the other hand, to think about other mediation proposals regarding the generalization of the *Polka Dot Sequence* pattern (Ribeiro & Ponte, 2019; Borko et al., 2014).

Thus, we note that the teacher educator provided a mathematical discussion about how to interpret the generalization of the sequence of balls present in the mathematical task and, at the same time, provided an opportunity for a mathematical discussion articulated with possible actions of the teacher to orchestrate discussions (Stein et al., 2008). So, we note that the teacher educator seeks to integrate the mathematical concept at play with the questions of teaching Mathematics, demonstrating their understanding of teaching and how to guide teachers in training based on research results (Jaworski & Huang, 2014).

Regarding the second excerpt, "*Connecting ideas*", the teacher educators intended to contemplate episode 6 of the class regarding the completion of the mathematical task performed by the teacher, in order to discuss with the participating teachers, the way in which Professor Felipe systematized his class. At the time of the formative process, we noticed that the teachers reflected on the way in which the systematization was not carried out and the possibilities of teacher Felipe's action so that he could carry out this action. On the other hand, the teachers identified that the development of the class, which sought to effect the moments of exploratory teaching (Ponte, 2005), was similar to the way in which the training process had been developed in each PTLT. This was an action planned by the teacher educators, as they wanted the teachers to have an exploratory teaching experience so that they could then carry out this approach in their classes, favoring that the teachers were able to question each step in a class (Ribeiro & Ponte, 2019). This analysis of the class, in our understanding, was enhanced using videos throughout the training process, an important tool to support and support the teacher educator's work (Borko et al., 2014).

In the third excerpt, "*Consolidation of Ideas*", the teachers highlighted the difficulty in developing exploratory teaching. At that time, the teacher educator took advantage of their statements to support the proposal to orchestrate discussions in classrooms (Stein et al., 2008) mediated by the resolution of a mathematical task following the precepts of exploratory teaching. Thus, it discussed issues related to time management and the organization of a class (Ponte, 2005), aspects that are part of the teacher's

didactical knowledge. Therefore, at that time, when he raised the existence of similarities between the two processes (the class in basic education and the training process) instead of being equal, he demonstrated that he knew that his role as an educator goes beyond promoting learning about mathematical content (Jaworski & Huang, 2014).

Seeking to value the moments of work in small groups and in the plenary, carried out in Professor Felipe's class, the teacher educator, even though he used the same "class" *design* during the formative process with the teachers, was clear that his objective was the professional development of teachers, which includes, in addition to mathematical knowledge about standards and regularities, reflections, teaching strategies and educational resources (Jaworski & Huang, 2014) with teachers. We can note here that the teacher educator recognized himself as an agent belonging to the second level of the Tetrahedron Model (Prediger et al., 2019), the level of professional development of teachers, by demonstrating knowledge about this, as well as about what happens in the elementary school classroom, as he referred, at all times, to the mathematical and didactic challenges experienced by Professor Felipe in his class and by the other teachers, throughout his training.

Finally, taking the analysis of the three excerpts extracted from the fifth PTLT in the formative process, together with the respective discussions between the teacher educators during the preparation of the PTLT, we verified the pertinence and relevance of the different domains and components of the PLOT model (Ribeiro & Ponte, 2020). In particular, those related to the role and actions of the teacher educator (RATE) in the conception, elaboration, implementation and evaluation of the training process stand out.

FINAL CONSIDERATIONS

Researches that take as their object of analysis the teacher educator of teachers who teach mathematics are still insufficient (Fiorentini et al., 2016), especially in Brazil. Thus, our study aimed *to understand what the role was and how the actions of the teacher educator took place, in a process of continuing education with basic education teachers about the teaching of patterns and regularities, with a view to providing learning opportunities for the teachers involved.* Thus, we understand that, based on the evidence and results presented in our analyses, we produce contributions that can promote and encourage necessary reflections for future referrals, regarding the professional development of teacher educators.

To operationalize the outlined objective, we seek to reflect on how the teacher educator favored the articulation of the mathematical and didactic dimensions of the teacher's professional knowledge and the way in which the teacher educator, during a formative process, orchestrated the mathematical and didactic discussions between the participants. Thus, we identified the importance of using videos as a tool in the teacher education space (Borko et al., 2014). We noticed, in our results, the potential of this instrument to favor the approximation with the classroom, since it takes more faithful portraits of reality.

In addition to the use of video, we investigated the knowledge evidenced in the teacher educator's practice, necessary to manage a educational space to provide opportunities for teachers' professional learning (Jaworski & Huang, 2014). We also highlight the role of articulator and mediator played by the teacher educator in the orchestration of mathematical and didactic discussions and in the articulation between mathematical knowledge (Moreira & David, 2008), providing opportunities for professional development and teachers' learning about the patterns and regularities in algebra of the elementary/middle/high school (Ribeiro & Ponte, 2019).

Thus, throughout our study, we identified and exemplified the features of the *PLOT* model (Ribeiro & Ponte, 2020), visualizing its potential both to unravel and understand the role and actions of the teacher educator and to support the use and organization of the entire PDR Cycle (Trevisan et al., 2020) in a formative process. However, even though the *PLOT* model, combined with the Tetrahedron model (Prediger et al., 2019), has allowed us to analyze the role and actions of the teacher educator at the level of the teacher's professional development (the second tetrahedron, Figure 1), we see a challenge to be faced in future research if we take the *PLOT* model in the organization and analysis of formative processes aimed at the professional development of teacher educators (the third tetrahedron, Figure 1). Here is an invitation to research.

Finally, we also raise the challenge of working with the *PLOT* model in initial education, as well as in other fields of mathematics, in addition to algebra. We conclude by inviting other researchers to help us understand the potential of using the *PLOT* model in formative processes, as well as to identify challenges and limitations that this model may present in future studies aimed at providing opportunities for professional learning for teachers who teach mathematics.

AUTHORS' CONTRIBUTION STATEMENTS

All authors actively participated in the discussion of results, reviewed, and approved the final version of the work.

DECLARATION OF DATA AVAILABILITY

The authors agree that data supporting the results of this study are available upon reasonable request, at the authors' discretion.

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REFERENCES

Aguiar, M., Ponte, J.P., & Ribeiro, A. (2021). Conhecimento Matemático e Didático de Professores da Escola Básica acerca de Padrões e Regularidades em um Processo Formativo Ancorado na Prática. *Bolema, 35*(70), Artigo 12.

Aguiar, M., Ribeiro, A. J., & Ponte, J. P. (2019). Mathematical and didactical knowledge about patterns and regularities mobilized by teachers in a professional learning task. In *Proceeding of the 11th Congress of the European Society for Research in Mathematics Education - CERME 11* (pp. 3815-3822). Utrecht.

Ball, D. L., & Cohen, D. K. (1999). Developing practice practitioners: Toward a practice-based theory of professional education. In L. Darling-Hammond, & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3-22). Jossey-Bass.

Borko, H., Jacobs, J., Seago, N., & Mangram, C. (2014). Facilitating video-based professional development: Planning and orchestrating productive discussions. In Y. Li, E. A. Silver, & S. Li (Eds.), *Transforming mathematics instruction* (pp. 259-281). Springer.

Coura, F. C. F., & Passos, C. L. B. (2017). Estado do conhecimento sobre o formador de professores de Matemática no Brasil. *Zetetiké*, *25*(1), 7-26.

Creswell, J. W. (2014). Pressupostos Filosóficos e Estruturas Interpretativas. In J. W. Creswell. *Investigação Qualitativa e Projeto de Pesquisa: Escolhendo entre cinco abordagens* (pp. 29-47). Penso.

Fiorentini, D. (2018, 04 a 08 de novembro). Mapeamento e Estado da Pesquisa sobre o Professor que Ensina Matemática como Campo de Estudo. In *Anais do 7º Seminário Internacional de Pesquisa em Educação Matemática*. SBEM.

Fiorentini, D., Passos, C. L. B., & Lima, R. C. R. D. L. (2016). *Mapeamento da pesquisa acadêmica brasileira sobre o professor que ensina matemática: período 2001–2012*. FE/UNICAMP.

Jaworski, B. (2012). Mathematics teaching development as a human practice: identifying and drawing the threads. *ZDM*, *44*(5), 613-625.

Jaworski, B., & Huang, R. (2014). Teachers and didacticians: Key stakeholders in the processes of developing mathematics teaching. *ZDM*, *46*(2), 173-188.

Lampert, M. (2010). Learning teaching in, from, and for practice: What do we mean? *Journal of teacher education*, *61*(1-2), 21-34.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge University Press.

Moreira, P. C., & David, M. M. (2008). Academic mathematics and mathematical knowledge needed in school teaching practice: Some conflicting elements. *Journal of Mathematics Teacher Education*, *11*(1), 23-40.

Passos, L. F., Teles, N. C. G., Gonçalves, M. O., & Reis, A. T. (2016). Desenvolvimento Profissional de Formadores de Professores Iniciantes dos Cursos de Licenciatura. *Reflexão e Ação*, *24*(3), 105-122.

Ponte, J. P. D. (2005). Gestão curricular em Matemática. In Grupo de Trabalho de Investigação da APM (Ed.), *O professor e o desenvolvimento curricular* (pp. 11-34). APM.

Prediger, S., Roesken-Winter, B., & Leuders, T. (2019). Which research can support PD facilitators? Strategies for content-related PD research in the Three-Tetrahedron Model. *Journal of Mathematics Teacher Education*, *22*(4), 407-425.

Ribeiro, A. J., Aguiar, M., & Trevisan, A. L. (2020). Oportunidades de aprendizagem vivenciadas por professores ao discutir coletivamente uma aula sobre padrões e regularidades. *Quadrante, 29*(1), 52–73.

Ribeiro, A. J., & da Ponte, J. P. (2019). Professional learning opportunities in a practice-based teacher education programme about the concept of function. *Acta Scientiae*, *21*(2), 49-74.

Ribeiro, A. J., & da Ponte, J. P. (2020). Um modelo teórico para organizar e compreender as oportunidades de aprendizagem de professores para ensinar matemática. *Zetetiké*, *28*, 1-20.

Romanowski, J. P., Cartaxo, S. R., & Martins, P. L. O. (2017). Práticas formativas de professores formadores articuladas com os anos iniciais da Educação Básica. *Revista Educação e Cultura Contemporânea*, *14*(34), 6-24.

Scheiner, T. (2019). If we want to get ahead, we should transcend dualisms and foster paradigm pluralism. In G. Kaiser, & N. Presmeg (Eds.), *Compendium for Early Career Researchers in Mathematics Education* (pp. 511-532). Springer.

Smith, M. S. (2001). *Practice-based professional development for teachers of mathematics*. NCTM.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical thinking and learning*, *10*(4), 313-340.

Superfine, A. C., & Li, W. (2014). Developing mathematical knowledge for teaching teachers: A model for the professional development of teacher educators. *Issues in Teacher Education*, *23*(1), 113-132.

Trevisan, A. L., Ribeiro, A. J., & Ponte, J. P. (2020). Professional Learning Opportunities Regarding the Concept of Function in a Practice-Based Teacher Education Program. *International Electronic Journal of Mathematics Education*, *15*(2), 1-14.

Webster-Wright, A. (2009). Reframing professional development through understanding authentic professional learning. *Review of educational research*, *79*(2), 702-739.