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Interpretative knowledge of Prospective Kindergarten and Primary Teachers in the Context of Subtraction

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ABSTRACT

Background: Teaching and learning with understanding leads to the adoption of "what" and "how" each individual knows as starting points, requiring from teachers a specialised knowledge, called Interpretative Knowledge (IK), which supports constructive feedback and pedagogical actions aimed at developing the students' mathematical knowledge and skills. **Objectives**: This paper aims to trace and discuss the problematic areas of the IK presented by prospective kindergarten and primary teachers' (PTs) when attributing meaning to students' productions within the scope of subtraction. **Design**: The focus is on the IK revealed by PTs when analysing and giving meaning to students' written productions on a task involving subtraction between natural numbers**. Setting and participants**: Twenty-six PTs attending the only course focusing on teaching and learning mathematics they have in their curriculum participated in the research. **Data collection and analysis**: The data collected and analysed, from the qualitative and interpretative perspective of content, come from the written productions of the PTs on a task for teacher education, together with the audio and video recordings of the group discussions. **Results**: The results show a "linear" interpretation of the students' productions, associated with a single way of understanding subtraction, limiting the PTs' development of constructive feedback and pedagogical interventions, so their practice is not based simply on "teaching how to do." **Conclusions**: The limited interpretation and, consequently, the limited development of constructive

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feedback stood out, showing the need for teacher education to focus on tasks specifically conceptualised for developing the specificities of (prospective) teachers' knowledge that would enable them to give meaning to students' production and assume the associated mathematical reasoning as a starting point for practice.

Keywords: Interpretative Knowledge; Kindergarten and Primary teacher education; Initial teacher education; Subtraction.

Conhecimento Interpretativo de futuros professores da Educação Infantil e dos Anos Iniciais no contexto da subtração

RESUMO

Contexto: O ensino e a aprendizagem com compreensão suscitam a adoção dos pontos de partida "o que" e "como" cada indivíduo conhece, demandando do professor um conhecimento especializado, denominado Conhecimento Interpretativo (CI), que sustenta o feedback construtivo e ações pedagógicas voltadas para os conhecimentos e capacidades matemáticas dos alunos. **Objetivos**: Busca-se aqui a identificação e discussão das áreas problemáticas do CI revelado por futuros professores dos anos iniciais ao atribuírem sentido a produções de alunos no âmbito da subtração. **Design**: Foca-se no CI revelado no conteúdo das resoluções e comentários apresentados por acadêmicos de Pedagogia no decorrer da implementação de uma tarefa envolvendo a subtração entre números naturais. **Ambiente e participantes**: Participaram da pesquisa 26 estudantes de uma disciplina do curso de Licenciatura em Pedagogia da UNICAMP. **Coleta e análise de informações**: Os dados coletados e analisados, a partir da perspectiva qualitativa e interpretativa de conteúdo, são provenientes das produções escritas dos acadêmicos sobre uma tarefa proposta, juntamente com o áudio e vídeo das discussões em grupos. **Resultados**: Os resultados revelam uma interpretação "linear" das produções dos alunos, associada a uma única forma de entender a subtração, limitando a elaboração, pelos futuros professores, de feedback construtivo e de intervenções pedagógicas para que não pautem suas práticas simplesmente na ação de "ensinar a fazer". **Conclusões**: Destaca-se a limitação interpretativa e, consequentemente, a elaboração de feedbacks construtivos, evidenciando a necessidade de processos formativos que foquem em tarefas e atividades relacionadas diretamente prática dos professores e nas produções dos estudantes da educação básica.

Palavras-chave: Conhecimento Interpretativo; Educação Infantil e Anos Iniciais; Formação Inicial de professores; Subtração.

INTRODUCTION

Teachers' mathematical practice is directly linked to students' mathematical learning; thus, the teachers' knowledge plays a central role in students' learning and results (Darling-Hammond, 2000; Nye, Konstantopoulos & Hedges, 2004). Although other factors can influence students' learning and results, teachers' performance – what and how the teacher says and does– and teaching objectives regarding the topic are shaped by teachers' knowledge, which are externalised by actions (Ribeiro & Carrillo, 2011) and reflect directly on the mathematical discussions in the classroom (Policastro, Almeida, Ribeiro & Jakobsen, 2020). Such knowledge also impacts the nature and focus of discussions expected to occur in teacher education contexts. In this sense, assuming that initial teacher education should be the "gateway" to quality mathematical practice, and that specialised knowledge is developed in and by training (Ball, Thames & Phelps, 2008; Carrillo et al., 2018; Ribeiro, Mellone & Jakobsen, 2013), it is essential to develop and implement tasks in teacher education that favour the access to and comprehension of the specifics of this knowledge, allowing the teacher to develop and practice pedagogical actions closely linked to the act of teaching mathematics, relating them to the students' reasoning and understanding.

Considering such a starting point, without imposing, beforehand, a specific way of doing mathematics – usually linked with giving the rule – requires from the teacher the mastery of Interpretative Knowledge – IK (Jakobsen, Ribeiro & Mellone, 2014; Ribeiro, Mellone & Jakobsen, 2016). Such knowledge supports the attribution of meaning and significance to verbal and written productions presented by students – especially those unexpected or with errors –, aiming to develop their mathematical knowledge, abilities and skills.

This mathematics teachers' knowledge, which allows them to take "what" and "how" students know as a starting point for the discussions, should therefore occupy a central place in a practice that promotes meaningful mathematical learning. However, since IK is based on the specificities of teachers' mathematical knowledge and due to its specialised nature, it is not developed in practice (Ribeiro et al., 2013). That is, teachers' IK is neither developed in teaching practice, in the course of the experiences in the classroom nor as a consequence of them.

On the contrary, such knowledge requires an educational process intentionally designed for this purpose.

In this sense, aiming at an effective improvement of the teaching practice, the primary function of the initial and continuous teacher education should be the development of (prospective) teachers' knowledge on each mathematical theme and topic they will have to address when in practice. More specifically, such teacher education processes must develop a knowledge that mathematically surpasses "knowing how to do," favouring the understanding of "what" is done in mathematics, of "why," "how," and "when" it is done. Moreover, the training process must stimulate the teachers' knowledge concerning the connections between the different topics and concepts, ensuring that the mathematics to be taught is understood by the teacher as a structured body of knowledge. Hence the importance of focusing on the specialised dimensions of teacher knowledge (Carrillo et al., 2018), since they imply promoting a practice focused on mathematical intentionality as one of the main goals.

Considering that "Numbers and Operations" occupy the largest space in the mathematical activities of students in primary school (Mandarino, 2009), and the subtraction with natural numbers is considered the first in which the students' difficulties are most evident (Kamii, Lewis & Kirkland, 2001), the focus on teacher knowledge regarding this topic becomes essential, since the students' difficulties often reflect the teachers' own difficulties. Likewise, discussing the content of the teachers' current and future knowledge on this topic becomes essential, especially when aiming to focus on the real difficulties presented by them when teaching mathematics.

Thus, seeking to contribute to the improvement of teacher education and, therefore, of the future pedagogical practices that allow them to take the students' production and reasoning as a starting point, both tracing these problematic areas of the teacher's knowledge and understanding more deeply the factors that make these areas problematic become essential. Here, we pursue the following question: *What Interpretative Knowledge do prospective kindergarten and primary school teachers reveal when they attribute meaning to the students' productions in the context of subtraction?*

SPECIFICITIES OF MATHEMATICS TEACHERS' KNOWLEDGE IN THE TOPIC OF SUBTRACTION

The National Common Curricular Base – BNCC (Brasil, 2018) directs significant attention to the teaching based exclusively on the memorisation of meaningless symbols, rules, and procedures, associated with calculations involving natural numbers, by emphasising that "the mathematical skills that students must develop cannot be restricted to learning the algorithms of the so-called 'four operations'" (Brazil, 2018, p. 232). In terms of research, several studies discuss students' knowledge of these operations, such as the strategies associated with mental calculation and algorithms, including subtraction (Clarke, Clarke & Horne, 2006; Loureiro, 2004). These studies are essentially concerned with the students' unawareness – what students do not know or their difficulties –; however, more recent studies found that the difficulties attributed to students are, in fact, also difficulties of teachers themselves (Ball, Thames, & Phelps, 2008).

In the context of learning to subtract, students' difficulties have featured in the literature for almost two decades (Kamii et al., 2001). Such strains are directly associated with the teaching process at school, based on conventional operation techniques and often associated with manipulative materials, but without the essential correspondence between the processes used in manipulating materials, the algorithms, and the expressions verbalised and recorded in this activity. That is, the processes used for the manipulation of physical resources, including forms of representation and registration, usually do not have the same meaning as those applied to the manipulation of algorithms for conceptualisations and calculations associated with operations with natural numbers (Faustino, Passos, 2013; Loureiro, 2004).

Despite their importance in processes associated with mathematical calculations, these algorithms – when understood simply as "rules" to be followed and replicated – must be assumed not as a starting point for pedagogical practice but rather as a destination point (Ribeiro, 2011). Thus, aiming at a teaching in which students understand "what," "why," "where," and "when" to do in mathematics and are not limited to "randomly" replicating a set of steps and rules, disregarding understanding, teachers must own knowledge that allows them to discuss the relations among those steps and processes inherent in operations and the algorithms associated with them, besides their connections with the structure of the decimal numbering system and with the properties of the operations (Muñoz-Catalan, Liñan, & Ribeiro, 2017). For students' understanding to be effective, teachers must have specialised knowledge that transcends the "know-how," effectively linked to the understanding of the different relationships, connections and implications that include, among other aspects, verbalisation and registers of the contents worked on, the use of resources and the questions that are formulated – and the way they are formulated, with the associated mathematical goals.

Although considering three meanings associated with subtraction – taking away, completing and comparing –, the traditional focus is on "taking away": given a set of elements, a portion of them is taken away. Reducing the understanding of subtraction to the act of taking away (removing) something from a set of elements becomes a barrier when, later on, the goal is to understand the relations among the four operations. Thus, working with the operations needs to consider the problems grounding the construction of the different meanings of such operation (Mcclain, Cobb & Bowers, 1998). Understanding these meanings is considered the ground for understanding the operation as a phenomenon and the correspondence between these meanings and the associated verbalisations with the algorithm, allowing navigating between the different representations to be used (Muñoz-Catalan et al., 2017; Ribeiro, 2011).

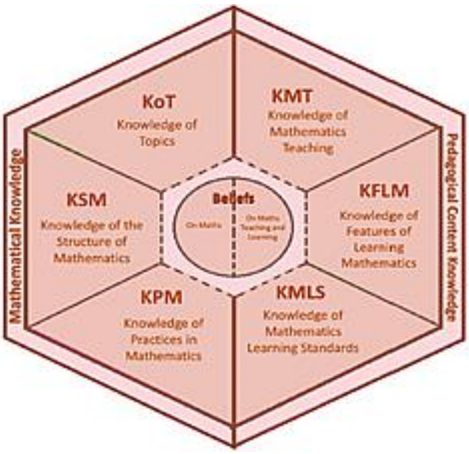
Teacher knowledge has assumed a prominent role in educational research and discourses in the last decade, driven mainly by the work of Lee Shulman (1986). However, frequently, in the case of mathematics education, this discourse and research have not directly considered the specificities of the mathematics teachers' knowledge, since most studies eventually focus on the general dimensions of teacher knowledge, associated mainly or exclusively with the pedagogical dimensions of teaching practices – dimensions that are common to teachers from the most different fields of knowledge –, which has contributed little to improve the mathematics teachers' education and practice (Ribeiro, 2018). In international contexts, a movement has sought to specify the domains of teacher knowledge associated with specific fields of knowledge. Moreover, in the case of mathematics, it has generated different conceptualisations of teacher knowledge, which consider the specificities of this knowledge for the teaching of mathematics (e.g., Ball et al., 2008; Carrillo et al., 2018).

In particular, the recent focus of these specificities has been on the knowledge of mathematical content and the need to clarify "how" such specificities contribute to a mathematical practice in the classroom that effectively leads to students' learning, so they understand "what" they do, "why," "when," and "how" they do in mathematics and even glimpse into future learning (Pinto & Ribeiro, 2013). Moreover, aspects of the teacher knowledge that allow them to assign meaning to the students' productions are also discussed, based on what they "should" know, how they should know it and what they actually know (Jakobsen et al., 2014), aiming at a pedagogical work that expands the students' mathematical knowledge, as expected for each educational stage.

Considering that the specificities of mathematics teacher's knowledge fall both in the domain of mathematical knowledge and in the domain of pedagogical knowledge to teach mathematics, we assume the specialisation of that knowledge according to the *Mathematics Teachers´ Specialised Knowledge* – MTSK (Carrillo et al., 2018), which also considers the significant interrelationships and feedback between the two domains in the development of the teacher's practice, in its various aspects, and also the relations of these dimensions – knowledge and practice – with the teachers' beliefs regarding mathematics and its teaching (Figure 1).

Figure 1

MTSK Model (Carrillo et al., 2018, p. 241)



Due to the context and focus of our work – see next section –, we address here the domain of *Mathematical Knowledge* (MK), which suits both teachers and prospective teachers. This focus stemmed from the understanding that the content of teachers' mathematical knowledge allows teachers to support giving meaning to students' productions and comments, thus enhancing the teacher's Interpretative Knowledge (Jakobsen et al., 2014; Di Martino, Mellone, & Ribeiro, 2020).

Therefore, aiming at contributing to the students understanding the "how" and the "whys" of mathematics, teachers are required to own a knowledge that goes beyond what students learn – which can be considered as "school know-how" – in the schooling process, associated, among others, with a knowledge related to "traditional" and alternative procedures or the different forms of representation of the different constructs and topics. It is also required to know the concepts and their relationships with real contexts from other areas of knowledge (e.g., geography, arts) or the mathematical content itself, representing the connections within the very mathematics (Montes, Ribeiro, Carrillo, & Kilpatrick, 2016). It also includes teacher knowledge on epistemological aspects involved in the use of examples that contribute to the discussion of the different meanings that they can attribute to the topic and the different contexts in which each topic can be situated (Knowledge of Topics – KoT). Regarding subtraction, it includes, for example, knowing the three distinct meanings of the notion of subtracting: "taking away", "completing", and "comparing" (Muñoz-Catalan et al., 2017); knowing the procedures (algorithms) – standard or non-standard – to determine the result of the subtraction operation (taking away, excess or difference) and the relationships and implications of these meanings associated with number sense (Brocardo et al., 2005); the different forms of representation associated with determining the result of a subtraction – pictorial, graphic, arithmetic – including modelling with resources such as the abacus.

Besides, teachers should have knowledge associated with the structure of mathematics, i.e., mathematical knowledge of each of the themes, assuming a perspective of their integration and relationship with broader mathematical structures and mathematical topics – global knowledge. This includes knowledge of the connections with more advanced and elementary concepts that enable teachers to work on elementary mathematics from a higher viewpoint and vice versa (Knowledge of the Structure of Mathematics – KSM). Note that this subdomain is not related to knowing how the concepts or themes are organised in curricular terms. Regarding subtraction, it includes teachers' knowledge on the connections between the meaning of completing and the addition operation, or the connections of subtraction with division, when associating division to the notion of sharing in equal portions.

Still included in the teachers' content knowledge, it is considered a mathematical knowledge associated with the ways of doing mathematics. Among them, there are the criteria to be established for a generalisation to be valid; the knowledge of mathematical syntax; the knowledge of different problem solving or modelling strategies, including the knowledge associated with the logical structure on which the solution is based (*Knowledge of Practices in Mathematics* – KPM). Concerning subtraction, it includes knowing the (im)possibilities of generalising the procedures used in the processes associated with finding the result of subtraction and its identification with distinct algorithms.

The content of this specialised mathematical knowledge supports the attribution of meaning to students' comments and productions, even when they contain mathematical errors or inadequacies, are guided by unconventional reasoning (Jakobsen et al., 2014; Ribeiro et al., 2013), and/or are not part of the so-called teacher's solution space (Jakobsen et al., 2014). To an effective meaning attribution, Interpretative Knowledge is required (Jakobsen et al., 2014; Di Martino et al., 2020), supporting the subsequent provision of *constructive feedback* to discuss and expand students' understanding and knowledge on the topic under discussion, grounded on their own initial understanding and knowledge.

The notion of solution space is associated with the knowledge the teacher has available and allows obtaining a set of possible answers and solutions to a "problem", even when there is only one solution. This set of possible answers and solutions includes, for example, different forms of representation and approaches to the same situation, from a mathematical and non-pedagogical point of view, which is related to problem solving strategies. In this sense, the solution space is necessarily linked to the nature of teacher's mathematical knowledge related to, among other aspects, mathematical definitions, concepts, approaches, representations and processes associated with the same topic and with connections involving different topics that can be involved in the search to solve each of the problems posed to students.

This specialised knowledge, which gives form and content to the elements of the solution space, will outline the type, form and nature of the mathematical connections aimed, allowing teachers to supplement their ways of knowing and establishing links with other knowledge that may be required to interpret responses different from their own (Jakobsen et al., 2014) – essentially those unexpected and quite distant from the expected, whether mathematically appropriate or not.

Broadening teacher's solution space requires, therefore, a broader, deeper, and more relational knowledge than knowing for oneself – knowing how to do/apply the rule – and is situated within the scope of content knowledge (MK), since this interpretation will contribute to subsequent pedagogical decisions (Jakobsen et al., 2014). Thus, activating and developing IK, expanding its content, requires knowledge that allows the teachers to overcome the boundaries of their own solution space. Such process can only be accomplished by teacher education, since this knowledge is not developed only with (mere) practice (Jakobsen et al., 2014), which brings to the centre of the discussion the need for teacher education to focus on its development. In this sense, the initial and continuous teacher education should assume as one of its roles the promotion and expansion of this knowledge, which demands and implies the design and implementation of tasks intentionally prepared for this purpose (e.g., Policastro, Almeida, & Ribeiro, 2017; Jakobsen et al., 2014; Ribeiro et al., 2021a, 2021b).

Thus, teacher education must contribute to activate "a real process of interpretation, advancing from an *evaluative listening* to a more flexible form of *hermeneutical listening*" (Di Martino et al., 2016, p. 4). This *evaluative listening* is associated with a process of lack of correspondence between what the student comments and/or produces and what the (prospective) teacher expects as a response. This expected response is part of the teacher's solution space for each of the situations, which is usually composed of a reduced number of elements – in most cases, a single element (Jakobsen et al., 2014) –, which considers the possibility of obtaining the "correct answer" in a single way. The development of an *hermeneutic listening* (Davis, 1997) will allow the teacher to redesign mathematical learning paths that incorporate the students' reasoning and knowledge (Di Martino et al., 2016), expressed in their productions or comments, taking them as a starting point for the construction and development of new mathematical knowledge.

Accordingly, Interpretative Knowledge differs from other ways of understanding the teacher's knowledge, insofar as it assumes as fundamental – and desirable – the understandings of the reasons that support the students' unconventional reasoning and errors. This aspect related to the IK leads us to consider the need to incorporate mathematically potent students' productions from contexts of practice in the tasks for teacher education (Ribeiro et al., 2021a).

However, IK is not restricted to the analysis of students' errors, even though this analysis is part of the teachers' knowledge, since the teaching practice requires (it is expected to) understanding the reasons that support this error – or the alternative reasoning, even if correct –, in addition to having them as a starting point to outline the discussions to be held with students, which can be real or simulated, depending on the context. At the same time, this redesign of the pathways to new learning cannot be guided by a feedback approach related to a "positive or negative reinforcement," on a personal level, continuing to leave aside the mathematical knowledge and its understanding by the students (e.g., "Congratulations! You reached the correct result!"), since this type of feedback does not contribute to improving mathematical learning.

Thus, to enable the (re)design of pathways that promote mathematical learning for students, developing teacher's IK is considered essential, requiring a different perspective for teacher education and, especially, for tasks for this educational purpose (Ribeiro, 2016; Ribeiro et al., 2021a, 2021b).

CONTEXT AND METHOD

This study is part of a broader research and teacher professional development project, with a cyclical methodological approach, in which both research and educational dimensions feed each other at each stage. The research dimension aims at obtaining elements that promote the understanding of the content of specialised knowledge and the IK of prospective kindergarten and primary school teachers (PTs) within the scope of some mathematical topics and of the relationships between elements that constitute these specialisations and the teachers' mathematical practices. Through this objective, we also intend to contribute to better conceptualising what we call “interpretative tasks” (Ribeiro et al., 2021b).

Here we focus on the IK revealed by 26 PTs in the context of subtraction of natural numbers. In particular, we discuss the knowledge mobilised when giving meaning to students’ productions to a task designed specifically for teacher education (Ribeiro et al., 2021a, 2021b) that aimed to discuss different approaches to subtraction, involving its multiple meanings and some of its properties. This task was applied in the course of a four-hour-long class of the only course PTs have in their degree[1]. Data concerns PTs’ written productions as responses to the proposed task, which comprise a larger set of information collected, together with audio and video recordings of group discussions and large-group discussions[2]. The PTs worked in groups of three or four.

Along the semester, an itinerary[3] of tasks for teacher education has been implemented, focusing on developing the IK and specialised knowledge of prospective teachers regarding natural numbers and the four operations. These tasks for teacher education, including the so-called interpretative tasks (Policastro et al., 2017; Ribeiro et al., 2021b; Ribeiro et al., 2013), have a particular structure, which has been developed and expanded by the studies conducted in the scope of the Research and Teacher Education group CIEspMat[4].

These tasks for teacher education are based on a conceptualisation that includes two complementary parts: Part I is associated with the development of (prospective) teacher’s specialised knowledge on the topic – in this case, subtraction. This first part, in the specific case we address here, is characterised by the anticipation of possible students answers, of a given educational stage, to problems

[1] The course, offered semiannually (with 60 hours, in 15 meetings of 4 hours each), is the only that is part of the framework of the degree for kindergarten and primary teachers in the scope of mathematics.

[2] All classes are usually recorded in audio and video – focusing on the Educator –, since this is considered a primary context for improving teacher education and the teacher educator practices.

[3] For an example of an itinerary see one of the books of the Collection Classroom and Itinerary at www.editora.cognoscere.com.br

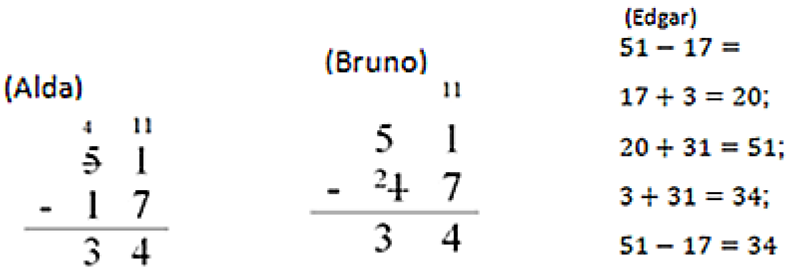
[4] CIEspMat is a Research and Teacher Education group focusing on mathematics teachers Interpretative and Specialised Knowledge. www.ciespmat.com.br

related to the topic. The second part aims at developing the solvers IK, based on the reflection and discussion about students' productions, regarding one of the problems proposed in the first part.

In the task discussed here, Part I begins with a question focusing PTs' attention on the three subtraction meanings; in the language used when we verbalise what we do when using an algorithm to perform 51-17 and its mathematical significance; in anticipating possible different grades students' answers and reasoning; and the knowledge associated with problem posing for 51-17. Part II asks solvers to interpret – analyse and comment –, in terms of mathematical adequacy and correctness, a set of students' productions and provide constructive feedback for each of these productions (Figure 2). Note that the students' names included here are fictitious and these productions come from previous research and work with students and teachers in the context of continuous teacher education programmes.

Figure 2

Students' productions included in Part II of the teacher education task



On the analysis, we will focus on prospective teachers' interpretations of Edgar's production. However, Alda's production was included in the task because it corresponds to the typical representation of how prospective teachers learned the subtraction algorithm when they were primary-school students in Brazil. Bruno's, in turn, was included because it represents the subtraction algorithm as traditionally approached in countries such as Portugal or Chile. These options allow, among other focuses, discussing and enhancing aspects of the history of mathematics and mathematics education.

In the specific case of Edgar's production, it was included in the task because it enhanced a discussion covering, for example, the mental calculation strategies, the mathematical property that underlies the procedure used – associative –, and the possibility (or not) of generalising this procedure. Particularly, this production allows discussing one of the meanings of subtraction (complete), favouring the expansion of PTs' knowledge, associated with the connections between the operations of addition and subtraction. Moreover, Edgar's reasoning and representation differ substantially from the "traditional" algorithm – the only one used by PTs when they solved the operation on their own, which was one of the questions in Part I –, and discussing them later seeks to enhance the expansion of the PTs' own solution space.

Regarding the analytical process conducted in this study, initially, the students' written productions for the task were organised into three tables, one corresponding to Part I of the task (PTs' own reasoning and ways of doing) and the other two corresponding to Part II. For the first table, PTs' productions were organised according to a criterion of correspondence with the question of the task to which they were associated and related to each of the six groups – "Group 1 to Group 6". This organisation provided a global overview on the types and nature of PTs' answers given for each question, and its analysis "to exhaustion" enabled the characterisation of particular aspects of PTs' specialised knowledge and on the elements constituting their solution space regarding subtraction, particularly involving the operation 51-17. Such aspects contributed to supporting the analysis of the information collected in the second part of the task.

The two tables elaborated corresponding to PTs' answers to Part II were constructed, respectively, from the PTs' written production when interpreting the students' productions and according to the feedback they provided for each of the interpreted productions. Both tables were constructed according to the organisation made by the correspondence between the response of each of the six groups and the production of the student in question. Of the six groups included for analysis, two did not provide feedback of any kind for the students' productions. In this phase of analysis, regarding the interpretation of the students' productions, readings were carried out until exhaustion, which allowed tracing two categories of interpretation for the PTs' Interpretative Knowledge:

(a) "descriptive-evaluative": identifies the correction or non-correction of the mathematical reasoning used, followed by a description of the steps/procedures employed by the student to solve the problem (51-17);

(b) "positive-evaluative": presents close relationships between own answer to the first part of the task and what they determined as corresponding to the students' productions and remaining focused exclusively on the result (34).

Regarding the comments to include feedback, we found similarities in the responses presented by the different groups, constituting a grouping of answers that emphasised only a positive or negative reinforcement on the production, without affecting any redesign for the learning pathways. Such group of responses was called "compliment".

RESULTS AND DISCUSSIONS

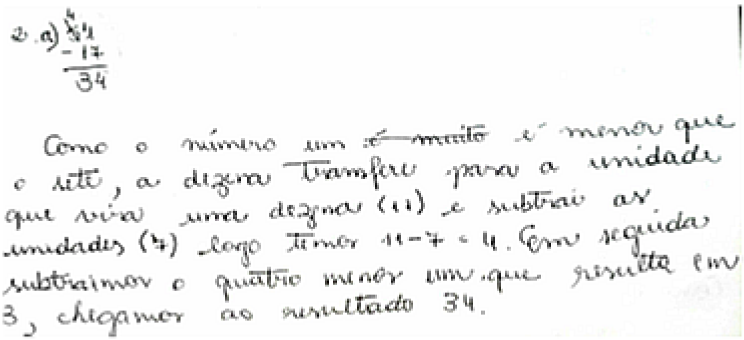
When answering the first question in Part I of the task ("What is subtracting?"), the six groups provided answers situated in the space where subtraction is considered exclusively associated with the sense of taking away (Mcclain, Cobb, & Bowers, 1998; Muñoz-Catalan et al., 2017). Their answers included "*It is to remove a quantity from another quantity*" or "*Subtraction is a type of calculation in which we find the difference between numbers by removing/decreasing a certain quantity*." Thus, these constitute evidence that the future teachers do not consider the other two meanings that can be attributed to this operation: that of comparing and that of completing (KoT – understanding the phenomena and the meanings of the operation).

Although one of the groups uses the word "*difference*", which, in terms of adequacy of the mathematical language, is the most appropriate to be used in contexts in which the sense of comparison is evoked (KoT – registers of representation for a construct), when associating this term to the meaning of taking away ("*we find the difference between numbers by means of removal/decrease*"), they evidence the non-correspondence between each of the meanings evoked for subtraction and the most appropriate type of verbalization to be used (KoT – registers of representation – verbalisation).

Figure 3 shows that in question 2 of Part I of the task, three of the six groups presented the “conventional” algorithm as a solution strategy when providing a personal solution for operation “51-17” and described the steps associated with the algorithm (KoT – procedures):

Figure 3

Example of prospective teachers’ answer to question 2 from Part I “determine the result of 51-17 and explain the steps involved”


2. a) 51
- 17
34

Como o número um ~~é muito~~ é menor que o sete, a dezena transfere para a unidade que vira uma dezena (11) e subtrai as unidades (7) logo temos 11-7 = 4. Em seguida subtraimos o quatro menor um que resulta em 3, chegamos ao resultado 34.

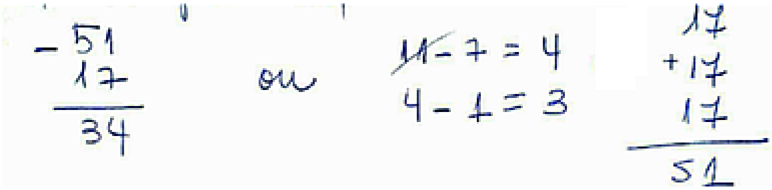
As the number one ~~is much~~ less than the seven, a ten is transferred to the unit that becomes a ten (11) and subtracts the units (7), so we have 11-7=4. Then we subtract the four minus one that results in 3, we get the result 34.

Note that the use of verbal language associated with the description of the procedure suggests that, although the PTs mention changing the order of the number in the process of “transferring” tens to units (“*the ten transfers to the unit which becomes a ten (11)*”) to perform 11-7, when verbalising 40-10, or 4 tens minus 1 ten, they just mentioned “*we subtract the four minus one*”, which shows the need to expand their specialised knowledge regarding the refinement of the adopted language, to maintain the mathematical adequacies (KoT – registers of representation for a concept).

We particularly highlight the solution of one of the groups (Figure 4), which is associated with the sense of completing, since, when doing 17+17+17=51, they reveal their own solution space (Di Martino et al., 2016; Jakobsen et al., 2014). Such solution space includes a procedure that relates to the addition of successive portions, starting from the lowest value, until the highest value of the operation is reached (KoT – procedures). And, although this group revealed knowledge on the topic related to a type of procedure different from that usually used in employing the “traditional” algorithm, due to the fact that, like the other groups, they related subtraction only to the sense of taking away, it is evident the need to expand their specialised knowledge as to the different meanings of the operation (KoT – understanding the phenomena and the senses of the operation) and to the need for a correspondence to be made between each one of them and the procedure used, in association with the registers of representation and the language employed (KoT – registers of representation).

Figure 4

Solution presented by one of the groups for the operation “51-17”

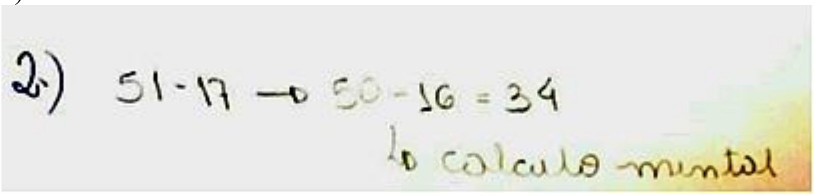


We also highlight the answers provided by two other groups (Figure 5), since they are grounded on mental calculation procedures/strategies and are implicitly related with the notion of comparing (5a) or with comparing and completing (5b).

Figure 5

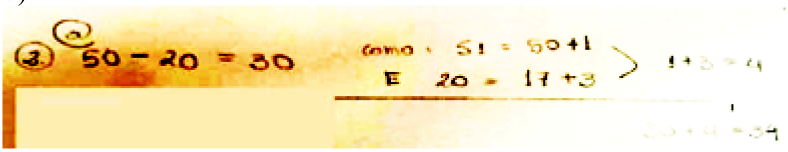
Examples of answers related to the senses of comparing and completing

a)



51-17→50-16=34 (mental calculation)

b)



50-20=30, since 51=50+1 and 20=17+3, 1+3=4, 30+4=3

Such solutions also allow us to affirm that these PTs, besides showing knowledge related to ways of doing to resolve the operation (KoT – procedures), have in their solution space types of procedures not associated with standard procedures (Ribeiro et al., 2013). It is important to notice the need to discuss in teacher education the validity of the procedures employed (KoT – procedures) as well as the (im)possibility of generalising these types of procedures (KPM – processes for validation). Such discussions are needed and required to occur aligned with the aim of developing PTs interpretative knowledge in order also to expand their own solution space.

The fact that the solution space presented by a portion of these prospective teachers (three groups) regarding the very notion of subtraction contains only one element – an element that corresponds to the sense of taking away, commonly associated with the verbalisation used in the “traditional” algorithm – conditions the nature of possible future interpretations (Di Martino et al., 2016; Jakobsen et al., 2014) that they may conduct on the students’ productions. This solution space with a single element is associated with a restricted content of their specialised mathematical knowledge, which limits the scope of their interpretations and subsequent pedagogical options to consider the mathematical reasoning expressed by these productions as a starting point for future discussions (Di Martino, Mellone, & Ribeiro, 2020).

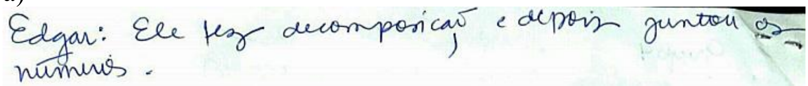
The analysis of the prospective teachers' interpretations of students' productions (Part II of the task) allowed to relate the knowledge they revealed when solving the subtraction for themselves – on the senses of the operation and procedures –, which relates to their solution space, and the content of their Interpretative Knowledge.

In fact, three of the six groups provided interpretations for Edgar's production that are classified into the category we call "descriptive-evaluative," since they were limited to presenting a description of what they determined as having been the procedures used by the student to solve the problem. (Two members of the same group made different records, so they were included in the same table cell – Figures 6c and 6d)

Figure 6

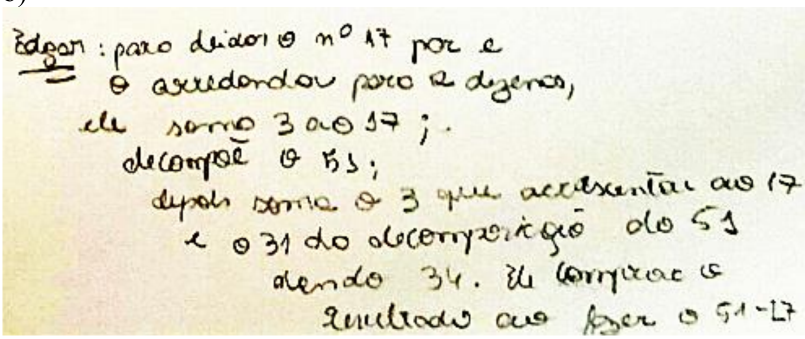
Interpretation provided by four groups on Edgar's production

a)


Edgar: Ele fez decomposição e depois juntou os números.

Edgar: He did the decomposition and then put the numbers together.

b)


Edgar: para deixar o nº 17 par e
o arredondou para 2 dezenas,
ele soma 3 ao 17;
decompõe o 51;
depois soma o 3 que acrescentou ao 17
e o 31 da decomposição do 51
dando 34. Ele comprova o
resultado ao fazer o 51-17

Edgar: to make number 17 even and round it up to 2 tens, he adds 3 to 17; decomposes the 51; then he adds the 3 he added to 17 and the 31 from the decomposition of 51 resulting in 34. He proves the result by doing 51-17.

c)

Para chegar ao resultado esperado **EDGAR** realizou o cálculo a partir de uma relação comum entre os dois algarismos da equação apresentada. No caso, ele utilizou o número 20 para trabalhar com cada algarismo e, das numerações diferentes daquelas já apresentadas, obtidas das operações realizadas ([17+3 =20] e [20+31=51] em que obteve-se "3" e "31" como novidades aos números já existentes nas equações e na relação entre eles), Edgar estabeleceu uma nova relação entre os números encontrados realizando a somatória entre eles.

To reach the expected result, EDGAR performed the calculation based on a common relationship between the two digits of the presented equation. In this case, he used the number 20 to work with each digit and, by the different numbers from those already presented, obtained from the operations performed ([17+3=20] and [20+31=51] in which "3" and "31" were obtained as new to the numbers already existing in the equations and in the relation between them), Edgar established a new relation between the numbers found, making the sum between them.

d)

Edgar V Edgar decompõe a Conta somando aos poucos e fazendo linha por linha como se fosse "Completando" até Chegar ao final

Edgar decomposes the calculation by adding gradually and doing it line by line as if he were "completing" until reaching the end.

It is important to note that only one of the groups (Figure 6d) interprets Edgar's production as associated with the sense of completing, attributed to subtraction, although they do not state explicitly that, in fact, they understand it as the use of a procedure associated with such operation sense (KoT – meanings of the subtraction and KoT – procedures;). It is worth mentioning that, in the first part of the task, although they answered the question about "what is subtracting?", describing the operation exclusively as "taking away", when they were asked to solve the operation themselves, they provided a solution associated with the sense of comparing (Figure 5a).

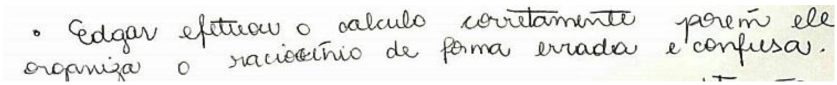
Besides, when asked to formulate two distinct problems for which the operation 51-17 would constitute the required solution process (one of the questions in Part I), the group proposed a problem that evoked the meaning of taking away and another that evoked the meaning of completing (KoT – understanding the phenomena and the meanings of the operation). In any case, this group interpretation was included in the "descriptive-evaluative" category exclusively because it was not possible to find a relationship between what the group itself revealed to know about the subtraction meanings and procedures, i.e., their own solution space, and the interpretation they provided for Edgar's production. This fact leads us to reinforce the need for explicit and intentional work proposals in teacher education contexts that favour the conscious establishment of the relationships between what students produce and/or comment and what (prospective) teachers themselves know about each of the topics (e.g., Ribeiro et al., 2021a, 2021b; Ribeiro et al., 2016).

The other three groups presented interpretations classified as "positive-evaluative", since they were focused only on the result of the operation (34). Although their interpretations contain links with their own answers for the first part of the task, expressions such as "*correct*", "*adequate*", and "*reached the expected result*" stood out in the arguments used by PTs to justify the mathematical adequacy and correction of Edgar's production and associated reasoning (Figure 6).

Figure 7

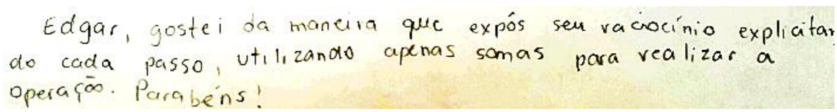
Interpretation provided by three groups for Edgar's production

a)


• Edgar efetuou o cálculo corretamente porém ele organiza o raciocínio de forma errada e confusa.

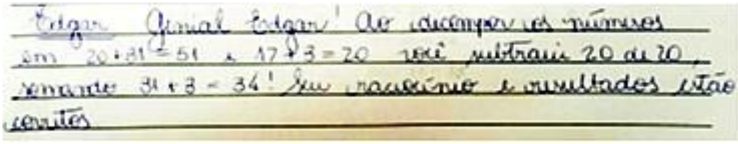
Edgar performed the calculation correctly, but he organises the reasoning in a wrong and confusing way.

b)


Edgar, gostei da maneira que expôs seu raciocínio explicitando cada passo, utilizando apenas somas para realizar a operação. Parabéns!

Edgar, I liked the way you exposed your reasoning by explaining each step, using only sums to perform the operation. Congratulations!

c)


Edgar Genial Edgar! Ao decompor os números em 20+31=51 e 17+3=20 você subtraiu 20 de 20, somando 31+3=34! Seu raciocínio e resultados estão corretos

Edgar Brilliant Edgar! By decomposing the numbers into 20+31=51 and 17+3=20 you subtracted 20 from 20, adding 31+3=34. Your reasoning and results are correct.

Some of the PTs do not recognise connections between the operations of addition and subtraction and, in particular, they do not recognise the meaning of completing that can be attributed to subtraction. When they do not recognise that Edgar's type of record is associated with that which can be performed with the support of a number line, they classify it as being correct but confusing ("*he performed the calculation correctly but he organised the reasoning in a wrong and confusing way*") –Figure 7a.

We also highlight the interpretation provided by one of the groups (Figure 7c) that inadequately finds, in Edgar's production, two mathematical procedures that they call "*decomposition of numbers*" and a subtraction operation. In this case, the group interprets that the student would have decomposed the quantity 51 into a sum of 20+31, just as he would have done with 20, decomposing it into 17+3. Then, the prospective teachers interpret that the student would have subtracted "*20 from 20, adding 31+3=34.*" However, mathematically, this reasoning is not consistent with what Edgar's record presents, since the operation 20+31=51, performed by Edgar, does not correspond to the decomposition of 51 into a sum of portions (20 and 31). This record is associated with a continuation of the register made previously (17+3=20), in which Edgar shows that he is adding quantities to 17 (lowest value in the subtraction), "completing it", until it reaches the largest quantity in the operation, 51.

In the case of the comment of the group of prospective teachers that found such decomposition, followed by a subtraction, seeking a mathematical validation (KPM – forms of validation) for this interpretation would clarify the non-correspondence of the procedures they consider to have been employed. In fact, from the expressions 20+31=51 (I) and 17+3=20 (II), if it were the case that Edgar had considered a decomposition in expression (I), seeking some correspondence with what the group interpreted from the production of the student, then, the expression II would have been used to effect a "substitution" of the value 20, which could have been recorded as: 20+31=51 and 17+3=20 → 17+3+31=51→3+31=51-17.

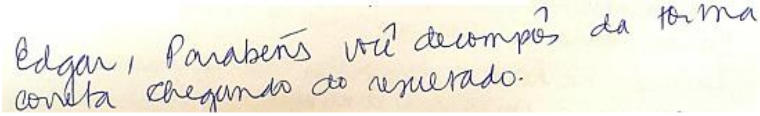
In general, the students seek to relate the procedures presented by Edgar to their own ways of understanding subtraction – only associated with the notion of "taking away" (a single element in their solution space) – and to a type of procedure linked to the traditional algorithm or the use of a type of representation associated only with the sense of taking away.

Regarding the question concerning providing feedback to the students' production, only three of the six groups presented what they considered to be a feedback. It was possible to determine, by the type of feedback they provided, that the prospective teachers did not activate a "hermeneutic listening" (Di Martino et al., 2016). The type of feedback provided by the groups ranged from a "reinterpretation" of the student's production – remaining at the descriptive or positive evaluation level, in correspondence with the students' own solution space – to a type of feedback that remained within the scope of "compliment", without focusing on the mathematical content involved, which proves ineffective for the expected process of (re)designing the students' learning pathways (Figure 8).

Figure 8

Production of the PTs to provide a feedback to the student

a)



Edgar, Congratulations you decomposed the correct way reaching the result.

b)

Edgar acertou a conta porém errou o
símbolo matemático, ao colocar o sinal de
soma ao invés de subtração, e ele organi
zou o raciocínio de forma confusa.

Edgar did the calculation correctly but the mathematical symbol was incorrect; by putting the plus sign instead of the subtraction sign he organised the reasoning in a confusing way.

c)

EDGAR, você chegou ao resultado esperado de modo interessante, a partir de um algarismo en
comum entre os números 17 e 51, e , da diferença entre eles, realizou a soma de 3 e 31. Também
importante estabelecer a conta com sua estrutura tradicional e estabelecer relações sobre a
unidades e dezenas de cada algarismo.

***EDGAR**, you reached the expected result in an interesting way, starting from a common digit between the numbers 17 and 51, and, from the difference between them, you added up 3 and 31. It is also important to establish the calculation with its traditional structure and to establish relations about the units and tens of each digit.*

We highlight, in particular, the comment of one of the groups in the feedback provided (Figure 8c), suggesting that the student resorted to a "*traditional structure"* in the procedure used to solve the operation. This comment is associated, on the one hand, with the way the prospective teachers themselves understand subtraction – just as taking away – and, on the other hand, with the type of procedure used, which does not correspond to what they have in their own solution space.

Therefore, there is further evidence that the single element of the prospective teachers' solution space does not allow them to attribute meaning to Edgar's answer and, consequently, does not allow them to develop a redesign for the new learning pathways for the student.

FINAL CONSIDERATIONS

The results show that prospective teachers' specialised knowledge (Carrillo et al., 2018) and solution space (Di Martino et al., 2016; Jakobsen et al., 2014) influences the content of their Interpretative Knowledge. Such influence reveals the importance of the content of PTs mathematical specialised knowledge and elements of solution space when interpreting and providing a feedback to students' productions (what they revealed about the meanings of subtraction; procedures and properties associated with the procedures associated to subtraction; procedures and registers of representation and connections with the addition operation). The fact that none of the PTs groups provided constructive feedback proves the need to develop a more refined type of "listening" that goes beyond what they expect as a response from students (Di Martino et al., 2016). This corresponds, among other aspects, to the development and expansion of the PTs' own solution space in mathematics teacher education.

The nature of the feedback provided by the prospective teachers, supported by the mathematical dimensions of their specialised knowledge, brings to the fore the need to expand their solution space and expand and deepen the aspects that make their specialised mathematical knowledge for teaching. Such expansion and development require considering the tasks for teacher education as being specialised for such an endeavour (e.g., Ribeiro et al., 2021a, 2021b) and to focus both on (prospective) teachers' mathematical and pedagogical knowledge, thus refining the focus that have been assumed mainly on the pedagogical dimensions, as we can now affirm that such foci did not improve students' mathematical knowledge, understanding, and results (e.g., Nye et al., 2004; Ribeiro, 2020; Ribeiro et al., 2021a).

In fact, Interpretative Knowledge, being based on (prospective) teachers' mathematical knowledge, requires teachers to be able to evaluate the students' productions critically – comments, questions – and, eventually, generalise the type of reasoning and the procedures used (KPM – forms of validation and generalisation); furthermore, recognise in the student's production potential different forms of representation (KoT – registers of representation) and connections with other topics or constructs – in this case, the connections between subtraction and addition beyond knowing they are inverse operations – (KSM – auxiliary connections). Only by mobilising the content of their own mathematical knowledge will teachers be able to advance from an "evaluative listening" to a "hermeneutical listening" (Di Martino et al., 2016) and, thus, propose a redesign of the teaching pathways aimed at students' mathematical learning.

The way (prospective) teachers understand their own role and knowledge for the work of teaching in each of the topics they will have to address shapes the pedagogical approaches they may use in their practice and the goals they pursue in such practices. These mathematical dimensions of teachers' specialised knowledge, and its content, influence the nature and focus of the provided feedback – focusing on reinforcing personal self-esteem or on *constructive feedback*, contributing to mathematical learning (Jakobsen et al., 2014). Thus, teacher education must prepare and implement tasks specifically designed to, and implemented in particular ways for, developing teachers' professional knowledge that can allow them to implement mathematical practices focusing on exploring and valuing the students' mathematical productions and associated reasonings, instead of just praising them for having reached the correct calculation result.

In this sense, the tasks for teacher education need to be of a nature and focus substantially different – but complementary – to the tasks for students (e.g., Policastro et al., 2017; Ribeiro, 2016; Ribeiro et al., 2021a). Considering that most of us cannot teach with different goals as the ones we have been taught in a specific moment, the need to focus explicitly and more intensively on developing the specialised mathematical dimensions of teacher's knowledge emerges. This change in how the work of teaching is seen demands that we recognise and assume the need to focus on teacher education (Ribeiro, Gibim, & Alves, 2021), unlike the perspective that has been in force for the last 30 years (at least). Thus, a shift of focus from the PCK dimensions to the specialised knowledge associated with interpreting students' productions is required because what has been done did not improve students' mathematical learning – see the results from national and international tests. Besides, we already know that this specialised knowledge grounding the interpretation does not develop over time in practice (Ribeiro, Mellone, & Jakobsen, 2013).

One dimension we have not discussed here, but that needs to be addressed, concerns the moments before the implementation of these tasks for teacher education – the Interpretative Tasks – in terms of the conceptualisation of such tasks and the teacher educators' knowledge required to implement them pursuing the educational goals associated. Tasks for teacher education and the development of teachers Interpretative Knowledge have intrinsic relationships. Thus, at least four research questions guiding our next steps – both in terms of research as well as educational practices - remain open:

(i) What are the levels of teachers' Interpretative Knowledge, and how can we develop such IK through the use of Interpretative tasks?

(ii) Which are the characteristics of the tasks for teacher education leading to steering change in teachers' mathematical practices and developing the specificities of teachers' knowledge?

(iii) What are the critical elements for designing teacher education tasks that will contribute to sustainable mathematical practices over time?

(iv) What are the critical elements of teacher educators' knowledge, and how can it be developed for them to prepare and implement teacher education tasks that promote effective knowledge development?

AUTHORS' CONTRIBUTIONS STATEMENTS

MR conceived the tasks' ideas presented. MR and MSP developed the theory and adapted the methodology to this context, performed the activities, and collected the data. MSP analysed the data in the first round. All authors actively participated in the discussion of the following rounds of analysis and results, reviewed, improved the theoretical discussion included and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data that support the results of this study will be made available by the corresponding author MR, or MSP, upon reasonable request via e-mail.

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