


Transitional-Apprehending Mental Model for Junior High School Students in Understanding the Concept of Integers

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ABSTRACT

Background: A concept that exists in students' minds and is used to describe and explain a phenomenon is called a mental model. **Objective:** This study aims to describe the transition-apprehending mental model of students in understanding the concept of integers. **Design:** This study used a qualitative approach and the type of research conducted was descriptive. **Setting and participants:** The subjects of this study were 35 students in grade VII Junior High School. Subjects were given a test to determine their understanding of the concept of integers. **Data collection and analysis:** data collection in this study, using test questions and interviews. Data analysis used five steps, namely 1) data transcoding; 2) reviewing data; 3) data reduction; 4) presenting data; 5) analyze the process of forming mental models; and 6) verifying the findings. **Results:** showed that grade VII students were at the transition-Apprehending level, as evidenced by students being able to compare negative integers and positive integers with the same magnitude symbol. **Conclusion:** based on the results of research that has been found, students already understand positive integers and negative integers using a number line.

Keywords: Mental models, Students, Concepts, Integers.

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Modelo mental de apreensão de transição para alunos do ensino médio na compreensão do conceito de números inteiros

RESUMO

Contexto: Um conceito que existe na mente dos alunos e é usado para descrever e explicar um fenômeno é chamado de modelo mental. Objetivo: este estudo tem como objetivo descrever o modelo mental de apreensão da transição de alunos na compreensão do conceito de inteiros. **Desenho:** Este estudo utilizou uma abordagem qualitativa e o tipo de pesquisa realizada foi descritiva. **Cenário e participantes:** Os sujeitos deste estudo foram 35 alunos do 7º ano do Ensino Médio. Os sujeitos foram submetidos a um teste para determinar sua compreensão do conceito de números inteiros. **Coleta e análise de dados:** coleta de dados neste estudo, por meio de perguntas-teste e entrevistas. A análise de dados usou cinco etapas, a saber: 1) transcodificação de dados; 2) revisão de dados; 3) redução de dados; 4) apresentar dados; 5) analisar o processo de formação de modelos mentais; e 6) verificar os achados. Resultados: mostraram que os alunos da VII série encontram-se no nível de transição-Apreensão, evidenciado pelo fato de os alunos poderem comparar inteiros negativos e inteiros positivos com o mesmo símbolo de magnitude. **Conclusão:** com base nos resultados da pesquisa encontrada, que os alunos já entendem inteiros positivos e inteiros negativos usando uma reta numérica.

Palavras-chave: Modelos mentais, Alunos, Conceitos, Inteiros.

INTRODUCTION

Understanding the concept is very important to be instilled in students; this is the first goal in learning mathematics (Permendiknas no.22 of 2016). Mathematical understanding is placing information or knowledge that is being studied and then linking it with previously owned knowledge (Hiebert & Carpenter, 1992). In studying mathematics, an understanding of concepts is needed to continuously become new knowledge for students (Subanji, 2015; Karadeniz et al., 2017). In addition, Duffin & Simpson (2000) explain that the concept can be interpreted as the ability of students to re-express what has been communicated to them. The concept exists in a person so there is a conceptual change. Research on conceptual change has been reviewed by several researchers (Pimthong, 2015; Hall et al., 2012). Pimthong (2015) explains that conceptual change is a person's new understanding resulting from a rational process as a substitute for ideas that already exist in one's mind. Meanwhile, Hall et al. (2012) explained that conceptual. Change is part of individual and collective professional learning. Conceptual change can be initiated when someone conducts discussions between groups (Hall et al., 2012). Students need conceptual changes to understand concepts in depth to be helpful in their

lives (Pebriyanti et al., 2015). Meanwhile, according to Sagala (2016) that understanding mathematical concepts is the ability to carry out mental activities in the form of actions (actions), objects (objects), and schemes (schema) when constructing the concept, as well as the ability to memorize and draw conclusions from specific rules into mathematical relationships that more general.

Anderson (2001) explains that students can understand a concept by constructing meaning from oral, written, or image communication. An idea exists in students' minds and is used to describe and explain a phenomenon called a mental model (Vosniadou & Brewer, 1992; Jansoon, 2009; NRC, 2012). Because the formation of mental models is influenced by the experience and prior knowledge of students, their attitudes and beliefs, and the problems they face. Regarding learning, mental models are built through a process of observation, interpretation, explanation, imagination, and understanding of scientific discourse (Chittleborough, 2007; Jansoon, 2009). Therefore, mental models are fundamental in learning knowledge and require deeper understanding (NRC, 2012).

Mental models can show students' ways of thinking. This has been investigated by several researchers (Senge, 2007; Solaz-Portoles & Lopez, 2007; Wang, 2007). Senge (2004) describes a mental model as a person's internal images and thoughts, images that limit a person to act. Solaz-Portoles & Lopez (2007) explain that mental models describe individual reasoning processes in problem-solving. A person is often unaware of realizing his mental model or the influence of mental models on one's behaviour. Meanwhile, Wang (2007) states that education generally requires research that studies mental models by focusing on conceptual aspects.

Mental model research can be found in many scientific studies. Among them is the study of cognitive psychology, about how humans understand, learn, remember, and think about information (Stenberg, 2008). The study of science, especially in physics, is to construct, revise, and justify mental models that are built by themselves, not just adapting mental models imposed by others and taken for granted (Lehrer, 2009). A study in mathematics discusses understanding the concept of negative integers (Bofferding, 2014). Furthermore, Utami et al. (2018) studied mathematics, which addressed the understanding of integers. In addition to the many findings of mental model research in several studies, mental model research also has several levels that several previous researchers have investigated. The levelling is divided into specific groups according to the results of each researcher. Vosniadou (1992)

divides mental models into three levels: initial mental models, synthetic mental models, and formal mental models. Barsalou (1992) levelled the mental model into two, namely the structural mental.

Model and the conceptual mental model. Vosniadou & Brewer (1994) divides mental models into three levels: initial mental models, synthetic mental models, and scientific mental models. Coll, R. K., & Treagust, D. F. (2008) also explained that scientific mental models could be in the form of oral, written, and image. Park & Light (2009) levelled the mental model into five levels: the initial mental model that was formless or unclear, the intermediate mental model 1, the intermediate mental model 2, the intermediate mental model 3, and the target mental model. Then Bofferding (2014) divides the mental model into five levels, namely the initial mental model, the mental transition I model, the synthetic mental model, the mental transition II model, and the formal mental model. Furthermore, Utami et al. (2018) developed the mental model into six levels: the pre-initial mental model, the initial mental model, the transition I mental model, the synthetic mental model, the mental transition II model, and the formal mental model.

In contrast to the many studies of mental models in material science, it turns out that there are only a few studies of mental models in the field of mathematics. Among them is the study of students' mental models on understanding the concept of integers by Bofferding (2014) and Utami et al. (2018). Mental models are essential to do, let alone refine students' mental models in understanding the concept of integers. This is because the construction of mental models is the core of meaningful learning; besides that, refinement of the level of mental models is needed because it accommodates all levels of mental models that exist in students in understanding the concept of integers. By knowing the mental model of students in understanding the concept of integers by their understanding, the teacher will more easily determine the right strategy for carrying out learning.

Several researchers have researched integers (Karantzis, 2010; Bofferding, 2014; Enzinger & Tobias, 2015; Utami et al., 2018; Nicole, 2019; & Sari et al., 2020). Karantzis (2010) researched addition and subtraction with the same two digits. Bofferding (2014) examines the comparison of two negative integers and sorting negative integers. Enzinger & Tobias (2015) researched the addition and subtraction of positive integers and negative integers, except for the expansion of positive integers and positive integers. Then Utami et al. (2018) investigated the sequence of negative integers on the number line, compared negative integers (−) and positive integers (+), and

compared two positive integers (+). Furthermore, (Nicole 2019; Sari et al., 2020) researched addition and subtraction operations on integers.

Based on research on integers that previous researchers have carried out, none of these researchers have discussed anything related to comparing negative integers and positive integers with the same magnitude symbol and comparing two negative integers with the same magnitude symbol. This mental model research needs to accommodate and provide a forum for students who have a different understanding from previous theories. In addition, the mental model is hoped to be used to develop assessments and detail the categories of mental models. Based on this, it is necessary to research mental models to understand the concept of integers that can accommodate all levels of students' understanding of integers. So, it is required to do a pre-research first to answer the research assumptions.

The researcher conducted pre-research on Junior High School students in Indonesia's Maduran sub-district. Furthermore, the researchers analyzed the results of the answers to the pre-research that had been carried out, and the results obtained that several students experienced mental models, including four students who participated in the pre-initial mental model level, two students who experienced the initial mental model level, two students who experienced the initial mental model level. Mental transition II model, and one student who shared a formal mental model. Students in the pre-initial mental model can only understand the concept of positive integers but do not understand the concept of negative integers. While the initial mental model, the student understands that negative integers are the same as positive integers. Furthermore, students in the mental transition II model can already understand the order of integers from the largest to the smallest and vice versa. Students who are at the level of formal mental models, these students can sort integers on the number line, compare positive integers and negative integers, and sort positive and negative integers from the smallest to the largest and vice versa.

Based on the results of the pre-research that has been done, the researcher found that there were students who were able to answer correctly when comparing positive integers and negative integers for the same magnitude symbol and explained that positive integers were always greater than negative integers. In addition, the student can compare two negative integers with the same magnitude symbol. In terms of mathematical conditions, the student is classified as very good with the ability of his mental model Chiras (2008). The mental model can be known by looking at the students' thinking processes in building concepts (Subanji, 2015).

The pre-research results that have been analysed show that there are still students whose mental models cannot be classified at the levels that have been developed by Utami et al. (2018). Because in certain aspects, the student already understands the comparison of positive integers and negative integers and can produce almost exact final categories. However, the subject cannot be said to be at the level of the mental transition I model or the synthetic mental model. However, they have been able to compare positive integers and negative integers with the same magnitude symbol. To accommodate these conditions, the researcher intends to study deeper by adding a mental model level that refers to the mental model of Utami et al. (2018). The part that will be developed is to insert one level between the transition-I mental model level and the synthetic mental model, called the transition apprehending mental model.

Thus, the theoretical level indicator of the mental model still needs improvement because it turns out that not all students can be categorized into the mental model level that the researcher has made. This study focused on students' understanding of the concept of integers when comparing positive integers and negative integers with the same magnitude symbol and comparing two negative integers with the same magnitude symbol. The purpose of this study was to describe the level of mental model transition apprehending junior high school students in understanding the concept of integers.

METHODOLOGY

This study was inspired by the research of Utami et al. (2019), who investigated the pre-initial mental model of first-semester students related to the function concept material. However, this study investigates junior high school students understanding the concept of integers. The approach used in this research is using a qualitative approach. In his study, Sugiono (2016) explained that a qualitative approach is based on phenomenology and constructivism paradigms to develop science. At the same time, the type of research used in this study is to use a phenomenological approach. According to Creswell (2015), the phenomenological approach is a method researchers use to find out phenomena that occur naturally. The phenomenological method emphasizes more on human experience and how humans interpret their experiences. The phenomenological approach can describe the meaning of life experiences for several people about a concept or phenomenon. People who are involved in dealing with a phenomenon explore the awareness of a life experience they face.

Participants

The subjects of this research were 35 students (14 boys and 21 girls) of Junior High School in Lamongan district, Indonesia. The issues of this study were seventh-grade students who had obtained the whole number material. This material was chosen because integers are universal. In addition, this material has been studied by all students while sitting in elementary school.

Furthermore, all subjects were given a test consisting of the first two questions related to comparing positive and negative integers for symbols of the same magnitude in obtaining scores of children playing games. Second, the subject is required to put a "<" (less than), ">" (more than), or "=" (equal) sign in the comparison of two integers and be accompanied by a reason for the answer.

From a total of 35 students' answers, 28 students can compare positive and negative integers for the same quantity symbol, and seven students cannot compare positive and negative integers for the same quantity symbol. Of the 28 who were able to compare positive and negative integers for the same magnitude symbol, it was seen that eight students had good communication, so interviews were conducted to gain an in-depth understanding of the concept of integers. This can be shown in Table 1.

Table 1

Classification of Student Answers

No	Total Students	Student's answers
1.	28	Students can compare negative integers and positive integers using the number line
2.	7	Students can compare negative integers and positive integers using the number line.

Based on the seven answers, students cannot compare negative integers and positive integers; there are two different reasons. The reasons can be seen in the following table.

Table 2*Reasons for Student Answers*

No	Total Students	Student's answers
1.	5	Both integers have the same magnitude, so $-11 = 11$
2.	2	It is difficult to distinguish the signs " $<$ " (less than) and " $>$ " (more than), so $-13 > 13$ and $7 < -7$.

Data Analysis

The process of data analysis in this study was carried out in steps (Huberman & Miles, 2012; & Creswell, 2015): (1) Transcoding the data collected, the transcribed data were the results of tests and interviews with the subject in this study; (2) We are reviewing available data from test results and interview transcripts; (3) Perform data reduction by selecting, focusing, and classifying similar data, then simplifying it by removing unnecessary things. The researcher selects the data generated from the test by the indicators that have been previously decided, then classifies the results based on the level of the mental transition model apprehending students in understanding the concept of integers and discarding unnecessary things; (4) presenting research data. In this step, the researcher presents. Research results from tests of students who are at the level of mental transition apprehending models as well as results of interviews to explain the results of their answers; (5) analyzing the process of forming the mental transition model arresting students' models in understanding the concept of integers and then connecting them to a constructive framework of reflective-abstraction knowledge mechanisms, (6) verifying findings and drawing conclusions. In this step, the researcher confirms the results, the mental transition model apprehending students understand the concept of integers, then concludes.

This study intends to investigate how students build their knowledge in understanding integers according to Piaget's framework. Piaget (in Gray and Tall, 2007) state that the development of cognitive structures is caused by reflective abstraction. To explain the theory and relate concepts in mathematics, Arnon et al. (2014) said that there are four constructions in reflective abstraction, namely interiorization, coordination, encapsulation, and generalization. The definition of construction in this study is presented in Table 3.

Table 3*Definition of Construction in Research* (Adapted from Arnon et al., 2014)

Construction Process	Definition of Construction
Interiorization	Thinking to explore enough information (differentiating positive and negative integers).
Coordination	Builds a new process from two or more other processes (Replaces integers into other forms of equations)
Encapsulation	Building mental objects from mental processes (drawing on a number line)
Generalization	Applying existing schemas to a wider set of phenomena (inferring with signs less than " $<$ ", greater than " $>$ ", and equal to " $=$ ")

Researchers observed student activities when completing the test questions that had been given. Furthermore, researchers conducted interviews with selected subjects. The data in this study were obtained from test results and transcripts of interviews with selected topics. In this study, the results of video interviews and written test results were collected. To ensure the validity and reliability of the study, data analysis was carried out by triangulation by displaying video results and comparing them with students' written test results. See in detail the results of the video interviews and written student test results for knowing the level of transition-apprehending students' mental model in understanding the concept of integers after determining the characteristics of the transition-apprehending mental model, then processing the data according to the characteristics of students in understanding the initial idea of integers. This categorization is an essential step in data analysis because it facilitates data interpretation. This categorization comes from a literature review, identifying each interview answer and student test results that represent the literature review concept.

RESULTS AND ANALISES

This study discusses the level of mental models that have been previously studied by Bofferding (2014) and Utami et al. (2018). Furthermore, in this study, researchers developed a mental model of Utami et al. (2018) to understand the concept of integers.

Research result

The following explains each answer that shows the level of mental model transition apprehending students in understanding the concept of integers and has gone through reflective abstraction. In the students' answers when answering the first question, we present four subjects who experienced the mental transition apprehending model and represent each of the four reflective abstraction processes.

The first question aims to compare positive integers and negative integers. Following are the answers of Subject 1 (S1), who experienced a transition apprehending mental model that has gone through the interiorization process, and the explanation is shown in Figure 1 below.

Figure 1

Answer subject 1

1. Dua anak bermain sebuah permainan dan mencoba memperoleh skor tertinggi
- a. Bayun mendapatkan skor 4 sedangkan zyan mendapatkan skor -4 . Siapa pemenangnya? Berikan alasannya.
- Jawab : Bayun

Translation:

- a. Bayun got a score of 4 while Zyan got a score of -4 . Who is the winter? Give the reason.
- Answer: Bayun

In picture 1 above, S1 answered that the winner was Bayun. It can be seen that S1 understands the purpose of the questions given without an explanation as to why Bayun is the winner, not Zyan after the researchers dig deeper information. Subjects can give reasons that positive integers are always greater than negative integers. This can be seen in the following interview transcript:

P: *Why do you answer this question, so! (pointing to the question) Who will be the winner between Bayun or Zyan?*

S1 : *This is sir, (pointing to the answer sheet) The one who got the highest score was Bayun!*

P : *Why is that? Can you give a reason!*

S1 : *The winner is Bayun. The reason is because Bayun scored 4, while 4 is a positive integer. Next Zyan gets a score of -4, but -4 is a negative integer!*

P : *Do you know the difference between positive and negative integers?*

S1 : *Yes... sir you know!*

P : *Can you explain it?*

S1 : *Yes...sir, the positive integers consist of 1,2,3,4,5,... and so on. Whereas negative integers are -1, -2, -3, -4, -5,... and so on, so positive integers are always greater than negative integers.*

P : *Is it the same with this question? (while pointing to the next question)*

S1 : *Yes, sir, the same!*

For the next question, S1 also experienced a transition apprehending the mental model, leading to the coordination process when comparing positive integers and negative integers with the same magnitude symbol. In contrast to Figure 1 above, for the following question, in this process, students organize their understanding structure related to the concept of integers by connecting the answers to questions accompanied by explanations of the answers. The answers can be seen in Figure 2.

Figure 2

Subject answer 1

b. Tika mendapatkan skor 10 sedangkan Yanti mendapat skor -10. Siapa pemenangnya? Berikan alasannya.
Jawab: Tika, karena skor 10 itu lebih besar dari pada skor -10

Translation:

b. Tika got a score of 10, while Yanti got -10. Who is the winner? Give the reason.

Answer: Tika because a score of 10 is greater than a score of -10

Furthermore, S1 also experiences a transition apprehending mental model which goes through an encapsulation process when comparing two negative integers with the same magnitude symbol. The answers can be seen in Figure 3.

Figure 3

Subject answer 1

c. Amin mendapatkan skor -8 sedangkan Alfin mendapatkan skor -8 . Siapa pemenangnya? Berikan alasannya. $-8 = -8$
Jawab :

Translation:

- c. Amin got a score of -8 , while Alfin got -8 . Who's the winner? Give the reason.

Answer: $-8 = -8$

It is the same with the answers in Figure 1 and Figure 2, as shown in Figure 3 that S1 has also been right in determining the winner between Amin and Alfin. S1 gives the reason why Amin and Alfin are not the winners because the scores obtained by both are the same, which is a negative integer -8 . It can be seen that S1 has gone through an encapsulation process that can determine the type of the two numbers of the acquisition score, namely negative integers so that the correct symbol is " $=$ ". The following interview transcript also supports this:

P : *Why do you put an equal sign like this? (pointing to the answer)*

S1 : *Yes... because both of their scores are the same, namely negative integers, sir; so the correct symbol is " $=$ ".*

In the next stage, subject 1 (S1) also undergoes a generalization process that requires understanding the concept by concluding the end when comparing two positive integers. So the winners are both Cika and Nesa because they get the same score. The answers can be seen in Figure 4.

Figure 4

Subject answer 1

- d. Cika mendapatkan skor 15 sedangkan Nesa mendapatkan skor 15. Siapa pemenangnya? Berikan alasannya.
Jawab: cika dan nesa, karena skor nya sama

Translation:

- d. Cika got a score of 5, while Nesa got 15. Who is the winner? Give the reason.
Answer: Cika and Nesa are the winners because their scores are the same.

From the test results and interview transcripts, it can be seen that S1 has used four constructs in reflective abstraction. Subject 1 (S1) tried to dig up information on the given problem, which compares positive integers and negative integers, two negative integers, and two positive integers for the same magnitude symbol. After S1 understands and reads the problems, it gives. Furthermore, subject 1 (S1) carried out the encapsulation process, which students by setting the fitting symbol which is the same as " $=$ ". This can be seen when subject 1 (S1) compares two negative integers. S1 understands that when comparing two equal integers, the symbol used is the same as " $=$ ".

Subject 1 (S1) can understand the problem given by writing down what is known and requested. Subject 1 (S1) can also describe strategies for solving the problem of comparing negative integers and positive integers on the same magnitude symbol well. In Subject 2 (S2) and an explanation of the answers is shown in Figure 5 below.

Figure 5

Subject answer 2

- Dua anak bermain sebuah permainan dan mencoba memperoleh skor tertinggi
- a. Bayan mendapatkan skor 4 sedangkan zyan mendapatkan skor -4. Siapa pemenangnya? Berikan alasannya.
Jawab: $4 > -4$ Karena Bayan skornya paling terbesar dari pada zyan
- b. Tika mendapatkan skor 10 sedangkan Yanti mendapatkan skor -10. Siapa pemenangnya? Berikan alasannya.
Jawab: $10 > -10$ Karena Tika skornya paling besar dari pada Yanti

Translation: Two kids are playing and trying to get the highest score.

- a. Banyu gets a score of 4, while Zyan gets -4. Who's the winner? Give the reason.

Answer: $4 > -4$. Because Bayun's score is bigger than Zyan.

- b. Tika got a score of 10, while Yanti got -10. Who's the winner? Give the reason.

Answer: $10 > -10$. Because Tika's score was bigger than Yanti.

In Figure 5, it can be seen that subject 2 (S2) has gone through coordination by composing two forms of explanation. In a way, subject 2 (S2) is arranged by giving the symbol more than " $>$ " and less than " $<$ ", and accompanied by the reason. Subjects answered that 4 was greater than -4 , and 10 was more significant than -10 . Furthermore, he gives the sense that Bayun's score is higher than Zyan's, which is true for Tika's and Yanti's scores. This can be seen in the following interview transcript:

P : *Why is 4 greater than -4 and 10 greater than -10 ?*
(pointing to the answer)

S2 : *Because 4 and 10 are positive integers while -4 and -10 are negative integers.*

P : *Are positive integers always greater than negative integers?*

S2 : *Yes sir.*

Next, subject 2 (S2) goes to the encapsulation process when comparing two negative integers with the same magnitude symbol. The answers can be seen in Figure 6.

Figure 6

Subject answer 2

c. Amin mendapatkan skor -8 sedangkan Alfin mendapatkan skor -8 . Siapa pemenangnya? Berikan alasannya.
Jawab: $-8 = -8$

d. Cika mendapatkan skor 15 sedangkan Neza mendapatkan skor 15. Siapa pemenangnya? Berikan alasannya.
Jawab: $15 = 15$

Translation:

- c. Amin got a score of -8 , while Alfin got -8 . Who's the winner? Give the reason
Answer: $-8 = -8$
- d. Cika got a score of 15, while Neza got 15. Who is the winner? Give the reason.
Answer: $15 = 15$

Similar to the answer to picture 3, in picture 6, it is also seen that subject 2 (S2) has correctly determined the winner between Amin and Alfin, Cika and Nesa. S2 gives the reason why Amin and Alfin are not the winners because the scores obtained by both are the same, which is a negative integer -8 . Likewise, when comparing two positive integers. It can be seen that S2 has gone through an encapsulation process that can find out the types of the two scores on integers. So setting the correct symbol is " $=$ ". The following interview transcript also supports this:

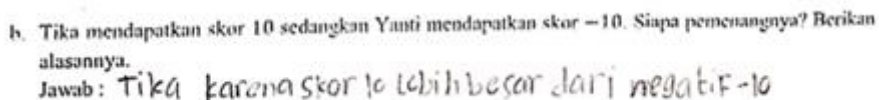
P : *Why do you put an equal sign like this? (pointing to the answer)*

S2 : *Because the two scores are numbers with the same magnitude symbol, so the correct one is " $=$ ".*

Furthermore, the answers to subject 3 (S3) and their explanations are shown in Figure 7.

Figure 7

Subject answer 3



b. Tika mendapatkan skor 10 sedangkan Yanti mendapatkan skor -10 . Siapa pemenangnya? Berikan alasannya.
Jawab: Tika karena skor 10 lebih besar dari negatif -10

Translation:

b. Tika got a score of 10, while Yanti got a score of -10 . Who's the winner? Give the reason.

Answer: Tika, because a score of 10 is greater than negative -10 .

In Figure 7 above, it can be seen that S3 has undergone encapsulation after the researchers dug deeper information. It is the same with subject 2 (S2) in figure 5. Subject 3 (S3) also gives the reason that positive integers are always greater than negative integers. This can be seen in the following interview transcript:

P : *Why do you answer this question, so! (pointing to the question) Who got the highest score between Tika and Yanti?*

S3 : *The highest score is Tika sir!*

P : *Why is that? Can you give a reason!*

S3 : *Because Tika's score is bigger than Yanti's score which -10 is a negative integer!*

P : *Are positive integers always greater than negative integers?*

S3 : *Yes... sir!*

In the next question, subject 3 (S3) can answer the question correctly while the answers and explanations are shown in Figure 8.

Figure 8

Subject answer 3

c. Amin mendapatkan skor -8 sedangkan Alfin mendapatkan skor -8. Siapa pemenangnya? Berikan alasannya.
jawab : seri. Karna sama sama mendapatkan skor -8 dan berada di titik yang sama

Translation:

- c. Amin got a score of -8, while Alfin got a score of -8. Who's the winner? Give the reason.

Answer: A tie, because they both get a score of -8, and if the number line is at the same point

Subject 3 (S3) can determine no winner between Amin and Alfin, so both are a draw. It can be seen that subject 3 (S3) experienced generalization, which was able to know that the two scores were at the same point. After the researcher dug deeply regarding the answer, subject 3 (S3) gave why he described it on a number line. The completeness can be seen in the following interview transcript:

P : *Can you explain this answer? (pointing to the answer)
What do you mean by being at the same point?*

S3 : *I mean that, because if they are on the number line, then Amin and Alfin scores are at the same point, sir, which is -8!*

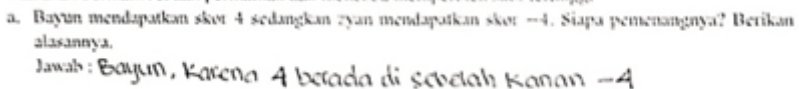
P : *Are you sure about your answer?*

S3 : *Yes sir I'm sure*

Furthermore, the answers to subject 4 (S4) and their explanations are shown in Figure 9.

Figure 9

Subject answer 4



a. Bayun mendapatkan skor 4 sedangkan zyan mendapatkan skor -4. Siapa pemenangnya? Berikan alasannya.
Jawab: Bayun, karena 4 berada di sebelah kanan -4

Translation:

- a. Bayun got a score of 4, while Zyan got a score of -4. Who's the winner? Give the reason.

Answer: Bayun, because 4 is to the right of -4 in the number line

Subject 4 quickly determines that 4 is to the right of -4. It can be seen that subject 4 experiences generalizations that can determine who the winner is. After the researcher dug deeply into the answer, subject 4 gave the reason he described it on the number line. This can be seen in the following interview transcript:

P : *Can you explain this answer? (pointing to the answer)
What do you mean 4 is on the right of -4!*

S4 : *I mean that, because if it is on the number line, then 4 is to the right of -4 sir!*

P : *Then how do you determine that Bayun is the winner?*

S4 : *Because Bayun's 4 scores are on the right of Zyan's -4 score, so Bayun's 4 score is bigger than Zyan's -4 score.*

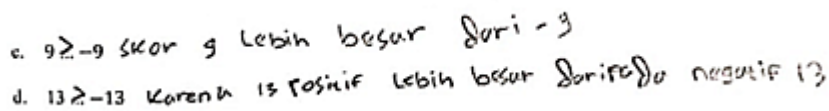
In the results of the student's answer to the first question, the mental transition model for students in understanding integers is seen from the reflective abstraction process that each subject has represented. Furthermore, when answering the question in number two, the issue of who experienced the mental transition apprehending model was also reviewed from the reflective abstraction process he experienced. From each of the four deliberative

abstraction processes, one subject was chosen to be explored in-depth on the answers given to conduct an interview.

The following answers to subject 5 (S5) and their explanations are shown in Figure 10.

Figure 10

Subject answer 5



c. $9 > -9$ skor 9 lebih besar dari -9
d. $13 > -13$ karena 13 positif lebih besar daripada negatif 13

Translation:

- c. $9 > -9$, Because 9 is greater than -9
- d. $13 > -13$ Because positive 13 is greater than negative 13

In Figure 10, it can be seen that subject 5 (S5) has undergone interiorization after the researchers dug deeper information. Subject 5 (S5) in Figure 10 understands the problem that has been given that positive integers are more significant than negative integers. So subject 5 (S5) quickly reads that 9 is more critical than -9 , and 13 is more powerful than -13 . This can be seen in the following interview transcript:

P : *Why did you choose the bigger symbol “>”? (while pointing to the answer)*

S5 : *Since 9 is a positive integer and -9 is a negative integer, so 9 is greater than -9 pack.*

P : *Are you sure about your answer?*

S5 : *Yes sir, I'm sure.*

Furthermore, subject 5 (S5) also experienced a process of coordination. When comparing positive integers and negative integers with the same magnitude symbol. In contrast to Figure 10 above, subject 5 (S5) organizes the structure of his understanding related to the concept of integers by connecting the answers to the questions accompanied by explanations for the answers. The answers can be seen in Figure 11.

Figure 11

Subject answer 5

- b. Tika mendapatkan skor 10 sedangkan Yanti mendapatkan skor -10 . Siapa pemenangnya? Berikan alasannya. $10 > -10$
- Jawab: KARENA Tika mendapatkan skor yg banyak dari pada skor Yanti -10
- c. Amin mendapatkan skor -8 sedangkan Alfin mendapatkan skor -8 . Siapa pemenangnya? Berikan

Translation:

- b. Tika got a score of 10, while Yanti got a score of -10 . Who is the winner? Give the reason.

Answer: $10 > -10$ because Tika got a bigger score than Yanti's score -10

Continued on subject 6 (S6), which tries to compare positive integers and negative integers by drawing a number line to solve the problems it faces. This can be seen in Figure 12.

Figure 12

Subject answer 6

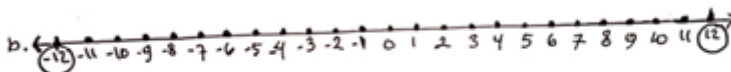


Figure 12 shows subject 6 (S6), building a new concept by making a number line. So that subject 6 (S6) easily determines the ratio between -12 and 12 . Subject 6 (S6) builds the concept that 12 is to the right of -12 on the number line. So on the number line the greater the right, so subject 6 (S6) easily determines that $-12 < 12$. This can be seen in the following interview transcript:

P : Which is greater between -12 and 12 ?

S6 : Bigger 12 pack!

P : Why is that?

S6 : Because 12 is to the right of -12 on the number line.

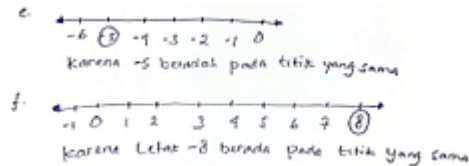
P : Why did you answer this question using a number line?
(pointing to the answer)

S6 : *Because it is easier and faster to determine the greater and lesser of two integers.*

Next, subject 6 (S6) tries to coordinate the difference between positive and negative integers by making a number line. This can be seen in Figure 13.

Figure 13

Subject Answer 6



Translation:

- e. Because -5 is at the same point
- f. Because 8 is at the same point

Figure 13 shows S6, building a new concept by creating a number line. So that subject 6 (S6) easily decides comparing -5 with -5 , and 8 with 8 . S6 starts constructing the concept because -5 with -5 , and 8 with 8 lie on the same point on the number line. Then the value obtained is equal to, so S6 easily determines that $-5 = -5$, and $8 = 8$.

Furthermore, subject 7 (S7) experienced a mental transition apprehending model in the encapsulation process of subject 7 (S7) answer's and the explanation is shown in Figure 14.

In Figure 14, it can be seen that the S7 has experienced encapsulation. After the researchers dig deeper information. Subject 7 (S7) in figure 6 determines that -7 is less than 7 , -12 is less than 12 , 9 is greater than -9 , and so on and so easily establishes that $-7 < 7$, $-12 < 12$, $9 > -9$, etc. This can be seen in the following interview transcript:

P : *Why do you answer that?* (pointing to the answer)

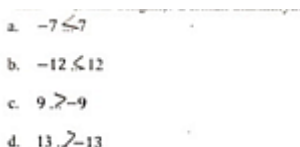
S7 : *Yes, sir.*

P : So how? Can you explain it!

S7 : -7 is a negative integer and 7 is a positive integer, so -7 is less than 7 . Likewise this one sir (while pointing to the answers to b, c, and d)

Figure 14

Subject answer 7

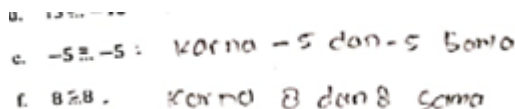


a. $-7 < 7$
b. $-12 < 12$
c. $9 > -9$
d. $13 > -13$

Then, subject 7 (S7) underwent construction in the process of generalizing the subject's answers, which can be seen in Figure 15.

Figure 15

Subject answer 7



e. $-5 = -5$ karena -5 dan -5 sama
f. $8 = 8$ karena 8 dan 8 sama

Translation:

- e. $-5 = -5$ Because -5 and -5 are the same
- f. $8 = 8$ Because 8 and 8 are the same

Subject 7 (S7) was able to know that both integers had the same value. After the researcher asks how do you know that the two are the same. S7 reaffirms that on the number line both are at the same point so that the symbol " $=$ ". Subject 7 (S7) illustrates the comparison of two negative integers and two positive integers using a number line. So that subject 7 (S7) underwent a generalization process, which applies the number line as a solution in determining the ratio of two integers for the same quantity symbol. Based on the results of Figure 15 made by S7, he tries to generalize by concluding that

two negative integers and two positive integers have the same value. This is also confirmed in the following interview transcripts:

P : *Why do you answer this question, then? (while pointing to the question)*

S7 : *Yes, sir!*

P : *Why is that? Can you give a reason?*

S7 : *Because -5 and -5 are both negative integers, sir!*

P : *How do you know that they are both negative integers?*

S7 : *both have a negative symbol ($-$) before the number 5 pack!*

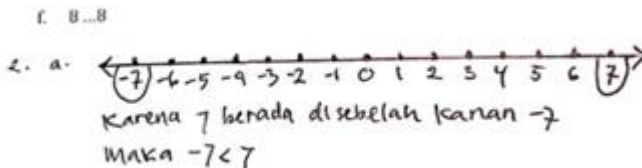
P : *Is there another way to explain it?*

S7 : *The two numbers are at the same point on the number line*

Furthermore, subject 8 (S8) experienced a mental transition apprehending model in the generalization process of the subject's answer, which can be seen in Figure 16.

Figure 16

Subject answer 8



Translation :

Since 7 is to the right of -7 , then $-7 < 7$

Subject (S8) seems to illustrate the comparison of these positive and negative integers using a number line. So that subject 8 (S8) underwent a generalization process. which applies the number line as a solution in determining the ratio of two integers for the same quantity symbol. After the researcher asked why he used the number line like that. He simply replied that the negative integer -7 is to the left of the positive integer 7. Based on the results of Figure 12 made by S8, he tried to generalize by concluding that

negative integers are less than positive integers, so $-7 < 7$. This can be seen in the following interview transcript:

P : *Why do you answer this question, so! (pointing to the problem) Which is bigger than -7 by 7 ?*

S8 : *Bigger 7 pack!*

P : *Why is that? Can you give a reason!*

S8 : *Since -7 is to the left of the positive integer 7 , so -7 is less than 7 !*

P : *Do you know where the positive integers and negative integers are on the number line?*

S8 : *Yes... sir you know!*

P : *Can you explain it?*

S8 : *The location of positive integers on the number line is to the right of the zero point, while negative integers are to the left of the zero point.*

Unlike the previous subjects, subject 8 (S8) also underwent an encapsulation process. After the researchers dig deeper information. Subject 8 (S8) in Figure 17 can distinguish positive integers and negative integers, after constructing using the symbol “ = ” subject 8 (S8) explains that -5 with -5 are both negative integers and 8 with 8 are both numbers. positive round. The complete explanation related to the answers to subject 8 (S8) can be seen in the following interview transcript:

P : *Why do you answer that? (pointing to the answer)*

S7 : *Yes, sir.*

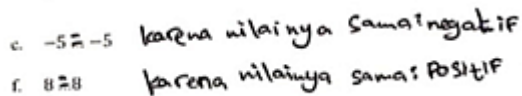
P : *So how? Can you explain it!*

S7 : *Since -5 and -5 are both negative integers, then 8 by 8 are both positive integers. (pointing to the answer)*

The explanation related to the interview is supported in Figure 17.

Figure 17

Subject answer 7



e. $-5 = -5$ karena nilainya sama: negatif
f. $8 = 8$ karena nilainya sama: positif

Translation :

e. $-5 = -5$. Because the value is the same as negative

f. $8 = 8$. Because the value is the same as positive

DISCUSSION

The research results by Vosniadou & Brewer (1992) show that there are three levels of students' mental models in understanding the concept of the shape of the earth, namely initial mental models, synthetic mental models, and formal mental models. Then Bofferding (2014) developed a Vosniadou (1992) level mental model from three levels to a five-level mental model in understanding the concept of negative integers by adding a transition mental model level I between the initial mental model level and synthetic mental model and adding a transition mental model level. II between the groups of artificial mental models and formal mental models. Furthermore, Utami et al. (2018) developed the five mental models into six mental models, namely the pre-initial mental model; Initial mental models; transition I mental model; Synthetic mental models, transition II mental model; and formal mental models.

In this study, the researcher refers to the level of the mental model developed by Utami et al. (2018); the researcher is to look for updates from the story of the mental model that several previous researchers have extended. However, based on the results of the research that has been done, the researchers found that there were students whose mental models were different from the level of the mental models that had been developed. These students cannot be classified at the stories of the mental model that has been developed by Utami et al. (2018). In certain aspects, they already understand the comparison of positive integers and negative integers and can produce final and almost exact categories, but cannot be categorized at the transition I mental model or synthetic mental model. Although they could compare positive integers and negative integers with the same magnitude symbol, they were also able to compare two negative integers with the same magnitude symbol. To

accommodate these conditions, the researcher breaks down the transition I mental model level into a mental transition apprehending model.

The term mental model level refers to Bofferding (2014) and Utami et al. (2018), who breaks down the transition I mental model. The subject has not been consistent in identifying a particular concept. The term apprehending has the meaning of understanding. Then the speculative theory at the level of an apprehending mental model is that the subject can understand a particular concept so that students can compare negative integers and positive integers with the same magnitude symbol and compare two negative integers with the same magnitude symbol.

The emergence of a new mental model level is because the concept of integers in Junior High School is discussed more broadly than in Elementary School. Based on the theoretical study and the research results, the researchers developed indicators regarding students' mathematical mental models in understanding the concept of integers. In the form of mental models at this level, children can understand a particular image. Subjects can understand negative integers and positive integers. Indicators at the level of mental model transition apprehending subject can solve problems comparing negative integers and positive integers for the same magnitude symbol.

At the Elementary School level, integer material is essential to learn at the Junior High School level. When understanding integer material, students must have the ability to understand a concept. A prerequisite for understanding the idea of ordering positive integers and negative integers, students must understand the concept of comparing positive integers with negative integers. Teaching the concept of integers to students should be associated with real contexts to help students understand integers (Akyus & Stephan, 2012; Jessica, 2016). So the researchers designed a contextual test of questions, namely comparing the highest score scores obtained from two children playing games. And compare two integers by giving the symbol " $>$ " more than and less than " $>$ ". The researcher found that most students could compare negative and positive integers with the same quantity symbol. From the results of this analysis and study, it will be possible for students to be able to compare whole numbers with different quantity symbols (Sukiyanto et al, 2022).

Based on the research results, students who are at the level of mental transition apprehending models have tried to carry out a reflective abstraction process to describe the process of understanding the concept of integers. The abstraction ability in understanding concepts is also essential, in line with this according to Goodson-Espy (2005), which states that the results of a person's

mental abstraction are schemes used to understand things, find solutions, or solve problems. Judging from the research results, subject 1 (S1) has undergone four constructions in reflective abstraction, namely Interiorization, Coordination, Encapsulation, and Generalization. Subject 2 (S2) has experienced two structures in reflective abstraction, namely Coordination and Encapsulation. Subject 3 (S3) has experienced two buildings in reflective abstraction, namely encapsulation and Generalization. Subject 4 (S4) only experienced one construction, namely Generalization. Subject 5 (S5) has experienced two buildings in reflective abstraction, namely Interiorization and Coordination. Subject 6 (S6) only experienced one construction in Coordination reflective abstraction. Subject 7 (S7) and 8 (S8) only experienced two buildings in reflective abstraction, namely Encapsulation and Generalization.

Subject 1 (S1), in Interiorization, is the construction of internal processes to understand the perceived phenomenon. The activity of thinking in this construction is the activity of digging up the necessary information. Subject 1 (S1) already understands the problem and explores information about comparing positive integers and negative integers with the same magnitude symbol and comparing two negative integers with the same magnitude symbol. A process is said to have been interiorized if the subject has compared two integers with the same magnitude symbol but without thinking about it during the process. In the results of research that has been carried out, subject 1 (S1) has been able to understand the comparison of two integers with the same magnitude symbol and determines that positive integers are always greater than negative integers. Subject 1 (S1) can determine that the highest score between 4 and -4 is a positive integer, which is 4. So that subject 1 (S1) can know that Bayun is the winner. It can be seen that subject 1 (S1) already understands positive integers. According to Whitacre et al. (2012), to understand the concept of positive and negative integers, it takes a very long time to equate the knowledge obtained with the experience previously obtained.

In addition, subject 1 (S1) has also determined the winning score between Amin and Alfin. So subject 1 (S1) can choose that $-8 = -8$, because both are negative integers. In the activity of understanding a situation, students often connect the move to the following understanding case. So that students will be more creative in dealing with and solving further problems. This can be seen from the completion of the following subject 1 (S1), S1 quickly determines that $-8 = -8$. In this step, S1 can compare positive integers and negative integers and can compare two negative integers. In this step, the abstraction activity is carried out in full. The interiorization stage plays a vital role in the

subsequent stages because the next stage can only be carried out if the interiorization stage has occurred.

After the interiorization process was passed, some subjects continued to the second process, namely Coordination, a composition consisting of two or more methods for reconstructing a new technique or coordinating the results of interiorization. At this stage, subject 1 (S1) is building a new concept, namely the completion process, by describing it in the form of a number line, even without explaining the answer sheet. However, subject 2 (S2) can provide new ideas or concepts that the number line as an alternative solution in determining the comparison of two integers.

In the next stage, students carry out the encapsulation process. Subjects 3 (S3) and 7 (S7) began to understand and solve problems by distinguishing between two integers. The subject constructs mental objects from mental processes and concludes two comparisons of positive integers and negative integers using the symbols more diminutive than " $<$ ", more than " $>$ ", and equal to " $=$ ". Furthermore, students who try to do a generalization process by using a number line as a solution to determine more than " $>$ " and less than " $<$ " when comparing positive integers and negative integers on the same magnitude symbol, and comparing two negative integers on characters the same magnitude. The study results were found in subject 4 (S4) and subject 8 (S8). Subject 4 (S4) compared the scores obtained by two children who were playing games. At the same time, subject 8 (S8) reached the negative integer -7 with the positive integer 7 . Using the number line, subject 8 (S8) quickly found out where -7 was to the left of 7 , and vice versa, 7 was to the right of -7 at the number line. Thompson & Dreyfus (1988) stated that the number line could improve students' understanding of integers. Furthermore, according to Stephan & Akyuz (2012) which explains that the number line can also be used to interpret operations on integers. In addition, number lines can also be used to sort integers (Stephan & Akyuz: 2012). Peled et al. (1989) concluded that students used a number line mental model to support their thinking about negative integers.

The results of this study, students who are still at the mental transition apprehending models have tried to carry out the interiorization process by extracting information from the problem. Students at the change apprehending mental model level assume that positive integers are always greater than negative integers. After the interiorization process, students proceed to the coordination process by replacing more diminutive than " $<$ ", more than " $>$ ", and equal to " $=$ ". Students carry out the encapsulation process by

representing the answer into a number line in the next stage. After making a number line, students then continue the generalization process by concluding that -4 is to the left of the integer 4 on the number line.

Based on the knowledge construction process through the reflective abstraction mechanism, the characteristics of students at the level of transition apprehending mental model are that the subject understands the difference between negative integers and positive integers for symbols of the same magnitude. The issue substitutes the comparison of negative integers and positive integers by using a number line as a process of understanding. The subject makes the conclusion by using the sign less than " $<$ ", more than " $>$ ", and equal to " $=$ ".

CONCLUSION

Based on the results of the research on the transition-apprehending mental model of class VII students, it was concluded that the transition-apprehending mental model could be seen from the information held by students stored in long-term memory before they were confronted with the concept. It can be seen that students already understand positive integers and negative integers using a number line. The characteristics of students' mental models in understanding the concept of integers are appropriate in using the sign less than " $<$ ", more than " $>$ ", and equal to " $=$ ". The researcher recommends for future researchers, so that they can dig deeper into whether other levels of mental models have the possibility to be developed. This is related to before the synthetic mental model there is a level that precedes it, namely the transition-apprehending mental model level.

AUTHORS' CONTRIBUTIONS STATEMENTS

All authors, Sy, TN, Sm, and IMS, have contributed to the design and development of this study and to the writing of this manuscript.

DATA AVAILABILITY STATEMENT

The data presented and supporting this research results are available at a reasonable request to the corresponding author, TN.

REFERENCES

- Akyuz, D., Stephan, M., & Dixon, J. K. (2012). The role of the teacher in supporting imagery in understanding integers. *Education and Science*, 37(163), 268-282.
- Arnon, I., Cottrill, J., Dubinsky, E., Oktac, A., Fuentes, S.R., Trigueros, M., & Weller, K., 2014. *APOS Theory. A Framework for Research and Curriculum Development in Mathematics Education*. Springer.
- Barsalou, L.W. (1992). *Cognitive Psychology: An Overview for Cognitive Scientist*. Lawrence Erlbaum. (410 p.).
- Bofferding, L. (2014). Negative Integer Understanding: Characterizing First Graders' Mental Models. *Journal for Research in Mathematics Education*. 45(2), 194-245.
- Chittleborough, G.D. & Treagust D. F. (2007). The modeling ability of non-major chemistry students and their understanding of the sub-microscopic level. *Chem. Educ. Res. Pract.*, 8, 274-292.
- Coll, R.K., (2008). Chemistry learners' preferred mental models for chemical bonding. *Journal of Turkish Science Education*, 5(1), 22 – 47.
- Creswell, J. W. (2015). *Qualitative research and research design: choosing from five approaches*. Pustaka Belajar.
- Enzinger, N. & Tobias, J.M. (2015). Preservice teachers' temperature stories for integer addition and subtraction. *Proceedings of 39th Psychology of Mathematics Education conference* (vol. 4, pp. 289-296). PME.
- Gray, E. & Tall, D. 2007. Abstraction as a Natural Process of Mental Compression. *Mathematics Education Research Journal*, 19(2), 23-40.
- Jansoon, N. (2009). Understanding Mental Models of Dilution in Thai Students. *International Journal of Environmental & Science Education*, 4(2), 147-168.
- Pierson Bishop, J., Lamb, Lisa L., Philipp, Randolph A., Whitacre, Ian, & Schappelle, Bonnie P. (2016) Leveraging Structure: Logical Necessity in the Context of Integer Arithmetic. *Mathematical Thinking and Learning*, 18(3), 209-232.
<https://doi.org/10.1080/10986065.2016.1183091>

- Karantzis, I. (2010). Mental arithmetic calculation in the addition and subtraction of two-digit numbers: the case of third and fourth-grade elementary school pupils. *International Journal of Mathematics in Education*, 3, 3-24.
- Lamb, L. L., Bishop, J. P., Phillip, R. A., Schappelle, B. P., Whitacre, I., & Lewis, M. L. (2012). Developing symbol sense for the minus sign. *Mathematics Teaching in the Middle School*, 18(1), 5–9.
- Liebeck, P. (1990). Scores and forfeits: An intuitive model for integer arithmetic. *Educational Studies in Mathematics*, 21(3), 221–239. <https://doi.org/10.1007/BF00305091>
- National Research Council. (2012). A framework for K-12 science education: Practices, crosscutting concepts, and core ideas. National Academies Press.
- Park, E.J. & Light, G. (2009). Identifying atomic structure as a threshold concept: student mental models and troublesomeness. *International Journal of Science Education*, 31(2): 233–258.
- Peled, I., Mukhopadhyay, S., & Resnick, L. B. (1989). Formal and informal sources of mental models for negative numbers. In G. Vergnaud, J. Rogalski, & M. Artique (Eds.), *Proceedings of the 13th Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 106–110).
- Paris, France. promote relational understanding of chemical reactions. *International Journal of Science Education*, 34(7): 973–998.
- Senge, P.M. (2004). *“The Fifth Discipline. The Art and Practice of The Learning Organization*. Doubleday Dell. (405 pages).
- Solaz-Portolés, J. J. & Lopez, V.S. (2007). Representations in problem-solving inscience: Directions for practice. *Asia-Pacific Forum on Science Learning and Teaching*, 8(2), 4.
- Subanji. (2015). *Teori kesalahan konstruksi konsep dan pemecahan masalah matematika* (The theory of concept construction error and mathematics problem solving). Universitas Negeri Malang.
- Sukiyanto, Nusantara, T., Sudirman, S., & Sulandra, I. M. (2022). Model mental transition-attempting: The case of beginners students in understanding the concept of integers. *Pegem Journal of Education*

and Instruction, 12(4), 81–88.

<https://doi.org/10.47750/pegegog.12.04.09>

- Sunyono & Yulianti, D. (2015). Introductory study on student's mental models in understanding the concept of atomic structure (case study on high school students in lampung indonesia). *The Online Journal of New Horizons in Education*, 5(4), 41-50.
- Utami, A. D., Sa'dijah, C., Subanji, & Irawati, S. (2018). Six Levels of Indonesian Primary School Students' Mental Model in Comprehending the Concept of Integer. *International Journal of Instruction*, 11(4), 29-44. <https://doi.org/10.12973/iji.2018.1143a>
- Utami, A. D., Sa'dijah, C., Subanji, & Irawati, S.. (2019). Students' Pre-Initial Mental Model: The Case of Indonesian First-Year of College Students. *International Journal of Instruction*, 12(1), 1173-1188. <https://doi.org/10.29333/iji.2019.12175a>
- Vosniadou, S., & Brewer, W. F. (1992). Mental models of the earth: A study of conceptual change in childhood. *Cognitive Psychology*, 24(4), 535–585. [https://doi.org/10.1016/0010-0285\(92\)90018-W](https://doi.org/10.1016/0010-0285(92)90018-W)
- Vosniadou, S. & Brewer, W. F. (1994). Mental Models of the Day/Night Cycl. *Cognitive Science* 18, 123-183.
- Wang, Ch. Y., (2007). *The role of mental modelling ability, content knowledge, and mental model in general chemistry students' understanding about molecular polarity*. Dissertation presented to the Faculty of the Graduate School University of Missouri Columbia.