ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.7024

Appealing to Creativity Through Solving and Posing Problems in Mathematics Class

Paulo Miranda [a]
Ema Mamede [a]

[a] CIEC, Universidade do Minho, Braga, Portugal

Received for publication 9 Mar. 2022. Accepted after review 22 May 2022
Designated editor: Claudia Lisete Oliveira Groenwald

ABSTRACT

Background: Creativity should be a key issue in mathematics learning. However, mathematics class rarely provides opportunities for students to experience it. Problem solving and posing can play a leading role in promoting creative thinking in mathematics. **Objectives:** This study aims to have an insight into 6th-graders understanding of problem solving and posing, analyse their solving strategies, their ability to pose problems, and their difficulties when doing so. **Design:** Qualitative methods were used in a case study approach. An intervention of five sessions comprising five problem-solving and four problem-posing tasks was implemented in mathematics class. **Setting and Participants:** Participants were thirty 6th-graders (11–12-year-olds) from a public supported school in Braga (Portugal). **Data collection and analysis:** Data collection used photographs, audio recordings, students' written productions, and researcher field notes. **Results:** Students conceptualised strategies such as building schemas and tables, solving from the end to the beginning, making attempts, and reducing to a simpler problem. Students faced problem posing positively, creating problems adjusted to the requirements, with a wide variety of creative contexts. Students' difficulties in problem solving rely on the interpretation of statements, recognition of previous similar problems, and mathematical communication; on problem posing, difficulties regarding the complexity of the formulated problems and a weak diversity of problems were identified. **Conclusions:** Problem solving and posing tasks can promote mathematical creativity and knowledge, therefore should be used more often in mathematics class, allowing the construction of solid mathematical skills and enthusiasm.

Keywords: Creativity in mathematics; Problem-solving; Problem-posing.

Corresponding author: Paulo Miranda. Email: paulomfmiranda@gmail.com

Apelando à criatividade através da resolução e formulação de problemas na aula de Matemática

RESUMO

Contexto: A criatividade é uma componente fundamental da aprendizagem matemática. No entanto, apenas esporadicamente são dadas oportunidades aos alunos para a desenvolver. A resolução e a formulação de problemas podem apresentar-se como uma resposta a esta lacuna. **Objetivos:** Este estudo procura perceber como alunos do 6.º ano compreendem a resolução e formulação de problemas. Analisam-se as suas estratégias de resolução, a capacidade de formular problemas e as suas dificuldades na resolução e formulação de problemas. **Design:** Utilizou-se uma metodologia qualitativa, numa abordagem de estudo de caso. Desenvolveu-se uma intervenção com cinco sessões, em aulas de matemática, compreendendo cinco tarefas de resolução e quatro de formulação de problemas. **Ambiente e Participantes:** Participaram trinta alunos do 6.º ano (11-12 anos) de uma escola pública em Braga (Portugal). **Coleta e análise de dados:** Utilizaram-se fotografias, gravações de áudio, produções escritas dos alunos e notas de campo do investigador. **Resultados:** Os alunos concetualizaram várias estratégias, como construir esquemas/tabelas, resolver do fim para o princípio, tentativas com conjetura e redução a um problema mais simples. Encararam a formulação positivamente, criando problemas ajustados aos requisitos, com uma grande variedade de contextos criativos. Na resolução de problemas, foram identificadas dificuldades na interpretação de enunciados, no contacto prévio com problemas semelhantes e na comunicação matemática; na formulação surgiram dificuldades na complexidade dos problemas formulados e na diversidade de tipologias. **Conclusões:** A resolução e a formulação de problemas podem promover a criatividade e o conhecimento matemático, pelo que devem ser utilizadas com mais frequência nas aulas de matemática.

Palavras-chave: criatividade em matemática; resolução de problemas; formulação de problemas.

INTRODUCTION

Contrary to what prevails in common sense, creativity is not just a privilege of the arts or people associated with the arts. Creative thinking can and, desirably, should be encouraged and demonstrated in all curriculum areas, as long as the pedagogical approach allows the expression of creative thinking and imagination (Kampylis & Berki, 2014; Kozlowski & Si, 2019).

Intellectually and scientifically, mathematics is considered one of the academic areas in which creativity should play a preponderant role (Ayllón, Gómez, & Ballesta-Claver, 2016; Silver, 1997). However, although a genuine mathematical activity is closely intertwined with creativity, the school environment seldom provides opportunities for students to experience this aspect (Ayllón, Gómez, & Ballesta-Claver, 2016; Silver, 1997).

In this paper, creativity can be understood as the conception of original ideas to produce something and creative thinking as the ability to consider something in a new way, and both cannot be taught directly (Kampylis & Berki, 2014). Still, a conscious and intentional educational practice can provide the means, opportunities, and a fertile environment for students' creative minds to flourish. School mathematics is not limited to calculation. There are concepts, representations, procedures, and processes, which can manifest themselves in different ways, oral and written, each of which has its own time and space (Ponte, 2003). Thus, it is essential to adopt behaviours that promote creative thinking, such as actively encouraging students to question, make connections, predict and explore ideas, and encourage and reward imagination and originality (Pound & Lee, 2011).

Problem solving and problem posing can play a leading role in the operationalisation of this change in perspective (Boavida et al., 2008; Dante, 2009; Kilpatrick, 1987; National Council of Teachers of Mathematics [NCTM], 2007, 2014; Palhares, 1997; Pólya, 1995; Silver, 1997; Vale & Pimentel, 2004; Van Harpen & Presmeg, 2013, Kozlowski & Si, 2019), enabling students to develop a new view on the construction of active, engaging, and creative mathematical learning. Through this process, it becomes possible to build a learning environment that is close and relevant to students, promoting interest and motivation for mathematics and the scientific process (Liel & Bayer, 2016).

This article aims to have insight into how 6^{th}-graders understand problem-solving and posing, identifying their solving strategies and their abilities to formulate problems. It addresses three questions: 1) What strategies do students use to solve problems? 2) How do students understand problem-posing? 3) What difficulties do students reveal when solving and posing problems?

THEORETICAL BACKGROUND

Mathematics and creativity

The elaboration of a consensual concept of creativity has not proved to be a feasible task for the academic community, which per se attests to its difficulty (Agić & Rešić, 2015; Akgul & Kahveci, 2016; Aktaş, 2016; Pound & Lee, 2011) and the need for investment in studies in this area.

One of the most used definitions, suggested by the National Advisory Committee on Creative and Cultural Education, defines creativity as an imaginative activity designed to produce original and valuable results (National Advisory Committee on Creative and Cultural Education [NACCCE], 1999). To this end, one associates developing a cognitive process that seeks new (Kandemir & Gür, 2007) innovative and insightful processes, which allow creating and solving new problems, or conceptualising new solutions and perspectives for solving known problems.

Creativity is closely related to the ability to persist, determination, and a risk-taker attitude. Gardner (2009) emphasises this perspective, highlighting that the students must feel safe and comfortable enough to take risks and not fear failure, as this process hides the key to innovation. In the same context, Kampylis and Berki (2014) add that students are more likely to express their creative potential when involved in meaningful, authentic, and intellectually challenging activities that suit their interests and abilities. Students also recognise the importance of creativity in mathematics, as shown in a study with 10^{th} to 12^{th}- graders (Lee, Kim & Lim, 2021), in which they felt they lacked opportunities to work creatively.

Focusing on creative learning in mathematics, Pound and Lee (2011) suggest that it emerges as a way of thinking, an innovative way of elaborating reasoning that allows us to look at a given process through a new prism. The same authors defend a set of characteristics for constructing the profile of a creative student (Pound & Lee, 2009), essential for promoting mathematical creativity. Some examples are adopting an attitude that allows taking risks in bold productions, flexibility in thinking, commitment, teamwork, and a stimulus to the practice of conjectural thinking, that is, constructing action schemes that enhance the constant search for the 'what if?'.

To promote this profile, it is essential that teachers consciously promote an open and welcoming atmosphere of innovative processes and reasoning, and develop tasks that promote autonomy and the creative spirit (Bezerra, Gontijo & Fonseca, 2021; Haddad, 2012; Kozlowski & Si, 2019). It is also relevant to look for ways to make learnings more relevant for students (Bezerra, Gontijo & Fonseca, 2021; Liel & Bayer, 2016; Schoevers et al., 2019) by conceptualising themes close to students' daily lives and reality and, when possible, integrating different curricular areas.

Problem-solving

Problem solving has been asserted as a fundamental competence in today's societies, becoming indispensable for constructing a complete profile of children (Pound & Lee, 2011), as students and as citizens.

George Pólya, considered as the father of the current study of problem solving, recognised this capacity as the specific achievement of intelligence (1981), explaining that problem solving is related to the ability to circumvent an obstacle, to take an indirect path to a resolution when no direct path becomes evident (1981).

Problem solving, as well as its posing, has emerged over the years as a natural vector for the development of mathematical learning (Boavida et al., 2008; Dante, 2009; Kilpatrick, 1987; Liel & Bayer, 2016; Palhares, 1997; Pólya, 1995; Silver, 1997; Vale & Pimentel, 2004; Van Harpen & Presmeg, 2013). They are crucial for developing mathematical reasoning and communication, presenting the relevance of mathematics in students' daily lives. Moreover, they can be catalysts for the construction of a positive attitude (Boavida et al., 2008; Dante 2009; Krulik & Rudnick, 1993; Liel & Bayer, 2016; Lupinacci & Botin, 2004; Mamede, 2009; Palhares, 1997), which can be essential in the development of students' creative skills.

Regarding problem solving, it is vital to highlight the relevance of the typology of problems and the strategies used in each study, which may vary depending on the authors and the specificity they intend to emphasise. Regarding the first, the use of the typology presented by Boavida et al. (2008) categorises problems into three groups: calculation problems, promoters of opportunities to apply previously learned concepts and skills; process problems, which enhance the development of more complex and creative solving strategies, not solvable only by the selection of appropriate operations. These can be used to develop different skills, introduce different concepts, or apply previously learned knowledge and mathematical procedures; and open problems, also called investigations, which can present more than one route to the solution and more than one correct answer. Through these, students have to explore regularities and formulate conjectures, thus appealing to reasoning, critical thinking, and reflection capacity.

Concerning problem solving strategies, the perspective of Vale and Pimentel (2004) is highlighted, who suggest the following: discover a pattern/rule or building law, explaining that this strategy focuses on specific steps of the problem and the solution is found by generalisations of specific solutions; make attempts/conjecture, suggesting that the solution has to be guessed according to the data of the problem, and confirm or not the conditions of the problem; working from the end to the beginning, starting the process of resolution by the end or by what one wants to prove; using logical deduction/doing elimination, facing all hypotheses and eliminating one by one, those that are not possible; reduce to a more straightforward problem/simplification, solving a particular case of a problem; make a simulation/experimentation/dramatization, using objects, creating models or dramatizing a situation that translates the problem to be solved; make a drawing, diagram, graph or schematic; make an organised list or table.

Still regarding resolution strategies, and as Boavida et al. (2008) and Dante (2009) emphasise, the teacher must provide tasks that enhance the emergence of strategies, underlining the importance of the typology of problems used, leaving it to the students to discover and build new action schemes. As they become familiar with the effectiveness of a series of strategies, they are more likely to choose the strategy that has worked best for them to solve a similar problem in the past, becoming more proficient (Hopkins, Russo & Siegler, 2020). However, the final role of identification and systematisation to cement the learning developed cannot be devalued.

Regarding assessment, in the context of problem solving, it is essential to diversify experiences and assessment tools (Ponte, 2005, 2008; Krulik and Rudnick, 1993). Students themselves can participate in this assessment process, carrying out their self-assessment and reflecting on the assessment carried out by the teacher (Ponte, 2005). Krulik and Rudnick (1993) highlight three fundamental instruments so that it is possible to monitor and reflect on the learning developed: observations, diaries or reflective paragraphs and tests. Concerning observations, the teacher's active role is emphasised, as a mediator of learning, elaborating guiding questions and taking mental notes about the students' behaviour for future reflection (Krulik & Rudnick, 1993). Regarding diaries or reflective paragraphs, the metacognitive processes associated with this procedure stand out, which allow teachers and students to understand better the reasoning elaborated in solving problems (Krulik & Rudnick, 1993). Finally, about tests, the importance of questions in which students are asked to solve problems and, if possible, explain their reasoning is emphasised (Krulik & Rudnick, 1993).

Problem-posing

Silver (1997) considers that problem posing can refer to either the creation of new problems or the reformulation of a given problem. Palhares (1997) also states that this occurs when an individual invents or discovers a problem, which may arise in articulation with problem solving (Boavida et al., 2008; Dante, 2009).

According to Pólya (1995), Kilpatrick (1987), Dante (2009), Oliveira and Santana (2013), and Van Harpen and Presmeg, (2013) the articulation between solving and posing problems enhance the success of the mathematical learning process, contributing positively to the development of problem-solving skills while promoting the deepening of the mathematical concepts involved, stimulating thought and reasoning.

Regarding typology, Stoyanova and Ellerton (1996), and later Stoyanova (1998), identify three categories of problem posing: free situations, in which students formulate problems without restrictions and or indications; semi-structured situations, where students pose problems similar to others they know or based on figures, diagrams or other types of indicated data; structured situations, when students create problems by reformulating problems they have already solved or changing conditions or questions of a problematic situation they already know. The development of diversified tasks enhances the construction of new learnings (Stoyanova, 1998; Stoyanova & Ellerton, 1996), being essential that these are always adequate and personalised to the context.

Regarding problem-posing strategies, several authors have proposed different typologies. Abu-Elwan (2002) starts from the structure of the typology of problems to list different solving strategies. In free situations, the following is proposed: invent a simple or more complex problem; build a problem for a math/test competition; come up with an enjoyable problem. In semi-structured situations, it is proposed the elaboration of open problems, for example, mathematical investigations; problems similar to a given problem; problems with situations similar to a previous problem; problems related to specific theorems; problems derived from figures; word problems. In relation to structured situations, it is possible to modify the statement and propose a new problem or keep the data and change what is requested.

Boavida et al. (2008) provided a more accessible view of problem posing strategies, concentrating the different strategies on two global types: Accepting data and *What if?*. The first suggests the formulation based on a specific static situation, such as a definition, a condition, an object, a figure, a table, etc.; the second encourages a formulation based on a concrete situation, in which its properties are identified, one of them is denied, and then questions are asked which, in turn, can give rise to the denial of another property and more questions.

Concerning the assessment of problem posing, the general assessment criteria postulated by Silver (1997) and by Silver and Cai (2005) stand out, with three categories that can be used to analyse the creative productions of students (Leikin, Koichu & Berman, 2009): fluency, or quantity, flexibility, or complexity and originality. Pinheiro and Vale (2013) interpret that fluency corresponds to the number of problems raised that fit the task requirements, in the sense of the number of problems and situations idealised by students who respect the requested requirements (Silver & Cai, 2005); flexibility corresponds to the number of different types of problems posed (Pinheiro & Vale, 2013), from a perspective of evaluating the complexity of formulations, both from a perspective of different types of posed problems, as well as in the mathematical or linguistic complexity that these can demonstrate (Silver & Cai, 2005); it originality corresponds to the number of problems raised that are unique or rare (Pinheiro & Vale, 2013), identifying cases of formulations that stand out as atypical or non-obvious, in the face of a task common to a group (Silver & Cai, 2005).

Problem-solving and posing in mathematics class

Problem-solving and posing play an essential role in stimulating mathematical reasoning and communication and as a catalyst for building a subject's positive attitude (Boavida et al., 2008; Dante 2009; Pound & Lee, 2011). These make mathematical learning more appealing, active, and dynamic, becoming challenging vectors for students (Pound & Lee, 2011).

The ability to create or pose and solve problems must be at the core of the construction of students' mathematical learning (Pound & Lee, 2011; Liel & Bayer, 2016; Van Harpen & Presmeg, 2013). Students should be given opportunities to develop procedures through iterative processes in problem-solving tasks, providing prospects for collective negotiation of what constitutes an appropriate and efficient strategy (Choppin, McDuffie, Drake & Davis, 2020). Thus, problem solving and posing emerge as unique means of jointly promoting mathematical and creative skills. Problem-solving practices should be challenging enough to leverage the construction of new learnings but should not assume too much difficulty. Therefore, the student must acquire as much experience through independent work as possible, not being left unaided or insufficient assistance to experience progress (Pólya, 1995). Thus, it is up to the teacher to adapt their teaching practice to encourage students in mathematics classes to become competent in formulating and solving problems (Bezerra, Gontijo & Fonseca, 2021; Oliveira & Santana, 2013; Vale, Pimentel & Barbosa, 2015).

Mamede adds that the teacher must discuss the processes and solutions found by the students, providing them with opportunities to confront their strategies, results, and reasoning involved in solving problems (Mamede, 2009). Asking children to explain their reasoning allows them to promote an unusual and valuable way of thinking, reasoning, and metacognition but can take time to develop (Boavida et al., 2008). Another related point is the importance of utterance interpretation skills, which are fundamental for problem solving (Costa & Fonseca, 2009), and which should be recurrent protagonists in the tasks performed by students.

In the context of problem solving and posing in the curriculum in Portugal, the importance of problem solving, reasoning, and mathematical communication skills are emphasised, emerging objectives and learning practices that relate these components to the different themes and contents to be addressed (see DGE, 2018). At the international level, documents that emphasise the relevance of problem solving as a fundamental process for mathematical learning are highlighted (NCTM, 2007, 2014), including the application of mathematics to everyday situations (Cockcroft, 1982; NCTM, 2014).

The literature already presents some studies on the scope of solving and posing problems in elementary school. Pinheiro and Vale (2013) studied the development of mathematical creativity through solving and formulating problems in a 5^{th}-grade class (10-11year-olds), finding out that students were very receptive to open tasks, showing enthusiasm and interest. Regarding the formulation of problems, they report that students were not used to this type of task. Some cases revealed disorganised statements that lacked information and were difficult to understand.

Martins (2016) carried out a study on problem posing with 4^{th}- and 5^{th}-graders (9-11-years-old) to analyse the types of problems formulated, students' ideas of problem posing, and the characteristics of creativity inherent in the formulations. The author found out that the students formulated problems evolved in complexity, creating plausible situations and contexts, and that problem formulation enhances the development of mathematical creativity.

Miranda (2019) also studied the resolution and posing of problems in two classes, 1st- and 6th-grade (6–7-year-olds and 11–12-year-olds), through an exploratory study, in a systematic approach to this type of task. The author observed that the students demonstrated new skills and competencies for solving and posing problems, building a more open and positive perspective in this area.

METHODOLOGY

The research used qualitative methods (Bogdan & Bicklen, 2013) in a case study approach (Yin, 2014) to understand and analyse students' performance in problem-solving and problem-posing tasks.

The participants were thirty 6th-graders (11–12-year-olds) from a public school in Braga (Portugal), integrated into an essentially urban environment. For reasons of ethics and confidentiality, all participants' names in this study are fictitious.[1]

The intervention comprised five sessions, taking as a global theme the Sustainable Development Goals (SDG), recommended by the United Nations (UN) (UN, 2015), which was chosen based on the interests of students and integration of the discipline of mathematics with the subject of Education for Citizenship. Problem-solving tasks were proposed, using basically process and open problems (see Boavida et al., 2008), enhancing the discovery and exploration of different problem-solving strategies and problem posing through structured and semi-structured situations (see Stoyanova & Ellerton, 1996; Stoyanova, 1998). When structuring the intervention, there was a concern to contextualise tasks and integrate them into what the students were addressing in the mathematics class, articulating the resolution and posing problems with the content taught.

Five problem-solving tasks and four problem-posing tasks were developed, with individual, pair, and group solving situations being provided assuming a diverse, motivating character, enhancing the spirit of research,

[1] All those responsible for the participants signed a Free and Informed Consent Form (ICF), and ethical evaluation was waived by the appropriate councils of the research project from which this work arises, assuming and explicitly exempting Acta Scientiae from any consequences arising therefrom, including full assistance and eventual compensation for any damage resulting to any of the research participants, in accordance with Resolution No. 510, of April 7, 2016, of the National Health Council of Brazil.

creativity, motivation for mathematics, and autonomy and cooperation. During task resolutions, students were supported by the researcher, one of the authors of this article, allowing them to learn through action. At all times, students were free to interact with each other and with the researcher.

Data collection was carried out with photographs, audio recordings, students' written productions, and field notes from the researcher.

RESULTS AND ANALYSES

This section presents the results obtained in an intervention in which nine tasks were applied, distributed over five sessions on solving and posing problems in the 6^{th} grade.

Session 1

The first session began with an introduction to the global theme chosen for the interventions in the class, the Sustainable Development Goals. These were outlined by the United Nations (UN) (2015) and proposed by the Ministry of Education of Portugal (DGE, 2015) as a possible subject to be addressed across different curriculum areas. Being a sufficiently broad theme, close to the students, and enhancing the construction of problematic statements with meaning, this was the starting point for the different activities in the mathematics class.

After a brief presentation (Figure 1) and viewing a video on the topic, we proposed a brainstorming on what they have learned and the importance of the subject. When asked, among other contributions, Pedro, one of the participants, managed to systematise his learning, stating that these goals were "sustainable development goals, in various areas of society, that the UN intends to develop with the goal in 2030".

Then, three tasks were elaborated: two related to problem solving and one related to problem posing. The first task was to solve a calculation problem (Figure 2) based on the contents that were being developed by the class. Thus, and realising that these were based on calculating areas and perimeters of figures, a problem related to the two contents was proposed. This problem also provided a straightforward approach to the different phases of Pólya's problem-solving method.

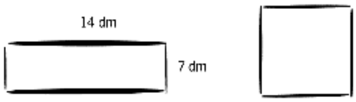
Figure 1

Presentation of the Sustainable Development Goals to the class.

Figure 2

Statement of the first problem in session 1. (Adapted from the Mathematics National Assessment Test, 2nd Cycle, 2001)

Mr. José cultivates a plot of land in an urban vegetable garden in Braga. He decided to organize his garden into squares. Your neighbour chose rectangles, with the following measurements:



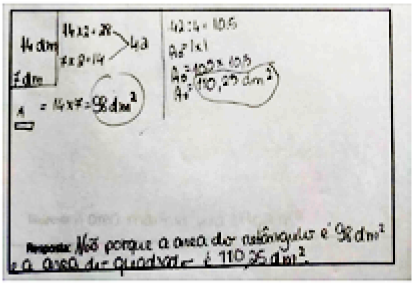
Mr. José was pensive. In a conversation with the neighbour, he discovered that the two figures had the same perimeter.
Do they also have the same area? Justify your answer.

After a class debate about the solving method, each student could solve the problem individually (Figure 3). Subsequently, the resolutions were debated on the board in a group class.

Analysing the problem-solving examples, the strategies used did not vary as all students presented the same type of reasoning. In fact, being a computation problem, it did not enhance a great diversity of strategies. However, it took on the role of introducing and contextualising the theme of problem solving in students' daily lives, making a bridge between routine and innovation situations.

Figure 3

Problem solving performed by Amelia.

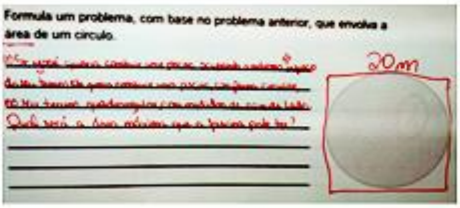


No, because the area of the rectangle is $8dm^2$ and the area of the square is $110.25dm^2$.

Then, students were challenged to pose problems. The task of formulating problems emerged as the first approach to the topic, which was why it was elaborated on in the group class (Figure 4). From the previous problem, in a structured situation, we suggested the creation of a new problem involving the area of a circle. The students jointly discussed the elaboration of the statement and the situation involved, reaching a consensus in the end.

Figure 4

The joint problem posed by the class.



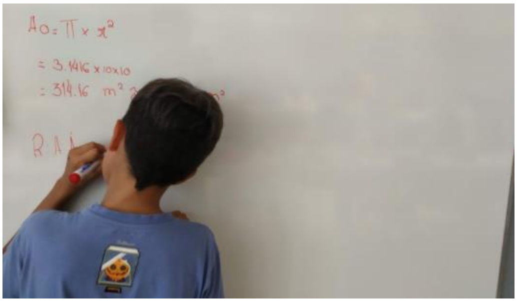
Mr. José wanted to build a swimming pool occupying as much space as possible on his land. He wanted to build a swimming pool, with a circular shape in its quadrangular terrain, measuring 20m on the side. What will be the maximum area that the pool can have?

In the first phase, the students reflected on the topic that the problem would address. Since the statement indicated that the formulation should involve the circle area, the students started the debate by deciding on the context in which it would be inserted. It was decided that the circle would represent “a swimming pool”, so this should be placed on the land. At this stage, two suggestions emerged: João suggested that the land should be square, while Miguel thought about a rectangular shape. Note the confusion between the terms and concepts of quadrangular and square, which emerged in the formulation process. The class preferred João's proposal, having decided on the shape and measure on the side of the square. Then, the problem question was decided, and only the suggestion of calculating the possible maximum pool area emerged. Finally, the conceptualisation of the particular situation was carried out, involving "Mr José" and his desire to build a swimming pool "with a circular shape".

In the posing process, some doubts arose since the students were not used to performing problem posing tasks, such as the different elements needed for the formulation, the data that needed to be present in the utterance, or the adequate sentence construction. Thus, the option for a structured formulation situation also emerged as a starting point, enabling students to have their first contact with this type of task. Afterwards, the problem was solved individually and later discussed (Figure 5).

Figure 5

Presentation to the class of João's resolution.



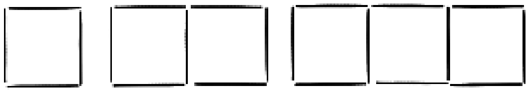
Once again, since this is a calculation problem, the diversity of solving processes was poor since all students used the known formulas and algorithms. No difficulties were reported by the students in the interactions they had with the teacher.

Following the theme, the teacher challenged the students to solve a process problem (Figure 6) related to sequences.

Figure 6

Statement of the second problem from session 1.

To keep your garden organized, Mr. José builds a new square, for each different species he has, with wooden beams. The following figure shows the procedure as he constructs the squares.



How many wooden beams will the garden have when Mr. José cultivates 6 different species?

Each student chose their solution strategy autonomously, after which there was a debate and confrontation about the different solutions found. Some students chose to solve the problem by drawing and identifying a pattern (Figure 7), while others decided to take a more formal approach to sequences, either by analysing the numerical pattern, starting by reducing it to a simpler problem or through the discovery of its formation law (Figure 8). In general, students showed few difficulties in solving this problem. The few doubts that arose in the resolution process were quickly clarified by the investigator.

Figure 7

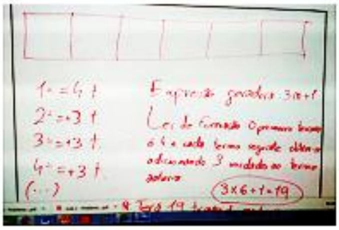
César's resolution, with drawing and pattern identification.

He will have 19 wooden beams!

During the debate, the students discussed the different resolutions, having the opportunity to learn about new strategies through contact and confrontation of the different ones used and to explain their own, enhancing the development of skills in the field of mathematical communication. The students did not establish a consensus regarding the most efficient strategy, and they concluded that, sometimes, the effectiveness and efficiency of the strategies depend on the solver and their preferences.

Figure 8

João's resolution, alluding to the law of formation and generative expression.



Generative expression: 3n−1

Law of formation: the first term is 4 and the following terms are obtained by adding 3 units to the previous term.

In this session, the students used different problem-solving strategies, such as drawing a picture, identifying patterns, and reducing it to a simpler problem. Besides, according to their comments, they had their first contact with problem formulation tasks through a structured situation. The students showed doubts about the posing process regarding the elements and data necessary for constructing a statement, doubts regarding mathematical concepts and terms, and the phrasing construction.

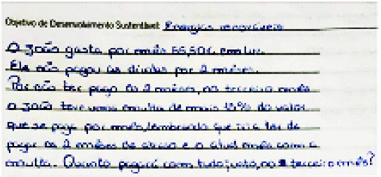
Session 2

The second session began with the exposure and analysis of the image alluding to the Sustainable Development Goals (SDG) (Figure 1), already known to the students, as the basis for formulating a problem. In this task, the aim was to develop a situation of semi-structured formulation, in which students, in pairs, could use one of the SDG to build an utterance autonomously (Figures 9-10).

In this statement, Luísa and Mateus decided to choose the SDG related to Renewable Energies to formulate a problem related to the cost of electricity. Some points of analysis stand out: the choice of content pertaining to percentages since it was content recently discussed in the class, which may suggest that students already felt comfortable with posing problems on this topic and, implicitly in the process, manipulate it; the construction of sentences and presentation of the data, in which there was a lot of concern that all the data are present, which led to a phrasing repetition, and even an almost explanation in the utterance, as it would become a little confusing without the clarification given.

Figure 9

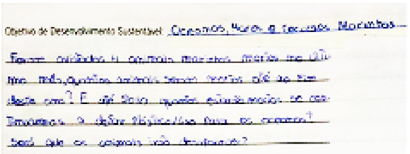
Example of problem formulation by Luísa and Mateus.



Sustainable Development Goal: Renewable energy
João spends €55.50 per month on electricity. He has not paid his debts for 2 months. For not having paid the 2 months, in the third month, João had a fine of 15% more than the amount paid per month, remembering that he would have to pay the two months of delay and the current month with the fine. How much will you pay with everything together in the third month?

Figure 10

Example of problem formulation by Beatriz and Ana.



Sustainable Development Goal: Oceans, Seas and Marine Resources

Four dead marine animals were sighted last month; how many animals would be killed by the end of this year? And by 2030, how many will be dead if we continue to dump plastic/trash into the oceans? Will animals disappear?

In this statement, Beatriz and Ana chose to formulate the problem based on the SDG related to the Oceans, Seas, and Marine Resources. Analysing the statement, the construction of a problem poses several questions, the last one being close to what can be considered an investigation, being the only one that presented this characteristic. On the other hand, there is no temporal contextualisation in the statement, so when they ask “until the end of this year?” there was no awareness that the solver would not indicate the starting date.

Throughout the task, the wide variety of contexts and statements constructed by the students was noticeable. This emerged at different SDGs and, within the same, diverse and interesting proposals of posed problems. In general, students showed great originality in terms of the choice of theme and the situation developed.

During the task, the students raised some doubts concerning the formulation process. We found a low diversity of problem typologies and approaches/content used. Cases of unsolvable statements, which would be analysed in a later session, were registered.

The second task developed was related to a situation of semi-structured problem formulation based on an image (Figure 11) and subsequent resolution.

Figure 11

Image used in the formulation task. (Credits: Dreamstime image bank)

Interesting formulations emerged in this task, with the students being able to conceptualise individually different situations that could be problematised based on the same image (Figures 12-13).

In the statement formulated by Inês (Figure 12), the conceptualisation involving percentages stands out since it was the content that was being taught. As this is a production in which this content is used and discussed in a correct and pertinent way, it can be a positive indicator in relation to the understanding of this content and its form of use.

In the statement formulated by Joel (Figure 13), the construction of a problem involving areas is highlighted. Thus, in this formulation, Joel correctly elaborated a problem involving contents developed in previous weeks, taking advantage of this opportunity to resume previous learning.

Figure 12

Formulation and problem solving by Inês.

Sara has three plots of lands. One plot had 15 carrots, another had 18 sprouts and the other had 12 corn ears. The animals ate 60% of her crop. How many pieces of foods did she get?

Figure 13

Problem posing and resolution by Joel, related to areas.

Mr. José of Cabbages has a plot of land with 7000m² and wants to plant 1 field of corn, 1 of carrots and one of cabbages, all with 240m². But he also wants to build a 240m² playground. Will there be room left to graze the cows?

Once again, the originality of the formulated statements stands out, with a great diversity of imagined situations and evolution in terms of the variety of mathematical themes. On the other hand, when asked about the choice of content, students stated that they tended to build statements that enhanced the use of content in which they felt more comfortable. The problems formulated by the students fit into calculation problems, without exception, since this is the simplest type of problem and the one that students were most used to dealing with in their daily school life.

In this session, a lack of variety of problem-solving strategies was verified since the solutions were based on calculus problems. About problem posing, the case of the formulation that indicated the presence of a small investigation is highlighted, as well as the examples of contents used by the students, some more recent and others resuming subjects discussed earlier. Regarding the identified difficulties, the presence of some statements that omit some important information for the solver is emphasised, and others, in which the concern that all information is present, led to the statements being disorganised or repetitive.

Session 3

The third session started with the problems formulated in the previous session, distributed among the pairs in an alternating way, i.e., each pair of students posed a problem (in the previous session) and now solved another problem, formulated by a different pair.

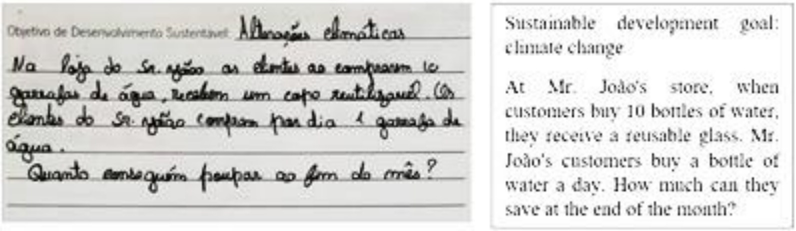
In the first phase, a discussion about the different statements took place. They were given time to interpret the problem and understand whether their situation was solvable and whether they had all the necessary data to solve it. This metacognitive activity emerged as a way for students to consciously analyse the feasibility of the problems, as the statements were subjected to an a priori analysis and some problems that did not have all the necessary data for their resolution were identified.

Thus, when one of the pairs realised that they did not have all the data, they asked for help, and, depending on the cases analysed before, we asked questions to improve understanding, to indicate the data that were not so explicit or to perceive how they would do it differently if they were to pose the problem so that that situation could be resolved (Figure 14).

In this statement, the problem question is not explicit as to the object they manage to "save", nor is it clear as to the month and, consequently, the number of days it presents. We agreed with the students that they would submit two answers, according to the month had thirty or thirty-one days.

Figure 14

Statement that needed a reformulation of the question.



Objetivo de Desenvolvimento Sustentável: Alterações climáticas

Na loja do Sr. João os clientes ao comprarem 10 garrafas de água, recebem um copo reutilizável. Os clientes do Sr. João compram por dia 1 garrafa de água.

Quanto conseguirão poupar ao fim do mês?

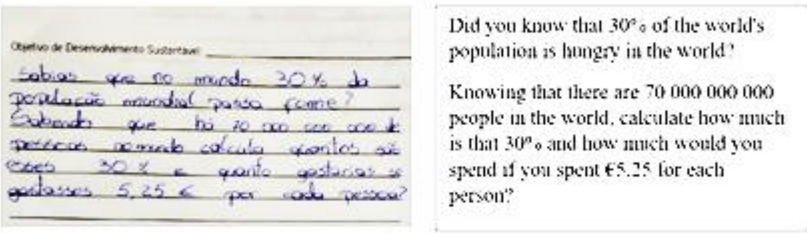
Sustainable development goal: climate change

At Mr. João's store, when customers buy 10 bottles of water, they receive a reusable glass. Mr. João's customers buy a bottle of water a day. How much can they save at the end of the month?

In a second phase, in which all pairs were already comfortable interpreting the different statements, each team continued with its resolution process. Here, difficulties of a different kind arose: some students decided to use, in their formulations, very high values (Figure 15) (for example, the value of the world population), so calculation difficulties arose, also already to be expected.

Figure 15

Example of problem formulation that used the value of world population.



Objetivo de Desenvolvimento Sustentável:

Sabias que no mundo 30% da população mundial passa fome? Sabendo que há 70 000 000 000 de pessoas no mundo calcula quantos são esses 30% e quanto gastarias se gastasses 5,25 € por cada pessoa?

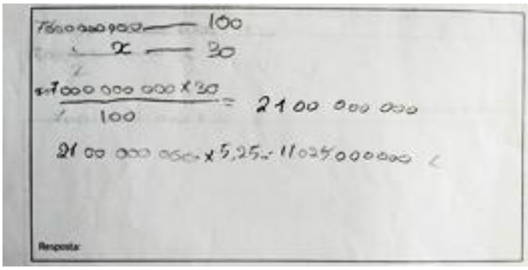
Did you know that 30% of the world's population is hungry in the world?

Knowing that there are 70 000 000 000 people in the world, calculate how much is that 30% and how much would you spend if you spent €5.25 for each person?

For its resolution, and realising that not even with the help of the calculator would it be possible to resolve the situation in a way that students could understand (since the results would appear in scientific notation, which they were not familiar with), two procedures were adopted: a brief explanation, within the appropriate level, of the value that the calculator presented to them, as they had doubts that it would not make sense to be ignored; since this is a problem related to percentages (the resolution process chosen by them was a simple rule of three), it was possible to elaborate an explanation based on an analogy with lower values (using a conversion strategy from one problem to another simpler) and then relate to the values of the original problem (Figure 16).

Figure 16

Problem involving the value of the world population.



Throughout the session, the students successfully solved the problems, mostly using rules of three (Figure 17), without a wide variety of different solving strategies (Figure 18). This may be because the students are discussing this subject in the mathematics class, applying the knowledge they had just learned to their resolutions.

Figure 17

Problem solved by Gonçalo using a rule of three

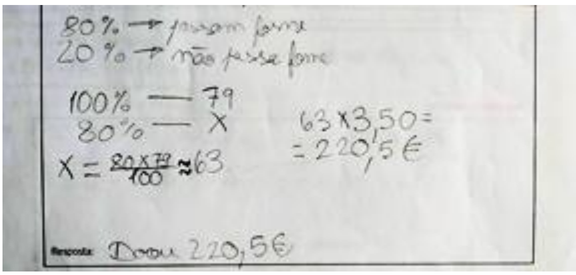
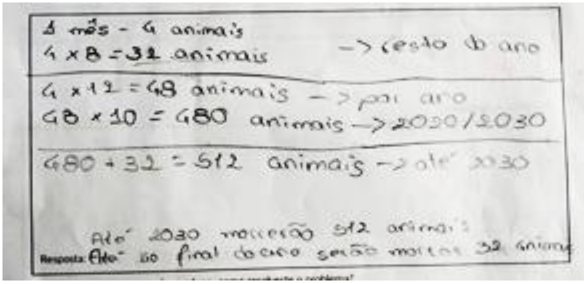


Figure 18

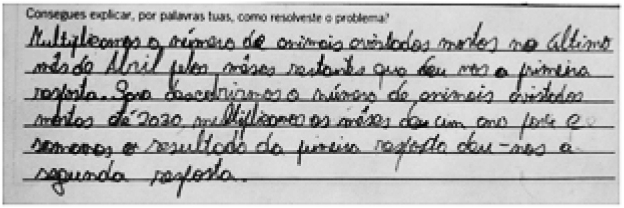
Problem solved by Isabel using a logical sequence of steps.



The exploration of mathematical communication also played a preponderant role throughout the interventions, which took special importance in this session. In this class, special attention was given to this component in written form. After solving all the problems, some examples of explanations were discussed (Figure 19) so that everyone could get to know the different problems and their resolution.

Figure 19

Explanation of the resolution process by Barbara and Joel.



We multiplied the number of animals sighted dead last April by the remaining months, which gave us the first answer. To find out the number of animals sighted dead by 2030, we multiplied the months of the year and added the result of the first answer, which gave us the second answer.

One of the most significant examples involved a pair of students who decided to explain the process individually. Thus, it was possible to obtain two different versions of the explanation of the same resolution, which was interesting for the activity. In one, student Renata explained: “To solve the problem, I did it seven times six”. In another, student Irene said that “To solve the problem, I multiplied the number of plastic bottles by the number of days, to get the result of bottles he managed to save. It was seven bottles times six days that I went to the beach”.

Taking advantage of the exposed differences, the students could build a debate about which would be the correct form, the one that would best describe and explain the resolution elaborated by the pair. Everyone recognised that the second proposal was better suited to the goal of the task, even the student who wrote the first proposal.

In this session, some students used different problem-solving strategies, such as the sequence of steps, while others solved the problems by applying content developed recently in the mathematics class, for example, simple three rules. With regard to problem posing, this session highlights the analysis and reformulation of statements, filling gaps identified by the students. At the level of difficulties, it is emphasised the resolution of problems whose formulation involved the use of very high values and verbal explanations of the resolutions of the problems. At this level, we highlight the work of analysis developed by the class and the students’ awareness of the mathematical importance of this process.

Session 4

The fourth session was based on the resolution of an open problem (Figure 20). However, when the students started to read the utterance, they were aware of having already contacted a similar problem.

Figure 20

Statement of the first problem in session 4.

At a meeting, a tribe leader decided to test his partners' reasoning ability by telling them an ancient riddle:
"A boy was returning to his homeland with a fox, a sheep and a cabbage. However, to get home, he had to cross a river. The boat that was on the bank only had the capacity for 2 passengers (the boy and one of his remaining possessions). The boy cannot leave the fox alone with the sheep, nor the sheep with the cabbage, otherwise he would loose one of his belongings."
How many trips are needed for everyone to safely cross the river?

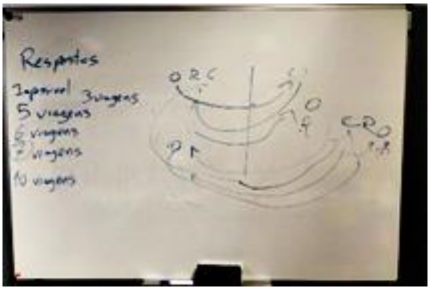
Normally, this would be one of the guiding questions that would come up, so this time the answer came innately. Some students managed to propose answers to the problem in the resolution, leading to some conversations and explanations among the students. It is necessary to consider that problem solving in the classroom, especially at the level of investigations, enhances discussion among students and that dialogue and confrontation of ideas can promote learning.

In addition to analysing the results of the resolutions, the strategies used by different students could also be investigated. Having multiple strategies, some students had difficulties in choosing the strategy they considered the most effective. Thus, it could be interesting for students to compare and analyse the variety of strategies presented in the classroom. Therefore, first, the different solutions they reached were discussed. Then then, the resolution strategies were analysed.

However, before it was possible to move on to this phase of the session, it was critical that everyone realised that the problem could be solved. Mateus did not reach this conclusion and asked to explain his point of view to the class. During the explanation, the student realised the mistake made, helping himself draw up a scheme on the board (Figure 21) while explaining his point of view to the class.

Figure 21

Scheme elaborated by Mateus in his explanation.



After this moment, there was an opportunity to discuss the different verbal explanations and the resolution strategies used by the students:

organisation by table (Figure 22); elaboration of a scheme; enumeration list of the different steps (Figure 23).

Figure 22

Table presented by Mário.

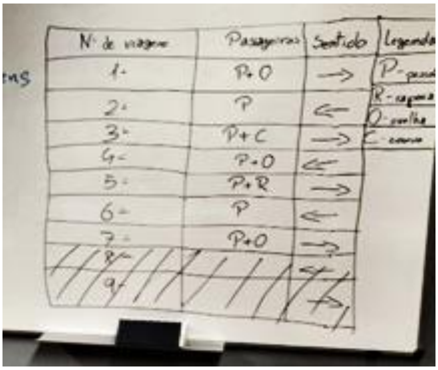
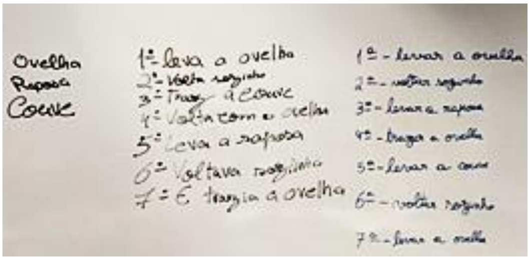


Figure 23

List of steps presented by Raquel and Irene.



Different investigations led to two different solutions to the problem being found. This fact triggered the debate in the class, analysing whether, in fact, this was possible, as well as the search for different resolutions that could satisfy the premises of the problem. Students concluded that the two solutions discussed used the least number of trips possible but that others satisfied the problem's premises (with redundant steps).

Students recognised the usefulness of the different strategies, having used the one they considered most appropriate according to their style of reasoning and resolution.

In this session, students experimented and discussed the use of different problem-solving strategies, such as step enumeration, schema construction, and table elaboration, among others. Concerning problem posing, this session did not develop tasks in this area. Regarding the identified difficulties, the situation in which a student considered, in a first phase, the problem solving as impossible, was highlighted, having reviewed his resolution process in its explanation in discussion with the class. Some students also experienced difficulties choosing the strategy they would use to solve the problem. At this level, it is considered that the low frequency of process and open problem solving, which allow multiple approaches and strategies, may have been an essential factor, emphasising the importance of the presence of several problem typologies in mathematics classes.

Session 5

The fifth session began with resolving a process problem (Figure 24), a rare task in students' daily lives. Initially, they were a little apprehensive, as they did not know any direct way to answer the problem. So, they were asked to think of a possible strategy to help them resolve it.

Figure 24

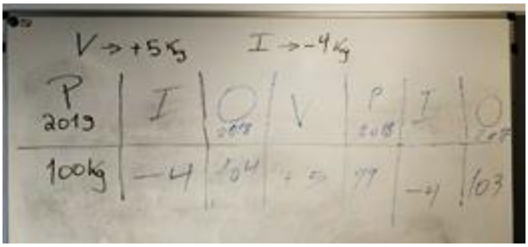
Statement of the first problem in session 5.

> In the forests of Finland, a bear fattens 5 kg in summer and loses 4 kg in winter. In spring and autumn, it always maintains its weight. In the spring of 2019 it weighs 100 kg how many kilograms did the bear weigh in the fall of 2017?

In this process, the students had many doubts and difficulties, as it was not a problem that presented an intuitive situation. The most suitable strategy for its resolution was the procedure of thinking, from the end to the beginning, a task the students were not used to solving. After the resolution time and noticing that part of the class was having difficulties, we decided to solve the problem together, on the whiteboard, using a table (Figure 25).

Figure 25

Joint resolution of the problem on the board.



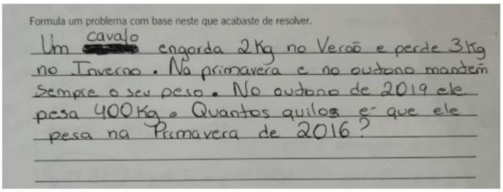
The resolution was based on questions directed to the class, followed by reflection and evolution to the next step, depending on the answers given by the students. From a certain point onwards, Mário took the lead in the explanation, realising the rub of the problem: the resolution was based on inverse operations from those that appear as intuitive, since it is an end-to-beginning process, stating that "for the bear to reach 100 kg, and if it loses 4 kg in winter, in autumn it has to weigh more than 100 kg, not less". As simple as it may seem, this reasoning is not intuitive, so it took a while for everyone to understand it. However, in the end, everyone concluded that they realised the process involved.

Then, a moment of formulation related to the extension of the previous problem was suggested, with the students free to choose the strategy followed. Thus, different approaches emerged: some of the students modified the problem data, maintaining its structure, making it possible to identify the What if strategy (Figure 26); others decided to depart from the context of the problem to create extensions of the problem (Figure 27). Finally, and after everyone had elaborated their formulation, there was a conversation about the different formulations, thus sharing the different statements and the students' observation of the diversity of elaborated proposals and the problems that stood out by their originality.

Renata decided to keep the original structure of the problem, modifying the animal and the values involved (Figure 26). However, the structure and reasoning that underpin the problem remain the same.

Figure 26

Problem posed by Renata, keeping the original structure.

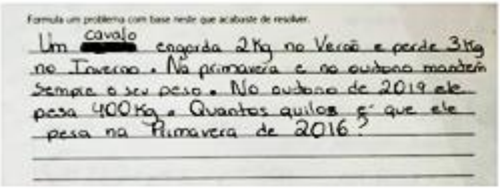

Formula um problema com base neste que acabaste de resolver.

Um [redacted] cavalo engorda 2Kg no Verão e perde 3Kg no Inverno. Na primavera e no outono mantém sempre o seu peso. No outono de 2019 ele pesa 400Kg. Quantos quilos é que ele pesa na Primavera de 2016?

A horse fattens 2 kg in summer and loses 3 kg in winter. In spring and autumn, it always maintains its weight. In the fall of 2019 it weighs 400 kg. How many kilograms does it weigh in spring of 2016?

Figure 27

Problem formulated by Filipa, through an extension.


Formula um problema com base neste que acabaste de resolver.

Um [redacted] cavalo engorda 2Kg no Verão e perde 3Kg no Inverno. Na primavera e no outono mantém sempre o seu peso. No outono de 2019 ele pesa 400Kg. Quantos quilos é que ele pesa na Primavera de 2016?

A horse fattens 2 kg in summer and loses 3 kg in winter. In spring and autumn, it always maintains its weight. In the fall of 2019 it weighs 400 kg. How many kilograms does it weigh in spring of 2016?

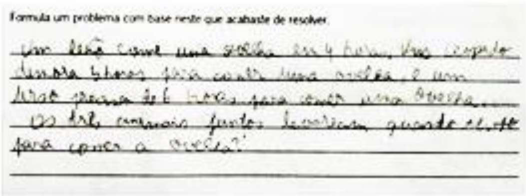
Filipa built an extension of the problem. As this is an extension, the student does not indicate all the data necessary to solve the problem, explicitly referring to the "previous problem". Thus, she assumed that the solver would already know some data about the problem, not having explained them in her statement. The omission of the bear's initial weight, the central premise of the problem, is an example of this.

It also highlights the formulation of two problems that were highlighted by the complexity of their formulation. In one of the cases (Figure 28), Celso decided to go from the context of the problem to creating a totally new situation, keeping only the context of the animals.

In his statement, Celso was only inspired by the theme of the previous problem, conceptualising an idea and a completely new situation. It is noteworthy that, when this problem was discussed, the student added very relevant information for solving the problem, which is not present in the statement: the student admitted that the three animals would eat the same amount.

Figure 28

Problem formulated by Celso, starting from the general theme of the previous problem.

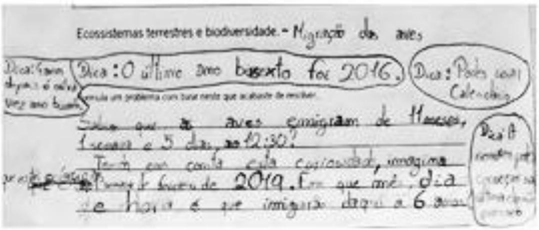


A lion eats a sheep in 4 hours, a leopard takes 5 hours to eat a sheep and a bear needs 6 hours to eat a sheep. The 3 animals together would take how long to eat the sheep?

In another case (Figure 29), Mário decided to build a problem that was also based on the theme of animals but which dealt with periodic intervals of time.

Figure 29

Problem formulated by Mário, starting from the general theme of the previous problem.



Did you know that birds emigrate from 11 months a week and 5 days at 12:30h? Given this curiosity, imagine that you are at the beginning of the first week of February 2019. In which month, day and time will they migrate in 6 years?

In the discussion, Mário explained that he intended to make a “difficult problem”, and after constructing his statement, he realised that it did not indicate enough data to become a solvable problem. So, he decided to add different data and suggestions to help the resolver in this mission. Even so, in the analysis of the statement, it is clear that this is not yet possible to resolve since the student did not indicate the date of the last migration to have the starting point for counting time.

In this session, students experimented and discussed using the end-to-beginning strategy of working through the construction of a table. With regard to problem posing, the elaboration of problem extensions and the conceptualisation of new problems are highlighted, based on a previous problem, developing the What if strategy. Regarding the difficulties, the process of solving problem stands out. Since this was not a problem with an intuitive solution, the students resisted looking for innovative strategies that could be used. We believe that the low frequency of these types of moments in mathematics class may explain this phenomenon.

Regarding problem posing, the difficulties related to including the necessary data to solve the problems in the statement should be highlighted. There were cases in which students perceived this difficulty autonomously and tried to include the required information by reformulating their statements or adding tips for the solver. Other students became aware of the lack of information only in the group discussion, realising that their statements would become irresolvable.

CONCLUSIONS

This study sought to provide students with conscious and intentional contact with tasks related to problem solving, identifying their solving strategies, their posing skills, and the difficulties demonstrated.

Throughout the different sessions, students conceptualised and used different problem-solving strategies. With the three global types of problems present (calculation, process, and open) (Boavida et al., 2008), students could experience opportunities that enhanced the use of different problem-solving strategies, contrary to what is common in their daily school life. Thus, they could develop new problem-solving strategies, such as building schemas and tables, working from the end to the beginning, trying with conjecture, and reducing it to a simpler problem, among others, in a natural way, such as argued by Vale and Pimentel (2004).

The need for students to creatively discover their own resolution processes was a cross-cutting factor throughout the study, moving away from the vision of prescribed teaching strategies (Oliveira & Santana, 2013; Boavida et al., 2008). Thus, through the idealisation of tasks that favour the emergence of these strategies, we could help students become active and participative agents in the construction and structuring of learning (Oliveira & Santana, 2013). There has always been a concern about promoting a learning process that encourages autonomy in problem solving (Pólya, 1995), focusing on students who, through reasoning, debate, and discussion, managed to develop different resolution strategies closer to those described in the literature (Boavida et al., 2008; Vale & Pimentel, 2004).

Regarding problem posing, this study was a means that provided students with initial contact with this type of task. Not being an ordinary activity in mathematics classes, the formulation of problems, as a component of problem solving, has numerous potentials for the development of students (Boavida et al., 2008; Dante, 2009; Kilpatrick, 1987; NCTM, 2007, 2014; Palhares 1997; Pólya 1995; Silver, 1997; Vale & Pimentel, 2004; Van Harpen & Presmeg, 2013;). The focus of the development of activities and research was directed towards the ability to pose problems according to two general strategies: the creation of extensions of existing problems, taking into account structured situations (Stoyanova, 1998; Stoyanova & Ellerton, 1996) that enhanced the strategy what if (Boavida et al., 2008); and the design of problems based on data or contexts provided, in semi-structured situations (Stoyanova, 1998; Stoyanova & Ellerton, 1996), which was in line with the strategy of accepting the data (Boavida et al., 2008).

Students demonstrated a creative spirit in the elaboration of different contexts and statements of problematic situations, both in structured and semi-structured cases. Overall, they could highlight the three dimensions postulated by Silver (1997) and Silver and Cai (2005) to analyse creativity in formulating problems: fluency, flexibility, and originality. Thus, regarding fluency, it was possible to identify that, in general, the participating students formulated problems that fit the proposed requirements, presenting themselves, in their entirety, as solvable. Concerning flexibility, that is, the different types of problems created, there was a less accentuated development, which will be the object of further analysis. About originality, we emphasise the variety of unique and creative contexts, one of the potentials identified in this study, but a low variety of typologies and unusual mathematical productions. This last aspect is documented in the literature and may be related to the low frequency of contact with this type of task (see Miranda, 2019; Pinheiro & Vale, 2013).

Regarding the difficulties the students manifested throughout the study, those related to solving and posing problems are distinguished. In relation to problem solving, we identified some difficulties, and it is essential to underlie some at the level of interpretation of statements, which leads to problems becoming even more complex to solve since understanding the initial situation is a crucial point for the construction of a resolution plan. Costa and Fonseca (2009) also identified difficulties of this nature in a study with 4th-grade students focused on interpreting mathematical statements in the context of problem solving. On the other hand, prior contact with a wide range of problems also allows for a more experienced and efficient approach to their resolution (see Boavida et al., 2008), a factor where there were also some gaps. Another difficulty was related to mathematical communication, namely in the oral and written explanation of the constructed reasoning. Assuming crucial importance for the integral development of mathematical skills (Boavida et al., 2008; Mamede, 2009), we identified difficulties in this area, which disappeared throughout the study, supporting the idea of the relevance of the intentional, systematic, and frequent execution of this type of tasks.

Regarding the main difficulty identified in the problem posing, one distinguishes two main ideas: the complexity of the posed problems that tended to be slightly below what was expected; and the observation that the typology of conceptualised problems was not very varied. These aspects have also been previously identified in the literature (see Pinheiro & Vale, 2013). Somehow, we consider that one of the reasons that may explain this is the low frequency of this type of task in students' daily lives, which makes them feel uncomfortable about risking bolder proposals. However, it has already been possible to identify some cases in which they left their comfort zone and ventured into formulating more complex problems. In this study, students who risked more tended to be more participative and committed to the different tasks proposed. Despite a regular school performance, some students managed to stand out in this type of task. This observation can support the role of stimulus and motivation in mathematics as a crucial issue for mathematical learning (Boavida et al., 2008; Dante, 2009; Pound & Lee, 2011).

Another aspect of the students' difficulties identified in this study leads us to the construction and articulation of statements, an element previously identified by Miranda (2019) and Pinheiro and Vale (2013). After identifying some difficulties in structuring the statements and organising the information presented, we believe that the low frequency of this type of task in students' daily lives can influence this aspect.

Thus, and in general, it is possible to conclude that the students in this study could develop new skills in terms of problem solving and posing.

Corroborating the ideas of Boavida et al. (2008), Costa and Fonseca (2009), Dante (2009), Miranda (2019), and Pound and Lee (2011), this study highlights the promotion of frequent contact with problem solving and formulation tasks, for mathematical learning that allows the construction of solid mathematical skills and the promotion of enthusiasm for mathematics. In this way, it is possible to promote proficiency in applied problem-solving strategies, in problem formulation strategies used, and in the explanation of reasoning, enabling students to feel comfortable to invest and risk in innovative processes and reasoning, which may lead to the flowering of mathematical creativity.

AUTHORS' CONTRIBUTIONS STATEMENTS

PM and EM conceived the presented idea. PM and EM developed the theory. PM and EM adapted the methodology to this context, created the models, and PM performed the activities and collected the data. PM and EM analysed the data. All authors actively participated in the discussion of the results, and reviewed and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The authors agree to make their data available upon reasonable request from a reader. It is up to the authors to determine whether a request is reasonable.

ACKNOWLEDGEMENTS

This work is financed by national funds through FCT – Fundação para a Ciência e a Tecnologia, I.P., within the scope of the project «2021.05895.BD».

REFERENCES

Abu-Elwan, R. (2002). Effectiveness of problem posing strategies on prospective Mathematics teachers' problem-solving performance. *Journal of Science and Mathematics Education in Southeast Asia*, *25*(1), 56–69.

Agić, H., & Rešić, S. (2015). Some outlooks on creativity in Mathematics teaching. *Human*, *5*(2), 11-22.

Akgul, S., & Kahveci, N. G. (2016). A Study on the Development of a Mathematics Creativity Scale. *Eurasian Journal of Educational Research*, 62, 57-76.

Aktaş, M. (2016). Turkish High School Teachers' Conceptions of Creativity in Mathematics. *Journal of Education and Training Studies*, *4*(2), 42-52.

Ayllón, M., Gómez, I., & Ballesta-Claver, J. (2016). Mathematical thinking and creativity through mathematical problem posing and solving. *Propósitos y Representaciones*, *4*(1), 169-218.

Bezerra, W., Gontijo, C., & Fonseca, M. (2021). Fostering Mathematical Creativity in the Classroom through Feedbacks. *Acta Scientiae*, *23*(2), 88-112.

Boavida, A. M. R., Paiva, A. L., Cebola, G., Vale, I., & Pimentel, T. (2008). *A Experiência Matemática no Ensino Básico – Programa de Formação Contínua em Matemática para Professores dos 1.º e 2.º Ciclos do Ensino Básico.* Direcção-Geral de Inovação e de Desenvolvimento Curricular.

Bogdan, R., & Biklen, S. (2013). *Investigação Qualitativa em Educação – uma introdução à teoria e aos métodos.* Porto Editora.

Choppin, J., McDuffie, A.R., Drake, C., & Davis, J. (2020). The role of instructional materials in the relationship between the official curriculum and the enacted curriculum, *Mathematical Thinking and Learning*. http://doi.org/10.1080/10986065.2020.1855376 .

Cockcroft, W. H. (1982). *Mathematics counts*. HMSO.

Costa, A.M., & Fonseca, L. (2009). Os números na interface da língua portuguesa e da matemática. *Actas do XIXEIEM*. Sociedade Portuguesa de Investigação em Educação Matemática.

Dante, L. R. (2009). *Formulação e resolução de problemas de matemática: teoria e prática.* Ática.

DGE. (2015). *Objetivos de Desenvolvimento Sustentável, 2016-2030 (ODS).* Ministério da Educação/Direção-Geral da Educação.

DGE. (2018). *Aprendizagens essenciais de Matemática – 1.º ano.* Ministério da Educação/Direção-Geral da Educação.

Ernest, P. (1991). *The philosophy of mathematics education*. Falmer.

Gardner, H. (2009). The Five Minds for the Future. *Professional Voice*, *7*(2), 18-25.

Hopkins, S., Russo, J., & Siegler, R. (2020). Is counting hindering learning? An investigation into children's proficiency with simple addition and their flexibility with mental computation strategies, *Mathematical Thinking and Learning*. http://doi.org/10.1080/10986065.2020.1842968 .

Kampylis, P., & Berki, E. (2014). *Nurturing Creative Thinking*. International Academy of Education.

Kandemir M. A., & Gür H. (2007). Creativity Training İn Problem Solving: A Model of Creativity İn Mathematics Teacher Education. *New Horizons in Education*, *55*(3), 107-122.

Kilpatrick, J. (1987). Problem formulating: Where do good problems come from? In: A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 123- 147). Lawrence Erlbaum.

Kozlowski, J. S., & Si, S. (2019). Mathematical Creativity: A Vehicle to Foster Equity. *Thinking Skills and Creativity*, 100579. https://doi.org/10.1016/j.tsc.2019.100579

Krulik, S., & Rudnick, J. (1993). *Reasoning and problem solving – A Handbook for Elementary School Teachers*. Allyn and Bacon.

Lee, K., Kim, Y., & Lim, W. (2021). Risks of aiming to kill two birds with one stone: the affect of mathematically gifted and talented students in the dual realities of special schooling, *Mathematical Thinking and Learning*, *23*(4), 271-290. http://doi.org/10.1080/10986065.2020.1784696 .

Leikin, R., Berman, A., & Koichu, B. (2009). *Creativity in mathematics and the education of gifted students*. Sense.

Liel, C., & Bayer, A. (2016). Problem Solving with Environmental Themes in Math Classes. *Acta Scientiae*, *18*(3), 692-706.

Lupinacci, V., & Botin, M. (2004). Resolução de Problemas no Ensino de Matemática. *Anais do VIII ENEM*.

Mamede, E. (2009). *Matemática – Tarefas para o novo programa – 1.º Ciclo*. AEME – Associação para a Educação Matemática Elementar.

Martins, B. (2016*). Formulação de problemas na aprendizagem de tópicos matemáticos do 1º e 2º ciclos do ensino básico* [Unpublished master's thesis]. Universidade do Minho.

Miranda, P. (2019*). Estratégias de resolução de problemas e formulação de problemas: um estudo nos 1.º e 2.º ciclos do ensino básico* [Unpublished master's thesis]. Universidade do Minho.

National Advisory Committee on Creative and Cultural Education. (1999). *All Our Futures, Creativity, Culture and Education*. DfEE Publications.

National Council of Teachers of Mathematics. (2007). *Princípios e Normas para a Matemática Escolar*. APM & NCTM.

National Council of Teachers of Mathematics. (2014). *Principles to Actions: Ensuring Mathematics Success for All*. National Council of Teachers of Mathematics.

Oliveira, S., & Santana, M. A. (2013). Jogos e Resolução de Problemas na Formação Continuada e em Aulas de Matemática nos Anos. *Acta Scientiae*, *15*(1), 76-92.

ONU. (2015). *Transforming our world: the 2030 Agenda for Sustainable Development*. UNDoc. A/70/L.

Palhares, P. (1997). Histórias com problemas construídas por futuros professores. In D. Fernandes. F. Lester, A. Borralho & I. Vale (Coord.). *Resolução de Problemas na Formação Inicial de Professores de Matemática: Múltiplos Contextos e Perspectivas* (pp. 159-188). GIRP.

Pinheiro, S., & Vale, I. (2013). Formulação de problemas e criatividade na aula de matemática. In: Fernandes, J. A., Martinho, M. H., Tinoco, J., & Viseu, F. (Orgs.) (2013). *Atas do XXIV Seminário de Investigação em Educação Matemática.* APM & CIEd.

Pólya, G. (1981). *Mathematical discovery: on understanding, learning, and teaching problem solving.* Wiley.

Pólya, G. (1995). *A arte de resolver problemas* (2nd Ed.). Interciência.

Ponte, J.P. (2003). O ensino da Matemática em Portugal: Uma prioridade educativa? *O Ensino da Matemática: Situação e Perspectivas* (pp. 21-56). Conselho Nacional de Educação.

Ponte, J. P. (2005). Gestão curricular em Matemática. In GTI (Ed.) *O professor e o desenvolvimento curricular* (pp. 11-34). APM.

Ponte, J. P. (2008). A investigação em educação matemática em Portugal: Realizações e perspectivas. In R. Luengo-González, B. Gómez-Alfonso, M. Camacho-Machín & L. B. Nieto (Eds.) *Investigación en educación matemática XII* (pp. 55-78). SEIEM.

Pound, L., & Lee, T. (2011). *Teaching Mathematics Creatively*. Routledge.

Schoevers, E. M., Leseman, P. P. M., Slot, E. M., Bakker, A., Keijzer, R., & Kroesbergen, E. H. (2019). Promoting pupils' creative thinking in primary school mathematics: A case study. *Thinking Skills and Creativity*, 31, 323–334.

Silver, E. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM*, 3, 75-80.

Silver, E. A., & Cai, J. (2005). Assessing students' mathematical problem posing. *Teaching Children Mathematics*, *12*(3), 129-135.

Stoyanova, E. (1998). Problem posing in mathematics classrooms. In A. McIntosh & N. Ellerton (Eds.), *Research in Mathematics Education: a contemporary perspective* (pp. 164-185). MASTEC.

Stoyanova, E., & Ellerton, N. F. (1996). A framework for research into students' problem posing in school mathematics. In: P. C. Clarkson (Ed.), *Technology in mathematics education (Proceedings of the 19th annual conference of the Mathematics Education Research Group of Australasia)* (pp. 518-525). Mathematics Education Research Group of Australasia.

Vale, I., & Pimentel, T. (2004). Resolução de problemas. In P. Palhares (Coord.), *Elementos de Matemática para Professores do Ensino Básico* (pp. 7-51). Lidel.

Vale, I., Pimentel, T., & Barbosa, A. (2015). Ensinar Matemática com Resolução de Problemas. *Revista Quadrante*, *XXIV*(2), 39-60.

Van Harpen, X. Y. & Presmeg, N. C. (2013). An investigation of relationships between students' mathematical problem-posing abilities and their mathematical content knowledge. *Educational Studies in Mathematics*, *83*(1), 117-132.

Yin, R. K. (2014). *Case study research: design and methods* (5th Ed.). Sage.