

# Conception of Learning Objects with Feedback for Self-Regulation of Learning Mathematical Concepts Necessary for Differential and Integral Calculus

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## ABSTRACT

**Background:** Due to the high failure rates in engineering courses, it is of fundamental importance to seek alternatives that minimise the difficulties that students entering Brazilian universities have in the disciplines of the area of mathematics. An alternative is to use learning objects (LO) that allow students to be aware of their errors and encourage them to formulate hypotheses and solve problem situations, aspects that are fundamental for the self-regulation of learning. **Objectives:** To present the conception and evaluation of an LO with feedback that allows the student to self-regulate their learning with regard to the concepts of mathematics necessary for differential and integral calculus. **Design:** The research is based on the design science research - DSR methodology. In the first stage, detailed in this article, we sought to present the relevance of the problem and the artefact's design proposal and the first phase of assessing this design. **Setting and Participants:** The LO was submitted to an online assessment by seven mathematics teachers and two mathematics education researchers. **Data collection and analysis:** It was carried out through the transcription of the videos recorded during the assessment via Google Meet. **Results:** The experts pointed out the potentialities of the LO to diagnose and remedy/intervene before possible errors made by higher education newcomers. **Conclusions:** After the assessment, the artefact was restructured and improved following the specialists' notes. The following steps of the research will be the implementation of the complete version of the Los, with questions that explore the strata of numerical, algebraic, and functional knowledge. We expect that the educational product of this research will help students with their mathematical difficulties in DIC.

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**Keywords:** Learning objects; Feedback; Learning self-regulation; Differential and integral calculus.

## Concepção de Objetos de Aprendizagem com Feedbacks para a Autorregulação da Aprendizagem de Conceitos Matemáticos Necessários para o Cálculo Diferencial e Integral

### RESUMO

**Contexto:** Devido aos altos índices de reprovação nos cursos de Engenharia, é de fundamental importância buscar alternativas que minimizem as dificuldades que os estudantes ingressantes nas universidades brasileiras apresentam nas disciplinas da área da matemática. Uma alternativa é o uso de Objetos de Aprendizagem (OA) que possibilitem a conscientização dos erros cometidos de modo a incentivar a capacidade de formular hipóteses e resolver situações-problemas, aspectos esses, fundamentais para autorregulação da aprendizagem. **Objetivos:** Apresentar a concepção e a avaliação de um OA com *feedbacks* que possibilitem ao aluno a autorregulação da sua aprendizagem, no que diz respeito aos conceitos de Matemática necessários para o Cálculo Diferencial e Integral. **Design:** A pesquisa está fundamentada na metodologia Design Science Research – DSR e, na primeira etapa, detalhada neste artigo, procurou-se apresentar a relevância do problema e a proposta de design do artefato, bem como uma primeira fase de avaliação desse design. **Ambiente e participantes:** O OA foi submetido a uma avaliação *online* por sete professores de Matemática, e dois pesquisadores da Educação Matemática. **Coleta e análise de dados:** Foi realizada através da transcrição dos vídeos gravados durante a avaliação, via *Google meet*. **Resultados:** Os especialistas apontaram potencialidades na utilização de OA para diagnosticar e remediar/intervir frente aos possíveis erros cometidos por alunos ingressantes no Ensino Superior. **Conclusões:** Após a avaliação o artefato foi reestruturado e melhorado seguindo os apontamentos feitos pelos especialistas. Os próximos passos da pesquisa serão a implementação da versão completa dos OA, com questões que explorem os estratos do conhecimento numérico, algébrico e funcional. Espera-se que, ao término da pesquisa, tenha-se um produto educacional capaz de auxiliar os alunos em suas dificuldades matemáticas em CDI.

**Palavras-chave:** Objetos de Aprendizagem; *Feedbacks*; Autorregulação da Aprendizagem; Cálculo Diferencial e Integral.

### INTRODUCTION

Problems related to the learning of mathematical concepts in basic education, in the case of higher education newcomers in exact sciences (Borges & Moretti, 2016), is a common topic in discussions in the academic environment nationally and internationally. Especially in mathematics education events, such as the International Seminar on Research in

Mathematics Education (SIPEM), the National Meeting of Mathematics Education (ENEM), the Brazilian Meeting of Graduate Students in Mathematics Education (EBRAPEM), the Psychology of Mathematics Education (PME) and the International Congress on Mathematics Education (ICME), the works presented point to high and growing failure rates in mathematical disciplines, also causing a significant number of dropouts, mainly in engineering courses, a fact that worries managers and teachers who teach classes in the first periods of undergraduate courses (Feitosa et al., 2020; Homa, 2020).

Research on the knowledge gap in college newcomers focuses mainly on what is usually called “basic mathematics content” without, however, having much clarity or consensus on what those “contents” are (Menestrina & Moraes, 2012).

In line with Zarpelon et al.’s (2017) findings, at the Federal University of Technology of Paraná (UTFPR), the authors’ work and research context, the highest failure rates in engineering courses occur in the subjects of Differential and Integral Calculus I (DIC I) and Analytical Geometry and Linear Algebra, offered in the first semester of the curriculum. Preliminary reports and institutional pedagogical initiatives indicate that this is one of the reasons for the high failure and dropout rates in these subjects. Zarpelon et al. (2017) support this hypothesis, pointing out studies highlighting the deficiency in basic mathematics education as a relevant factor to justify the lack of success in DIC I.

In addition to the factor mentioned above, academics face an unknown and often hostile environment when starting higher education. In particular, in the first semester, the engineering student faces definitions, demonstrations, and properties associated with the different concepts explored in mathematical disciplines (Christo et al., 2018). Such elements are unknown, creating an abyss in the transition from high school to higher education.

In this context, it is essential to reflect on the traditional approach to teaching mathematics in force at the university (Cabral, 2015), in which the teacher exposes the content, gives examples, and then applies tests to verify whether the student can reproduce the content that was taught (Mendes et al., 2018). One should think of ways of working in which the student has a more active role in their knowledge development process.

Some proposals in this direction involve the use of digital information and communication technologies (DICT) (Borba & Penteadó, 2019), work with

exploratory tasks (Trevisan & Araman, 2021) and collaborative work (Rodrigues et al., 2018). In addition, self-regulation emerges as an alternative for the student to be more active in the teaching and learning process, as it refers to the degree to which the individual acts at the metacognitive, motivational and behavioural level, in relation to their own learning processes and in carrying out school activities (Zimmerman, 1986; Casiraghi et al., 2020).

Therefore, proposing a model that allows the self-regulation of the learning of mathematical content for academics entering higher education is relevant in this context. The main objective of the model is to minimise the mathematical deficiencies of the students who will take the DIC. Thus, we expected an improvement in students' performance and, consequently, an increase in the number of approvals. The proposal conceived in this work is characterised as a form of self-regulation of learning based on the theory of multiple external representations (MER) in learning objects (LO), discussed by Leite et al. (2013).

The use of an LO aims to promote individual learning because it helps students to become aware of their errors, which may expand their ability to think and solve problem situations and create hypotheses –aspects that are fundamental for the self-regulation of their learning– and reach new knowledge (Hadwin & Oshigh, 2011; Mendes, 2014).

This work aims to present the conception and initial evaluation of an LO with immediate feedback that allows the students to self-regulate their learning of the mathematics concepts necessary for DIC 1. First, to support the conception of the LO and the relevance of the proposal, we conducted a bibliographic survey, summarised in the following section. Then, we elaborated a prototype of the LO and the first phase of the assessment of this design, carried out by seven mathematics teachers and two mathematics education researchers, whose results are presented and analysed below.

## **THEORETICAL REFERENCE**

The reality of Brazilian universities has changed over the years, which can also be observed at the Federal University of Technology of Paraná (UTFPR), especially in the DIC 1 discipline. Among many factors,

students with difficulties in basic mathematical concepts, high failure rate, full classrooms, and the high number of students who seek to enrol in a subject that does not offer seats for

everyone (many students, few classrooms and/or an insufficient number of teachers). This description is well known to many professors of Differential and Integral Calculus (DIC) from many Brazilian universities (Borsoi et al., 2017, p. 460).

Turning our eyes to dropouts in engineering courses, the report made by Tonini and Pereira (2019) for ABENGE – Brazilian Association of Engineering Education highlighted some of these causes. These are related to the newcomers' personal issues and can also be influenced by the students' school education background, i.e., they enter higher education with a knowledge deficit, and cannot keep up with the intense routine of studies required, culminating in successive failures.

Along with the increasing dropout and high-failure rates in these courses, several factors can be raised regarding this issue in the engineering area, such as the socioeconomic factor and the way of entering higher education, cited by Gomez et al. (2015) in an analysis of dropouts in engineering courses at the Federal Technological University of Paraná. Godoy and Gerab (2018), in a study carried out with teachers from an engineering school in the ABC of the state of São Paulo, point out that, besides the factors mentioned above, students show some discouragement due to the difficulties encountered in basic education mathematics and the difficulties of adaptation in the transition from high school to higher education.

Cunha and Carrilho (2005) emphasise that despite the difficulties of adaptation and academic performance of students, in the transition from high school to higher education, a successful adaptation process at the beginning of graduation can be a determining factor in the persistence and success of students throughout their academic life. This points to the importance of a more careful look at the transition between these two very different levels of education.

Trevisan e Mendes (2018), in turn, state that when starting the DIC 1:

our student usually has characteristics arising from his routine of studies in basic education, such as: lack of previous experience with investigative tasks; expectation of lectures, followed by the resolution of tasks similar to the examples presented by the teacher; misconceptions about some mathematical concepts (often arising from the focus on the mechanisation of processes, instead of understanding and

attributing meaning); habit of working, most of the time, individually, having difficulty exposing and discussing their ideas in a group or to the whole room. (p. 213)

For Godoy and Gerab (2018), it is necessary to take a closer look at this critical moment in the initial phase of the academic life of a higher education newcomer. The authors emphasise that teachers should frequently revisit their teaching practice to contribute to the academic success of these young people entering a new phase in their educational life.

### **DICT in DIC Teaching and Learning 1**

The use of DICT as a way to establish a relationship between the teacher and the student has been discussed since the late 1980s (Faria et al., 2018). In mathematics education, studies on teaching using DICTs refer to the same time and, according to Borba et al. (2020), the use of fast and quality internet is “democratising” the publication and dissemination of digital materials in mathematics on the great network.

In higher education, in particular, in DIC teaching, there is much discussion about the use of DICT and the changes they are generating in recent years, such as new methodological approaches and the best ways to take advantage of these tools in the classroom as didactic resources (Santos et al., 2020). The authors point out that the DIC/DICT intersection as a didactic-pedagogical resource in the classroom can favour the teaching and learning processes of the subjects’ contents, enhancing and giving new meaning to the construction of mathematical knowledge.

These aspects are in line with the guidelines present in the National Curriculum Guidelines for the Undergraduate Course in Engineering (Resolution CNE/CES No. 2/2019) (Brazil, 2019). In this document, the profile of the graduate of the undergraduate engineering course is described, which must, among others, comprise the following characteristics:

Communicate effectively in written, oral and graphic forms: a) be able to express yourself properly, whether in your native language or a language other than Portuguese, including through the consistent use of digital information and communication technologies (DICT), always keeping up to date in terms of available methods and technologies. (Brasil, 2019, p. 2)

It also emphasises that the use of methodologies for active learning should be encouraged in the course to promote a more student-centred education.

Based on the profile of its entrants, Resolution CNE/CES No. 2/2019 in Art. 7 defines that the Pedagogical Project of the Course (PPC) must:

Provide for reception and levelling systems, aiming at reducing retention and evasion, when considering: I. the basic knowledge needs that are prerequisites for entering the activities of the undergraduate engineering course; II. pedagogical and psychopedagogical preparation for monitoring the activities of the undergraduate engineering course; III. guidance for newcomers, aiming to improve their conditions of permanence in the higher education environment. (Brasil, 2019, p. 5)

With these guidelines and technological advances, the use of DICT has become increasingly necessary in teaching and learning processes in different areas of knowledge. In particular, issues related to higher education mathematics have been taking shape within the scope of research, and several authors deal with the use of DICT in this context, especially in DIC subjects.

An example of authors who highlight the use of DICT is Borba and Penteadó (2019), who describe the use of DICT as allies to the teaching of DIC, but often, they are related only to the fragmentation of content and replacement of the teacher by a platform of virtual learning. Certainly, its use can be seen by many with fear with regard to changes in the way of teaching, as they believe that they are not yet prepared, while others show a certain distrust. Thus, for the teacher to start using new technologies, it is necessary for him/her to reflect on the subject and have as a principle that everyone can produce mathematics in its different expressions. Which leads us to reflect on a change in the teaching and learning process and the use of DICT, particularly in the DIC 1 subject.

As discussed by Trevisan et al. (2018), DICTs can contribute in this direction, as they enable visualisation, reflection, and deductions to refine knowledge. The authors warn, however, that no technology guarantees such processes by itself. Therefore, careful planning is necessary to use it, in particular, in the elaboration, application and refinement of tasks that make use of this resource, which configures the importance of the student's active role in these tasks (Trevisan et al., 2018b; Nonato & Costa, 2021).

As authors of this work, in a search on the discussed topic, we found a lack of research focusing on the use of LO with feedback for self-regulation of DIC and Pre-Calculus learning. This context justifies the need for a closer look at this issue, and a more in-depth study on this relevant topic for the teaching and learning of DIC.

## **Learning Objects**

In this work, the LO are defined as a cognitive resource to assist and expand teaching and learning with peculiar characteristics, among which reusability, granularity, accessibility and interoperability stand out (Marczal et al., 2015). Reusability is linked to its size: the smaller, the easier to be reused. However, care should be taken to keep them small and relevant in terms of content. The granularity aims for the LO to be a modular unit that can be combined to form larger units. Accessibility aims to determine how an LO can be found and accessed, and it is desirable to do so over the internet. Interoperability concerns the use of the LO on different platforms and operating systems (Marczal, 2014).

With this definition of LO, one can think of better interaction between the student and the mathematical content in the DIC discipline. Inspired by Marczal's (2014) work, this study proposes the construction of LO at FARMA as an alternative to the difficulties and needs mentioned above regarding the students' performance in DIC.

According to Marczal et al. (2015), FARMA allows constructing tasks aimed at learning concepts of analytical induction that involve arithmetic and/or algebraic expressions. Its main feature is to be a simple and objective tool for the development of highly interactive LOs and to promote learning from errors, which are recorded from the moment they occur. With this, students can later explore their own mistakes, allowing teachers to have integrated or individual access to their results.

The tool has already been investigated and adapted in works such as Pereira's (2018), who developed LO at FARMA, covering essential geometry content for students' learning in the 9<sup>th</sup> grade of elementary school. In higher education, Kutzke and Direne (2018) studied the mediation of error in computer programming teaching, describing the proposal of a system framework for the manipulation of error records (FARMA-ALG) that promotes the mediation of the error with effective teacher participation. In turn, Silva et al. (2018) present ADAPTFARMA, a modified version of FARMA, which consists of a sequence

of exercises after an introduction, which is the theoretical part of an LO in which concepts are defined through text, images, audio, and videos. In the same vein as FARMA-ALG and ADAPTFARMA, the work intends, in a second phase, to adapt a version of FARMA that is specific for error remediation and self-regulation of the learning of DIC concepts, called FARMA-CALC.

### **Error Remediation and Learning Self-Regulation**

Error remediation is a fundamental element in the construction of concepts. From errors, the students can expand their knowledge base further, i.e., to remedy means, in this case, to help the learner recover from an error (Leite et al., 2013; Ferreira & Pimentel, 2016).

Through errors, two main forms of learning can be offered: error remediation, in which the student receives immediate feedback on the failure, allowing its recovery and continuation of studies; and error feedback, in which the learner can explore their errors and, from there, try to redo the interaction in which the error was made to understand its reason and then solve it (Marczal et al., 2015).

In this work, the term “remediation” is understood as a set of interventions made by the teacher in a teaching process associated with the regulation of learning, which has the role of providing feedback. The feedback can optimise and regulate learning, giving the student full knowledge of the content. One of the annexed functions of regulation is feedback, which should provide necessary information on overdue stages and difficulties encountered during an activity (Mendes, 2014; Mendes et al., 2018).

Feedback is defined as a set of information provided to the student about their performance, which shapes their knowledge and helps in their learning process (Costa et al. 2016). The authors refer to feedback as a pedagogical resource that can arouse a reflective action in students and contribute to their learning.

In this context, several authors explore the idea of using feedback in virtual environments to teach mathematics. Melo et al. (2018) present a proposal for using video-feedback in activities developed in the discipline of mathematics in high school. The objective of using immediate video-feedback was to promote the improvement of student performance in a personalised way. Cordeiro et al. (2021) evaluate the use of feedbacks in digital educational games (DEGs) aimed at teaching basic mathematics operations. They did not observe

the use of effective feedback in the four mathematics DEGs evaluated. They emphasise, however, that its use could improve student performance in this area. Regarding remote teaching, Nóbriga and Dantas (2021) presented a proposal for an activity to be carried out in GeoGebra and containing exercises with automatic feedback. The article aims to provoke reflections and debates in the academic community about the use of didactic materials with automatic feedback in mathematics teaching.

Leite et al. (2013) propose the remediation of errors through feedback that uses the multiple external representations in learning objects implemented in FARMA. The authors present the application of the Pythagoras learning object and point out that it allowed students to deepen their conceptual knowledge using the connections acquired by manipulating the LO itself.

Based on these works, the types of feedback used in the LO are: mathematical definition of the concept addressed in the question; submission of another representation record for the same concept; short explanation of the issue; video suggestion, approaching the concepts, from existing repositories.

## **METHODOLOGY**

### **Research characterisation**

The investigation that gave rise to this article is qualitative, of an interpretive nature (Bogdan & Biklen, 1994), guided by the design science research - DSR (Dresch et al., 2015; Brocke et al., 2020). The DSR is a methodological process to conduct research with a more rigorous character. In addition to presenting ways of conceiving knowledge and carrying out scientific research, DSR can also be seen as a set of analytical techniques that allow the development of research in several areas (Lacerda et al., 2013).

Although DSR was initially applied in engineering, its increasing use in several research areas can be observed. In information systems, for example, Rocha et al. (2014) describe the process and results of a survey conducted on the influence of group size on participation in educational chat, in which the DSR guided the conduct of a behavioural survey and the production of an artefact (mathematical model to estimate the size of the chat group).

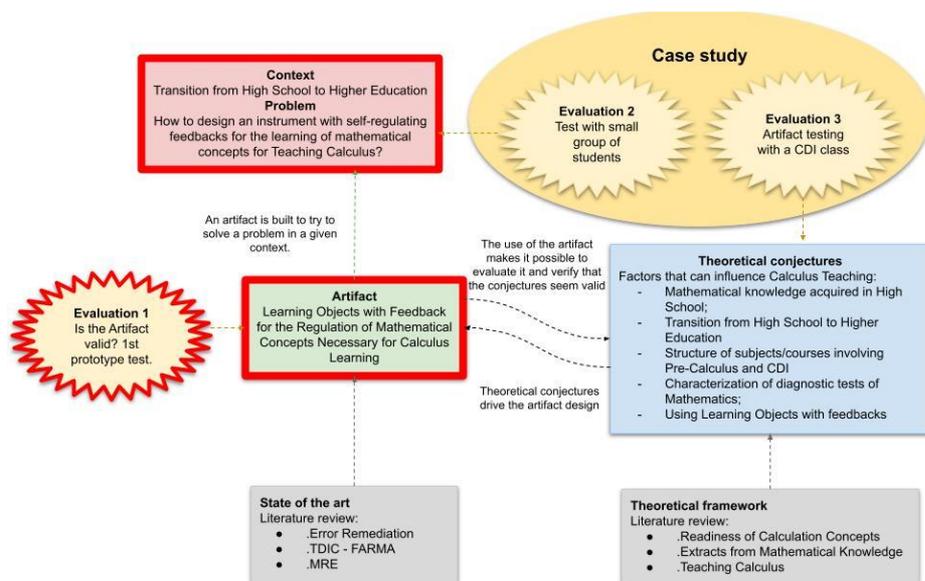
In education, Pimentel et al. (2020a) present research on smart music mats, in which the DSR method is linked to the thinking-doing process. In the area of education, Szesz Junior (2021) presents an assistive technology, called MATH2TEXT, which allows blind students to access mathematical

expressions through the computer. DSR is also one of the methodologies suggested by the Special Committee on Informatics in Education (Pimentel et al., 2020b).

In DSR, the artefact is designed based on theoretical conjectures, and its assessment provides new data on the conjectures elaborated to increase theoretical knowledge (Rocha et al., 2014). Figure 1 shows a conceptual map of the principles of DSR, applied to the research that gave rise to this article, and which follows the principles of Hevner et al. (2004). In particular, this article deals with the design and first assessment of the artefact (OA), highlighted in red in Figure 1.

**Figure 1**

*Elements of DSR in the construction of LO. (Hevner et al., 2004)*



## LO design

The proposed artefact was conceived as an LO that uses immediate feedback, which should allow the student to self-regulate their learning in relation to the mathematics concepts necessary for the learning of DIC. It

consisted of a set of questions elaborated from the calculus concept readiness instrument - CCR, proposed by Carlson et al. (2015), organised based on the taxonomy of 23 items distributed into five categories (Table 1), as well as other validated questions (ENEM, Prova Brasil, Pisa) that cover the different strata of mathematical knowledge (Cuevas-Vallejo et al., 2018).

**Table 1**

*CCR Taxonomy (Carlson et al., 2015, p. 216, our translation)*

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**Reasoning skills**

R1 - Proportional Reasoning: Observe that two quantities that are changing together are related by a multiplicative constant, and that as the two quantities change together, the ratio of one quantity to the other remains constant; then use that knowledge to determine new values of one quantity for specific values of the other quantity;

R2 - View of Function as Process: View a function as a process that maps input values in the function's domain to output values in the function's image;

R3 - Quantitative and Covariational Reasoning: Conceptualise quantities in various situations and think about how two quantities in one situation change together.

**Understand, represent and interpret patterns of function growth**

F1 - Linear;

F2 - Exponential;

F3 - Nonlinear polynomial;

F4 - Rational;

F5 - Periodic.

**Understand and use the following concepts or ideas**

U1- Magnitude;

U2 - Variable;

U3 - Slope / Constant rate of change;

U4 - Average rate of variation;

U5 - Function composition;

U6 - Inverse function;

U7 - Function graph translations (horizontal and vertical displacements).

**Understand central ideas of trigonometry**

T1 - Angle measurement;

T2 - Radian as a unit of measurement;

T3 - The sine and cosine functions as the covariation of the length of an arc (measured in units of the radius of the circle) and the horizontal and vertical coordinates of the end of the arc (measured in units of the radius of the arc).

T4 - The sine and cosine functions as a representation of the relationship between an angle measure and the sides of a right triangle.

### **Other skills**

H1 - Solve equations;

H2 - Represent and interpret inequalities;

H3 - Use and solve systems of equations;

H4 - Understand and use function notation to express one quantity in terms of another.

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For the first prototype, we chose two questions involving the concept/idea of composition of functions (U5), associated with nonlinear growth patterns (F3) and the ability to understand and use function notation to express one quantity in terms of another (H4). The CCR test, as a whole, is not in the public domain, and the questions used here are inspired by the work of Carlson et al. (2015), Carlson et al. (2010) and others.

The LO was developed in such a way that, for each question, for each error made by the student in one of the resolution steps, a different type of feedback is presented. The variation of feedback aimed to contemplate types of intervention that are more effective in a new attempt to resolve the same item. The following types of feedback were adopted: mathematical definition of the concept addressed in the question; submission of another representation record for the concept; short explanation of the issue; video suggestion, approaching the concepts, from existing repositories (YouTube Channels, University Sites, etc.); presentation of similar examples.

The examples used in the LO (Figure 2) explore the concept of a composite function, whose objective, according to the CCR instrument, is to verify whether the student: i) can visualise the function as a process; ii) understands what it means to evaluate a function; iii) understands how to compose two given functions in a graphical representation context.

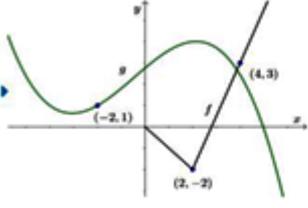
Figure 2 presents (a) a question about the concept of composite functions, and the feedback (b) shows the graphic representation of the image of a function for Item 1. Feedback (c) presents an example of a composite function like the one proposed in the question.

## Figure 2

Example of an LO question with feedback.

**(a) Question 1.**

Use os gráficos de  $f$  e  $g$  para calcular o valor de  $g(f(2))$ .



**Item 1:**  
Calcule o valor de  $f(2)$ ?

Resposta:

Clique aqui para responder

**Item 2:**  
Para qual valor de  $x$  temos  $g(x) = 17$ ?

Resposta:

Clique aqui para responder

**Item 3:**  
Usando a definição de função composta e os resultados obtidos nas questões anteriores, calcule o valor de  $g(f(2))$ .

Resposta:

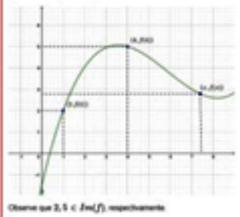
Clique aqui para responder

**(b) Feedback with graph.**

Resposta: 5

Resposta incorreta, analise a dica e tente novamente.

**Dica 2**  
Veja um exemplo na imagem abaixo:



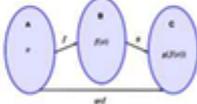
Observe que 2, 5 e  $f(2)$ , respectivamente.

**(c) Conceptual feedback with similar example.**

Resposta: 4

Resposta incorreta, analise a dica e tente novamente.

**Dica 2**  
Observe que o domínio de  $g$  é igual a imagem de  $f$ .



**Exemplo:**  
Determine o  $pgf(x)$  e  $fpg(x)$  nas funções  $f(x) = 3x + 2$  e  $g(x) = 5x$ .  
 $pgf(x) = g(f(x)) = g(3x + 2) = 5(3x + 2) = 15x + 10$ .  
 $fpg(x) = f(g(x)) = f(5x) = 3(5x) + 2 = 15x + 2$ .

Thinking about possible answers given by the students and based on Table 2 of the CCR, it is possible to elaborate feedback for each question elaborated in the LO. An LO prototype (artefact) was built for testing at FARMA with two questions inspired by the CCR test and with feedback that relate various representations, which are based on the use of techniques to represent, organise, and present knowledge.

## LO design evaluation procedures

Braga et al. (2013) propose an iterative methodology called INTERA (Intelligence, Educational Technologies and Accessible Resources), which considers the LO development process as a project divided into several components (phases, roles, stages, and artefacts). INTERA adequately addresses pedagogical and validation issues in the development of an LO, satisfying the principles of DSR-based research. The authors state that a collaborative effort by a multidisciplinary team is required to produce a high-quality, reusable LO. One of the proposed steps is the Tests and Quality, which verifies the acceptance of the LO by the users.

Thus, for the evaluation of the prototype of the LO, two groups were invited, one of them formed by two teachers of DIC, also mathematics education researchers (called specialists - E1, E2), and another formed by seven teachers of basic education and higher education, who took a course on DIC Teaching offered by the postgraduate programme, of which the author is a member (referred to as professors only – P1, P2, ..., P7). It is noteworthy that the research was approved by the Ethics Committee of (omitted for evaluation) (process n° 08957619.3.0000.5547; opinion n° 4.683.713). We proposed that the two groups test and evaluate the LO as if they were new students in DIC.

### **Data production and analysis**

The data to be analysed in this article was taken from interviews we carried out with both groups to determine their impressions about the LO, focusing on the feedback proposed in the prototype. A script for semi-structured interviews (Bogdan & Biklen, 1994) in each of the groups contained: (i) a presentation of the research objective and (ii) a presentation of the LO at FARMA; (iii) questions about the OA's perceptions regarding the arrangement of elements, clarity, division of questions into steps and their structure; (iv) questions about the overall evaluation of the LO design; (v) open space for suggestions for improving the LO and (vi) analysis of the proposed feedbacks and perception of which would be more appropriate to the different concepts addressed in the questions. We told the evaluators that they were free to make suggestions about what types of feedback could be used and to make criticisms regarding the usability of the LO.

We collected data from videos recorded during the interviews via Google Meet, fully transcribed by one of the authors.

## RESULTS AND ANALYSES

We started with a comment on the LO overview from one of the experts, who highlighted some important points about its design.

*“I made three errors and with each error, he gave me feedback, on the fourth attempt he gives me an answer. I liked it a lot, but when talking about a learning object, the idea of dynamic comes to me, and I found it a little static... The video feedback could be more focused on the prerequisite of the composite function”.* (E1)

One of the points raised by E1 concerns the lack of dynamism of the LO. Another is the need to rethink the prerequisites necessary for the mathematical concept. Regarding the order presented in the feedback, E1 adds that

*The first error led to the definition, if we're going to think the way you think calculus, maybe this was the last feedback, not the first! ... I think the feedback should be more provocative, as letting go of the definition provokes very little!”* (E1)

In the same direction, E2 suggests rethinking the proposal to bring a definition as initial feedback, as this could “provoke” little or no reflection on the error made:

*“The definition seems more complex than the exercise itself!”* (E2)

The specialists had difficulties in recognising the expected answer to one of the LO questions due to the way the statement had been prepared and the need to provide an algebraic expression as an answer. In addition, there were doubts about the terms used in the formulation of the question, which led us to work with banks of already validated questions (such as ENEM, Prova Brasil, and Pisa) and to rethink how students were asked to respond.

*“I think you should improve the wording of the question!”* (E1)

*“I couldn't see the answer that I think is the correct one! Is it the answer? It must be some problem with the platform!”* (E2)

One question raised by the experts was about the target audience of the research. It was not very clear to them whom the LO was aimed at.

*“Should the questions be formulated with a pre-calculus audience in mind? Does the type of question you are asking suit*

*the Calculus student? These questions may not reflect a direction in the reality of Pre-Calculus students!” (E1)*

*“Within the context of UTFPR, I think that other people would buy into this idea of having the instrument that everyone can apply here, to rethink their teaching practices, but would not use it within a classroom context... could be a diagnostic protocol with feedbacks guided to basic concepts. As a study guide taking advantage of the tool structure”. (E1)*

This dialogue with specialists in mathematics education was very productive, as it led us to reflect on some unclear aspects of the LO and rethink it, especially in defining the target audience and reviewing the formulation of the questions.

Let us now look at some reports from teachers about their perceptions of the LO. A positive point highlighted about the design of the LO was the presence of feedback with varied configurations, especially the video.

*“I found it very interesting, I put in incorrect answers to see how far it would go. I saw that there are videos and there comes a point that gives the answer, on the fourth attempt. I think the student would be self-sufficient with the use of this platform”. (A1)*

*“It’s quite interesting because it gives feedback and at the end a video that helps to complement and then gives the answer”. (A3)*

Notes were also made about some limitations of the LO, linked to the FARMA platform, involving its layout.

*“What bothered me the most was the two blue arrows, because I thought I had more help, but nothing appeared, only after another error that the next help appeared... It bothers things to be right in the corner. The layout should be more user-friendly.” (A2)*

*“On the fourth error, it already gives me the answer, so why do I need to type the answer again?” (A4)*

*“I tried about twelve minutes, and it gave an error in  $4\pi$ , because it is  $4 * \pi$  right?” (A5)*

This group also commented about the target audience for which the LO is intended and the order in which the feedback was presented.

*“My question was, is this more like tutoring? Because the first tips were definitions with symbologies and such, then I realised that the student must have knowledge of the definitions to remember. So, I deduced it was like reinforcement! Is the definition as the first tip the most efficient? Wouldn't using the graph be more interesting? (A3)*

*“I think the definition at the end gets more interesting!” (A2)*

*“I also agree because, with the numerical and graphic tips, he builds the concept and definition at the end”. (A5)*

Regarding the answers of the two groups, the potential of the LO will be analysed in relation to three categories that emerged from the analytical process of organising the data: LO Design, Type of Feedback, and Order and Formulation of Questions.

The LO design was generally well accepted. However, one of the points the teachers raised was that it was too static, which led us to rethink the formulation of the questions to attribute a more dynamic character and make them more understandable for the user. They found it challenging to fill in the answers with a formula, which made us suppress this type of question in a future version of the LO.

The diversity of feedback, with emphasis on the video, was pointed out as a positive point, but the order of presentation must be revised, since the students must receive the feedback in such a way that they build the concepts until reaching the formal definition.

They did not find the formulation of the questions very attractive, as it presented a very technical character, which could discourage the student. Thus, the issues must be restructured, and a more in-depth study must be done for their expansion.

## **DISCUSSION OF DATA AND FINAL CONSIDERATIONS**

This article presented the first results of broader research with an interface in the areas of mathematics education and computing in education. Its main objective was to present the conception and evaluation of an LO with feedback that allows the students to self-regulate their learning regarding the

mathematics concepts necessary for the DIC. To support the conception of the LO, we carried out a bibliographic study on the difficulties faced by the newcomer students entering engineering courses, in particular at UTFPR, in relation to the concepts of mathematics necessary for the teaching of DIC, as well as the use of DICT in mathematics teaching and the self-regulation of learning through LO with immediate feedback.

This article focused on presenting the evaluation of the design of an LO prototype to validate a model based on the readiness of DIC concepts, adapted and implemented in a platform for the remediation of mathematics errors (FARMA). This model was tested with different types of feedback to verify which interventions, in the reviewers' opinion, would be most appropriate for different contexts.

From the results obtained from this investigation, we could analyse the potential of the LO in relation to three categories that emerged from the analytical process of organising the data: the first deals with the design of the LO, which was generally well accepted. Nevertheless, the questions must be improved to have a more dynamic character and their formulation be clearer for the user.

The second refers to the types of feedback and their order of presentation. The specialists indicated that the students should receive feedback to gradually elaborate on concepts until they reach the formal definition. The last category is related to the formulation of the questions, whose evaluation indicated the need to reformulate the statements.

Making the necessary adjustments and expanding the range of questions, we intend to move on to the next stages of the research (Figure 1), which include organising the total set of questions that address the mathematics concepts necessary for learning DIC and that explore the strata of numerical, algebraic, and functional knowledge, as well as the application in the classroom (Case Study), with a group of beginners in DIC. We expect that the research educational product can help students with their difficulties in math concepts necessary for DIC learning.

## **AUTHORS' CONTRIBUTION STATEMENTS**

ASC, ALT, and DM conceived the idea presented in the article. ASC prepared the learning object, collected the data, and performed the analyses

under the guidance of ALT and DM. The three authors discussed the results and contributed to the final version of the manuscript.

## DATA AVAILABILITY STATEMENT

The data supporting this article are in the custody of ASC and may be made available upon reasonable request.

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