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Geometry in Mathematics Teaching in Undergraduate Courses at Brazilian Federal Universities

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ABSTRACT

Background: Geometry has been an essential point of reflection in Mathematics Education, especially in teaching. **Objective**: We conducted a numerical calculation to quantify the study load of Euclidean geometry concerning the total course load. From this data, we established different pathways to explore the Geometry subjects of these courses and their approaches. We aimed to outline an understanding of Geometry teaching in Mathematics teaching courses in the selected universities. **Design**: We consulted the CPPs through the official websites of the universities. We catalogued the total study load, the subjects of Geometry, and a comparison of the Geometry subjects and their occurrence. Finally, we analysed the syllabuses and bibliography of one subject listed. **Context and participants**: We surveyed 68 Brazilian Federal Universities, all institutions that offer the Mathematics Teaching course. **Data collection and analysis**: We treated the data through qualitative analysis and presented them in a large table subdivided into smaller tables to present the data. **Results**: After analysing the tabulated data, its result corroborated the research of Crescenti (2005), Lorenzato (1995), Lovis (2009, 2013), Pavanello (1989, 1993), Perez (1991, 2000), Serralheiro (2007) that Geometry is little taught in educational institutions, be them K-12 or higher education. **Conclusions**: We have identified a significant variation in each course's study load percentage dedicated to Geometry. Even among those that reached the highest values, it is still possible to question if those numbers are enough to solve the issues pointed out by K12 teachers. We have also identified divergencies in the approach suggested in the syllabuses and bibliographies of the subject Plane Geometry. Such numerical and approach deviations need to be deepened by other works.

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Geometria nos cursos de licenciatura em Matemática das universidades federais brasileiras

INTRODUCTION

Mathematics[1] can be considered a set of activities, knowledge, and social practices. It is traditionally divided into branches, among them: Algebra, Geometry, and Arithmetic. Though national education has gathered them for almost 100 years, their division is evident, for example, in the high-school "fronts" in private schools, chapters in K-12 didactic books, and in the subjects of Mathematics Teaching undergraduate degrees. Other subjects on pedagogical and, more recently, practical knowledge were added to the latter, creating other organizational blocks[2]. Studies such as Pavanello (1989), Pais (2019), and Ramassotti (2015) point out the lack or the distancing of Geometry in K-12 education. Each of these authors highlights different movements that might have led to this scenario, flexible guidelines, the Modern Mathematics Movement (Pavanello, 1989), and the "revaluing of Geometry" (Pais, 2019). Ramassotti (2015), in particular, refers to the focus of our study, the training of Mathematics teachers:

> considering that the problem starts in the training of Mathematics teachers, in this study, we heard the arguments of teacher trainers, aiming to understand their opinions and remarks on Geometry teaching in the Mathematics Teaching undergraduate degree. This seems to directly reflect on teachers' actions in the public education system. According to Perez's (1991) study, teachers focus on arithmetic and algebra, needing more knowledge and methodology to teach Geometry. (Ramassotti, 2015, p. 09)

[1] We opted to use the term Mathematics instead of just Math to indicate and defend the plurality encompassed under this name, not restricting it to the classic objects of the subject of mathematics.

[2] Souza (2022) analyzes the divisions and subdivisions of five Mathematics Teaching undergraduate courses at Universidade Federal de Mato Grosso do Sul.

Even within Geometry, there are very particular subdivisions and work tendencies. Pavanello (1989), over 30 years ago, indicated an “abandonment of Geometry" mainly due to the Modern Mathematics Movement and the flexibilization of school content in the 1970s. In this movement, there was a general paradigmatic change in mathematical knowledge, mainly supported by algebra processes to ground mathematical knowledge. Thus, a different Geometry was presented, less based on axioms and Euclidian-based theorems and more on transformations (Catunda, 1990; Detoni & Oliveira, 2018). The Euclidian axiomatic model has lasted since the 4th century A.D. and has been going through systematic questioning and improvement. For instance, the reformulation proposed by David Hilbert (Greenberg, 1993), or even the non-Euclidean geometries that work similarly to the Euclidean model, though refuting or altering its postulates/axioms. The influences of this axiomatic deductive model can be seen in other sciences and the creation of Mathematics Education. Imenes (1989) show how the Mathematics didactic books continued to perpetuate such operation mode. More general statements about how little Geometry is taught in Brazilian schools (Crescenti, 2005; Lorenzato, 1995; Lovis, 2009, 2013; Pavanello, 1989, 1993; Perez, 1991, 2004; Serralheiro, 2007) need, therefore, to consider what Geometry is not taught because there seem to be diverse possibilities to this comparison. To illustrate this point, sticking only to the axiomatic deductive model, we observed (Moreira, 2018) how choosing a support book for the undergraduate subject Geometry Elements can substantially change the axiomatic proposed. It is impossible to compare two books at certain moments to understand and delineate a safe way to solve the proposed problems. If we consider the endless possibilities to approach the subjects of geometric character, the field becomes overly dry and lacking studies that can indicate what geometries and their measures are being used in K-12 education and in teacher training.

Thus, we intend to investigate how and to which extent the Mathematics Teaching undergraduate degrees in Brazilian federal universities present the subjects of Geometry teaching. Based on this survey, we analyze the primary bibliography of one subject in several institutions.

LACK OF UPDATED BIBLIOGRAPHY TO TEACH GEOMETRY IN HIGHER EDUCATION

The main difficulty in raising the bibliography was finding updated studies on the theme. Beyond not having all national studies catalogues in the same database or repository, a severe problem faced by Brazilian research on Mathematical Education is the lack of studies on higher education, as most results were focused on K-12 education –mainly high school.

Many of them present a problem and/or a didactic situation involving Geometry in high school. They eventually cite higher education, suggesting it as a possible origin of the problem or as a place for intervention. The studies bring to the discussion some authors, such as Vianna (1988), Imenes (1989), and Pavanello (1989), who have been considered precursors of this debate in Brazilian Mathematical education. In our research, since 1995, there has been a sharp fall in the publications about Geometry, discretely reappearing with Perez (2000), with an article about the reality of Geometry teaching in elementary, middle, and high school in the state of São Paulo[3]. Since 2000, rare studies on the investigation of Geometry in Higher education were pulverized within several research repositories. Thus, it is difficult to find them using the same search mechanism. However, between 2004 and 2015, there was a new surge, though limited to two to three publications a year. Since 2015 we can see a renewal in the studies about Geometry. Nonetheless, these studies are focused on K-12 education, mainly high school. For example, Barros & Mendes (2017) conducted a bibliographical survey between 1990 and 2010 on the dissertations and thesis on Spatial Geometry in high school. In the same direction, the article by Souza; Almeida & Madruga (2022) surveyed ten years of studies available in two scientific databases, the *Catálogo de Teses de Dissertações da CAPES* (Theses and Dissertations Catalog of CAPES) and the *Banco Digital de Dissertações e Teses* (BDTD) of Instituto Brasileiro de Informação, Ciência e Tecnologia (Ibict- Brazilian Institute of Information, Science, and Technology). Their mapping sought studies that used the Problem-Solving methodology to teach and learn Geometry in K-12 education.

[3] It is important to highlight that this conclusion was established based on this survey – we bring below the problem raised by several studies that do not appear.

Oliveira & Cristovão (2022) work analyzes the Spatial Geometry questions on the Exame Nacional do Ensino Médio (ENEM- National High School Exam). Souza's (2016) dissertation presents an experience with Problem- Solving for learning and teaching Geometry supported by support material composed of touchable geometric solids with senior students in a public high school in Campina Grande (state of Paraíba). Campos & Ponte (2022) approach the thought process of 9th Year students who won medals in the *Olimpíada Brasileira de Matemática das Escolas Públicas* (OBMEP- Brazilian Mathematics Olympics for Public Schools) regarding arithmetic/algebra exercises and Plane Geometry. Another dissertation that emerged in our research was Manoel's (2019) research. It proposes a Geometry teaching model through 11 thematic axes of analysis (curriculum, history, other areas of knowledge, nature, everyday life, affectivity, problem-solving, cognitive abilities, critical thinking, aesthetic appreciation, and creativity), presenting the importance of Geometry as a learning base.

Santos (2015) focuses on Information and Communication technology by using the digital tool SketchUp to teach Spatial Geometry to high schoolers. Damasceno (2021) analyses another digital tool (the software Geobra) to teach spherical and non-hyperbolic Non-Euclidean geometries. Zorzin & Silva (2022) also used the software Geogebra, though this time as a pedagogical tool for in-service Math teacher training. The study of Benito; Silva & Casabò (2022) approaches the teaching of conic in high school based on the Anthropological Theory of the Didactic (ATD) methodology, grounded on the principles of Didactic Engineering for the Teaching of Conic (parable, ellipsis, and hyperbole) for senior high schoolers. Didactic Engineering also appears in the article by Kiefer; Santos & Bisognin (2022), which maps the dissertation and theses in the repositories between 1996 and 2021. Focusing on Geometry, the research presented 69 theses, among them 15 on Analytical Geometry.

Lourenço (2014) reports on Geometry teaching through the construction of mosaics in paving, using the geometry properties of the polygons. The proposal is seen as fruitful, creating a bridge between Geometry definitions and postulates with students' touchable learning. Another study in this direction is Becker's (2009) dissertation in which he creates a didactic sequence to teach the Geometry of solids through their unfolding. Becker (2009) highlighted an activity he called Becker's box in which there is an interaction with solids through touch. Kiefer & Mariani (2021) have also done a bibliographic survey in research databases seeking works that developed some didactic sequence on plane figures using software in K-12 education.

These studies above appeared in our survey on research databases and repositories[4]. They bring the discussion on Geometry; however, they are not directly focused on the theme of Higher Education. They, at most, make some remarks about this education level. The studies presented below focus on Higher Education, mostly on Mathematics Teaching undergraduate degrees. However, none directly approaches the issue of teaching and learning Geometry broadly, as we intended.

Guerato's (2012) dissertation presents an approach to teaching Analytical Geometry based on the use of vectors in comparison with the classical Cartesian approach. Through the Registers of Semiotic Representation (TRSR) and Didactic Engineering, Guerato (2012) applies the methods in a Mathematics Teaching undergraduate class at a Brazilian public university.

Pereira (2019) proposes a methodology to teach Plane Euclidean Geometry with the aid of GeoGebra, a software of dynamic geometry, for undergraduates on a Mathematics Teaching degree. Based on Didactic Engineering, Pereira (2019) proposes a methodology that potentializes the learning of several topics in Plane Geometry, be it the Axiomatic Treatment and/or the Study and Development of Polygons. Martins (2018) subtly gets closer to our research aim by analyzing university students' knowledge of Analytical Geometry. The difference was that the survey was conducted with undergraduates from the Physics Teaching degree. By analyzing the dissertation, we can perceive that the measurements were algebraic and non-axiomatic. There was a care to understand the learning of Analytical Geometry through solving algebra problems involving geometry, which is farther from our aim.

Vieira (2017) conducted a case study with six graduates with a Mathematics Teaching degree who, during their undergraduate studies, received axiomatic training in Geometry which, according to the research reports, has helped novice teachers in their pedagogical practice – regarding the understanding of what was taught. However, in their professional practices, such an approach was little used, considering that

[4] We searched in Google Academic, Scielo, *Banco Digital de Dissertações e Tese* (BDDT). After the search, we created a table with all the results raised.

the approach of K-12 didactic books occurs intuitively, with no space for demonstrations or argumentations. Vieira (2017) quotes Lorenzato (1995) and Pavanello (2002) and recurrently states the co-dependence between K-12 and higher education for teaching Geometry.

Lopes (2019), similar to Martins (2018), approaches Geometry study for Higher Education. However, he refers to Analytical Geometry focusing more on Algebra and vector relation. We can highlight Souza's (2016) work with seven professors of the Mathematics Teaching degree and keeps her analysis to Higher education, not correlating them with K-12 education, making it unique in the studies on Geometry in Higher Education. Cunha (2010) conducted a fascinating investigation for our research as, in his hypothesis, he investigates a possible affinity relationship of an Online Education tutor with his teaching method. To Cunha (2010), the greater the affinity with certain content, the greater the ability to teach it. This relation affinity/learning also appears in Lorenzato (1995), Serralheiros (1999), Ramassotti (2015), and others. Cunha's research was held with 20 tutors of the Mathematics module of the *Curso Normal Superior de Educação a Distância* at *Universidade Estadual de Maringá* (UEM).

Lutz (2010) presents robust research aiming to insert notions of Fractal Geometry into the Mathematics Teaching degree of *Instituto Federal de Educação, Ciência e Tecnologia Farroupilha* (IFFar). We highlight that, according to Lutz (2010), the theme was chosen by noticing that the institution develops in the curriculum only Euclidean Geometry, similar to the initial assumption of our research. We also stress that Lutz (2010) promoted workshops with undergraduates to train them to teach Fractal Geometry in their respective pedagogical practices. There is a correlation between what is learned and what is taught, thus agreeing with several authors already mentioned.

The studies above are the result of systematic searches on the database platforms. We highlight this because other works we know were, for some reason, not listed on the search results, which has called our attention. In the *Grupo História da Educação Matemática e Pesquisa*[5] (HEMEP- History of Mathematics Education and Research Group), for example, three dissertations and one thesis on Geometry in Higher Education were defended in the last years. However, they did not appear

[5] www.hemep.org

in the results. Therefore, there may be other works that, until this moment, were not available in the repositories used. Besides them, we can highlight the chapter written by Leme da Silva (2010) which also does not appear in the searches but is relevant because it diverges from Pavanello's (1989) considerations when relating the Modern Mathematics Movement (MMM) with the "supposed" abandonment of Geometry in Brazilian K-12 education. Her data shows the movement's concern in introducing "another" form of approaching Geometry. This model would be grounded more on the introduction of Geometric Transformations and less on incorporating the language of Set Theory and the Euclidean axiomatic, which had already shown negative effects in most elementary education levels.

In the international scenario, we sought broader works. We found the conference by Keith Jones on the *XXIII Seminário de Investigação em Educação Matemática* (SIEM- Research Seminar in Mathematics Education), entitled "Geometrical and spatial reasoning: challenges for research in mathematics education" (Jones, 2012). In this conference, he defends the need for two closely connected aspects in high school education: deductive/geometric thought (based on Euclidean axiomatic) and spatial thought (connected to visualization and projection of bi and tridimensional figures). "These twin aspects of geometry, the spatial and the deductive, I argue, are not separate; rather, they are interlocked." (p. 3-4). Though Jones's assumptions are relevant to other educational levels by pointing out and distinguishing these two aspects, it is possible to consider Geometry from these perspectives or others. Though the author is precisely pointing out the need for a comprehensive treatment, it is impossible to imagine that such disconnection might take place. Jones's (2012) statement seems to dialogue with Pavanello (1989) regarding a possible "abandonment" not of Geometry itself, but of a way of working with it exclusively through its axiomatic structure.

WHAT WE DID AND HOW WE DID IT

As pointed out, our interest was to understand how Geometry is dealt with in the training of Mathematics teachers. However, to conduct the research, we needed to establish some limits to our work scope. Souza's Souza (2022) work has given us evidence of the power of analyzing the curriculum frameworks of the courses to raise questions and notes. We have opted to map the curriculum framework of the

Mathematics Teaching degrees in all Brazilian federal universities – at least one course of each university, if it is offered in multiple *campi* (as is the case of UFMS, which currently offers six Mathematics Teaching courses). We limited ourselves to Brazilian federal universities for some reasons: 1) the law of information access imposes the free availability of documents; 2) the service bulletins are all available online, as well as a webpage of the course in which we can see the Pedagogical Curriculum Proposal (PCP) and Curriculum framework; 3) last, but maybe more importantly, there are federal universities in all Brazilian states, thus, giving us the broad panorama we wanted. The project "Mapping of the formation and practices of Math Teachers in Brazil" (Garnica, 2018) has been showing, for more than 15 years, how different regions of Brazil, and even states in the same region, have particularities in teacher training, induced by migrations flows and local politics:

> The studies on the History of Education and the investigations on Mathematics Teacher training that use (mostly episodically and incidentally) a historical panorama – intentionally or not – consider the *Universidade de São Paulo* and the *Universidade do Brasil* as the centres of teacher training studies. These were the first Brazilian institutions with higher education for teacher training in the field of Mathematics, implying that these centres would disseminate guidelines that would circumscribe teacher training in several points of the country, which is, we believe, a partial –if not mistaken- view of this movement. (Garnica, 2018)

Besides this, as states and municipal systems regionalize K-12 curricula, these curricula can induce proposals for teacher training, as it is a professional demand.

Thus, we listed 69 federal universities, not considering their *campi*, as we would reach 280 unities, surpassing our data treatment and analyses. With this delineation, we know that we have left behind some important Brazilian universities. In some states, such as São Paulo and Paraná, there are many state universities with significant historical and current relevance for teaching mathematics teachers. The same can be said about private and religious universities, which we also excluded from our survey.

Out of the 69 Brazilian federal universities, only the *Universidade Federal de Ciências da Saúde de Porto Alegre* (UFCSPA) does not offer the Mathematics Teaching undergraduate degree. Therefore, there are 68 courses.

Brazilian federal universities usually have more than one *campus*. For example, UFMS has ten *campi* strategically spread in the state of Mato Grosso do Sul. In our research, we did not investigate these *campi* separately but the *Cidade Universitária*, the biggest *campus* of the institution.

We raised all information on the institutions' official websites in the Course Pedagogical Project (CPP) and the Curriculum Framework (a part of the CPP). When we did not find them on the websites, we directly contacted the course coordinator *via* email.

With the 68 CPPs as our starting point, we treated the material and raised the desired information. Even though we initially concentrated on numerical data, our analysis as a whole aimed to produce qualitative information because, among other factors, we are in a qualitative research paradigm in which all decisions taken involved a broad discussion and dialogue with other research aspects:

> To make science is to simultaneously work with theory, method, and technics, considering that the components of this tripod mutually determine each other: the way of doing depends on what the object demands and the answer to the object depend on the questions, the instruments, and the strategies used in data collection. (Minayo, 2012, p. 623)

Working with a large number of data demands a much preparation. Defining what we intended to take away from the source and correctly choosing the questions to be done is critical because each work step is long and demanding. Poorly formulated questions can lead to rework for endless hours. Thus, the treatment is also connected to the researcher's subjectivity because the pathways are chosen based on what one wants to know. Our questions also change depending on each new data that seems relevant to the work.

> Qualitative research aims to understand the phenomena and not simply to explain them. Anadón (2005) also mentions that in this approach, the emphasis is on the process, not the product. The subjects' interpretations, actions, behaviours, feelings, and senses are considered as

a whole. Silva (1998) and Anadón (2005) highlight that the researcher needs to create a research design, formulate the problem, and select adequate methodological strategies. (Leite et al. 2017, p. 45)

In this first phase, we opted to organize all the subjects of these 68 courses on a table, counting the study load of each one. After we grouped, in each course, the subjects focused on Geometry, such as Geometry I, II and III; Euclidean Geometry; Non-Euclidean Geometry; Analytical Geometry; Descriptive Geometry, among others. So, we could see the percentage of this group in the total study load.

We have analyzed only the compulsory subjects, i.e., those offered in a regular training trajectory, because the list of option subjects listed on the CPP often does not match the real offer over time. Though they are listed, there is no evidence that they were or are offered.

Considering the sizable volume of information and the number of pages needed to visualize this data, in the following table, we present the 68 courses with their state; university and acronym; total study load of the course; total study load of Geometry topics or subtopics; and the percentage that Geometry subjects represent in the course total. The courses were ordered from the highest to the lowest percentile:

Table 1

Percentage of Geometry in the Mathematics Teaching course in Brazilian Federal Universities

State	University	Acronym	Degree S.L.	Geo. S.L.	% Geo. Degree
Ceará	Universidade Federal do Ceará	UFC	2830	420	14.8%
Santa Catarina	Universidade Federal de Santa Catarina	UFSC	3000	396	13.2%
Minas Gerais	Universidade Federal do Triângulo Mineiro	UFTM	2925	375	12.8%
Distrito Federal	Universidade de Brasília	UNB	2820	360	12.8%
Pernambuco	Universidade Federal do Pernambuco	UFPE	2955	360	12.2%
Mato Grosso	Universidade Federal de Rondonópolis	UFR	3200	384	12.0%
Minas Gerais	Universidade Federal de Itajubá	UNIFEI	3240	384	11.9%
Minas Gerais	Universidade Federal de São João Del-Rei	UFSJ	2816	324	11.5%

Espírito Santo	Universidade Federal do Espírito Santo	UFES	3200	345	10.8%
Paraná	Universidade Tecnológica Federal do Paraná	UTFPR	3708	396	10.7%
Minas Gerais	Universidade Federal de Ouro Preto	UFOP	2550	240	9.4%
São Paulo	Universidade Federal de São Carlos	UFSCAR	3230	300	9.3%
Rio de Janeiro	Universidade Federal do Estado do Rio de Janeiro	UNIRIO	3230	300	9.3%
Mato Grosso do Sul	Universidade Federal da Grande Dourados	UFGD	3894	360	9.2%
Bahia	Universidade Federal do Sul da Bahia	UFSB	2958	272	9.2%
Sergipe	Universidade Federal de Sergipe	UFS	1965	180	9.2%
Rio Grande do Sul	Universidade Federal de Pelotas	UFPEL	3225	288	8.9%
Amapá	Universidade Federal do Amapá	UNIFAP	3360	300	8.9%
Rio de Janeiro	Universidade Federal Fluminense	UFF	2910	255	8.8%
Bahia	Universidade Federal do Recôcavo da Bahia	UFRB	3226	272	8.4%
Mato Grosso do Sul	Universidade Federal de Mato Grosso do Sul	UFMS	3230	272	8.4%
Rio Grande do Sul	Universidade Federal de Santa Maria	UFSM	3215	270	8.4%
Minas Gerais	Universidade Federal de Alfenas	UNIFAL-MG	3605	300	8.3%
Acre	Universidade Federal do Acre	UFAC	2900	240	8.3%
Tocantins	Universidade Federal do Tocantins	UFT	2955	240	8.1%
Tocantins	Universidade Federal do Norte do Tocantins	UFNT	2955	240	8.1%
Goiás	Universidade Federal de Goiás	UFG	3200	256	8.0%
Goiás	Universidade Federal de Catalão	UFCat	3200	256	8.0%
Pará	Universidade Federal do Pará	UFPA	3015	240	8.0%
Piauí	Universidade Federal do Piauí	UFPI	3075	240	7.8%
Rio Grande do Sul	Universidade Federal do Rio Grande do Sul	UFRGS	2800	210	7.5%
Minas Gerais	Universidade Federal de Viçosa	UFV	3210	240	7.5%
Alagoas	Universidade Federal de Alagoas	UFAL	3220	240	7.5%
Minas Gerais	Universidade Federal de Uberlândia	UFU	3230	240	7.4%
Roraima	Universidade Federal de Roraima	UFRR	3240	240	7.4%
Rondônia	Universidade Federal de Rondônia	UNIR	3336	240	7.2%
Minas Gerais	Universidade Federal dos Vales do Jequitinhonha e Mucuri	UFVJM	3010	210	70%
Maranhão	Universidade Federal do Maranhão	UFMA	2595	180	6.9%
Pernambuco	Universidade Federal do Agreste do Pernambuco	UFAPE	3150	210	6.7%
Paraná	Universidade Federal da Integração Latino Americana	UNILA	3893	255	6.6%
Paraíba	Universidade Federal da Paraíba	UFPB	2805	180	6.4%
Rio Grande do Sul	Universidade Federal do Pampa	UNIPAMPA	2810	180	6.4%
Rio Grande do Norte	Universidade Federal do Rio Grande do Norte	UFRN	2820	180	6.4%
Bahia	Universidade Federal Lusofonia Afro-Brasileira	UNILAB-BA	3590	225	6.3%
Ceará	Universidade Federal Lusofonia Afro-Brasileira	UNILAB-CE	3590	225	6.3%
Pará	Universidade Federal do Sul e Sudeste do Pará	UNIFESSPA	2992	187	6.3%

Pernambuco	Universidade Federal Rural de Pernambuco	UFRPE	2895	180	6.2%
Goiás	Universidade de Jataí	UFJ	3304	192	5.8%
Mato Grosso	Universidade Federal do Mato Grosso	UFMT	3328	192	5.8%
Paraná	Universidade Federal da Fronteira Sul	UFFS-PR	3135	180	5.7%
Rio Grande do Sul	Universidade Federal da Fronteira Sul	UFFS-RS	3135	180	5.7%
Santa Catarina	Universidade Federal da Fronteira Sul	UFFS-SC	3135	180	5.7%
São Paulo	Universidade Federal do ABC	UFABC	3216	180	5.6%
Rio de Janeiro	Universidade Federal Rural do Rio de Janeiro	UFRRJ	3220	180	5.6%
Minas Gerais	Universidade Federal de Juiz de Fora	UFJF	3240	180	5.6%
Rio Grande do Sul	Universidade Federal do Rio Grande	FURG	3290	180	5.5%
Rio Grande do Norte	Universidade Federal Rural do Semi-Árido	UFERSA	3335	180	5.4%
Bahia	Universidade Federal da Bahia	UFBA	3158	170	5.4%
Rio de Janeiro	Universidade Federal do Rio de Janeiro	UFRJ	2880	150	5.2%
Amazonas	Universidade Federal do Amazonas	UFAM	3240	150	4.6%
Paraíba	Universidade Federal de Campina Grande	UFCG	2835	120	4.2%
Paraná	Universidade Federal do Paraná	UFPR	3200	135	4.2%
Minas Gerais	Universidade Federal de Lavras	UFLA	3026	102	3.4%
Pará	Universidade Federal do Oeste do Pará	UFOPA	3328	102	3.1%
São Paulo	Universidade Federal de São Paulo	UNIFESP	3280	72	2.2%
Ceará	Universidade Federal do Cariri	UFCA	3288	64	1.9%
	National Average		**3126.5**	**236**	**7.55%**

The percentile variation of the Geometry subjects in the course greatly varies from 1.9% to 14.8% (from 64 to 420 hours).

We need to consider the structure of Mathematics Teaching undergraduate courses in Brazil, its historical aspect (Moreira, 2012) and the current K-12 – as PCNEM (1999) and BNCC (2018) -, and Higher Education legislation (Resolução n.º 3, 2012; Resolução n.º 2, 2015), which impose at least 400 hours of school placement and practical subjects (practice as a curriculum component). Initially, we tried to compare different areas (Geometry, Algebra, and Arithmetic). However, this endeavor was unsuccessful due to the difficulty in categorizing all subjects in this area's courses. For instance, to know if the subject "Teaching practice I" approached the objects of these areas and which. Maybe it would be necessary to change the strategy to observe only some courses instead of all. We worked to create a general ranking, an indication of how they are present in the courses, though not pointing out how they are effectively worked and discussed in each course. Here we bring Ramassotti's (2015, p. 33) statement:

> The collocations are representative of the point of view of teacher training. Considering the quality of the professionals leaving the Mathematics Teacher degree, we expect they can make a formal demonstration, accepted in the current mathematical standards. However, we know that axiomatic geometry is only sometimes studied, being left behind in some training courses.

Ramassotti (2015) clearly shows his interest and points out the need for Geometry subjects that bring the axiomatic aspect. We also had this interest (Moreira, 2018) though, initially, before the formalization of the project that led to this study, we wanted to analyze what other geometries were present in the training degrees for Mathematics teachers, a clear approximation to Silva’s (2019) interests.

Paying attention to the highlighted percentiles, we wanted to understand if regional factors were connected to this discussion. We knew that the regional K-12 curriculum had different contents in some states (in Paraná, for example, there were contents of Non-Euclidean geometry) or the presence of certain groups and professionals in certain regions (such as Omar Catunda in the state of Bahia). However, when observing the UFC and UFCA courses in Ceará, we have a discrepancy of 14.84% versus 1.94%, respectively. The same happens when we compare the course of UFSCAR with 9.28% and UNIFESP with 2.19%, both in São Paulo. Similarly, in Minas Gerais, UFTM has 12.08% and UFLA 3.37%. Thus, the factor 'state' where the course is held does not seem relevant to this percentage. Therefore, we have opted to combine the courses by Geoeconomic regions.

This way, we separated the information in Table 1 into the five regions of the country (South, South-East, Center-West, North-East, and North), creating Table 2. By doing so, we aimed to see possible discrepancies in the percentage results when comparing the Brazilian regions. We also present the average for each region.

Table 2

Survey per region

Region	Average S.L.	Geo.Average	% Geo. In Degree
Center-West	3272	284	8.8

South-East	3118.6	240.2	7.8
South	3211.9	234.5	7.3
North-East	3108.1	221.1	7.2
North	3132.1	217.9	6.4

Though the difference seems small in the percentiles, we must highlight that there is a 2.4 percentage difference, equivalent to 66.1 hours of work, approximately one 6-month subject[6].

As stressed above, though working with quantitative data, our research is generally aligned to the qualitative paradigm, mainly interested in researching, making questions, and investigating the phenomenon or study object, in our case, Geometry in the Mathematics Teaching undergraduate degrees in Brazilian Federal universities. Knowing that some courses operate with 1.9% and others with more than 14% is essential data for our panoramic perspective. Undoubtedly, there are historical reasons and processes connected to these choices, which only quantitative data would not be enough to show.

As we have previously stated, there are different Geometries that can be approached in these courses: Euclidean, Non-Euclidean, Analytic, Geometric Construction, Descriptive Geometry, and even distinctive approaches to the same subject, as pointed out by Jones (2012) regarding high school. Souza (2021) highlights how subjects related to geometric construction can have different focuses and approaches. Our data do not allow us to go into these details. However, we can see the subjects studied in these courses, which is the survey we present below.

In the table, we show the titles of the subjects, the number of degrees offering this subject in their obligatory curriculum framework, their respective institutions and, in the last column, the percentage of courses in our sample that offer this subject.

[6] We were based on the *Universidade Federal de Mato Grosso do Sul* curriculum, which uses module 17, with subjects of 34, 68, and 102 hours.

Table 3

Geometry subjects in the degrees

Subject	Number F.U.	F.U. offering subject	% of degrees
Analytical geometry	44	UNB; UFRB; UNILAB-BA; UFC; UNILAB-CE; UFAL; UFPE; UFRPE; UFAPE; UFS; UFMA; UFPI; UFERSA; UFRR; UFAC; UFAM; UNIFES-SPA; UNIFAL-MG; UNIFEI; UFJF; UFSJ; UFU; UFV; UFTM; UFVJM; UFES; UNIRIO; UFF; UFRRJ; UTFPR; UFFS-PR; UNILA; FURG; UFFS-RS; UFPEL; UFSM; UNIPAMPA; UFSC; UFFS-SC.	64.7
Spatial geometry	30	UFGD; UFG; UFCAT; UFC; UFAL; UFPE; UFMA; UNIR; UFRR; UFAC; UNIFAP; UFT; UFNT; UNIFES-SPA; UNIFAL-MG; UNIFEI; UFJF; UFLA; UFSJ; UFU; UFMT; UFUJM; UFSCAR; UFES; UFFS-PR; FURG; UFFS-RS; UFSM; UNIPAMPA; UFFS-SC.	44.1
Plane geometry	23	UFGD; UFC; UFAL; UFPE; UFAPE; UFMA; UNIR; UNIFAP; UFPA; UNIFAL-MG; UNIFEI; UFJF; UFOP; UFSJ; UFMT; UFABC; UFES; UFFS-PR; FURG; UFFS-RS; UFSM; UNIPAMPA; UFFS-SC.	33.8
Euclidean Geometry I and II	20	UFMT; UFR; UFSB; UNILAB-BA; UNILAB-CE; UFCG; UFRPE; UFS; UFPI; UFERSA; UFRR; UFT; UFNT; UFOP; UFVJM; UFSCAR; UFRJ; UFRRJ; UNILA; UFPEL.	29.4
Geometric drawing	13	UFBA; UFSB; UFRB; UFAL; UFRPE; UFAPE; UFPI; UNIFEI; UFSJ; UFU; UFV; UFTM; UFPR.	19.1
Vectors of Analytical Geometry (VAG)	12	UFMS; UFG; UFCAT; UFMT; UFCA; UFPB; UFRN; UNIR; UFPA; UFOP; UFSCAR; UFRGS.	17.6
Geometry I, II and III	10	UNB; UFMS; UFG; UFCAT; UFJ; UNIRIO; UFF; UFPR; UTFPR; UFRGS; UFSC.	14.7
Geometric constructions	10	UFMS; UFMT; UFAC; UNIFAP; UFPA; UNIFAL-MG; UNIFES; UFF; UFRRJ; UFTPR.	14.7
Differential Geometry	8	UFG; UFCAT; UFR; UFPB; UFPE; UFOP; UFTM; UFES.	11.8
Linear Algebra and Analytical Geometry	7	UFGD; UFBA; UFSB; UFCG; UFOPA; UNIFEI; UFLA	10.3
Fundaments of Geometry I and II	6	UFRN; UFAM; UNIFES-SPA; UFV; UFRJ; UFPEL.	8.8
Analytical Geometry I and II	6	UFJ; UFR; UFPI; UNIFAP; UFT; UFNT.	8.8
Plane and Spatial Geometry	5	UFRB; UNIFEI; UFU; UNILA; UFPEL.	7.4
Geometry for teaching I and II	5	UNB; UFVJM; UFES; UNIRIO; UFPR.	7.4
Topic on Geometry I and II	4	UFGD; UFS; UFAC; UFSCAR	5.9
Non-Euclidean Geometry	2	UFABC; UFSC	2.9
Differential Geometry of Curves	2	UNILAB-BA; UNILAB-CE.	2.9

Descriptive Geometry	2	UFR; UFC.	2.9
Laboratory for Geometry teaching	2	UFPA; UFOPA.	2.9
Symmetries in the Euclidean plane	1	UFABC	1.5
Graphic Geometry	1	UFPE	1.5
Dynamic Geometry	1	UFRB.	1.5
Topics on Elementary Geometry	1	UFPE	1.5
Mathematics for teaching Geometry, Magnitudes, and Measurements	1	UFRR	1.5

We must reaffirm that we listed in Table 3 only the compulsory subjects. This table questions the heterogeneity of the curriculum frameworks in the Mathematics Teaching degrees in Brazilian federal universities. Indeed, this pluralism is foreseen and defended by the scientific and teaching community because institutional autonomy should prevail to attend to the wishes and needs of the local community. However, this heterogeneity needs to be further understood by us in future studies because we cannot determine, only by the curriculum framework, if the subject Plane Geometry, for example, in the 23 universities listed, has the same syllabus and bibliography. Furthermore, we need to find out if the work in class is similar and focus on the same approach. Similarly, we cannot certify in this table if what is worked on the subjects entitled "Geometry I" is akin to, for example, "Fundaments of Geometry I". However, the presence of subjects such as Analytical Geometry in 64.7% of courses, but not all, can indicate that this subject characterizes these Mathematics courses. This number is even more relevant when adding the subject "Vectors and Analytical Geometry", moving up to 56 courses and more than 80% of the sample. In this direction, we envision a future work that exclusively focuses on these subjects.

On the other hand, we can see a small number of words and expressions related to K-12 teaching, around 10% of the sample. As we said before, it is possible that the subjects of Teaching Practice approach Geometry contents. These are not part of our survey, except those in which the word Geometry was relevant in the title.

There are also some subjects offered in only one university: Symmetries in the Euclidean Plane at UFABC; Graphic Geometry at UFPE; Dynamic Geometry at UFRB – which do not seem to have equivalent with other names; Topics of Elementary Geometry at UFPE; and Mathematics for teaching Geometry, Magnitudes, and Measurements, offered at UFRR – might be offered with other names. We also did not see a nominally present subject in all courses.

Aiming for a deeper understanding and analysis, we initially tried to regroup the subjects into subgroups:

- Axiomatic – Geometry with an axiomatic bias, with postulates and axioms, proofs, and corollaries.
- Hybrid– Geometry is studied through other Mathematical branches, such as Analytical Geometry, Linear Algebra Linear, and others.
- Shape Geometry – subjects toward drawing and the construction of geometric shapes, for instance, Geometry Drawing.
- School Geometry – subjects on Geometry teaching in the classroom, the study of angles, area, perimeter, volume, etc.

However, this categorization was unsuccessful, as the titles were either too synthetic or could hide the aspects we raised. For such classification, we would need to study the CPP, and check the course syllabus and primary and complementary bibliography so that then, we could reason their approximations and deviations. As our initial questions focused on observing the presence of Non-Euclidean Geometries and our survey led us to understand their presence and the possible hegemony of Euclidean Geometry in these courses (as we hypothesized), we turned our attention to a less pretentious exercise. Hence, we selected the subject **Plane Geometry**, the focus of our previous research (Moreira, 2018) present in 23 of the 68 federal universities in Brazil, and analyzed their syllabuses and primary and complementary bibliography. These data resulted in a table with approximately 12 pages, making it impossible to present here. The average study load in the 23 courses with this subject is 65.73 hours, with most institutions ranging between 60 to 65 hours and some from 70 to 80 hours, increasing the average. In this scenario, we highlight UFSM, whose 90-hour offer surpasses the total amount of

Geometry subjects in some institutions, as is the case of UFC, with 64 hours total.

Returning to the main bibliographies of these 23 courses, we observe that 15 (65%) present the work: BARBOSA, J. L. M., *Geometria euclidiana plana.* Rio de Janeiro: Sociedade Brasileira de Matemática, de João Lucas Barbosa (1995, 1997, 2000, 2001, 2003, 2004, 2005, 2006 e 2012) as a primary reference[7]. Highlighting that the basic bibliography is usually composed by at least three titles and, even so, João Lucas Barbosa's work is suggested by most CPP, in a total of 15 institutions

After, the work Dolce O. & Pompeo J. N. *Fundamentos de Matemática Elementar: Geometria Plana.* 8. ed. São Paulo: Atual, 2005. 9 v is indicated in 11 federal universities. This work is part of a collection organized by Gelson Iezzi, to whom normally is mistakenly given the authorship, which, indeed, happened in some CPP.

The book Rezende, E. Q. F. & Queiroz, M. L. B. *Geometria Euclidiana Plana e Construções Geométricas.* Editora da Unicamp, (2000, 2008) also stand out. Ten federal institutions suggested it as a basic bibliography. After, the works: Lima, E. L. *Medida e forma em Geometria*. 4. ed. Rio de Janeiro: SBM, (1991, 2000 e 2009), indicated by six institutions, and Moise, E.E. & Dows JR. F.L. *Geometria Moderna:* Parte 1 e Parte 2. São Paulo: Editora Edgard Blücher Ltda, 1971, used by five institutions.

In descending order, we have: Muniz Neto, A. C., *Tópicos de Matemática elementar: Geometria Euclidiana Plana,* SBM, 2012. Coleção do Professor de Matemática com três ocorrências; Dante, L. R., *Matemática: contexto e aplicações.* Volume único. 3.ed. São Paulo: Ática, 2009; Wagner, E. *Construções geométricas,* Rio de Janeiro: SBM, Coleção do Professor de Matemática, (1993, 2007), and Edgar de Alencar, *Lições de Geometria Plana*, vol. 1 e vol.2, Livraria Nobel S.A. São Paulo, 1968; with two mentions each.

Finally, the works indicated by only one university: Castrucci, B., *Lições de Geometria Plana*, Editora Nobel, 1976; Euclides. *Os elementos: Euclides*. São Paulo: UNESP, 2009. 600 p; Machado, A. S. *Matemática:*

[7] The institutions: UFGD; UFAL; UFMA; UNIFAP; UNIFEI; UFJF; UFSJ; UFTM; UFABC; UFFS-PR; FURG; UFFS-RS; UFSM; UNIPAMPA, and UFFS-SC.

Áreas e Volumes. São Paulo: Atual, 1988; Lima Netto, S. *Construções geométricas exercícios e soluções.* Rio de Janeiro: SBM, 2009. (Coleção do professor de matemática, 22); Pogorelov, A.V., *Geometria Elementar*, Mir, Moscou, 1974; Posamentier, A. S. & Salkind, C. T. *Challenging Problems in Geometry*. New York: Dover Publications Inc, 1996, and Rich, B. Geometria Plana. São Paulo: McGraw-Hill do Brasil, 1972.

Though we have not profoundly analyzed the manuals in these syllabuses, there are substantial approach differences. In a previous master's dissertation, we compared two books: *Geometria Euclidiana Plana*, from João Lucas Barbosa (2006) and *Geometria Euclidiana Plana e Construções Geométricas*, from Eliane Quelho Frota Rezende and Maria Lúcia Bontorim de Queiroz (2000). In this analysis, we could perceive how two manuals with very similar titles diverged in the axiomatic presented[8]. This material diversity reaffirms the difficulty of creating the previously mentioned groups.

This diversity in the materials reaffirms the difficulty of creating the before-mentioned subgroups and points out a possible diversity in the approaches of each subject, according to the manual used. However, the axiomatic approach or logic deductive is present in both, with demonstration exercises and conjectures from axioms/postulates and theorems.

When comparing the syllabuses and bibliographies, we noticed that the institutions that opted to use João Lucas Barbosa's book have more axiomatic aspects in their syllabuses, with a focus on Euclid's fundaments

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[8] We can bring the example of Axiom III_3 (Barbosa, 2006), summarizing it states that: $AB + BC = AC$, i.e., having three collinear points A, B, and C, consecutively, the distance between points A and C is given by the sum of the distances between A and B and between B and C. A similar idea is expressed by Rezende and Queiroz (2000). However, to them, such statement is presented in a definition (1.4): "Be A, B, and C three collinear points and distant two for two. If $AB + BC = AC$, we say that B is between A and C, what we denote by A – B – C" (Rezende & Queiroz, 2000, p.17). It calls our attention that for one author the statement does not have the same relevance as to the others, i.e., they are used with different purposes. Barbosa (2006) presents it as an axiom to talk about distances, while Rezende and Queiroz (2000) use the idea of distance to define "between".

and postulates. The institutions that chose Dante or Dolce and Pompeu sought a Geometry teaching more grounded on school Geometry with solving algebraic and arithmetic problems (with magnitudes and measures) with Geometry as a background. They briefly mention Euclid's axiomatic but carry on with the resolution of measurement problems. Here we could again bring Jones (2012) to the discussion, though in a different context, the analyzed subject seems to also diverge in the approaches.

Ramassotti (2015), when interviewing several professors to know how Geometry teaching takes place in Mathematics Teacher Training, also came across Barbosa's book. Besides the aims and the focus of the work, he adds:

> The text contains 15 axioms, 19 definitions, 41 propositions, and 23 theorems. Among the ten chapters, we can find 128 exercises and 97 problems. Between them, we can see that in 59, the author asks the reader to prove a specific result, and a demonstration is asked in eight. In many other exercises or problems, the author prefers to use the term "show" when asking to check a result. We can also see 83 figures, apart from those in the problems or exercises. They seek to visually help explain the definitions, propositions, axioms, and, mainly, the proofs presented by the author [original emphasis]. (p. 79)

Castrucci's production is also referenced in this work and considered as not focused on teacher training:

> According to Castrucci (1968), this book was ideal for those who had finished, at the time, the Science Degree, currently High School, who would have had the opportunity to study the theme more axiomatically. The target public was also those who would join higher education or novice teacher. It is not a textbook we are used to, that is, with an explanatory theory before or between the definitions and some results. The whole book is basically composed of the enunciation of postulates, definitions, and theorems in four chapters, without titles, divided into paragraphs. (p. 79)

These two works, as well as Rezende and Queiroz, point toward axiomatic Geometry, focused on problems of proof and demonstrations, very distant from the official documents on K-12 guidelines. Based on his interviewees, Ramassotti (2015), in the final remarks, signs the need for this axiomatic work in the Mathematics Teaching degree, including the works of Euclides e de Hilbert (Ramassotti, 2015, 89).

We could not access Antonio Caminha Muniz Neto's (2012) work. However, one of Ramassotti's interviewees presents it as an exciting work to connect Geometric Drawing and Euclidean Geometry. According to him, the work builds an axiomatic justification for the construction build, in which the student can see an application, a real result (Ramassotti, 2015, pp. 110-111).

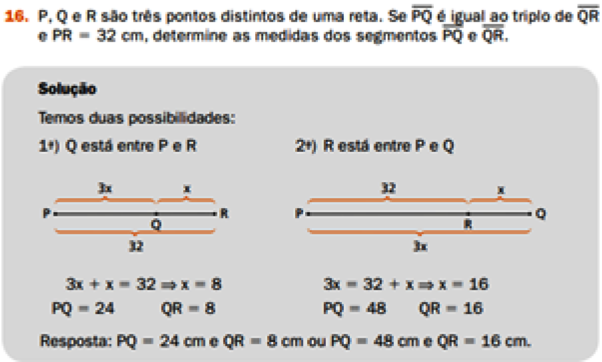
The other works identified here (Dante, 2009; Dolce & Pompeo, 1993, 2005) have a different approach from the previous ones, closer to what is proposed in K-12 education. Both do not appear in Ramassotti's study; however one of his interviewees indicated the work of the collection Iezzi (Dolce & Pompeo) as a possibility of **complementary study** as an introduction, not taken exclusively, because it does not have the same axiomatic character of Barbosa's work:

> We always talk about Gelson Iezzi, because he wrote a significant part of this collection, right? It's an old collection, a collection that was…was used in high school a long time ago, but it is a book that, though not having an axiomatic character, it has many exercises that can give the students some ideas. He also works some concepts in a lighter way, a softer way, more malleable and accessible. So, with the book "fundaments of Elementary Mathematics", I advise my students on some specific topics to seek in this book or start to do some exercises in this book, thinking about what was discussed in class. After, they go to Barbosa's exercises.(Ramassotti, 2015, pp. 168-169)

We inferred that this other approach, or other exercises brought by Iezzi, can be connected to a process of "algebrization" or "arithmetization" of Geometry teaching. Problems or exercises obligatorily resort to algebraic or arithmetic operations to be solved, sometimes focusing more on these than on the geometry properties. (Figure 1).

Figure 1

Geometry exercise with an algebra focus[9] (Dolce & Pompeo, 2013, p. 1)



16. P, Q e R são três pontos distintos de uma reta. Se $\overline{PQ}$ é igual ao triplo de $\overline{QR}$ e PR = 32 cm, determine as medidas dos segmentos $\overline{PQ}$ e $\overline{QR}$.

Solução

Temos duas possibilidades:

1ª) Q está entre P e R

$3x + x = 32 \Rightarrow x = 8$

PQ = 24 QR = 8

2ª) R está entre P e Q

$3x = 32 + x \Rightarrow x = 16$

PQ = 48 QR = 16

Resposta: PQ = 24 cm e QR = 8 cm ou PQ = 48 cm e QR = 16 cm.

By bringing these adjectives to the word Geometry, we are not defending that the "pure" Geometry is that of axioms because we could also point out here another adjective, an "Axiomatic Geometry" over a "Geometry of Constructions" or "of transformations". We only bring these movements/adjectives as potentialities to look at the scenario we found and produce some marks that can help us understand it.

Hence, even the "same" subject" can have different focuses, approaches, and ways to approach Geometry(ies). Returning to the work of Crescenti (2005) on Geometry in K-12 education, he affirms that it is structurally connected to Algebra and Arithmetic. However, its teaching in educational institutions has a slight disadvantage because, typically, their topics are displaced to the end of the teachers' annual planning:

> Geometry "is interconnected with arithmetic and algebra because their objects and relations correspond to the

[9] Translation note: P, Q, and R are three points in a line. If $\overline{PQ}$ is equal to the triple of $\overline{RQ}$ QR and PR= 32 cm, determine the measures of $\overline{PQ}$ and $\overline{RQ}$ segments.

Solution

We have two possibilities

1st) Q is between P and R

2nd) R is between P and Q

Answer: PQ= 24 cm and QR=8 cm or PQ= 48 cm and QR+ 16 cm.

> others”. However, when taught, Geometry teaching is isolated from Arithmetic and Algebra, usually left behind as the last content to be taught by teachers [original emphasis]. (Crescenti, 2005, p. 27)

Crescenti’s statements on K-12 education connect with Ramassotti’s (2015) text which points out that one of the causes of gaps in K-12 Geometry teaching would be mainly connected to the abandonment of Geometry in Higher Education in the undergraduate degrees of Mathematics Teaching.

> When observing the curriculum frameworks to train Mathematics teachers, we perceive, beyond the subjects offered, the order they are offered in the general context of the courses. We are not interested in directly discussing the ideal curriculum framework or pointing out a presentation order of the several subjects in these courses. However, we have seen in the professors' testimonies, considering Geometry specifically, that a change of order could change the way of working the axiomatization and formation and that this could influence the perspective of future teachers toward geometry. (Ramassotti, 2015, p. 36)

Even though the results presented in Table 2 ratify Ramassotti’s (2015) statements, as we have found an average of 7.6% of the study load of Teaching degree in Geometry (an average of only 236 hours), we also need to observe their multiple possibilities of work/focuses.

FINAL REMARKS

Initially aiming to problematize Geometry teaching in K-12 education and the training of Mathematics teachers, we have delineated a study that sought to understand the current Brazilian scenario, focusing on the federal universities. From a broad survey, we could perceive, through the compulsory subjects, some understandings (or distributions) of Geometry in these courses. If the data does not explicit the positions, they show different scenarios depending on the regions and even within each Brazilian state. For instance, we can see that in the state of Ceará, the *Universidade Federal do Ceará* reserves 14.84% of its study load for Geometry teaching. Conversely, at the *Universidade Federal do Cariri*, in the same state, only 1.94% is used for Geometry teaching. Such discrepancy allows us to infer that the regional aspects do not delineate this offer option. They are more related to aspects of each institution, which we could not analyze in this study. We believe that other studies would be possible using other approaches to decipher these numbers.

When listing the several subjects that compose the group we called "Geometry subjects", we perceived some variations and similarities. There are subjects offered in only one course, such as Symmetry in the Euclidean Plan, by UFABC; Graphic Geometry at UFPE; Dynamic Geometry at UFRB; and Topics of Elementary Geometry at UFPE. On the other hand, Analytical Geometry and Vectors and Analytical Geometry are present in 64.7% of Brazilian federal universities, reaching a total of 44 out of 68 institutions. We also highlight the presence of the subjects Plane Geometry, Spatial Geometry, and Euclidean Geometry I and II.

Analyzing the syllabuses and bibliographies of the subject called Plane Geometry – present in 23 of the 68 institutions- we perceive that, despite having the same name in several universities, their topics vary in the execution of contents. While in some places we have identified an Axiomatic Geometry, in others, the approach was towards a Geometry of solving metric problems involving Geometry and Algebra and Arithmetic.

Our investigation also identified an expressive use of the book *Geometria Euclidiana Plana*, from João Lucas Barbosa (2006) – out of the 23 institutions offering the subject Plane Geometry, 15 present this book as a primary bibliography. In 2018, we presented a study that did a Wittgenstein bibliographic therapy on the language games in two manuals of Plane Euclidean Geometry in Mathematics Teaching degrees, one of the books was Barbosa's (2006). The book is composed of ten chapters. From the first to the sixth, the author deals with Euclid's Postulates[10], from the seventh to the tenth, Barbosa (2006) summarizes a study on triangles, circles, trigonometric relations, and areas. Considering this, we believe that, though the syllabuses varied the content topics in the subjects, most federal universities apply Axiomatic Geometry in the subject Plane Geometry. Ramassotti (2015) interviewed some professors in the

[10] As presented in Moreira's 2018) research, when we say that Barbosa (2006) deals with Euclid's Postulates, we are alluding to what appear to be Euclid's Postulates because, in Moreira's (2018) remarks, the book only presents the first and the fifth postulates, the others do not appear, except the third which, to Barbosa (2006), is understood as a definition.

Geometry area, such as Irineu Bicudo, concluding that there is almost a standard bibliography to teach Geometry in Brazilian federal institutions.

> A basic bibliography is used in the training of Mathematics teachers approaching the theme of Geometry. In a way, this bibliography seems connected to the history of the professors, the fact that they used it during their training and undergraduate degrees, and the standard bibliography in the institution. However, there are some reservations regarding the bibliography. We presume there is no consensus on the books used by the professors. (p. 48)

Summing up, we believe that the difficulty of working with Geometry in K-12 education,[11] as pointed out by Ramassotti (2015) is directly related to teacher training. Thus, it is far from being overcome or fully explained, even because we walk away from restrictive perspectives of cause and effect. However, a study such as ours can help understand how Geometry is being treated in the Mathematics degrees of Brazilian Federal institutions and its study load, incentivizing research continuity and helping to improve the quality of K-12 and higher education in the country.

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DECLARATION OF AUTHORS' CONTRIBUTIONS

P. G. S. M. conceived the idea of the research presented. P. G. S. M. collected the data. Both authors (P. G. S. M. and T. P. P.) actively

[11] As pointed out by: Crescenti (2005), Lorenzato (1995), Lovis (2009, 2013), Pavanello (1989, 1993), Perez (1991, 2000), Serralheiro (2007) among others.

participated in developing the theory, methodology, organization and data analysis, result discussion, and approval of the final version of this work.

DECLARATION OF DATA AVAILABILITY

The data supporting this study's results are available under reasonable questioning with the author T. P. P.

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