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Contextualised Approach to Mathematics in Engineering from the Perspective of Cognitive Dysfunctions

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ABSTRACT

Background: The relevance of enabling beginning students of engineering courses to solve problems directly related to their future professional activities, already in the initial subjects of mathematics, implies the need to elaborate such problems, implement them and analyse their potential in terms of construction and application of mathematical knowledge in different contexts. **Objectives**: In this article, we analyse the cognitive dysfunctions evidenced by the students during the process of solving a problem articulating the real functions of a real variable to the study of the characteristic curve of a semiconductor diode, content linked to the analogical electronics. **Design**: This qualitative-nature investigation is characterised, as field research, of exploratory-descriptive combined type. **Environment and participants**: the research subjects, seven students in the first semester of an engineering course offered by a private institution in the state of São Paulo, with an interest in following the control and automation qualification, participated voluntarily in the study. **Data collection and analysis**: three synchronous meetings were held in remote modality. For data collection, we used audio and video recordings of the discussions and the written productions of the students in such moments. In this article, we present the analysis of the first meeting. **Results**: We evidenced, among other aspects, cognitive dysfunctions that can lead to obstacles in the transposition of mathematical knowledge to extra-mathematical contexts. **Conclusions**: The results obtained allow, in the future, the planning of adequate mediations so that the teacher can help the students to convert the identified dysfunctions into sufficiently developed cognitive functions.

Keywords: Articulation of mathematics with engineering; Theory of mathematics in the context of sciences; Cognitive phase; Theory of structural cognitive modifiability; Cognitive dysfunctions.

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Abordagem contextualizada da matemática na engenharia sob a perspectiva das disfunções cognitivas

RESUMO

Contexto: a pertinência de possibilitar aos estudantes iniciantes de cursos de Engenharia que, já nas disciplinas iniciais da área de Matemática, resolvam problemas diretamente relacionados às suas futuras atuações profissionais, implica na necessidade de elaborar tais problemas, implementá-los e analisar suas potencialidades em termos da construção e aplicação de conhecimentos matemáticos em diferentes contextos. **Objetivos**: neste artigo, analisamos, as disfunções cognitivas evidenciadas pelos estudantes durante o processo de resolução de um problema articulando as funções reais de uma variável real ao estudo da curva característica de um diodo semicondutor, conteúdo, vinculado à Eletrônica Analógica. **Design**: a investigação realizada, de natureza qualitativa, caracteriza-se como uma pesquisa de campo, do tipo exploratória-descritiva combinada. **Ambiente e participantes**: os sujeitos da pesquisa, sete estudantes do primeiro semestre de um curso de Engenharia, ofertado por uma instituição privada do Estado de São Paulo, com interesse em seguir a habilitação Controle e Automação, participaram voluntariamente do estudo. **Coleta e análise de dados**: foram realizados três encontros síncronos na modalidade remota e para a coleta de dados recorremos a gravações em áudio e vídeo das discussões realizadas e às produções escritas dos estudantes em tais momentos. Neste artigo, apresentamos as análises relativas ao primeiro encontro. **Resultados**: evidenciamos, entre outros aspectos, disfunções cognitivas que podem levar a entraves na transposição de conhecimentos matemáticos para contextos extra matemáticos. **Conclusões:** os resultados obtidos possibilitam, futuramente, o planejamento de mediações adequadas para que o professor possa auxiliar os estudantes a converter as disfunções identificadas em funções cognitivas suficientemente desenvolvidas.

Palavras-chave: articulação da Matemática com a Engenharia; Teoria a Matemática no Contexto das Ciências; Fase cognitiva; Teoria da Modificabilidade Cognitiva Estrutural; disfunções cognitivas.

INTRODUCTION AND PROBLEM POSING

University professors, researchers, and students have debated nationally and internationally how important it is that engineering students are offered the opportunity to solve problems directly related to their areas of interest or future professional activities since graduation. These scholars call for an approach according to this orientation in the subjects that are part of their courses, especially in the first years, related to basic sciences, such as mathematics, physics, and chemistry. Therefore, it is essential to encourage students to engage in problem solving that allows them to study content in the basic areas and face situations closer to those they will experience in their future professions (Gomes, Bianchini, & Lima, 2021a).

Among the basic sciences, mathematics is the one that students find more challenging, perhaps because, as Camarena (2002) points out, mathematics is a supporting tool for engineering, while physics and chemistry constitute its cognitive bases, being naturally more linked to its central issues. This question may stem from the students' difficulty perceiving how to connect mathematics with specific problems in their areas of interest, which may owe to the education of some teachers who teach the subject in undergraduate engineering, making the procedural character the protagonist.

As condensed by Lima, Bianchini, and Gomes (2021), based on investigations by Pitt (2019), Pohjolainen (2018), Mercat et al. (2018), and Kapranos (2019):

> For courses to be attractive to students and effective in training future engineers, they should, as much as possible, from the beginning, provide opportunities for group work with situations in which basic sciences and mathematics can be immediately recognised by them as supports, foundations in their formations and not as disconnected content. [...] the motivation of engineering students to study mathematics can be expanded by explaining how the precepts of this science are used in industry, society, and especially in the student's future professional performance. [...] a student's perception of mathematics and its teaching impacts their academic performance in mathematics, and positive attitudes and insights towards this subject will encourage the individual to learn it better. [...] the development of the skills required of the professional in training is enhanced by the resolution, during the course, of problems closer to the reality they will face in their careers. (Lima, Bianchini, & Gomes, 2021, p. 187)

In Brazil, approaches aligned with this guideline have been targeted, especially after the approval, in 2019, of the current DCN - *Diretrizes Curriculares Nacionais* for engineering courses (Brasil, 2019). We have directed investigations to the perspective of approaching mathematical content in line with the *Modelo Didático da Matemática em Contexto* (MoDiMaCo) [Didactic Model of Mathematics in Context], concerning the theory *A Matemática no Contexto das Ciências* (TMCC) [Mathematics in the Context of Sciences], developed by the Mexican researcher Patricia Camarena Gallardo, with a particular focus on university courses to which mathematics serve, but is not the object of study. As argued by Lima et al. (2021), this model follows the DCN. Furthermore, as the authors, based on Graham (2018), argue, its aspects keep "a close relationship with characteristics that are linked to what is expected from the engineering education sector in the future and that will, therefore, differentiate institutions today called 'emerging leaders' from those that are currently leaders in the sector" (Lima et al., 2021, p. 812).

The main teaching tools in MoDiMaCo are the so-called *contextualised events*, conceived as "problems, projects, or case studies that behave as integrators between areas" (Camarena, 2021, p. 179) and that are not routine activities, but that "must cause a cognitive conflict in students when reading the statements, motivate and intrigue them so that they want to continue with the task" (Idem). Examples of contextualised events that were formulated or supervised in their productions can be found in Gomes et al. (2018a, 2018b), Lima et al. (2020), Lima, Bianchini, and Gomes (2021), Pinto (2021), and Silva (2022).

This article presents a contextual event linking mathematics to analogue electronics. Next, we analyse data collected through its application with seven students enrolled in the first semester of an engineering course offered by a private institution in the state of São Paulo, interested in pursuing the control and automation qualification. The event was implemented in three meetings. In the present work, we only focus on the analysis of the first event, developed from the cognitive perspective, from the point of view of the *cognitive functions* postulated by the Romanian psychologist based in Israel, Reuven Feuerstein, in the scope of the theory of Structural Cognitive Modifiability (SCM).

The focus of the discussions presented in the following sections is on the analysis of the *cognitive dysfunctions*, i.e., the functions that were not efficiently developed, revealed by the students during the process of solving two of the three guiding questions with which they worked in the highlighted meeting.

We chose to analyse the dysfunctions because it is important that the professor perceives them so that, through appropriate mediation, he/she can direct efforts to modify them and, consequently, provide the students with the opportunity to develop their respective functions properly.

THEORETICAL REFERENCES

The contextualised event studied in this article was prepared according to the procedures established in the theory of mathematics in the context of science, reference presented below.

Theory of Mathematics in the Context of Science (TMCC)

The TMCC was developed, according to Camarena (2021), within a line of investigation called social mathematics (of which Camarena is also the precursor), whose primary purpose is to consolidate, in a reasoned manner, educational actions to be developed to face problems related to students' feelings, such as mathematics being extremely abstract when compared to other basic sciences, the non-attribution of meaning to the mathematical contents in their daily practices, and that undergraduates must be trained to assume specific attitudes and behaviours for the current job market.

Therefore, Camarena (2021) assumes that the learning environment in courses that do not aim to qualify mathematicians and the TMCC determine a complex system that allows them to be approached in a non-isolated way. This system is composed of five subsystems, called *phases*, namely: *curricular*, *epistemological*, *didactic*, *teaching*, and *cognitive phase*. These are deeply intertwined, so any change in elements of a given phase will result in changes in elements of the others. Thus, it is evident that the problems above are related to this complex system in which the degree of connectivity between its subsystems is quite high, consequently, they cannot be managed by resorting to aspects of only one phase, mobilising questions inherent to some other phase is always necessary. The ideas presented in this paragraph are illustrated by Camarena (2021) through the scheme in Figure 1.

To explain the central objectives of each of the phases of the TMCC, Camarena (2021) produced a scheme we adapted in Figure 2. The texts in red indicate the main problem of each phase, and the texts in blue show how to face them according to the scope of the TMCC.

Turning attention to the future engineer's education, the production of teaching resources articulating mathematics with engineering, i.e., contextualised events, is the main objective of the epistemological phase, which includes specific procedures for this development, in addition to a fundamental theoretical construct of TMCC: the *contextualised transposition*.

Figure 1

Interactions between the phases of the TMCC (Camarena, 2021, p. 89)

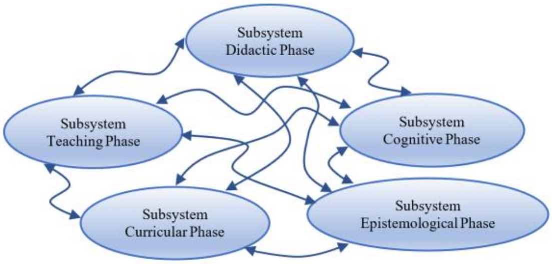
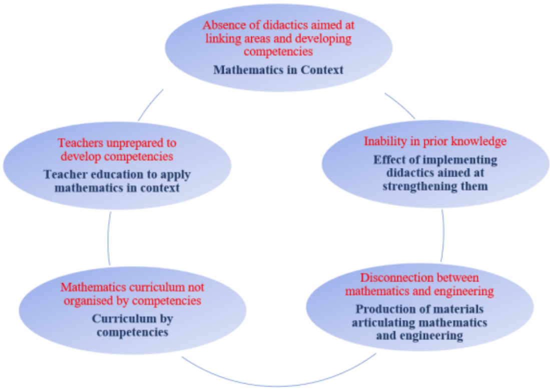


Figure 2

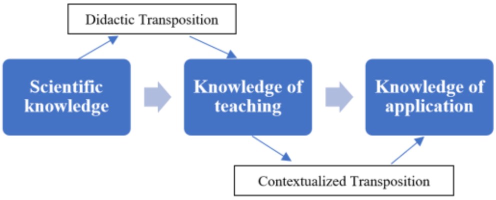
Problems present in the learning environment and coping proposals via TMCC (adapted from Camarena, 2021)



According to Camarena (2021), when we think about the education of an individual who will need to employ mathematical knowledge in their professional daily life, as illustrated in Figure 3, the contextualised transposition gives continuity to the *didactic transposition* – which, according to Chevallard (1991), refers to the transformations undergone by scientific knowledge so that it becomes teaching knowledge – since it is not always applied in the profession in the way it was presented in the classroom, requiring new transformation to become knowledge for applications. The mentioned construct is, therefore, directly related to the need to be clear about the difference between school mathematics and mathematics for applications in the professional field.

Figure 3

The contextualised transposition (inspired by Camarena, 2021)



The guidelines for the implementation of resources prepared in the epistemological phase are objects of the didactic phase, and the results, in terms of student learning, provided by the work with contextualised events, are analysed in the scope of the cognitive phase, the focus of this article. However, cognitive analyses of the results of a contextualised event require that it be elaborated and implemented, forcing us to also go through the other two phases mentioned.

For the construction of the event we will analyse, as explained in detail in Lima, Bianchini, and Gomes (2021) and Gomes, Bianchini, and Lima (2021b), we chose *a priori* the study of a semiconductor diode as an extra mathematical context and revisited the real exponential functions of a real variable –not as a review of what is studied in high school, but already connected with an engineering situation– as mathematical content in focus. We then analysed one of the main bibliographic references cited in the syllabuses of the subjects in which the diode theory is addressed: *Dispositivos Eletrônicos e Teoria dos Circuitos* [Electronic Devices and Circuit Theory], by R.L. Boylestad and L. Nashelsky, 2013 edition. In this text, we identified a situation about the study of the characteristic curve of a semiconductor diode, which inspired us to conceive the event.

In turn, the development of the event with the students took place according to the assumptions of MoDiMaCo, a constructivist model, which, according to Camarena (2017), is supported by Piaget's psychogenetic, Vygotsky's sociocultural, and Ausubel's meaningful learning theories. In this model, according to Lima et al. (2021, p. 798), "the contextualised events are solved by teams composed of students [...] each of them with complementary characteristics to carry out an effective collaborative work".

For the analysis of the effects of implementing a contextualised event, as indicated by Pinto (2021) from Trejo and Camarena (2011) and Camarena and Trejo (2011), in studies within the cognitive phase of TMCC, researchers have associated this reference to other cognitive theoretical frameworks to enrich their analysis of students' work with a contextualised approach to mathematics. We also resorted to this procedure since, in this article, like Zúñiga (2004), in a doctoral thesis supervised by Camarena, we associated the issue of cognitive functions studied in the theory of Structural Cognitive Modifiability (SCM) with the TMCC.

Theory of Structural Cognitive Modifiability (SCM)

The SCM was developed by Feuerstein at the heart of his investigations on how to evaluate and increase the intelligence of subjects in situations of sociocultural vulnerability and with low academic performance (Prieto, 1989). The researcher sought to answer the following questions, among others: Is it possible to cognitively modify a person to help them develop strategic skills that teach them to: identify problems and transform them into an opportunity for development; when necessary, shape their environments so that learning is more effective; and, consequently, go beyond learning a set of facts and procedures? Is it possible to modify a person's thinking, equipping them with essential tools for an adequate adaptation to life, even when these are lacking in some way? (Feuerstein, Feuerstein, & Falik, 2014).

Assuming that human beings can change during the course of their lives, and given the need to put thinking as a protagonist in people's lives, the main object of study of the SCM is "the capacity for cognitive modifiability that the human being has and how this ability of the brain/mind to change informs how we can help students improve their ability to think and learn" (Feuerstein, Feuerstein, & Falik, 2014, p. 17). Some core concepts in SCM are: *mediation*, *intelligence*, *change*, and *structural change.*

Mediation is understood as "an intentional interaction with the learner, to increase their understanding beyond immediate experience and help them to apply what is learned in broader contexts" (Feuerstein, Feuerstein, & Falik, 2014, p. 21). As the authors point out,

> the human mediator does not continually or constantly impose him/herself on the person being mediated and the world. It does not cover all the territory between them but leaves the mediated a large area of direct exposure to the stimulus. However, in the area where the mediating agent acts, it is active in several ways. It delivers to the mediated the components that will be responsible for its ability to understand phenomena, look for associations and connections among them and thus benefit from them and be modified. (Feuerstein, Feuerstein, & Falik, 2014, p. 65).

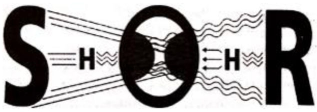
Intelligence is conceived as the "ability to think adaptively in response to changes in our environment. [...] an energetic agent or dynamic state that is unstable and responsive to a person's need to change to adapt to situations and successfully manage them" (Feuerstein, Feuerstein, & Falik, 2014, p. 49), being this modification or *change* understood as "a person's acquisition of quantities of knowledge or skills and new cognitive structures by which new areas not previously included in the set of knowledge and skills are opened" (p. 34).

Structural change is not random or limited in time or space but "will affect learning and behaviour in profound, sustainable, and self-perpetuating ways. [...] if a structural change is created, it will not be confined to the event alone, but will manifest itself in several additional events that have similar elements" (p. 43). In this way, "a structural change tends to continue operating even after the initial factor that caused it is no longer directly experienced" (p. 44). The structural changes are *permanent* (preserved over time), *resistant* (to changes in relation to the situations that gave rise to them), *flexible/adaptable* (can be adapted to new situations) and *generalisable/transformable* (the individual continues to be structurally modified through independent efforts).

But how can teachers bring about structural changes in their students? It is assumed that this is only possible through mediation, more specifically through a *Mediated Learning Experience* (MLE), which "occurs when a person (mediator) that has knowledge, experience, and intentions mediates the world, makes it easier to understand, and gives meaning to it by adding direct stimulus" (Feuerstein, Feuerstein, & Falik, 2014, p. 60). As the authors point out, MLE "creates in human beings flexibility and sensitivity, readiness and desire to understand what happens, and the ability to generalise beyond the isolated phenomenon being experienced" (p. 92). Figure 4 illustrates what happens in an MLE situation.

Figure 4

Mediated Learning Experience model (Feuerstein, Feuerstein, & Falik, 2014, p. 65)



Through Figure 4, we understand that: "in a mediated learning situation, the organism (O) being directly exposed to a stimulus (S) reacts and responds (R) with skill and completeness only after the stimulus characteristics have been organised, classified, differentiated, shaped, and adapted, and organised by a mature human mediator (H)" (Feuerstein, Feuerstein, & Falik, 2014, p. 71). In short:

> Mediated interaction introduces order into a human being's encounter with the world [...] The order will allow the mediation receiver to discover associations among stimuli by making comparisons and other mental operations. [...] The mediation interaction gives human beings tools to reflect on the phenomenon and understand the connections between them, and discover the system of laws that govern it. (Feuerstein, Feuerstein, & Falik, 2014, p. 76-77)

During the implementation of the contextualised event on screen in this article, we provided the students with the opportunity to experience an MLE: as mediators with knowledge of the subject, experience in working with contextualised mathematical problems, and clearly established intentions, we mediated the situation, attributing meaning to it by incorporating direct stimulus and making it easier for participants to understand it. The mediation took place through guiding questions previously elaborated to conduct the intervention and by proposing a series of complementary questions formulated from what we observed when the groups worked with what had been proposed.

The proposition of guiding questions is not initially foreseen in MoDiMaCo, but we have implemented it since we started working with this model to allow students to explore the mathematical notions intended and gradually approach the final resolution of the event (Gomes, Bianchini, & Lima, 2021c). We also understand that this procedure is in line with the need for the mediator, aiming to develop in the mediated the perception that he/she is competent, to offer the mediated tools that help them perform new tasks which, in turn, "are somewhat away from reach and, therefore, require effort" (Feuerstein, Feuerstein, & Falik, 2014, p. 95). Finally, working initially with these guiding questions makes it possible for students, while meeting challenges and dealing with new situations, to feel competent for such actions, "to control these situations, to overcome difficulties, to become familiar with the new and the unknown, and approach challenges expecting that they will be overcome" (p. 84).

The guiding questions fulfilled the role of catalysts for the so-called *mediating cycle*. Upon receiving the statement of the contextualised event, the students were given a series of stimuli and then, as mediators, we, little by little, proposed questions that made it possible to organise those stimuli. When formulating them, we analysed aspects of the set of stimuli that we considered convenient to be emphasised regarding the students' needs so that they could effectively respond to the event. In each question, some of these stimuli had their characteristics highlighted so they would become more significant for the student at that moment, who could then internalise "what was mediated, according to the aspects that were the focus of the mediation, and, when he/she returned to the set of stimuli, such aspects would be known, understood, remembered, and structurally assimilated" (Feuerstein, Feuerstein, & Falik, 2014, p. 84-85), regardless of whether they had undergone some changes in their most immediate particular characteristics.

> When I mediate my intention for a student, why I choose this stimulus, why I choose to emphasise this principle over another, and why I choose this method over another, I give them the means to mediate by themselves when the mediator is not available between them and the world anymore –the self-perpetuating quality of learning. (Feuerstein, Feuerstein, & Falik, 2014, p. 85)

When conducting the application of the event, we also aimed to mediate the meanings of what was being worked on for the students, allowing them to understand the importance of learning what was being studied.

> Meaning makes the mediator's message understood and rationalised, also for extension and application beyond the immediate situation. [...] The mediator must generate in the mediated the need to seek meaning for themselves. Not just the search for the specific meaning of what is being worked on but also the search for associations and connections between

events and phenomena, in the broader sense of cause and purpose. (Feuerstein, Feuerstein, & Falik, 2014, p. 89)

During the meetings to solve the event, we sought to develop in the student "the readiness to learn and move from known to unknown situations, in addition to the tendency to confront the challenging novelty, the complexity and not giving up are essential for our adaptation (Feuerstein, Feuerstein, & Falik, 2014, p. 104).

For this data analysis, we assume *mental operation* as "the set of internalised, organised, and coordinated actions, from which we carry out the elaboration of the information we receive" (Prieto, 1989, p. 54). And, based on this idea, in line with the SCM, we recommend that the teacher, in the role of mediator in an MLE, should have as main objective "to teach and equip the subject with a series of functions or prerequisites necessary for the use and the handling of the mental operation" (p. 54). Such functions, called *cognitive functions*, as indicated by Zúñiga (2004) from Feuerstein (1989), are "underlying mental operations, serve the internalisation of information, and allow the self-regulation of the organism. [...] cognitive functions as activities of the nervous system partially explain the individual's ability to use previous experience to adapt to new situations" (p. 34).

We also consider the premise of the SCM that every *mental act* (thought process) can be broken down into three phases: *input*, *elaboration*, and *output* which, as Prieto (1989) indicates, "are interrelated and each one of them has meaning insofar as it is in close relationship with the other. The phase is an important parameter in the analysis of a subject's mental act because it helps to locate the origin of an incorrect answer" (p. 55).

During each of those phases, different cognitive functions and dysfunctions are present, the latter interfering with learning, and may prevent the subject from learning effectively. These dysfunctions are considered by Feuerstein, Feuerstein, and Falik (2014) as deficient functions, but the authors do not attribute any negative or pessimistic connotation to this term. On the contrary, they are identified as deficient so that "we can direct our efforts to improve these functions. This expresses an essential optimism: that they can be changed and that we can target interventions for their modifiability" (p. 128).

According to the perspective brought by Zúñiga (2004) to the foreground, it is relevant to analyse, from a cognitive point of view, the entire process of solving a contextualised event, elaborated according to the precepts of the TMCC, considering such a resolution as a mental act that includes the understanding of the proposed problem (input phase, in which cognitive functions or dysfunctions are manifested related to the quantity and quality of data accumulated by the individual before starting to solve a problem), the very process of solving the problem (elaboration phase, in which cognitive functions and dysfunctions related to the organisation and structuring of the information available to solve the problem compete) and issuing an answer to the problem (output phase, in which cognitive functions and dysfunctions are directly related to the exact and precise communication of the problem solution).

Figures 5, 6, and 7 show the cognitive functions and dysfunctions related to the input, elaboration, and output phases of a mental act.

Figure 5

Cognitive functions and dysfunctions in the input phase of a mental act (inspired by Prieto, 1989)

	Cognitive function	**Cognitive dysfunction**
1. Clear perception	Exact and precise knowledge of the information received, according to parameters of simplicity and familiarity.	Obscure perception, which consists of a poor and inaccurate knowledge of information data.
2. Systematic exploration of a learning situation	Ability to organize and plan the information presented.	Impulsiveness in a learning situation, i.e., inefficiency to treat information in a systematic and planned way.
3. Language skill	Ability to discriminate and differentiate objects, events, relationships, and operations through verbal rules.	Linguistic inability to understand words and concepts.
4. Spatial orientation	Ability to establish relationships between events and objects located in space.	A lack of spatial orientation.
5. Time orientation	Ability to establish relationships between past and future events.	A lack of time orientation.
6. Conservation, constancy, and permanence of the object	Perceptual stability depends on the ability to preserve the invariability of objects beyond the possibilities of variations in some of their attributes and dimensions.	Irreversibility or rigidity of thought that, associated with the episodic perception of reality, prevents the establishment of relationships between objects.
7. Information organization	Ability to use different sources of information, establish relationships between objects and events, find coherence or inconsistency between different information.	Inability to relate and consider two or more sources of information at the same time, taking into account only one of several alternatives or dimensions.
8. Precision and accuracy in the collection of information	Ability to perceive and select information (all the data that would lead to the correct answer) in a rigorous, careful, and precise way.	Inaccuracy and in the collection of information.

Figure 6

Cognitive functions and dysfunctions in the elaboration phase of a mental act (inspired by Prieto, 1989)

	Cognitive function	Cognitive dysfunction
9. Perception and definition of a problem	Ability to understand what the problem asks for, which points should be delimited and how to check them.	Inability to elaborate the information, which makes the definitions meaningless for the subject, making it difficult for him to reflect, compare and combine elements.
10. Selection of relevant information	Ability to choose previously stored information relevant to the solution of the problem, establish comparisons and relationships between events that occurred in different activities and moments.	Inability to use the acquired information, leading the individual to perceive him/herself as a passive recipient of information.
11. Interiorisation and mental representation	Ability to use internal representation symbols.	Excessively concrete behaviour and without appropriate generalisation, causing a low level of abstraction due to the restricted use of symbols.
12. Mental breadth and flexibility	Ability to use different sources of information, establishing an appropriate coordination and combination between them to achieve operational thinking.	Narrowing or limitation of the mental field, implying inability to manipulate and process several units of information simultaneously.
13. Conduct planning	Ability to predict the goal to be achieved, using previously acquired information, and establish a plan that includes all the steps to reach the solution of the problem.	Inability to organize data in a more appropriate direction, manifesting a predisposition to respond to a stimulus in an episodic and fragmented way.
14. Perceptual organisation and structuring	Ability to guide, establish, and design relationships.	Episodic perception, i.e., difficulty in grouping and organising relationships between objects and facts of everyday life.
15. Comparative conduct.	Ability to carry out all types of comparisons, relate objects and events experienced prior to the situation, and handle previously acquired information as necessary.	Inability to establish relationships of similarity and difference between objects and events.
16. Hypothetical thinking	Ability to establish hypotheses and prove or reject them.	Inability to use strategies to relate hypotheses and intuit several alternatives when explaining a fact.
17. Logical evidence	Ability to justify the validity of answers through logical reasoning.	Inadequate formulation of reasons when exposing arguments and lack of perception of inconsistencies.
18. Cognitive classification	Ability to organise data into inclusive and superior categories. Comparative, summative conduct, the use of relevant dimensions and the establishment of virtual relationships.	A lack of conceptual repertoires and rules when explaining the transformation required for a classification.

Figure 7

Cognitive functions and dysfunctions in the output phase of a mental act (inspired by Prieto, 1989)

	Cognitive function	Cognitive dysfunction
19. Explicit communication	It consists of using clear and precise language to solve the problem, which assumes a certain level of understanding on the part of the subject.	Egocentric communication, that is, the lack of differentiation between the subject who speaks and the subject who listens.
20. Projection of virtual relationships	Ability to see and establish relationships that potentially exist, but not in reality, requiring restructuring and configurations of relationships when faced with new situations.	Inability to deduce and project relationships of a different kind.
21. Verbal rules to communicate answers	Ability to use, manage, and deduce verbal rules for problem solving.	A lack of vocabulary, concepts, and mental operations to communicate correct answers.
22. Elaboration and disinhibition in communicating the answer	Ability to express the answer quickly, correctly, and systematically.	Blocking that leads the subject to not issue any answer.
23. Trial-error answers	Conduct with a very limited value, not leading to systematisation in search of the final goal.	Difficulty in perceiving something accurately and completely, in assuming a comparative and summative behavior, in thinking reflectively, and in seeking causal relationships.
24. Precision and accuracy in answers	Ability to think and express the correct answer to a problem or general learning situation.	Imprecision leads the subject to not answer clearly, to inflexibility and lack of verbal fluidity.
25. Visual transportation	Ability to complete a figure and transport it visually.	Instability in the perception of a figure due to the vulnerable nature of the reference systems that support the perceived elements. Difficulty considering the relevant data of a piece of information, paying attention to the irrelevant ones.
26. Control of answers	Ability to reflect before issuing any type of answer, implying metacognitive processes.	The inability to self-control or impulsiveness, which manifests itself through vague answers.

Identifying functions, especially cognitive dysfunctions, in each of the phases of the mental act of solving a mathematical problem contextualised in engineering is a relevant activity for different actors. Teachers will be able to identify them not only in teaching and learning scenarios employing contextualised events but also in general contexts, in which aspects need to redirect their mediations with students to effectively assist them in their processes of structural cognitive modifiability. Based on knowledge of cognitive functions and dysfunctions (present in each phase of problem solving), researchers will be able to identify relevant focuses for their investigations to assist in the development of teacher education and teaching resources. As already highlighted in the previous section, in this article, we chose to present an analysis of the cognitive dysfunctions evidenced during the process of answering two of the three guiding questions proposed in the first of the three meetings held, due to the importance that the teacher perceives them so that, through appropriate mediations, they can then direct efforts to modify them and, consequently, provide students with the opportunity to develop their functions properly.

METHODOLOGY

The data presented and analysed in this article are part of those obtained through field research of a qualitative nature, which, in the sense of Lakatos and Marconi (2021), is combined exploratory-descriptive, which, globally, aimed to analyse from different perspectives the results obtained through the work of a contextualised event presented in Figure 8, linking mathematics (the study of piecewise-defined functions) to analogue electronics (the study of the characteristic curve of a semiconductor diode).

The didactic organisation[1] proposal for working with the event, as described in Gomes, Bianchini, and Lima (2021a), included prior preparation (carried out in a vrtual learning environment (VLE) asynchronously, including the production of a video by students); three synchronous meetings of two hour each via Zoom platform, when ten guiding questions were approached (three in the first, four in the second and three in the third meeting). At the end of the third meeting, the event was effectively solved and later (asynchronously), the students carried out two work-completion activities (answering an activity perception form and a producing a podcast).

[1]For more details about the didactic organisation of the event, access to the prior preparation activity, the guiding questions and their answers, consult: https://drive.google.com/file/d/1KEHPeUYPzxbRLNU5xBafvkU16ZXKKr65/view?usp=sharing

Figure 8

The contextualised event proposed to students (Gomes, Bianchini, & Lima, 2021a, p. 708)

Contextualised Event: A diode, like other electronic components, needs some time to change from its state of conduction to nonconduction; it is called diode recovery time. Many practical applications require diodes that "recover" easily, that is, that pass in the shortest possible time interval from the state of conduction to non-conduction. One of the silicon diodes with this characteristic is the 1N4148, one of the most used in electronics and which has a recovery time of 4 nA. The *datasheet* of the 1N4148 diode in which the electrical characteristics of this device are highlighted can be accessed at https://pdf1.alldatasheet.com/datasheet-pdf/view/551820/WINNERJOIN/1N4148.html.

Consider this 1N4148 diode subjected to a current of 30 mA and determine the forward voltage drop across it and the approximate values of its saturation currents at the following temperatures: $-45°C$, $50°C$ and $125°C$.

The study of concepts related to solid state physics demonstrates that the general characteristics of a semiconductor diode can be related, for the forward and reverse polarisation regions, by an equation called Shockley's equation: $I_F = I_R\left(e^{\frac{V_F}{nV_T}} - 1\right)$. In this equation, I_F represents the forward current passing through the diode, I_R represents the reverse saturation current, V_F represents the forward polarisation voltage applied to the diode, n: represents an ideality factor, which depends on the operating conditions and physical construction of the diode and V_T represents the thermal voltage, defined by:

$V_T = \frac{kT_K}{q}$ in which k is the Boltzmann constant, whose value is $1{,}38 \times 10^{-23}$ J/K, T_K is the absolute temperature in Kelvin, which is given by adding 273 to the measured temperature in degrees Celsius, q is the magnitude of the elementary electric charge, which is given by $1{,}6 \times 10^{-19}\ C$.

Seven students attending the first semester of an engineering course offered by a private institution in the state of São Paulo, with an interest in pursuing the control and automation qualification, voluntarily participated in the research. They signed a Free and Informed Consent Form. The study is part of a broader project aimed at detecting how future engineers perceive mathematics, duly registered on the Brazil Platform, identified with the number 48879421.3.0000.5482, and approved by the Research Ethics Committee at PUC-SP.

To collect data produced during the three synchronous meetings, we resorted to the audio and video transcripts of the discussions and the students' written productions at such times. In this article, we chose to present the analysis, from the perspective of cognitive functions and dysfunctions, of the data collected during the resolution of two of the three guiding questions that, as explained in Figure 9, were proposed in the first meeting.

Figure 9

Guiding questions of the first meeting (Lima, Bianchini, & Gomes, 2021, p. 184)

1. Does Shockley's equation explain a functional relationship? If your answer is yes, which is the dependent and which is the independent variable?

2. Is thermal voltage a function of any variable? Explain, and if your answer is yes, make a graphical representation of this function.

3. Knowing that the 1N4148 diode operates between −65°C and 175°C, determine the range of variation of the thermal voltage of this diode in this range.

In the meeting on screen in the analyses presented in this article, the students worked in two groups, one with three and the other with four members. We will identify these groups by *G1* and *G2*, respectively. G1 was supervised by two authors of this article, and G2 by the third author. In this text, the members of the groups will be identified as *AiGi*, meaning student *i* (*Ai*) of group *i* (*Gi*). The groups were constituted to contemplate students with different learning styles, identified through the answers given to a questionnaire specially prepared for this purpose, as detailed in Gomes, Bianchini, and Lima (2021b).

We transcribed the dialogues recorded during the reflections made by the two groups when solving the first two questions listed in Figure 9. The cognitive functions and dysfunctions (focusing on the latter) identified in the individuals' manifestations in the different phases of mental acts intrinsic to two other main ones – namely, answering guiding questions 1 and 2 – were thoroughly analysed.

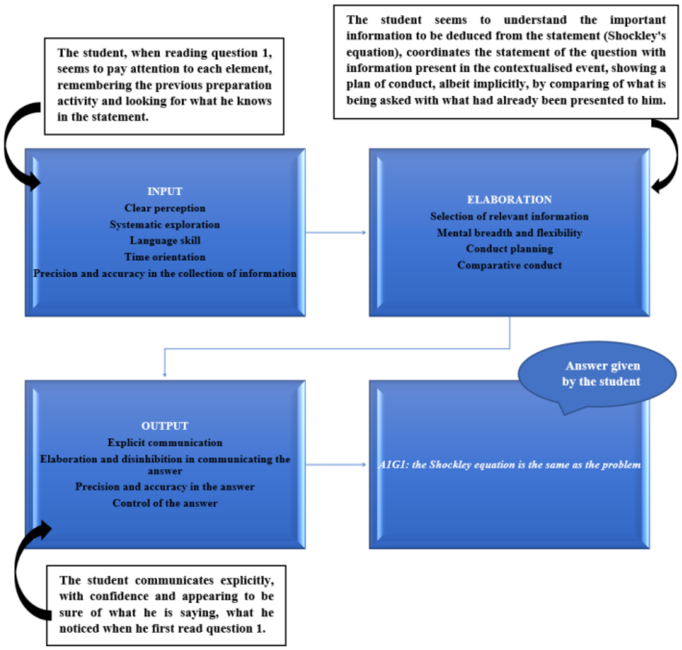
RESULTS AND ANALYSES

Upon receiving the first three guiding questions, the two groups showed quite different behaviour toward the mental act "*Carry out the first analysis of what was proposed*", as shown in Figures 10 and 11. In our view, this fact should also be highlighted due to how the groups were composed. In both, we sought to include the diversity of learning styles so that, in addition to other issues, we could work with more homogeneous groups. Although this aspect is not analysed in this article, it is important to highlight that there was no such homogeneity.

In *G1*, after receiving the statements of the questions, one of the students (*A1G1*) quickly and without any intervention from researchers or other colleagues stated: "*Shockley's equation is like the equation of the problem*" (of the contextualised event that had been proposed).

Figure 10

Group 1's reaction when receiving the questions

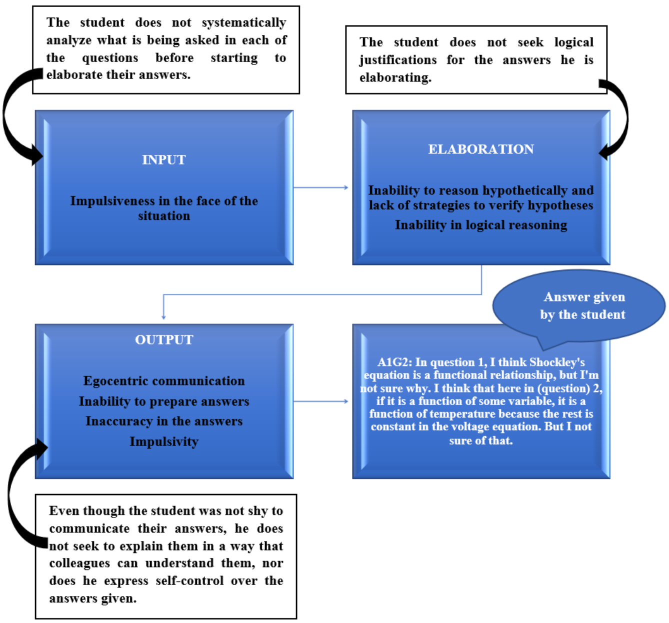


G2 members, in turn, spent almost two minutes in silence and did not seem to have perceived Shockley's equation in the statement of the contextualised event, with explanations about each element involved in it. The researcher needed to draw attention to this fact, and then, after this intervention, one of the students (*A1G2*) said: "*In question 1, I think Shockley's equation is a functional relationship, but I'm not sure why. I think that here in (question) 2, if it is a function of some variable, it is a function of temperature because the rest is constant in the voltage equation. But I'm not sure of that*".

As illustrated in Figures 10 and 11, the students' reactions when they come in contact for the first time with the questions they should answer, even if at first glance they may seem banal to the less attentive or inexperienced teacher, reveal some aspects of cognitive functions and dysfunctions.

Figure 11

Group 2's reaction when receiving the questions



Identified cognitive dysfunctions while solving question 1

The members of *G1* began discussing question 1: "*Does Shockley's equation explain a functional relationship? If your answer is yes, which is the dependent and which is the independent variable?*"

Although one of the students stated, "*I think it has a functional relationship*" (in Shockley's equation), the other colleagues did not question it, did not discuss this answer; not even the student who said so justified what he said. They immediately began discussing what the independent and dependent variables would be. Let us then observe the following dialogue, in which, in the analyses presented in Figure 12, we focus on *A1G1*'s statements, highlighting the cognitive functions and dysfunctions explained in them and focusing especially on the detailed analysis of the latter.

A1G1: Would −1 be the independent variable?

A2G1: I don't think so, because −1 is not a variable. It is always −1.

A3G1: It is fixed; I think the dependent is the I_F because it depends on I_R and the equation of T. Depending on the value of I_R, I_F changes.

A1G1: Now, I think the independent is n because it seems that it is the only one that is not related to others.

A2G1: Which one?

A1G1: n. The others are all that relate to each other.

(Transcription of an excerpt from the dialogue in *G1* when solving question 1)

Observing the students' difficulty in answering which the independent variable in Shockley's equation is, one of the researchers sought to encourage them to think about the physical context inherent to the situation. Question: "*In which magnitude existing in Shockley's equation do I need to intervene if I want current to pass through the diode?*" Upon hearing this question, the students immediately stated that it was necessary to interfere with the voltage (V_F) and, therefore, this is the independent variable in this situation. The researcher's mediation seems to have contributed to minimising the effects of *difficulty accurately using and properly understanding words and concepts* in the input phase, and that also reflected in the elaboration and output phases.

Figure 12

Cognitive functions and dysfunctions evidenced in the dialogue that occurred in G1 when analysing which is the independent variable in the function algebraically represented by Shockley's equation

Mental act: Analyse, in the function represented by the Shockley equation, which is the independent variable

INPUT

- Impulsivity
- A lack of time orientation
- Irreversibility or rigidity of thought
- Inaccuracy in the collection of information
- Linguistic inhability

The impulsivity manifested is of the epistemic type, i.e., a lack of attention when incorporating data, to solve the problem, he had already met, which also shows a lack of temporal orientation (inability to establish relationships with situations already faced in the past) and an irreversibility or rigidity of thought (translated by the inability to observe the conservation of the invariability of an object, in this case the notion of an independent variable). Impulsiveness is also associated with imprecision in collecting information: we were referring to the independent variable in the context of a function and not to the independent term associated with the equation of a line, for example. In addition, the term independent was not interpreted correctly, taking into account the context (linguistic inhability). Impulsiveness also made it impossible for him to carry out a systematic process of reflection on the information given about the n in the problem statement.

ELABORATION

- Episodic perception of reality
- Narrowing or limitation of the mental field
- Inability in comparative conduct
- Inability to select relevant information

An episodic perception of reality seems to cause students' difficulties in transposing the functional relationships – already studied – to a broader context. The narrowing or limitation of the mental field makes it difficult to work simultaneously with items of information, with some being made available externally and others to be fetched from memory. The inability in comparative behavior is evidenced by the fact that the student has not established a comparison between the meaning he associated with the independent word and the meaning with which it is being considered in the problem. The inability to distinguish between relevant and irrelevant data for the solution of the problem reveals itself in the lack of careful reflection about what the n depend on the operating conditions and physical construction of the diode, i.e., understand that, for each diode, n will be constant.

OUTPUT

- Inability to design virtual relationships
- Trial-error response
- Impulsivity (lack of answer control)

The inability to project virtual relationships is evidenced by the student' difficulty in projecting the new situation, in the context of the Analog Electronics, something that he has already studied in Mathematics. The lack of answer control is directly explained by the trial-error answers given by the student, who first says that -1 is the independent variable and then claims that such variable is n .

ANSWER

A1G1: Would -1 be the independent variable?
A1G1: Now, I think the independent is n because it seems that it is the only one that is not related to others.
A1G1: The n. The others are all that relate to each other.

Figure 13

Cognitive functions and dysfunctions evidenced in the dialogue that occurred in G2 when analysing the characteristics of a functional relationship

Mental act: Analyse what characterizes a functional relationship

INPUT

- Obscure perception
- Impulsiveness in the face of the problem situation
- A lack of time orientation
- Inability to understand words and concepts
- Irreversibility or rigidity of thought
- Inability to organise information
- Inaccuracy in data collection

The obscure perception about the function object is directly associated with the lack of understanding of words and concepts, since it was not taken into account that the term was being approached in the mathematical context. The student should have realized that the words present in the results of the research he carried out in the *web* were not related to the mathematical idea of function. Instead of exploring the given situation in a systematic way, the student performed a Google search, impulsively and without criteria. The irreversibility or rigidity of thought translates into an episodic perception of reality and a lack of temporal orientation that make it difficult to establish a relationship between objects and between the current situation and others that the student has already faced. The inability to organize the information and the imprecision in data collection are evidenced by the fact that the student resorts to Google but considers only one source among those obtained in the search - one that was unrelated to the mathematical context. And even so, the student did not perceive this error nor the inadequacy of the result obtained, which he himself expressed not having understood.

ELABORATION

- Inability to perceive a problem and define it
- Inability to select relevant information
- Narrowing or limitation of the mental field
- Inability to conduct planning
- Inability in perceptive organization and structuring
- Inability in comparative conduct
- Inability in hypothetical reasoning
- Inability in logical reasoning
- Inability to elaborate cognitive categories

The student's difficulty in perceiving and defining the problem to which he should respond seems to be because the definition of the function found in his search makes no sense to him. The inability to select the relevant information appeared from the student's posture of a passive receiver of information that is inadequate to the context he/she searched on the *web*, also revealing a narrowing or limitation of the mental field, ratified by the inability to process different sources of information. The inabilities in the conduction of planning and organization and perceptive structuring are revealed by the student's difficulty in organizing the data in the most appropriate direction and by their responses to stimuli in an episodic and fragmented way, evidencing an inability to establish relationships of similarity and difference between objects (in this case, the function object in Mathematics and the notion of function in the field in which the result of the search performed in the *Web*). The student assumes the definition found in his search as a hypothesis, but he/she is not able to explain it, argue about it, nor realize its inconsistencies in relation to the mathematical context. The student cannot also perceive the characteristics that allow classifying an object as being or not belonging to the cognitive function category.

OUTPUT

- Egocentric communication
- Lack of vocabulary, concepts, and mental operations to communicate correct answers
- Disinhibition to communicate the answer
- Inaccuracy in answer
- Impulsivity

Although in an uninhibited way, the student presents, impulsively, imprecisely, and uncritically an answer that even he/she does not understand (something beyond egocentric communication, in fact), which shows a lack of vocabulary and mastery of the mathematical concept function.

ANSWER

A3G2: I researched here: it is the relationship between the answer and the consequence.
A3G2: No, it is indicated by the previous condition. But I didn't understand what that previous condition is.

Let us now observe the following dialogue of the students who composed *G2* and carefully analyse, in Figure 13, only *A3G2'*s statements

> *A1G2: Now the 1st (question) here... I think it should be functional, I hope it is, but I'm not sure.*
>
> *Researcher: What characterises a functional relationship?*
>
> *A3G2: I researched here: it is the relationship between the answer and the consequence.*
>
> *A4G2: I think it's like this, as the x varies, the y varies too.*
>
> *Researcher: But there is a specific condition that has to be satisfied for a relation to be a function. What condition is this? Is any relation between 2 variables a function?*
>
> *A3G2: No, it is indicated by the previous condition. But I didn't understand that previous condition.*
>
> (Transcription of an excerpt from the dialogue in *G2* when solving question 1)

Let us now see one more dialogue that explains the mediation of the researcher who monitored *G2*, carried out to help students to understand the idea of a functional relationship properly and, subsequently, compare it to Shockley's equation. In this dialogue, in the analyses presented in Figure 14, our attention will be directed to *A2G2'*s statements.

> *Researcher: Think about Calculus, which you studied in high school... How did you define function in high school?*
>
> *A1G2: Each element in the domain has only one in the range. When you put a value, there will always be another single value.*
>
> *A2G2: Not necessarily one only; a x specific may have other y, for example, given a value of x we can have more than one value of y. For example, function $y = x^2$. For the same x we have two values.*
>
> *Researcher: Can there be a function such that for a domain value you have two different images?*
>
> *A1G2 and A3G2: No, that doesn't exist.*
>
> *A3G2: This is not a function. The opposite is possible, right?*

A1G2: It may be that two (elements) in the domain go to the same one in the range. The reverse is not possible.

A3G2: I think the opposite would not be a function.

(Transcription of an excerpt from the dialogue in *G2* when solving question 1)

Figure 14

Cognitive functions and dysfunctions evidenced in the dialogue in G2 when members were asked about how they defined function in high school

Mental act: Remember how they defined function in high school

INPUT

- Obscure perception
- Systematic exploration of the situation
- Language disabilities
- Information organisation
- Perception and accuracy in collecting information

The obscure perception is revealed by the imprecise way in which the student demonstrates to understand the mathematical object function. Even so, he systematically explored the situation, although he did so mistakenly due to the fragility of his knowledge regarding the notion of function. In this exploration, he even sought to set an example from what he was saying. Language disabilities relate to the difficulty of accurately using and properly understanding the concept of function. Likewise, despite mistakes arising from a misunderstanding of the definition of function, the student demonstrates the ability to organise information, perceive it, and collect it accurately.

ELABORATION

- Perception and definition of a problem
- Selection of relevant information
- Interiorisation and mental representation
- Mental breadth and flexibility
- Conduct planning
- Perceptual organisation and structuring
- Comparative conduct
- Hypothetical thinking
- Logical evidence
- Inability to elaborate cognitive categories

Although the student made a mistake because he did not understand the mathematical object in focus, he efficiently mobilised a series of cognitive functions at this stage. One of the possible causes for the result of their mobilisations not being successful is the evidenced inability to elaborate cognitive categories (in this case, the mathematical relations that could be inserted in the cognitive function category).

OUTPUT

- Explicit communication
- Verbal rules to communicate the answer
- Elaboration and disinhibition in communicating the answer
- Precision and accuracy in the answer
- Control of the answer

Although he does not adequately conceive the idea of function, the student seeks to make himself understood, through explicit communication with colleagues and the researcher, efficiently using verbal rules to communicate the answer. In addition, although the student undeniably has a mistaken understanding of what a functional relationship is, from the perspective of his understanding, the answer was elaborated and communicated precisely and without any inhibition. In the same way, the answer was not impulsive, it was well thought out, despite being wrong.

ANSWER

A2G2: Not necessarily the only one; a specific x may have other y, for example, given a value of x , we can have more than one value of$y$. For example, function $y = x^2$. For the same x , we have two values.

After this student debate about what characterises a functional relationship, a new mediation by the researcher was necessary for them to return their attention to answering question 1, as evidenced by the following dialogue:

Researcher: So, in relation to everything you guys are talking about, what does it take to characterise a functional relationship?

A3G2: Define what we are calling domain and range in this equation.

Researcher: You can do this. And then it will have to do with the second part of the answer: analysing how the variables in the equation are related.

Researcher: Put Shockley's equation on the screen and observe what conditions are necessary for it to represent a functional relationship. First, what quantities/variables does this equation relate to?

A1G2: Current.

A3G2: Voltage.

A1G2: Yeah, I think that's it. And the rest is constant, then?

Researcher: What is n*?*

A2G2: It is the ideality factor.

Researcher: So it's a constant, right? And the I_R *is constant, does it vary?*

A1G2: I think it's constant.

Researcher: And the V_T*?*

A1G2: At a given temperature it will be constant too.

A3G2: So, it is I_F *by* V_F*, right? (Meaning in relation to the variables).*

Researcher: So, the equation is relating which magnitudes?

A1G2: It is relating the polarisation voltage with the current that passes through the diode.

Researcher: And in this relationship, who depends on whom?

A1G2: The current depends on the voltage.

Researcher: Do the others agree?

A3G2: Yes, it's like we were talking about, I_F will be y and V_F will be x.

Researcher: If you agree that I_F depends on V_F, what must happen for this dependency to be a function?

A1G2: Each value of V_F must result in a I_F.

Researcher: Do the others agree? For a voltage of 10V can I have more than one possibility for the current?

A1G2, A3G2: No, just one.

Researcher: And that happens in this equation?

A1G2: No, for each voltage value, we have only one current value.

Researcher: So it's a functional relationship?

A1G2, A3G2: Yes.

Researcher: And this function looks like what kind of function you've already studied?

A1G2: Exponential.

Researcher: Since you concluded that it is a functional relationship, which is the dependent variable and which is the independent one?

A1G2: The dependent [variable] is I_F, I think... because it depends on V_F.

A3G2: Yeah, I think it is.

Researcher: And the independent one?

A1G2: It's V_F, right?

Researcher: So, although it's not even in the question, tell me: what is the domain and what is the range of this function?

A3G2: The domain will be V_F (the set of values that can be assumed by this variable).

A1G2: The range will be the current values.

(Transcription of an excerpt from the dialogue in *G2* when solving question 1)

This dialogue evidences that all the students' discussions with each other and with the researcher in the input phase allowed them –gradually, at least concerning the mental act to answer question 1– the elucidation of an unblemished perception; a systematic exploration of the situation; input-level linguistic skills; conservation, permanence, and constancy of the object (in this case, the mathematical function object); information organisation; and precision and accuracy in gathering information. In the elaboration phase, in turn, this dialogue reveals the following cognitive functions: perception and definition of a problem; selection of relevant information; internalisation and mental representation; mental breadth and flexibility; conduct planning; organisation and perceptual structuring; comparative conduct; hypothetical thinking and logical evidence. Finally, in the output phase, cognitive functions related to: explicit communication; projection of virtual relations (in this case, perceiving implicit relations between what you have already studied about function and the elements that make up Shockley's equation); verbal rules to communicate the answer; elaboration and disinhibition in communicating the answer; precision and accuracy and control of answers.

Cognitive dysfunctions identified during the resolution of question 2

Let us now observe a dialogue between the members of *G1* when starting the analysis of question 2: *Is thermal voltage a function of any variable? Explain, and if your answer is yes, make a graphical representation of this function.*

In this one, when analysing it in Figure 15, we will direct our attention to A2G1's statement

A3G1: Is thermal voltage a function of any variable? Yes, it is too. It says there (referring to the expression representing thermal stress) thermal voltage is a constant "times" a variable.

A1G1: Thermal stress is a function of temperature.

A2G1: But I think it also depends on the magnitude of the q.

A1G1: No, q *it is always the same; is the charge of the electron.*

(Transcription of an excerpt from the dialogue in *G1* when solving question 2)

The following dialogue, in which, in the detailed analyses in Figure 16, we turn our attention to the statements of *A1G1* and *A3G1*, reflects a moment of debate about how to sketch the graphical representation of the thermal voltage function.

A1G1: How will the graph be?

A2G1: I think it's going to be a line, isn't it? Because there's only one variable and it changes linearly.

A1G1: (using Word's drawing tool) Should we just put a straight line here?

A3G1: Yes, it's just a straight line; I think the inclination is irrelevant; it's okay if we don't put any.

A1G1: (draws an inclined line and asks) Is it like that or do I draw a line parallel to the horizontal axis?

A3G1: No, a straight line that starts from zero.

(Transcription of an excerpt from the dialogue in *G1* when solving question 2)

Figure 15

Cognitive functions and dysfunctions evidenced in the dialogue in G1 when members analysed on which variable the thermal voltage function depends

Mental act: Analyse which variable does the thermal stress function depend on

INPUT

- Impulsivity
- A lack of time orientation
- Irreversibility or rigidity of thought
- Inaccuracy in the collection of information

The student's impulsiveness, which can be classified as epistemic, manifests itself in answering that the thermal stress is also a function of the electron charge. Certainly, the student had already worked with the fact that the electron charge is constant in situations studied in High School in Physics subjects. However, we observe that he did not take adequate care to incorporate this information in this context of thermal stress, also revealing a lack of temporal orientation, translated by the inability to establish relationships between situations already experienced in his studies and is inherent to the question with which he was working. There is still an imprecision in information collection since it seems that the student, when faced with a "letter" in a mathematical expression (in this case,q) already binds it directly to a variable.

ELABORATION

- Inability to select relevant information
- Narrowing or limitation of the mental field
- Episodic perception
- Inability in comparative conduct
- Inability to elaborate cognitive categories

The inability to select the relevant information is evidenced by the fact that the student does not establish comparisons and relationships between this situation and others with which he has already worked in which the magnitude q of the elementary electric charge was present. The narrowing or limitation of the mental field is related to the difficulty in dealing simultaneously with external items of information (in this case, the expression that makes it possible to determine the thermal tension) with others that must be sought in memory (the meaning of the magnitude of the elementary electrical charge$q$). Linked to those two previously mentioned elements is the episodic perception, evidenced, in this case, by the student's difficulty in establishing relationships between the$q$, present in the expression that represents the thermal tension, and the meaning of the magnitude of the elementary electric charge q in other situations. In the same way, the inability in the comparative conduct becomes explicit as the student does not seek a meaning, in terms of the context that is being considered for the "letters" present in the expression that represents the thermal stress. The student shows that he does not understand whether the magnitude of the elementary electric charge q should be included in the cognitive category of physical constants or mathematical variables.

OUTPUT

- Explicit communication
- Verbal rules to communicate the answer
- Disinhibition in communicating the answer
- Impulsivity

Although I don't understand that q is a constant and not a variable, the student seeks to make himself understood through explicit communication with colleagues and the researcher, efficiently employing verbal rules to communicate the answer. Furthermore, while he undeniably does not understand the meaning of q in the context of the problem, which makes it impossible for him to answer accurately, the answer, although impulsive and without regulatory behavior, was communicated without any inhibition.

ANSWER

A2G1: But I think it also depends on the magnitude of the q.

Figure 16

Cognitive functions and dysfunctions evidenced in the dialogue in G1 during the analysis of the slope of the straight line that graphically represents the thermal voltage function

Mental act: Analyse the slope of the line that graphically represents the thermal voltage function

INPUT

- Obscure perception
- Impulsivity towards the situation
- A lack of time orientation
- Irreversibility or rigidity of thought
- Inability to organise information
- Inaccuracy in the collection of information

The obscure perception associated with an epistemic impulsiveness towards the situation and the lack of temporal orientation are revealed through the non-incorporation in this context, by the A1G1 and A3G1 students, of the knowledge they were used to mobilising when working with more usual algebraic representations, as they appear in mathematics classes, of 1st-degree polynomial functions, in which they could possibly identify the value of the angular coefficient. These elements may also be related, in our understanding, to an irreversibility or rigidity of thought, relative to the non-perception of the stability of a given object despite transformations in other attributes, in this case the algebraic representation of a 1st-degree polynomial function and the information present in such representation. We also noted an inaccuracy in the collection and an inability to organise the information that makes it difficult for the subject to use all the information they have at hand to answer to what is asked.

ELABORATION

- Inability to perceive and define a problem
- Inability to select relevant information
- Narrowing or limitation of the mental field
- Inability to plan the conduct
- Episodic perception
- Inability in comparative conduct
- Absence of logical evidence

The inability to perceive and define the problem, associated with the inability to select relevant information and the narrowing or limitation of the mental field, is explained by the difficulty of the two students based on that the information available in the statements of the contextualised event and question 2 reflect and combine such information to identify the angular coefficient of the line that graphically represents the thermal voltage function and, consequently, the slope of this line. Students reveal a lack of internalisation about how to obtain, from the algebraic representation of a function, the angular coefficient of the line that represents it graphically, which lead them to a lack of planning of conduct. Their knowledge about the idea of angular coefficient and algebraic representation of a function prove to be episodic and fragmented, which reflects in inabilities in the comparative conduct and in the absence of logical evidence, which can be perceived by the manifestation of A3G1 about the non-importance of the slope of the straight line being sketched.

OUTPUT

- Egocentric communication
- Verbal rules to communicate answers
- Disinhibition in communicating the answer
- Inaccuracy and inaccuracy in the answer
- Inability in visual transportation
- Impulsivity

Egocentric communication is revealed when A3G1 states: "I think that the inclination does not matter; it's okay if we don't put any," but doesn't explain his answer. However, despite this aspect, he presents his answer in an uninhibited way, although imprecise and inaccurate. Inability in visual transportation is made explicit, evidenced by the fact that the students did not pay attention to the given information that was relevant so that they could identify the angular coefficient of the line that would be drawn and an impulsiveness in the answer, indicated by the lack of self-control when A3G1 responds to A1G1 in a rather vague way.

ANSWER

A1G1: (using Word's drawing tool) Should we just put a straight line here?
A3G1: Yes, it's just a straight line; I think the slope is irrelevant; it's okay if we don't put any.
A1G1: (draws an inclined line and asks) Is it like that or do I draw a line parallel to the horizontal axis?
A3G1: No, a straight line that starts from zero will do.

It is pertinent to point out that later in the resolution of the third question, one of the researchers asks which is the angular coefficient of the line that graphically represents the thermal voltage function. The same student who had said that the slope of the line to be sketched was irrelevant (*A3G1*) promptly responded that the angular coefficient was $\frac{k}{q}$, which leads us to conjecture that, perhaps, what we assumed from *A3G1*'s statements as being cognitive dysfunctions directly linked to the ideas of angular coefficient and slope of a straight line were actually the result of a mistaken perception on the part of the students that, when answering question 2, they only needed to present a sketch of the curve that represents the thermal voltage function without any concern for accuracy or rigour. This conjecture is reinforced by the fact that *G1* used Microsoft Word drawing tools (a text editor) instead of working with a mathematics software to outline the graph.

Another noteworthy observation concerns the immediate identification established by students of a mathematical object with the way it is mostly denoted in their experiences in classrooms, an identification that, for applications in extra-mathematical contexts, can be limiting and hinder what Camarena (2013) calls the "contextualised transposition". From the moment they began their studies, relating a straight line in the Cartesian plane to the graphical representation of the points in the plane that satisfy a relation represented by a polynomial function of the 1st degree, the axis of abscissas (which, later, in the study of graphical representations of functions, will be understood as the axis related to the values of the independent variable of the function) is denoted by x, or, even more often, by *x-axis*, while the axis of the ordinates (which, in the context of the study of graphical representations of functions, it will be understood as the axis associated with the values of the dependent variable of the function) is denoted by y or, more commonly, by $y-axis$.

Thus, before a situation in which the independent variable is not denoted by x and the dependent variable is not denoted by y, students still sought to support their understanding in the x-axis and y-axis ideas with which they are more familiar. The dialogue below highlights this fact, also showing that although the association between the concept of abscissa and independent variable to the nomenclature x and between the concept of ordinate and dependent variable to the nomenclature y may lead the student to face obstacles in the process of transferring mathematics knowledge to a field of application external to this science, this was not the case at this time.

Researcher: But will you leave the graph without the names of the variables on the coordinate axes?

A1G1: But which axes?

A3G1: The y axis is the V_T and the x axis is T_K.

(Transcription of an excerpt from the dialogue in *G1* when solving question 2)

We focus now on a dialogue established between the members of *G2* and between them and the researcher when answering question 2. After concluding, without the intervention of the researcher, that the thermal voltage is a function of the temperature since all the other elements present in the expression that represents it algebraically are constant, the researcher indicated that to build the graphic representation of this function, they could use some software of their preference.

They then choose to use GeoGebra. The dialogue below reproduces the discussions after this decision and, in the analyses presented in Figure 17, we chose as the focus the moment when the students discussed how to insert the algebraic expression of the thermal voltage function into the GeoGebra input field, which they sought to do using the language with which they are most accustomed, i.e. *y* to denote the dependent variable and *x* to denote the independent one. The obstacles arising from this action confirm our statement about the limiting and difficult character that it can assume in the students' process of transferring knowledge from mathematics to an extra-mathematical context.

A2G2: So we will need to put (in the input field of GeoGebra) this expression (the expression that algebraically represents the thermal voltage function) with the constants only, right? But how can we put the T_k in the part of x?

A3G2: No! Just put in the numbers now.

A2G2: So let's transform T_k in a number?

A1G2: (writes in GeoGebra input field)

$y = 1{,}38 \times 10^{-23}$ ...

Researcher: (noting that the students were not sure about how to continue typing the algebraic expression of the function) Okay, you entered k, the Boltzmann constant, and now you need to enter T_k. Which one is the T_k?

A1G2: x (and then writes) $y = 1{,}38 \times 10^{-23} \cdot x$

Researcher: note that T_K is the absolute temperature in Kelvin, which is given by adding 273 to the temperature measured in degrees Celsius.

A1G2: (seeming not to give much importance to what the researcher had said, he continues his reasoning and says) Now, we must just divide by q, which is $1{,}6 \times 10^{-19}$ (and writes in the GeoGebra input field) $y = \frac{1{,}38\times10^{-23}\cdot x}{1{,}6\times10^{-19}}$.

Researcher: In this expression you wrote down, how is the fact that T_K is the absolute temperature in Kelvin, which is given by adding 273 to the temperature measured in degrees Celsius? If we wanted to obtain the range of this function at some value of x... give me an example of some value of x in which we could obtain the image of the thermal stress function.

A2G2: In this function here? (and points to the expression $V_T = \frac{kT_K}{q}$).

Researcher: Suppose, for example, that the diode will operate at 25 degrees.

A2G2: 25 Kelvin?

Researcher: No! I am considering 25°C. So, in this case, what value do you need to give to x in the expression you wrote?

A2G2: Ah, 25 plus 273. So, it must be $x + 273$ in the expression.

Researcher: Exactly

Researcher: (after students correctly construct the graphical representation of the thermal voltage function) what kind of function is this?

A1G2: Linear

(Transcription of an excerpt from the dialogue in *G2* when solving question 2)

Figure 17

Cognitive functions and dysfunctions evidenced in the dialogue that took place in G2 during the construction, with the help of GeoGebra, of the graphical representation of the thermal voltage function

Mental act: With the help of *software* GeoGebra, build the graphical representation of the thermal stress function

INPUT

- Clear perception
- Systematic exploration of the situation
- Language skill
- A lack of time orientation
- Irreversibility or rigidity of thought
- Information organization
- Inaccuracy in the collection of information

Despite the obstacles faced by the students because they chose to use other variables instead of working with those that were present in the expression representing thermal stress, we could detect a clear perception, a systematic exploration of the situation, linguistic skills, and information organization. A lack of temporal orientation is evident when A2G2 does not associate the term degrees with a temperature measurement on the Celsius scale. The irreversibility or rigidity of thinking is revealed when students face difficulties in understanding how to insert the algebraic expression of the function, especially the independent variable, in the input field of GeoGebra and, finally, the imprecision in collecting the information becomes clear when they do not consider that, if they wanted to work with the temperature directly in Kelvin, they would need to add 273 to the temperature in degrees Celsius.

ELABORATION

- Perception and definition of a problem
- Inability to select relevant information
- Narrowing or limitation of the mental field
- Conduct planning
- Episodic perception
- Inability in comparative conduct
- Hypothetical thinking
- Logical evidence

The inability to select the relevant information becomes explicit only when students should identify how to denote the argument of the thermal stress function depending on the scale to be assumed to measure temperatures and they do not. The narrowing or limitation of the mental field is also evident at this very moment. The episodic perception of reality, associated with an inability in comparative behavior, is revealed when students do not associate the term degrees with a temperature measurement on the Celsius scale, an idea with which they have already worked at different previous moments of their formative processes. Finally, it is noteworthy that the students correctly classified the thermal stress function in the cognitive category linear function.

OUTPUT

- Explicit communication
- Inability to project virtual relationships
- Verbal rules to communicate answers
- Elaboration and disinhibition in communicating the answer
- Precision and accuracy in the answer
- Control of answers

The obstacles faced seem to be due to the influence of the inability to project virtual relationships since they choose to denote by y the dependent variable in the thermal stress function and by x the independent one, but they do not adequately establish, at least in principle, the relationship between the values to be assumed by x depending on the scale of temperature measurements considered – Celsius or Kelvin. They do not show clarity, in the first answers they provide to the researchers, that, if they chose to directly consider the temperature in Kelvin as an element of the domain of the function that represents the thermal stress, they could write, according to the notation chosen by them, $y = \frac{kx}{q}$, but if the option were to determine the thermal stress in an element of the domain assumed as a temperature in Celsius, they would have to write, also in line with the notation they used to indicate the variables of the thermal stress function, $y = \frac{k(x+273)}{q}$.

ANSWER

A2G2: How can we place the T_k on the part of x?
A3G2: No! Just put in the numbers now.
A2G2: So let's transform T_k in a number?
Researcher: Which is T_k?
A1G2:x... $y = 1{,}38 \times 10^{-23} \cdot x$ ... now just divide by$q$, that is$1{,}6 \times 10^{-19}$ ... $y = \frac{1{,}38\times10^{-23}\cdot x}{1{,}6\times10^{-19}}$
Researcher: Suppose, for example, that the diode will operate at 25 degrees.
A2G2: 25 Kelvin?
Researcher: No! I am considering 25°C.
A2G2: Ah, 25 plus 273. So, it must be $x + 273$ in the expression.

Let us then move on to the final considerations that can be deduced from the study carried out.

FINAL CONSIDERATIONS

The data presented and analysed indicate, among other aspects, cognitive dysfunctions that significantly influence the ability to perform the contextualised transposition (Camarena, 2013, 2021).

In the input phase, the following are evident: a lack of temporal orientation (identified, above all, by the students' inability to establish relationships, in extra-mathematical contexts, with mathematical situations already faced in their school trajectories); irreversibility or rigidity of thought (translated by the episodic perception of reality and by the inability to observe the conservation of the invariability of a mathematical object, despite the possibilities of variations of some of its attributes and dimensions, variations resulting from its insertions in contexts that are not purely mathematical); and inaccuracy in collecting information (related to misunderstandings about mathematical objects that become even more evident when knowledge linked to them needs to be mobilised in extra mathematical situations).

In the elaboration phase, in turn, the following stand out: the episodic perception of reality (evidenced in moments when students face difficulties in transposing contents with which they had already worked to a broader and not purely mathematical context); the narrowing or limitation of the mental field (which becomes explicit in the obstacles the students face when they need to work with information explicitly presented simultaneously with others, especially those related to mathematical content, which they should recover in their memories); inability to compare and consequently respond to stimuli in an episodic and fragmented way (evidenced by the failure to establish comparisons between notions that, in different contexts – mathematical or extra-mathematical – acquire different meanings, due to the lack of perception of possible inconsistencies in relation to such meanings and for not seeking the specific meanings, in the considered context, for some notion with which they need to work in the process of solving a problem); and inability to identify the characteristics that allow classifying a mathematical object, whether it is inserted in an intra or extra-mathematical context, as belonging or not to a certain cognitive category.

Finally, in the output phase, the dysfunction with the greatest impact on carrying out the contextualised transposition that could be identified is: the inability to design virtual relationships (evidenced by the students' difficulty in designing in an extra-mathematical context something they have already worked with in a purely mathematical domain).

The results complement those coming from another perspective: that of the communication established by the researchers with the students, in this first meeting, regarding the types of questions we asked, the students' answers and the aspects they revealed regarding the transfer of knowledge from mathematics to an engineering situation and the mobilisation of mathematical competencies and general competencies that constitute the epistemological basis of engineering (Gomes, Bianchini, & Lima, 2021a).

The analysis of the cognitive functions ratifies conclusions obtained focusing on communication. We particularly stress the fact that even the students who performed well, were interested in mathematics, and had already revisited the real functions of a real variable at the time the contextualised event was implemented faced conceptual obstacles when transposing mathematical knowledge to an analogue electronics context, which reveals that such knowledge is fragmented and that, particularly regarding the functional relationship, they do not perceive it as a high-level abstract object.

The analyses underscore the pertinence of linking the precepts of the SCM to work with MoDiMaCo and the importance of allowing future engineers to link mathematics with the different areas of engineering. However, the way the professor explores such linkage must provide opportunities for structural change (as it is called in the SCM), so that the lessons arising from working with the event can be internalised, endure, and be available in the students' cognitive structures to be applied in other situations.

The teacher's mediation should enable the future engineer to build sufficiently solid cognitive structures so that, before new situations, elaborated from alterations in some elements of the original ones, these changes cannot confuse or distract him/her. As Feuerstein, Feuerstein, and Falik (2014) indicate, cognitive changes must be structured in such a way as to enable the learner to adapt to a new situation, in a flexible way, a successful behaviour when solving a previous problem. We must also remark that when the mediator finishes his/her work, the future engineer continues their development autonomously.

It is essential "to encourage the student to compare, collect and classify data and to give meaning to the current experience in relation to the previous experience" when working with contextualised events (Feuerstein, Feuerstein

& Falik, 2014, p. 71). These actions will allow experience to be transformed into learning.

We return to a position by Feuerstein, Feuerstein, and Falik (2014), with which we fully agree:

> The belief in the ability of individuals to increase their modifiability enables educators to look for signs of change when assessing students and make a more dynamic (and optimistic) prognosis that takes into account the changes that have occurred rather than relying solely on the existing operating level. (Feuerstein, Feuerstein, & Falik, 2014, p. 106)

We believe that students can change. Thus, we seek to identify cognitive dysfunctions so that, through adequate mediation work, subsidised by data from investigations such as the one presented in this article, these can be converted into functions. We do not accept that this situation cannot be modified because the student has not yet efficiently developed some cognitive functions at a given moment in their formative journey.

It is because of this non-acceptance that we insist on working with contextualised events, despite the obstacles pointed out by some colleagues: students do not have sufficient prior mathematical knowledge; at the beginning of their graduations in engineering, they still do not have specific knowledge that allows them to approach situations closer to the engineer's areas of activity; some of the professors do not have enough engineering knowledge to work with applications closer to reality, etc. We do not deny the existence of these obstacles, but, as students and professors are permanently modifiable beings, we argue that we can plan strategies to overcome them by being aware of the difficulties and identifying them in detail. If we accept the insurmountable impossibility of changing the mathematics teaching and learning processes in engineering, this insurmountability, in addition to increasing, will paralyse us, lead us to repeating old practices to obtain the same results that we do not like and to blaming previous education – those of professors and students – for all this inertia. To practice teaching, it is necessary, first of all, to believe in human modifiability!

AUTHORSHIP CONTRIBUTION STATEMENT

The three authors worked together on formulating the problem and the guiding questions that, from a didactic point of view, supported its resolution;

on implementing the situation with the research subjects, and analysing the resulting data. The writing, proofreading, and approval of the article for submission were also carried out jointly.

DATA AVAILABILITY STATEMENT

The authors agree to make their data available upon a reasonable request from the reader, it being up to them to determine whether a request is reasonable.

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