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Elements for the Design of a Teaching Experiment for Mathematics Teachers in Engineering Careers

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ABSTRACT

Context: One of the most relevant tasks of mathematics education is the teaching professional development (TPD) of mathematics teachers. In higher education, it is evident that the type of teaching in mathematics courses for engineering has not been transformed despite the changes that the curriculum has undergone. **Objective**: To show the design stage of a teaching experiment in relation to professional development programmes for higher education mathematics teachers who teach engineering careers, created under the principles that guide such effective improvements. **Design**: Through a qualitative approach, we chose the design-based research to design a TPD programme for university teachers who teach mathematics in an Engineering College, whose training is aimed at teachers of basic sciences and mathematics for engineering, attending to the changes required by the 21st-century teaching. **Environment and participants**: With the eight principles of effective TPD programmes and the ALaCT process, cycles of reflection are planned to train university teachers. **Data collection and analysis**: A teaching experiment for effective TPD is designed. This design considers situations created by the experts and those presented by the participating teachers. **Results**: The situations that the participants face will allow them to have a series of problems for their teaching, with their justifications in the field of mathematics didactics. **Conclusions**: The different stages of the ALaCT reflection cycle allow the experts to install the reflection in the teachers participating in the experiment. One of the main difficulties was counting on the process of teaching and learning linear algebra problems brought by the participants.

Keywords: Professional development, Effective programme, Modelling in teaching, Reflection on teaching practice, Deepening of knowledge.

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Elementos para el diseño de un experimento de enseñanza para profesores de matemáticas en carreras de ingeniería

RESUMEN

Contexto: Una de las tareas más relevantes de la Educación Matemática es el desarrollo profesional docente, DPD, de los profesores de matemáticas. En educación superior se evidencia que el tipo de docencia en cursos de matemática para ingeniería no se ha transformado a pesar de los cambios que ha tenido el currículo. **Objetivo:** mostrar la etapa de diseño de un experimento de enseñanza en relación con programas de desarrollo profesional para profesores de matemáticas de educación superior que imparten docencia en carreras de ingeniería, creado a la luz de principios que guían dichos perfeccionamientos efectivos. **Diseño:** Mediante un enfoque cualitativo, se escoge la Investigación Basada en el Diseño, con propósito de diseñar un programa DPD para profesores universitarios que realizan clases de matemáticas en una Facultad de Ingeniería, cuya capacitación apuntó a los docentes de Ciencias Básicas y Matemáticas para Ingeniería atendiendo a los cambios que requiere la enseñanza del Siglo XXI. **Entorno y participantes:** Con los ochos principios de programas de DPD efectivos y el proceso ALaCT se planifican ciclos de reflexión para la capacitación de docentes universitarios. **Recopilación y análisis de datos:** Se diseña un experimento de enseñanza para DPD efectivos, este diseño considera situaciones creadas por los expertos, así como las que presentan los profesores participantes. **Resultados:** Las situaciones que los participantes enfrenten, les permitirá disponer para su docencia de una serie de problemas, con sus justificaciones en el campo de la didáctica de la matemática. **Conclusiones:** Las diferentes etapas del ciclo de reflexión ALaCT permite a los expertos instalar la reflexión en los docentes participantes del experimento, una de las principales dificultades fue contar con problemáticas del proceso de enseñanza y aprendizaje del álgebra lineal que provengan de los participantes.

Palabras clave: desarrollo profesional docente, programas efectivos, modelación en la enseñanza, reflexión de la práctica, profundización en los conocimientos.

Experiência de ensino para professores de matemática em carreiras de engenharia com base em evidências de didática de matemática e desenvolvimento profissional

RESUMO

Contexto: Uma das tarefas mais relevantes da Educação Matemática é o desenvolvimento profissional docente, DPD, de professores de matemática. No ensino superior, fica evidente que o tipo de ensino nos cursos de matemática para engenharia não se transformou apesar das mudanças que o currículo passou. **Objetivo**: mostrar a etapa de projeto de um experimento de ensino em relação a programas de desenvolvimento profissional para professores de matemática do ensino superior que lecionam carreiras de engenharia, criado à luz dos princípios que orientam tais melhorias efetivas. **Design**: Através de uma abordagem qualitativa, a Pesquisa Baseada em Design é escolhida, com o objetivo de projetar um programa de DPD para professores universitários que lecionam matemática em uma Faculdade de Engenharia, cuja formação é voltada para professores de Ciências Básicas e Matemática para Engenharia. mudanças exigidas pelo ensino do século XXI. **Ambiente e participantes**: Com os oito princípios de programas de DPD eficazes e o processo ALaCT, estão previstos ciclos de reflexão para a formação de professores universitários. **Coleta e análise de dados**: É projetado um experimento de ensino para DPD eficaz, este projeto considera situações criadas pelos especialistas, bem como aquelas apresentadas pelos professores participantes. **Resultados**: As situações que os participantes enfrentam permitirão que eles tenham uma série de problemas para o seu ensino, com suas justificativas no campo da didática da matemática. **Conclusões**: As diferentes etapas do ciclo de reflexão ALaCT permitem que os especialistas instalem a reflexão nos professores participantes do experimento, uma das principais dificuldades foi ter problemas do processo de ensino-aprendizagem de álgebra linear que vêm dos participantes.

Palavras-chave: desenvolvimento profissional docente, programas eficazes, modelagem no ensino, reflexão sobre a prática, aprofundamento do conhecimento.

INTRODUCTION

Teacher professional development (TPD) is defined as a process of continuing and personal learning and growth in which the teacher participates of their own free will, gradually acquiring confidence, skills, and autonomy to improve their practice to address the students' learning challenges and deepening of their knowledge - whether disciplinary or in the didactics of the discipline— (Avalos 2011; Bautista & Ortega-Ruíz, 2015; Mizwell, 2010). This is naturally achieved in collaborative work among peers, individual or collective reflection processes, and voluntary participation (Korthagen & Vasalos, 2009), which begins in the initial education, advances throughout their careers and culminates in continuing training.

In the TPD, the teacher is inserted in the review processes of their practice (Ponte & Chapman, 2008), where the training programmes in which they are involved play a relevant role when it comes to evolving, defining the moment and level in which they are located (Ramos-Rodriguez, 2014). These training proposals "should offer professional development opportunities to teachers and motivate them to develop the knowledge, skills, and dispositions they need to teach mathematics well" (Swoder, 2007, p. 160-161). The changes in teachers' practices, which derive from the TPD programmes in which they are involved, depend to a large extent on the opportunities they offer to build (or reconstruct) the knowledge they have of the disciplinary, didactic, and pedagogical contents, in a context that supports and encourages them to leave their comfort zone (risk-taking) and reflect (Swoder, 2007), even more so if the new knowledge is valued.

The above fosters effective TPD programmes, which are defined according to the development of the practice of the teachers involved and those teachers' students' learning (Desimane & Pak, 2017; Montecinos, 2003; Timperley, Wilson, Barrar, & Fung, 2007). The underlying characteristics of those programmes offer opportunities to face teachers' continuing training better.

A wide range of research has focused on identifying principles or constituent elements of effective TPD programmes (Bautista & Ortega-Ruíz, 2015; Blank & de las Alas, 2009; Cockcroft, 1982; Guskey, 2003; Desimone & Pak, 2017; González-Weil et al., 2014; McNeill & Krajcik, 2008, among others) in which they repeat the concepts of collaborative work and peer reflection (Korthagen & Vasalos, 2009).

Conceiving an effective TPD programme is an exercise of creativity that needs to consider a diversity of information about the context and the participants, such as the degree of interest, country's culture, and local needs (Martínez, 2009). Effective training for university mathematics teachers is a country's need, which we will address more properly by being situated and supported by a university research centre that develops both the discipline and its didactics.

Regarding the TPD courses for the continuing training of mathematics higher education teachers, we highlight that they are scarce, and those that exist do not address specific contents of the mathematics curriculum. TPD courses must promote instrumentalisation processes (Trouche, 2018) so that the teacher integrates them in their mathematics classrooms to address mathematical calculations in modelling processes or simulations that are required in the training of engineers (Borromeo-Ferri, 2006) and their future professional development (Villa-Ochoa & Jaramillo, 2011).

Trejo et al. (2013) established a methodological strategy for the teaching of mathematics in engineering called Didactic Phase of Mathematics in the Context of Sciences, a strategy that requires teachers to be involved in the engineering career and have the knowledge to solve or model a problem like the ones that are inserted in the linear algebra course. In this regard, we underscore the role of linear algebra, whose contents permeate the different branches of mathematics: differential and integral calculus, calculus in several variables, numerical analysis, or differential equations and, therefore, have a unifying character in the mathematics training in engineering (Dorier, 2000) through, for example, the linear approximation. In this module, concepts are usually presented abstractly, which brings with it conceptual difficulties —due to the nature of the issues addressed - and cognitive difficulties - by the thinking that students must develop to understand them. These are attributed to the fact that these concepts are presented in different disarticulated abstract, algebraic, and geometric languages (Dorier & Sierpinska, 2001).

This work aims to show the design stage of a teaching experiment related to professional development programmes for higher education mathematics teachers who teach engineering careers, created in the light of principles that point to the effectiveness of said refinement. In this work, we use these central ideas (the principles) to design a TPD programme using technology oriented towards the continuing training of university mathematics teachers who teach engineering careers, considering a pilot implementation with a group of teachers with at least seven years of university teaching experience. This study is not focused on increasing the teachers' mathematical knowledge but on creating and studying appropriate tasks to develop competencies in content teaching and understanding mathematical models of engineering, thus disrupting the mathematics teaching in our university by emphasising the use of technology.

We believe relevant to show the stage of creation, considering the detail of the theoretical aspects (principles) that support TPD programmes in higher education and the specifications of the programme created in light of them.

Thurm and Bazel (2020) inform us of a professional development programme in terms of what they call guidelines for an effective programme using technology. The authors heed the frequent criticism of effective professional development (PD) publications as they insufficiently describe the design and content of the professional development programmes at stake (Driskell et al., 2015; Sztajn et al., 2017; Goldsmith et al., 2014). The professional development programme design they propose is around four modules of six hours each and with a gap of two months between them. The PD addresses the teaching of functions as content, both in the teachers' experimentation and in creating tasks to be developed with their students. The first two modules address the introduction to the effective use of technology to teach functions with examples, thus developing the sense of showing and reflecting on how technology supports the advancement of conceptual images and the making of a model. In the later modules (3 and 4), the teachers reflect on the role of the situations posed in the first modules to design tasks with technology so that students understand and develop modelling skills using technology. In a guided process, the teachers design tasks that allow their students to deepen concepts and technological procedures and, thus, develop skills that can be mobilised to other situations of modelling and understanding of mathematics. In module 4, the teachers create (supported by personal experience) and analyse the situations their students could address and reflect on the relationship between assessment and documentation that teaching with technology requires.

In the courses that focus on using technological resources, one of the products is that teachers increase their bibliographic resources. They are no longer limited to those found in the classic library. However, it opens the universal library that includes specific software for the different mathematical contents and examples and topics linked to the different disciplines of the training line in engineering, architecture, and economics, among others (Kayali & Biza, 2021).

It is essential to have information about the participants' and the identified leaders' beliefs to develop a TPD successfully - for example, course group coordinators (who are usually assigned for their leadership among their peers).

Thurm and Barzel (2020) identify three types of beliefs related to the use of technologies: epistemological beliefs, self-efficacy beliefs, and beliefs about teaching with technology. The TPD should enhance beliefs that mathematical calculations should be performed with technology, that technology supports discovery learning, and supports multiple representations. In this way, although time is indeed required to encourage students to use technology, this investment is projected in time and appropriate use, especially in courses of linear algebra, calculus in one or more variables, and those of ordinary differential equations or in partial derivatives, where many processes can be made more complex and understood by using suitable software. In this way, in classes, they can address modelling situations or problems of the career they are studying that use mathematics and that teachers have known in those instances of constant improvement.

THEORETICAL FOUNDATIONS

The conceptual framework of this study focuses on the concept of effective TPD programmes and the principles that underpin the concept. The study of TPD training proposals can be carried out by looking at the results of those teachers' students or by looking at the teachers' teaching practice (Ávalos, 2011). In other words, an effective TPD programme is one where there is a relationship between the programme and the improvement of the teachers' practice and their students' learning (Desimone and Pak, 2017; Montecinos, 2003; Timperley et al., 2007). The effective ones consider that teachers experience fulfilling the role of students and teachers so that they can face the difficulties that each of these roles entails. In this sense, it is a process focused on learning (Borko, 2004; Martínez, 2009).

Approaching the line of mathematics education, it highlights a conceptual framework that allows effective TPD programmes for mathematics and science teachers proposed by Desimone and Pak (2017), which considers five essential characteristics: focus on content, active learning, coherence, sustained duration, and collective participation. When studying the TPD process of mathematics teachers in years 2012 to 2017 in Mexico, Bulger-Tamez (2017) concluded that it is possible to integrate the TPD programme designed on a large scale into practice. The author mentions that those programmes integrated into the work context of the mathematics teacher are more effective than those traditional ones that are carried out in places other than the workplace. McCrary (2011) affirms that there is a relationship between the effectiveness of a TPD programme for mathematics teachers and the expectations of the teacher educator about the training itself. Akiba and Liang (2016) show that collaborative teacher-centred TPD activities to learn about mathematics teaching and learning (teaching collaboration and informal communication) seem to be more effective in improving students' performance in mathematics than the learning activities that do not necessarily involve such teacher-centred collaborative opportunities.

Focusing on the Chilean reality, we have that at the university and technical-professional level in general, mathematics teaching is carried out by graduates in mathematics, master's degrees in mathematics, and, in a few cases, PhDs, since they focus their teaching on licentiate degrees, master's degrees or PhDs in mathematics. This type of reality has been driven by accreditation processes of Chilean universities - for almost two decades - that demand the highest standards from professionals in mathematics training in different careers. The classes are traditional lectures, i.e., the contents are exposed, and students are not called to reflect on them, or reflection is not included (Santos & Tirado, 2019). Furthermore, technology is usually not used in teaching to generate knowledge, as can be seen in the in-depth works by Barzel and Biehler (2020) and Thurm and Barzel (2020).

One of the reasons that limit the TPD with technologies is the teachers' previous beliefs. Teachers usually do not see that technology saves time for the development and management of operations that teachers usually promote, a situation addressed by Thurm and Barzel's (2020) project.

A similar work focusing on the development of skills related to modelling processes, the use of representations, and problem-solving at the secondary level is by Barzel and Biehler (2020). Their study gives us six design principles for efficient professional development: competence orientation, participant orientation, stimulating cooperation, case-relatedness, use of various instructional formats, and fostering reflection.

In these studies, technology is fundamental, and its role turns from the typical use to representing someone who develops competence in the use and understanding of mathematical content inserted in modelling processes and problem-solving. These works give an account of how they address the development of teacher competencies of the German Centre for the training of mathematics teachers, which, among its objectives, is to establish new standards and prototypes for the professionalisation of the mathematics teachers.

As Barzel and Biehler (2020) mentioned, one way to address the quality of the TPD process is by identifying the principles that guide them. A principle allows to base the action in a given context; in this case, it allows us to base actions around the TPD of the mathematics teacher. As already mentioned, several studies focus on identifying principles of effective TPD programmes. This work considers the principles of effective TPD programmes for mathematics teachers defined by Ramos-Rodriguez et al. (2022) as detailed below:

Principle 1: *Teaching for learning*. It should focus on improving educational practices in favour of students' mathematics learning.

Principle 2: *Focus on knowledge*. They should focus on the specialised knowledge of the mathematics teacher and a vision of what to teach mathematics for and what is its effective teaching.

Principle 3: *Inquiry – reflection*. It requires constant investigation of the practice, promoting reflection on practice, on what they learned, and how to take it to their classroom.

Principle 4: *External links*. It must provide a balance between mathematics and the curriculum, providing links between other levels of the educational system, schools, universities, and students/teachers, where management leaders offer proactive support for the TPD.

Principle 5: *Time*. It involves the time teachers need to appropriate and change (beliefs, for example).

Principle 6: *Communities of practice*. It should focus on communities of practice through exchanging opinions and collaborative work rather than individual teachers.

Principle 7: *Classroom data collection*. It must have an instance of data collection from the classroom, experiment in the classroom, and action research, for example.

Principle 8: *Expert facilitation*. It considers the participation of experts (if possible, they are one of the teacher educators) who help model effective math teaching, valuing the authority of experience as a source of professional learning.

In the literature related to TPD, they refer partially or directly to the eight principles mentioned in one way or another. Thus, for example, in Barzel and Biehler's (2020) study, we note that the methodology focuses on mathematical content via a personal and shared experience with the participants, which is naturally done in a collaborative inquiry and reflection work. This type of activity, according to the proposed methodology, allows teachers to share previous experiences, which is very important to have an appropriate teaching situation base, but without a doubt, this process is superlatively enhanced with the collaborative action of experts [1] in the area of the didactics of mathematics, who guide the discussion on the different ways of approaching the teaching problem in terms of representation (Duval, 2004; Radford, 2014), mathematical work (Kuzniak & Richard, 2014), cognitive (Arnon et al., 2014; Sierpinska, 2000), the different modelling perspectives (Frejd & Bergsten, 2016), basic ideas of the different theoretical frameworks (or theories), which allow the analysis of successful tasks and the development

[1] In this study, there is a commitment to the participation of experts in different areas, who support the doctorate and master's degree in didactics of the same institution.

of instrumental genesis (Trouche et al., 2020), providing, in the reflective process, a dynamic of work to refine and increase the range of exercises or problems, and the proper use of software that, due to its constant changes, could alter the key didactic element of the original situation, thus producing an erroneous documentation process.

METHODOLOGY

The paradigm of the study is qualitative, where it is necessary to choose a research methodology that allows systematising the creation, implementation, and analysis of TPD programmes, highlighting the importance of showing reality as faithfully as possible, with the complexity that the training practice of math teachers implies (Plomp, 2010) and that allows moving towards principles for effective TPD programmes for university teachers. We chose design-based research or design research (Nieveen, 2010) since it involves a study during practice (the TPD programme) that corresponds to our case. Respecting the actual teaching conditions in the TPD programmes, we suggest a systematisation to facilitate studying the principles of effective TPD training proposals, looking at how theory articulates with practice in training mechanisms; furthermore, without intending to enhance the programme, but rather examine what qualities it contributes to its purpose, in this case, to design a course under the aegis of principles for effective TPD programmes. This study is applied educational design research, since we seek an alternative solution, a possibility of solving an educational problem (on effective TPD programmes for higher education math teachers) in pedagogical practice.

Plomp (2010) describes it as a process that involves three stages:

Preliminary phase: a study from mixed sources (primary and secondary) investigating the needs and analysis of the context, literature review that lead to the establishment of theoretical principles to design an effective TPD programme for mathematics teachers.

Prototype phase (pilot study): intervention prototypes are optimised through a pilot study, which includes design cycles, formative evaluation, and review, each being a micro cycle of research.

Evaluation phase: it concludes when the intervention meets the predetermined specifications (retrospective analysis). In this phase, recommendations will be made to improve the intervention in terms of the principles of effective TPD programmes, followed by the analysis of how those theoretical principles are put into play in the programme to establish coherence and complementarity between the theoretical and empirical proposal. Also, the relationship between the programme and the practice of two teachers and their students' learning is analysed.

Figure 1 illustrates the key elements of the design research phases, highlighting those that materialise for this study (prototype phase).

Figure 1

Methodological scheme of the study.

We must keep in mind that this study is part of a larger project that will give sequence to the other stages of the teaching experiment.

RESULTS

The results are based on the course description for university mathematics teachers in terms of the eight principles that support it.

The training programme aims to "improve university mathematics teachers' education in the acquisition of active teaching-learning methodologies using technologies for the training of engineers".

The estimated time for the course is one semester.

To participate in the training, the teachers had to meet the following conditions:

1. Have a master's degree in mathematics, taught engineering courses for at least seven years, and taught all mathematics courses in engineering, ranging from Fundamentals of Mathematics to Differential Equations.
2. Accept to participate after we informed them and discussed the professional development instance.
3. During the two years of the pandemic, when only virtual remote classes were held, those teachers promoted technological resources with their students and used software and apps in Linear Algebra.
4. They used the linear algebra material that is housed in the Virtual Classroom of the module, to which the teachers and students of the course have access.

Those conditions enabled us to identify a leader we examined and interviewed to verify whether his beliefs about using technologies in teaching were appropriate, according to Thurm and Barzel (2020).

We selected four experts in mathematics (mathematics researchers from Chile, two from inside and two from outside the institution) with two objectives: to analyse the teachers' assessments during a semester of the pandemic and determine the beliefs of those same evaluators, following the methodology of Thurm and Barzel (2020), to identify beliefs about using technology. The assessments were analysed by three experts in mathematics education.

The discussion above allows us to be clear about important elements for the regular development of the proposed professional development course.

On the one hand, the course is designed under a methodology that includes reflection cycles, in particular, the ALaCT cycle (Korthagen & Vasalos, 2009; Korthagen & Verkuyl, 1987). The description of those phases corresponds to:

1. Action: it is the starting point, the action, characterised by being subjected to processes with a high intuitive or unconscious component. The course involves addressing teaching-learning situations of linear algebra in its broad spectrum, which makes one reflect on the relevance of the content in the programme.

2. Review the action: it emphasises the analysis that is carried out on the action developed. One looks back, paying attention to the action that has been carried out to start a slow process of awareness. The situation developed will be analysed, contrasting one's vision with one's colleagues' about the students' difficulties or learnings, sharing experiences.
3. Become aware of essential aspects: to verbally become aware of those specific aspects of their performance that are most likely to be changed for improvement. In the collective reflection, there is the emergence of ideas to improve the situation and/or adjust its objectives according to their students and the future assessment processes of the subject.
4. Create alternative action methods: the teacher consciously investigates and seeks alternatives and creates and plans actions to improve what was analysed before on new forms of action, giving solutions to those aspects that make teaching problematic. The teacher plans a new situation considering the weaknesses that emerge in step 3, tending to connect with the following content.
5. Check a situation: this stage corresponds to the execution of the new methods developed in point 4. The results will be tested and evaluated so that a cycle is completed, starting from practice and returning to practice, having gone through the theoretical bases that support the decisions. From there, a new cycle begins. When posing the following situation, the teacher can execute the situation planned in the previous stage and advance with the contents in a more solid way since they have more information from the reflection cycle.

The reflection cycle considered will allow the development of principles 1, 5, 6, and 7 naturally, since the staging of these cycles aims that teachers focus on student learning. The data collection allows the study of the classes from the videos of classes where the teaching practices are, which involves time in the constitution of a practice unit where teachers are observed and observant in the practice of their peers. In this sense, they have time to improve student learning.

Naturally, teachers start from their knowledge in the discipline and based on practice without reflection. From the reflection cycle that we intend to promote, didactic aspects of the content and proposals for modifying their practices to achieve better learning will emerge, addressing principles 2 and 3. The reflection process will be mediated by the experts, who will participate in the different stages of the ALaCT cycle. The experts create the initial tasks for the transposition based on their experience and what the literature on mathematics education and modelling indicates, thus covering, as already stated, the need for the new mathematics curriculum in engineering, thus achieving principle 4.

The TPD course programme is developed based on problems that teachers raise or that are presented by researchers.

One of the examples selected for the course, which has been prepared to experiment with the participants in the cycle of reflection, can be found in Figure 2, which we analyse below.

Figure 2

Situation to be experienced with the reflection cycle.

¿En qué asignatura es adecuado trabajar situaciones como estas?

Considerar $g : U \to U$ con $U = \{ asen(x) + b\cos(x) / a, b \in \mathbb{R} \}$, y $g(f) = f'$

¿Esta función es biyectiva?

It seems necessary to note that the interesting thing about this problem (Figure 2) is that the content can be differential calculus, linear algebra, or differential equations, which allows us to look in a wide spectrum at the didactic transposition, see the role of linear algebra in the other contents, and how the technology can be used.

It is worth mentioning that teachers can address this situation regardless of the subject they are teaching. Since the TPD focuses on linear algebra topics, it requires that the problem be addressed with those contents and their respective transposition. This is where teachers will reflect on Harel’s (2000) principles of *Concreteness, Necessity, and Generalisation,*[2] which teachers seldom address for not knowing them, as they are not concerned about teaching

[2] Harel (2000), based on Piaget's reflective abstraction, indicates that these principles are crucial in the teaching of linear algebra and that the teacher generally transgresses them in practice.

for learning (principle 3), which allows linking the content with its transposition (Chevallard, 1997).

To address principle 4, *external links*, this TPD course for mathematics teachers who teach engineering courses has considered the recent curricular change in their university engineering careers. In addition to delivering the mathematical content, this situation demands that teachers use linear engineering models in their disciplines by analysing their characteristics, properties, and scope for use in engineering. The change in the mathematics programmes in engineering allowed us to address principle 4, external links. On the other hand, considering TPD courses such as the one we present, they were designed specifically for mathematics teachers in a duration suitable for the teachers involved to appropriate the knowledge and manage to change the presentation of the contents, which must be centred on the students.

The teachers know the discipline, but most do not have explicit training as mathematics teachers. Moreover, we do not see that a professional development policy of this nature is being encouraged. However, the accreditation processes require permanent innovation in teaching, which is why Chilean universities generally offer generic improvement courses, or diploma courses of a few hours of classes, aimed at all teachers from different careers and specialities. This TPD course here is aimed exclusively at mathematics teachers, projected over time considering other topics. Principle 2 will be considered insofar as the mathematical content (that they know) is transversal to the mathematics subjects in the engineering curriculum, such as the concepts embedded in linear algebra, and that this TPD will focus on the development of principle 3, which is related to the transposition of these topics as an object of teaching.

The problems that teachers face (such as the one in Figure 2) will allow them to have a range of linear algebra situations with their respective justifications in the field of mathematics didactics that they must collect as part of the evaluation of the TPD course, thus promoting the documentary genesis (Gueudet & Trouche, 2008), strengthening principles 6 and 7 of an effective TPD, and impacting the use of this material in other subjects, which results in an economical method of professional teacher development, especially in the use of technologies in the classroom to stimulate linear models (Trouche, 2018).

CONCLUSIONS

We have set out to show the design stage of a teaching experiment concerning professional development programmes for higher education mathematics teachers who teach engineering careers, created in the light of principles that guide such developments.

With this study, we could create a training device that visualised and specified the eight principles of effective programmes in its foundations. Each of them allows giving form and substance to the training intended to be delivered.

One of the first elements to consider before starting a teacher training process in a group of academics is knowing the conditions of those attending the course and determining their beliefs and conditions. Although it is true that the participants have their specific beliefs, they have usually been proactive in evolving from class-centred teaching centred, in the traditional sense, to student-centred teaching, relying on the use of technology. Therefore, the TPD provides an opportunity to fine-tune its evaluation systems in the face of students' use of technology, given that it is a learning activity that is currently added to all programmes of mathematics courses in engineering, a result similar to that reported by Quere (2019).

Another vital element for teachers is that the course can provide them with resources that are the basis for their evolution and be developed and assessed through technology.

The use of technology is a relevant element in constructing this TPD programme since it allows putting elements of the teachers' disciplinary knowledge into practice. For example, by using an app such as matrixcalc.org, calculations are simplified, and times are shortened to decompose a matrix, either diagonally or triangularly, where students can very accurately see the role of the eigenvalue (Wawro, Zandieh, & Watson, 2018). Also, the teacher reveal situations that turn out to be modelled by the matrix; situations that appear in updated literature and use technology (Lay et al., 2015; González-Martin et al., 2021)

Based on the consideration of the eight principles of effective programmes, we maintain that the TPD will encourage and make visible to the participants what is understood as a process of documentary genesis, since, from the TPD, they will have a broader range of resources, which they can modify and share with other teachers (Grenier-Boley et al., 2021).

The ALaCT reflection cycle will help the experts to carry out reflection cycles with the teachers. The hard part is having available linear algebra problems brought by them. For this, they have the assessments used for years and must be enhanced with didactic foundations as elements to raise the initial problems. In this sense, teachers will reflect on the contents of the assessments, the questions that they cover, and what dimensions, in relation to students' learning, according to Harel (2000), are addressed in the questions. The creation of new questions or situations to cover the deficiencies evidenced is also contemplated. We must consider that it is a tradition in the university that the assessments are prepared as a team, where everyone participates in formulating the questions. However, in the previous interviews with those selected, we have that their comments revolve around the classic assessments without considering using technology. Their comments are around the mathematical object, concerned mainly with whether the required algorithms have been developed. Generally, an a priori analysis of each problem is not carried out; only the expert response is considered. The selected group is clearly sensitive to the use of technology.

Once the TPD is finished, we expect teachers to develop content and reflect on the assessment analysis. In addition, they must consider the modelling situations, which will be another source of problematics to reflect, given the new curriculum, naturally forming a community of teachers who work and learn collaboratively. These elements put the principles of effective programmes into action.

On the other hand, in this TPD for university teachers, the documentary genesis (Trouche et al., 2020) plays a predominant role, based on the acquisition of resources to implement in their classes, given that university mathematics teachers only have the study programme and the guide text, unlike the resources available to teachers at the several school levels in our country, namely: programme and study plans, teacher's text, student's text, exercise booklet, teaching handbooks, teaching materials to work on each learning objective

In turn, applying mathematics in engineering requires that mathematics teachers interact with teachers of different specialities. However, it seems that they do not usually have instances for it, mainly because of the time they must spend on it. However, the resources provided by this TPD can help solve this situation (Camarena et al., 2013).

Since this is a TPD for higher education, this study helps solve the permanent tension between the service provider unit, the Institute of Mathematics, and its different academic units, recipients of the service, such as the Engineering, Marine Sciences and Geography, Agronomy and Food Sciences, Architecture and Urbanism, Philosophy and Education, and Sciences Colleges. This tension is a situation that is replicated, to a greater or lesser degree, in the different Chilean universities with maximum years of accreditation (seven years) and recognised internationally.

Finally, we cannot fail to mention how important it is to have effective PD principles, as they validate the creation and design of the course in a more coherent and higher quality way, since its impact is required to cause changes in university mathematics teachers (Winslow et al., 2021).

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DECLARATIONS OF CONTRIBUTIONS BY AUTHORS

The three authors conceived the presented idea. PV and JM adapted the methodology to this context. ER developed part of the theory. PV proposed the activities, and the data analysis. All authors participated in the analysis of the results and conclusions.

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