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The Surface Area of Plane Figures in Textbooks throughout Elementary School

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ABSTRACT

Background: The studies emphasise the importance of a kind of teaching that contributes to understanding the concept and the calculus of the area of plane figures, considering the textbooks that are often the main resource teachers use to prepare classes. **Objective:** To investigate how the topic *area of flat figures* is introduced and guided in elementary school textbooks and to verify the potential and limitations of the proposals to mobilise the figural apprehensions and acquire knowledge. **Design:** This is a documentary research with a methodology based on the content analysis proposed by Bardin. **Setting and participants:** Four collections approved by the PNLD in 2019 and 2020 were selected. **Data collection and analysis:** In these collections, the volumes and units that addressed the topic in question were identified. The data analysis, carried out in light of Duval's figural apprehensions and the four stages suggested by Clements and Stephan for constructing the concept of plane figure areas, seeks to identify whether the mathematical skills described by the BNCC can be favoured. **Results:** the proposals of the collections show the importance of using the grid for introducing the concept and calculation of the area of plane figures at the different levels of education; however, few activities mobilise reconfiguration processes. **Conclusions:** It is essential to propose activities that allow students to understand the area equivalence between the modified figures and lead them to understand the rectangular configuration and the formulas for area calculation.

Keywords: Area of plane figures; Textbooks; Figural apprehensions; Knowledge acquisition; BNCC.

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A Área da Superfície de Figuras Planas nos Livros Didáticos ao Longo do Ensino Fundamental

RESUMO

Contexto: estudos enfatizam a importância de um ensino que contribua para a compreensão do conceito e do cálculo de área de figuras planas, levando em consideração os livros didáticos que são, muitas vezes, o principal recurso didático utilizado pelo professor para a elaboração de suas aulas. **Objetivo:** busca-se investigar como é introduzido e orientado o tópico *área de figuras planas* no livro didático, ao longo do Ensino Fundamental, e verificar quais as potencialidades e limitações das propostas para mobilizar as apreensões figurais e obter a aquisição do conhecimento. **Design:** trata-se de uma pesquisa documental baseada na análise de conteúdo proposta por Bardin. **Cenário e participantes:** selecionou-se quatro coleções aprovadas pelo PNLD de 2019 e 2020. **Coleta e análise de dados:** nessas coleções identificou-se os volumes e as unidades que abordavam o tema em questão. A análise dos dados, realizada à luz das apreensões figurais de Duval e das quatro etapas sugeridas por Clements & Stephan para a construção do conceito de áreas de figuras planas, busca identificar se as habilidades matemáticas descritas pela BNCC podem ser favorecidas. **Resultados:** as propostas das coleções evidenciam a importância do uso da malha quadriculada para a introdução do conceito e do cálculo de área de figuras planas nos diferentes níveis de escolaridade, todavia poucas atividades mobilizam processos de reconfiguração. **Conclusões:** é essencial a proposição de atividades que possibilitem ao estudante o entendimento da equivalência de área entre as figuras modificadas e conduzam à compreensão da configuração retangular e das fórmulas de cálculo de área.

Palavras-chave: Área de figuras planas; Livros didáticos; Apreensões figurais; Aquisição do Conhecimento; BNCC.

INTRODUCTION

Teaching the concept and procedures for the calculus of the surface area of flat figures requires special attention and a deeper look at both the initial approach and subsequent discussions since “mathematics teachers, supported by textbooks, introduce the concept of area as a number associated with a surface and quickly move on to the calculation of area through formulas” (Facco, 2003, p. 31) without paying attention to the meaning of the concept of area. According to the National Common Core Curriculum (BNCC), geometry study cannot be restricted to applying numerical formulas for area calculation (Brasil, 2018). It is noteworthy, for example, that the equivalence of areas has been discussed for a long time, since the Mesopotamians and the ancient Greeks, without using formulas (Brasil, 2018, p. 272).

The BNCC also provides that, in the early years of elementary school, students should recognise that measuring is comparing a magnitude and a unit of measurement and that the value obtained is represented by a number (Brasil, 2018). On the other hand, research on this topic has been pointing to the benefit of developing sequences of activities that can help understand the calculation of area and its measurement. Authors such as Facco (2003), Cavanagh (2008) and Clements and Stephan (2004) discuss the use of the checkered grid and the possible reconfigurations of a figure as important tools in teaching the concept and calculating the surface area of plane figures.

In this study, we chose to analyse collections indicated in the 2019 National Textbook Program (PNLD) report, initial years, and in the PNLD of 2020, final years, to verify how each author approaches the concept of area of plane figures during those during elementary school. We considered authors who had textbooks recommended by the PNLD in the years 2016, 2017, 2019, and 2020. We verified that only two of them had works indicated in these four editions of the PNLD, so we chose the sources of those authors, whom we identified as A1 and A2 in our study.

Throughout the early years of elementary school, students are expected to be able to solve problems related to everyday situations that involve calculating the area of triangular and rectangular regions without using formulas. In the final years, they are expected to know how to relate the area as a measure associated with a geometric figure, solve problems that involve the use of more commonly standardised measurement units and determine expressions for the calculus of the areas of quadrilaterals, triangles, and circles (Brasil, 2018).

Thus, we present a study of how the textbooks approach the theme. The central objective of this research is to identify, in four collections of textbooks, the potentialities and limitations of approaching the area of flat figures for figural apprehensions (Duval, 1994) and the construction of knowledge (Clements & Stephan, 2004). To this end, we also rely on the mathematical skills and competencies highlighted by the BNCC (Brasil, 2018).

An analysis of the teaching of areas of plane figures in elementary school textbooks collections indicated in the reports of the PNLD 2016, initial years, and the PNLD 2017, final years, is presented by the first author in his dissertation (Imafuku, 2019). This study was the starting point for the research we report in this article, which aims to answer the questions: How is the area of plane figures introduced and guided in elementary school textbooks? What are the feasibilities and limitations of the proposals to mobilise figural apprehensions and acquire knowledge?

THEORETICAL BASIS

Clements and Stephan (2004) and Duval (1994) were the basis for the development of this analysis. Clements and Stephan (2004) guided our analysis of the construction of the concept of area. They emphasise the importance of developing the teaching of plane figure areas in a way that favours students' understanding, not merely restricted to the use of formulas. For the authors, it is fundamental that students know how to use a checkered mesh to structure a covering of the surface region whose area must be calculated.

This study describes at least five fundamental concepts in learning area calculation: partitioning, iteration, conservation, matrix structuring, and linear measurement. Based on these five concepts, Clements and Stephan (2004) highlight the importance of proposing activities that help students understand the concept of area. As a result, the authors suggest a sequence with four steps, which may contribute to the understanding of the concept of area and its measurement: 1) encourage observation of the entire region of the figure, using an adequate unit of measurement and verifying that the region needs to be all covered without gaps or overlaps; 2) promote the construction and understanding of a matrix structure of a rectangular surface; 3) lead to the perception that the length of the sides of a rectangle is associated with the number of measurement units that are supported on each side and the number of lines and columns in the structuring of the rectangle's coverage; 4) develop the understanding that, when multiplying the two dimensions, the total number of squares that make up the rectangle and, therefore, the area of the rectangle is determined.

We also based our analysis on Duval's theory (1994), pointing out that problems in geometry are original because they mobilise registers of figural representations associated with flat or spatial figures with autonomous interpretations. The author describes these as perceptive, discursive, sequential, or operative apprehensions.

According to Duval (1994), perceptive apprehension is immediate in the exploration of a figure, with an epistemological function of identification of two-dimensional and three-dimensional objects, enabling the identification or recognition of the form, being restricted to the findings.

On the other hand, discourse apprehension has an epistemological function of demonstration, in which the elements of a figure are evidenced through articulating the information contained in the subtitles or the hypotheses regarding the existing mathematical properties.

Sequential apprehension is associated with the act of building a model, as it is mobilised in activities that require construction through instruments, such as a ruler, compass, and software, among others, to reproduce or build geometric figures. When mobilising this apprehension, it is important to respect a specific order, which will depend on the required mathematical properties and the technical restrictions of each instrument (Duval, 1994).

Operative apprehension is centred on the possible modifications that a figure may undergo. Its development does not occur independently from the others, mainly from perceptual apprehension, since both use the same laws and organisational parameters that allow the recognition of a figure. This apprehension can be carried out mentally or physically, with different types of modifications (Duval, 1994): mereological, which expresses the division of a figure into parts for possible reconfigurations; optical, which presents a variation in the size of the figure through enlargement, reduction, or deformation, preserving its shape; and positional, which develops the displacement of the figure in relation to a referential, preserving its shape and size.

In general, the activities proposed in elementary school textbooks are formulated or accompanied by figures, for which reason we believe that observing and constructing the figures and their outline helps students understand the possible solving strategies because, according to Duval (1994), "[...] a figure gives a representation of a geometric situation that is easier to learn than its presentation in a verbal statement"[1] (1994, p.121, our translation). Guided by the theoretical view of Duval (1994), we will seek to identify the different apprehensions that these figures can provide and that the student can mobilise in the context in which they appear.

[1] "[...] une figure donne une représentation d'une situation géométrique plus facile à appréhender que as présentation dans um énoncé verbal" (Duval, 1994, p. 121)

METHODOLOGY

We conducted documentary research (Gil, 2002) as we developed an analytical treatment for selecting data in textbooks that had not yet been researched. For the development of this study, we adapted the content analysis method proposed by Bardin (1977), which evolves in three stages: pre-analysis, material exploration, and treatment of results.

In the pre-analysis, we organised the central ideas and selected the books to be analysed and the content to be studied. We also conducted a survey of studies on teaching the concept and calculation of the surface area of plane figures to establish the foundation that guided the entire analysis.

To verify the transition from teaching surface area of plane figures throughout elementary school, we analysed the collections approved by the PNLD 2016 - 2017 and 2019 - 2020. This choice owes to the fact that the works reviewed in the first period followed the National Curriculum Guidelines for Elementary Education Grades 1-9 (Brasil, 2016). The second review period was guided according to the mathematical skills and abilities described by the BNCC (Brasil, 2018). We reiterate that, at this stage, we checked all textbook authors whose works were reviewed and indicated in the PNLD in 2016, 2017, 2019, and 2020.

Based on this survey, we established as a criterion for selecting collections to compose our analysis those indicated by the PNLD in the years considered. We verified that only two authors had their works indicated in the PNLD of 2016, 2017, 2019, and 2020. For our analysis, we used all the works by these two authors, as shown in Table 1:

Table 1

Textbook collections chosen and reviewed for this article.

Author	Textbook Collection
Author A1	Early Years Textbook Collection – Author 1
Author A1	Final Years Textbook Collection – Author 1
Author A2	Early Years Textbook Collection – Author 2
Author A2	Final Years Textbook Collection – Author 2

In exploring the material, we identified the volumes and units of the selected collections that addressed the content related to the surface area of flat figures. Furthermore, we observed the articulations between topics related to this content all over the volumes of the collections.

When treating the results, we established categories that guided our analysis, as predicted by Bardin (1977). In the sequences proposed by the authors, we observed how the teaching of the area of flat figures is conducted, how the checkered grid is used and the possible reconfigurations of the figures, and how the area calculation is developed through formulas. We also highlight the expectations of mobilisation of possible figural apprehensions provided for constructing knowledge by the activities proposed in the collections.

DATA ANALYSIS AND RESULTS

We carried out our analysis by bringing selected data from the four textbook collections of two authors (author A1 and author A2) indicated by the PNLD of 2019, initial years, and by the PNLD of 2020, final years, to verify how teaching the area of plane figures is proposed along elementary school. In the collections, we verified how the idea of the area is introduced and how the sequences of the proposed activities can contribute to understanding the concept and, consequently, developing area calculation. During the analysis, we highlighted the possibilities of contributions to mobilising figural apprehensions (Duval, 1994) in the approach of activities. We also observed whether the organisation of the proposed activities favours the construction of knowledge according to the stages described by Clements and Stephan (2004), i.e., whether the proposals presented in the collections of the early years lead students to understand the calculus of the area of the rectangular region, how a resumption and deepening of the topics covered in previous years is carried out, and how the development of calculus is proposed through formulas in the collections of the final years.

According to the BNCC, teaching the area of flat figures should start in the 3rd grade of elementary school through activities that address the comparison between areas by covering and, thus, enabling the acquisition of the skill EF03MA21[2] (Brasil, 2018). Based on this guideline, we found that the

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2The alphanumeric code EF03MA21 has the following composition: the first pair of letters indicates the elementary school stage. The last pair of numbers indicates the position of the skill in the sequential numbering of year "21". The first pair of numbers indicates the year the skill refers to – 3rd grade. The second pair of letters indicates that the curricular component is mathematics (MA). The EF03MA21 skill is described in the document as the ability of the student to "Compare, visually or authors of the analysed texts chose to address this teaching as a topic in a chapter only from the 4th-grade volume onwards. However, we observed that they presented the first idea in the volume of the 3rd grade when addressing other subjects to develop the corresponding skill.

In author A2's collection, the 3rd-grade volume addresses the initial aspects of area teaching in the chapter on displacement, location, and symmetry, conducted from the proposal of four activities that precede the study of symmetry to work on the comparison of shapes and filling in the surface of the figures presented in the checkered grid. We found that, in these activities, he informally presents the concept of area in a way that leads to the notion of covering the surface and leading to the perception that figures, with different formats, can be composed of the same amount of squares. We emphasise that the mobilisation of perceptive apprehension is fundamental in this process, with the recognition of the surfaces of the figures and their comparison. We believe that those activities enable an initial work according to the first of the four steps suggested by Clements and Stephan (2004), which describes the importance of proposing activities that encourage students to analyse the entire surface of the figure, which an appropriate unit of measure must completely cover without overlaps or gaps.

To solve the activity in Figure 1, author A2 suggests that "counting the squares can be done one by one or you can think of a 3 by 4 rectangular arrangement in which 4 squares are missing" (Teacher's handbook, author A2, 3rd grade, p.186). However, we found that the rectangular arrangement is explored in the chapter on the multiplication of natural numbers, but it is not linked to the idea of area and seems to us to be suggested early in a more elaborate context in this introductory problem to the concept of area.

For this reason, we believe that the proposal for a resolution by rectangular layout, without providing an exploration of the concept and area calculation, may not favour a real understanding of the meaning of the process carried out. According to Clements and Stephan (2004), when this is not developed according to the sequence of proposed levels, it is common for students to confuse the concepts of area and perimeter and use area formulas without assigning meaning to them.

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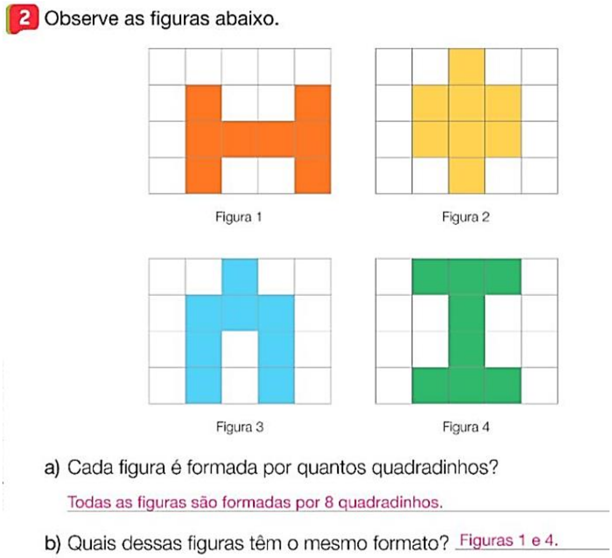
by superimposition, areas of faces of objects, of flat figures or of drawings." (Brasil, 2018, p.289)

Figure 1

Activity involving surface covering. (Author A2, 3rd grade, 2017, p. 186)



When analysing these activities, we understand that A2's intention was to promote the acquisition of the skill suggested for the year and help the understanding of the following content, and not to introduce the concept of area formally. The same treatment was observed in the volume of the third grade of author A1's collection, which proposes, in some units[3], activities that use checkered mesh or Tangram pieces to approach the idea of comparing surfaces, as we can see in Figure 2.

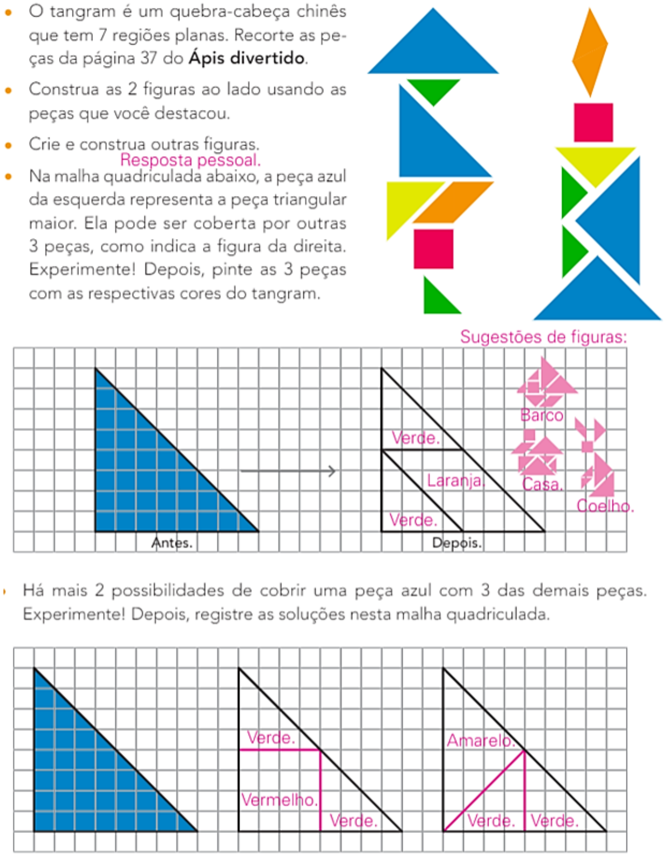
In the 4th and 5th-grade volumes, the authors' proposal for developing the concept and calculating the surface area of plane figures reinforces the idea of comparing surfaces. They present sequences of activities that favour the mobilisation of perceptual apprehension since figural representations are present in most activities, which we understand to be of paramount importance for teaching this content. Sequential apprehension is required in the construction of figures in checkered meshes, which are suggested mainly as a

[3]Such units were named by the author as: Flat regions and contours, multiplication and division with natural numbers, and numbers greater than 1000.

deepening of the topics discussed. However, at this level of education, operative and discursive apprehensions are not significantly emphasised and are only found in some activities.

Figure 2

Activity of comparison with Tangram pieces. (Author A1, 3rd grade, 2020, p.122)

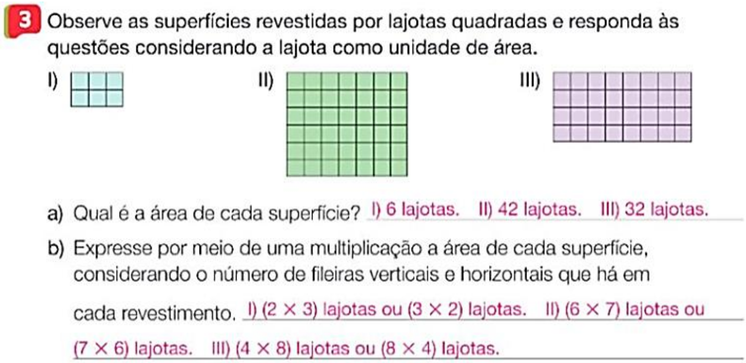


In the 4th-grade volume, the first activities proposed by author A1 use non-standard measurement units to present the standardised units then, always relating them to the surface of a small square. Author A2, on the other hand, chooses to work only with non-standard units. The use of the checkered grid and area calculation by counting squares are often found in the development of the proposed activities, which allows us to identify approximations with the ideas described by Clements and Stephan (2004), who highlight that for students to understand the area of a surface in a two-dimensional context, it is important that they know how to structure it in a checkered mesh.

However, we found that author A2 addresses the calculation of the area measure of rectangular regions by the multiplicative process (length x width) already in the third activity, as we can see in Figure 3.

Figure 3

Activity involving multiplication process. (Author A2, 4th grade, 2017, p.130)


3 Observe as superfícies revestidas por lajotas quadradas e responda às questões considerando a lajota como unidade de área.

I) II) III)

a) Qual é a área de cada superfície? I) 6 lajotas. II) 42 lajotas. III) 32 lajotas.

b) Expresse por meio de uma multiplicação a área de cada superfície, considerando o número de fileiras verticais e horizontais que há em cada revestimento. I) (2 × 3) lajotas ou (3 × 2) lajotas. II) (6 × 7) lajotas ou (7 × 6) lajotas. III) (4 × 8) lajotas ou (8 × 4) lajotas.

We observed that for the resolution of this activity, the use of perceptual apprehension favours the identification of the shape of the figures and the recognition of the horizontal and vertical rows. Discursive apprehension is necessary when describing, in item b), the area calculation through multiplication.

When analysing the sequence proposed by author A2 for the 4th- grade book, we verified that the calculation through the multiplicative process is presented early, since activities that could develop the covering of rectangular regions by the students were proposed only after this approach. We consider, like Clements and Stephan (2004), that activities in which students need to build matrix structures of rectangular regions constitute an essential step in conducting the understanding that the product of the dimensions of a rectangular region corresponds to the measure of its surface. For those authors, this is a fundamental step for the student to understand the area as truly two-dimensional, and they emphasise that it is a process that can take a long time to be apprehended.

Author A1 proposes only resolutions by counting squares. We highlight, supported by Duval (1994), that the figures presented in some activities not only mobilise perceptual but also the operative apprehension, characterised by the reconfigurations of two half units to compose a whole unit (a small square). This joint mobilisation of perceptive and operational apprehensions is common, as both have the same laws and organisational parameters that provide recognition of the figure. (Duval, 1994).

Among the activities, we observed that the construction of figures in the checkered mesh is requested, promoting a sequential apprehension of the figure, which will require an organisation in stages guided by the concepts mobilised in each. Despite the reduced number of activities, we understand that it is possible to work on the second stage[4] described by Clements and Stephan (2004).

It is essential to highlight that both authors present, along the sequences, activities that show figures built in checkered meshes and address the idea of equivalent figures, favouring the development of the skill EF04MA21[5] proposed by BNCC (2018) for the 4th grade of elementary school.

When analysing the 5th-grade volumes, we observe that the use of the checkered grid is again highlighted. Author A1 resumes and reinforces the work with non-standardised units, followed by a proposal involving standardised ones, while author A2 introduces the use of standardised units.

According to A1, the sequence of activities was organised in such a way as to lead students to calculate the area of rectangular regions using the multiplicative process (length x width). This sequence seeks to reinforce the area calculation using standardised units (cm^2 and m^2) and also relate them to the surface of a small square. The study of the calculation of the area of

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[4]Construction and understanding of a matrix structure of a rectangular surface. (Clements & Stephan, 2004)

[5]"(EF04MA21) Measure, compare, and estimate the area of flat figures drawn in a checkered mesh, by counting the squares or squares halves, recognising that two figures with different shapes can have the same area measure." (Brasil, 2018, p.293).

rectangular surfaces begins with the product of its dimensions associated with the count of the squares of the covering. The author also proposes an activity that discusses the calculation of the area of regions determined by right triangles, which is associated with half the measurement of the surface area of a rectangular region intuitively, through the joint mobilisation of perceptual, discursive, and operational apprehensions, in addition to the concept of congruence.

As in the 4th-grade volume, we highlight that some activities proposed for the 5th grade immediately provide a perceptive apprehension, with the recognition of the unit used and the figure represented in the plan, and may demand an operative apprehension. This occurs by carrying out mereological and positional modifications related to the possibilities of joining two halves, leading to understanding the composition of a small square to determine the measurement of the area as proposed in the activity in Figure 4.

Figure 4

Activity involving decomposition and reconfiguration of the unit. (Author 1, 5th grade, 2020, p. 334)

Author A2 proposes a sequence of activities that introduces and enhances the use of standardised units in calculating the area of flat figures, presenting figures contained in square grids. Despite having already approached the calculation of the area of the rectangular region through the product of its dimensions in the 4th-grade book, we verified that, in the 5th-grade book, the author presents only one activity that can lead the student to directly use this idea, enabling the development of the resolutions of the other activities through both counts of squares and the multiplication process.

The skills EF05MA19[6] and EF05MA20[7] indicated by the BNCC (Brasil, 2018) for the 5th grade of elementary school can be developed at different times along the sequences of activities presented by both authors. Proposals that make possible the discussion about the calculation of area and perimeter are found in the 4th and 5th-grade books. When these calculations are not directly addressed in the statement presented, the comparison is suggested in the teacher's support material through sequential apprehension, with the reproduction of figures using the checkered grid as a construction tool.

The collections aimed at the early years of elementary school present a way of teaching the surface area of flat figures that emphasises the importance of using the checkered grid and can provide the mobilisation of figural apprehensions, especially the perceptive and operative ones, in the formation of knowledge, as well as the development of the four stages described by Clements and Stephan (2004). However, we believe that more activities should be proposed for the 5th-grade students to develop the third[8] and the fourth[9] steps and thus enable (or lead to) a more consolidated understanding of the calculation of the area of the rectangular region.

In the final years of elementary school, teaching the concept and calculation of the surface area of plane figures is developed from the 6th grade onwards. To this end, both authors seek to emphasise the calculation of the area of the main plane figures through formulas.

In the two collections analysed, we found that the transition from calculating the area by counting squares to the multiplicative process is formalised at the beginning of the chapter in the 6th-grade volume. This passage is in an introduction that addresses the measurement of the area of a

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[6] "(EF05MA19) Solve and elaborate problems involving measurements of length, area, mass, time, temperature, and capacity, using transformations between the most common units in sociocultural contexts." (Brasil, 2018, p. 297).

[7](EF05MA20) Conclude through investigations that figures of equal perimeters may have different areas and that, also, figures that have the same area may have different perimeters." (Brasil, 2018, p. 297).

[8] Lead to the perception that the length of the dimensions of a rectangle can determine the amount of units in each line and the number of lines in the structuring. (Clements & Stephan, 2004).

[9] Develop the understanding that by multiplying the dimensions of the rectangular region, the number of units that covers the surface is determined. (Clements & Stephan, 2004)

rectangular region with the surface completely squared, in which the number of squares that covers the surface is initially highlighted to lead to the perception that this quantity can also be obtained by the product of the dimensions of the given rectangle (length x width). The development of this process is indicated through a joint mobilisation of the perceptive and discursive apprehensions of the figure, with the identification of the surface and the measured unit, and with the articulation between the existing mathematical properties and the presented figure. We can also observe that this approach contemplates two[10] of the steps described by Clements and Stephan (2004) – 1st and 4th.

Author A1 initially resumes the idea of area, in the volume of the 6th grade, presenting some activities similar to those proposed in the volumes of the initial years. It reinforces, therefore, the importance of using the checkered grid to understand this concept and expanding the study with standardised units of area measurement and their transformations. Author A2, on the other hand, rarely uses the checkered grid for the 6th grade and starts the topic by addressing the meaning of surface and area to, afterwards, deepen the study of the standardised area units, improving the transformations between them.

The transition to area calculation using formulas starts with the introduction of area calculation of the rectangular and square region, followed by the area of the parallelogram and the triangle. However, A2 extends the study by presenting the formulas for the area of the rhombus, the trapezoid, and the circle. We verified that the mobilisation of the operative apprehension is emphasised in the introduction of those formulas and uses mainly the operative apprehension, which allows performing mereological and positional modifications that occur through reconfigurations of the figures that lead to the calculation of the area of a rectangular region, as shown in Figure 5. We also observed the informal use of congruence or preservation of the area when the figures are congruent.

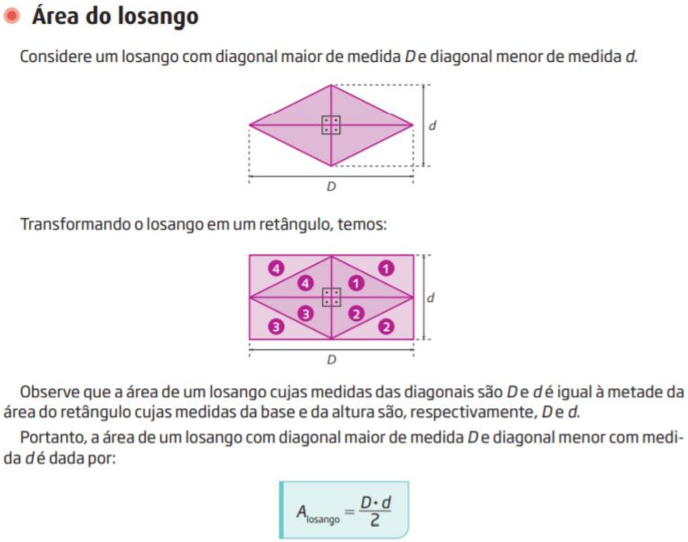
For the development of those introductions, we observed that perceptive and discursive apprehensions are also essential. Through them, some properties inherent to each figure are identified, allowing the conduction of possible modifications. However, the activities proposed throughout the chapter do not favour this practice since, after each formula presented, small sequences of activities are approached, simply aiming at its application, which

[10] First step: the analysis of the entire surface using a suitable unit of measurement that covers the figure without gaps or overlaps. (Clements & Stephan, 2004)

does not allow a study with more complex figures, which must change a little to obtain the measurement of the requested area.

Figure 5

Introduction of the rhombus area through decomposition and recomposition. (Author 2, 6th grade, 2019, p.336)



Área do losango

Considere um losango com diagonal maior de medida *D* e diagonal menor de medida *d*.

Transformando o losango em um retângulo, temos:

Observe que a área de um losango cujas medidas das diagonais são *D* e *d* é igual à metade da área do retângulo cujas medidas da base e da altura são, respectivamente, *D* e *d*.

Portanto, a área de um losango com diagonal maior de medida *D* e diagonal menor com medida *d* é dada por:

$$A_{losango} = \frac{D \cdot d}{2}$$

We understand that the process of reconfiguring a figure is fundamental for understanding the formulas for calculating the area of flat figures. Once the formula for the area of the rectangular region is understood, it is through decompositions and recompositions that obtaining the formulas for the area of the other plane figures is justified.

For this level of education, the BNCC suggests the development of two skills, EF06MA24[11] and EF06MA29[12] (Brasil, 2018). Both authors point out that the EF06MA24 skill is explored, emphasising the resolution of problems involving the calculus of the surface area of flat figures or activities that seek to use everyday situations in their statements. For skill EF06MA29, author A1 proposes an activity that presents a figure contained in a checkered grid and suggests that students conclude "that the area measure of a square region is not proportional to the length measure of the side of the region" (Teacher's handbook, author A1, 6th grade, p. 279). Author A2 indicates, in the teacher's handbook, that he intends to develop this skill when it comes to the length unit, approaching the idea that the perimeter of a square region is proportional to the measurement of the side. However, when analysing the proposed activities, we did not find such a discussion.

We understand that the sequences of activities proposed by the authors for the 6th grade may not give the student a meaningful understanding of why the product of the two dimensions of a rectangular region determines the measure of its surface. As we can see in the books of the initial years, calculus is introduced early and, in the books of the 6th grade, as soon as the formula for the area of the rectangular region is addressed, activities are organised to enhance the exercise of area calculation through the studied formulas. For these reasons, again, we emphasise that more activities could be proposed before this transition to better explore each of the steps suggested by Clements and Stephan (2004) before formalising the process by applying formulas.

In the analysis of the 7th-grade volumes, we verified that the sequences of activities presented by the two authors were developed in a way that would allow the development of the mathematical skills suggested by the BNCC (Brasil, 2018) for the grade. We believe that the purpose of acquiring skill

[11] "(EF06MA24) Solve and prepare problems involving the quantities length, mass, time, temperature, area (triangles and rectangles), capacity, and volume (solids formed by rectangular blocks), without using formulas, inserted, whenever possible, in contexts arising from real situations and/or related to other areas of knowledge." (Brasil, 2018, p.303).

[12] "(EF06MA29) Analyse and describe changes that occur in the perimeter and area of a square when expanding or reducing, equally, the measures of its sides, to understand that the perimeter is proportional to the measure of the side, which does not occur with the area." (Brasil, 2018, p.303).

EF07MA32[13] guided the entire process outlined and the development of the skill EF07MA31[14]. However, we understand that the sequence proposed in the 6th-grade volume already seeks the development of this skill.

Author A1 initially addresses three topics: the approximate area measurement of figures with diverse outlines, the relationship between area and perimeter measurements, and the equivalence of areas. We showed that, to approach the area, figures contained in checkered meshes are used, both in the introductions and in the proposed activities, which is not observed in the last two topics. In them, the author first resumes the explanations of the area formulas of the rectangular, square, and triangular region and limited by a parallelogram, studied in the previous year, and then addresses the discussion on calculating the area measure of plane figures by decomposing them in other simpler figures that make it possible to perform the resolution using the calculations of the areas of the decomposition figures, as shown in Figure 6.

We observe that the possibility of mobilising perceptual, discursive, and operative apprehensions is again present, providing discussions that can lead to the understanding of the equivalence of areas. Although we found a more in-depth approach in relation to the reconfiguration of flat figures, we believe that it would be essential to propose more activities that promote this practice since few of them favour the students' analysis, through which they can develop means to make those changes. This fact may contribute to one of the problems in teaching the area of plane figures described by Clements and Stephan (2004): the difficulty for the student to understand the conservation of the measure of the area when decomposing and recomposing parts of a figure.

For the 7th grade, both authors address the equivalence of areas through activities that reinforce using formulas already studied and the approximate calculation of the area of an irregular region using the checkered grid. This leads us to highlight the importance of this tool for calculating the area throughout the teaching, being essential for understanding the concept, understanding the equivalence between the figures, deducing the formulas of the areas, and deepening at the different levels.

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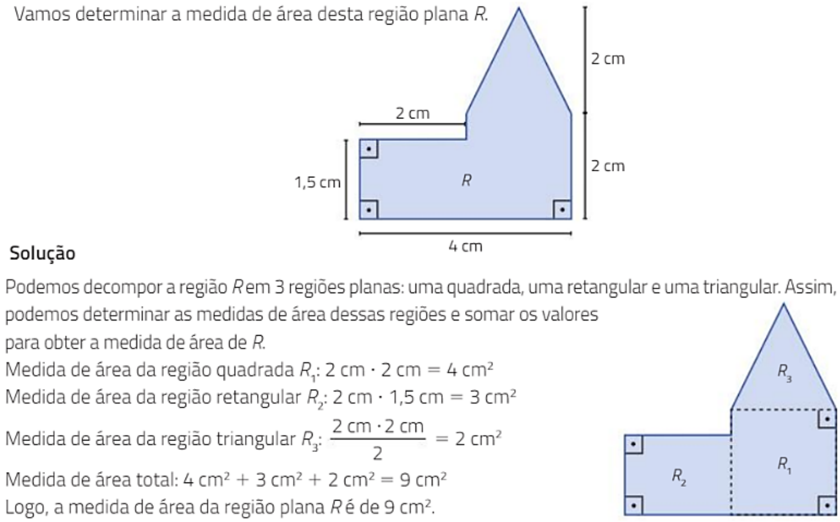
[13] "(EF07MA32) Solve and elaborate problems of calculating area measurement of flat figures that can be decomposed by squares, rectangles and/or triangles, using the equivalence between areas." (Brasil, 2018, p.309).

[14] "(EF07MA31) Establish expressions for calculating the area of triangles and quadrilaterals." (Brasil, 2018, p.309).

Figure 6

Introduction of area calculation through decomposition and composition of polygonal figures. (Author A1, 7th grade, 2019, p.289)

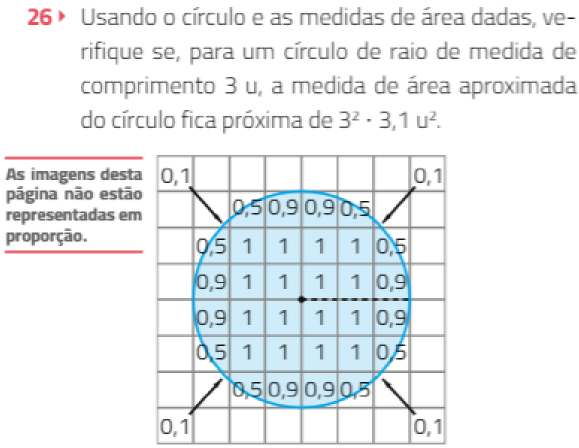


The sequence proposed by the authors for the 8th grade reinforces all the topics already discussed over previous grades, with explanations that resume the composition and application of area formulas for rectangular, square, triangular, and parallelogram-limited regions. Author A1 introduces and formalises the formula for calculating the area of the rhombus, trapezoid, and circular regions only in this volume, while author A2 deals with these formulas since the 6th-grade volume.

We found that A1 seeks to present different resources to justify and guide the understanding of the calculation of the surface area of the circle. First, it presents situations that promote the calculation through approximation, using the checkered grid or restructurings that make it possible to apply the exhaustion method but without specifying the name of the method. In Figure 7, we can observe that to develop the resolution of one of the activities that use the idea of the approximate area, the use of perceptual apprehension is essential.

Figure 7

Circle area by approximation. (Author A1, 8th grade, 2019, p. 175)



We believe that the author starts the explanation through this process to consolidate the discussions presented along the previous volumes and, thus, develop ways to justify the formula. The author also addresses a situation in which the surface of the circle is mereologically modified into congruent circular sectors by mobilising operative apprehension and uses perceptual apprehension to relate the area of the reconfigured region with the area of a parallelogram. Author A2 addresses the formula for the area of the circle only by decomposition, as we can see in Figure 8, as this process had already been presented in the 6th-grade volume.

We note that topics covered in the 8th-grade volume can promote skill acquisition EF08MA19[15], since many of the proposed activities are described through problem situations and allow practicing calculus.

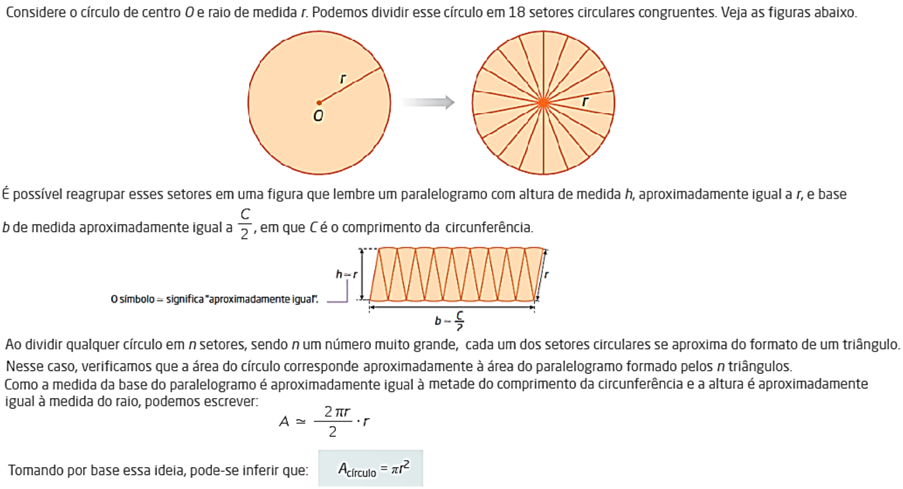
Author A1 ends the 8th-grade volume with a short sequence of activities addressing the flattening of geometric solids to obtain the measurement of the lateral surface area. For the 9th grade, the book presents a

[15] "(EF08MA19) Solve and elaborate problems involving area measurements of geometric figures, using area calculation expressions (quadrilaterals, triangles and circles), in situations such as determining land measurements." (Brasil, 2018, p.315).

discussion of the calculation of the area of polygonal figures contained in the Cartesian plane, in which the perceptive and discursive apprehensions are mobilised together with the identification of the figure and articulation of the present information that can help the development of the resolution, as we can see in Figure 9. Author A2, in the 9th-grade volume, does not propose any topic that directly addresses the teaching of area calculation.

Figure 8

Circle area by decomposition. (Author A2, 8th grade, 2019, p.162)


Considere o círculo de centro *O* e raio de medida *r*. Podemos dividir esse círculo em 18 setores circulares congruentes. Veja as figuras abaixo.

É possível reagrupar esses setores em uma figura que lembre um paralelogramo com altura de medida *h*, aproximadamente igual a *r*, e base *b* de medida aproximadamente igual a $\frac{C}{2}$, em que *C* é o comprimento da circunferência.

Ao dividir qualquer círculo em *n* setores, sendo *n* um número muito grande, cada um dos setores circulares se aproxima do formato de um triângulo. Nesse caso, verificamos que a área do círculo corresponde aproximadamente à área do paralelogramo formado pelos *n* triângulos.
Como a medida da base do paralelogramo é aproximadamente igual à metade do comprimento da circunferência e a altura é aproximadamente igual à medida do raio, podemos escrever:

$$A \simeq \frac{2\pi r}{2} \cdot r$$

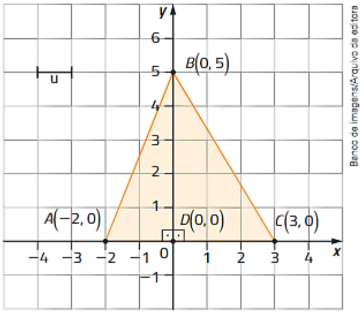
Tomando por base essa ideia, pode-se inferir que: $A_{círculo} = \pi r^2$

We verified that, for teaching the surface area of flat figures, both analysed authors initially soughtto help students understand the concept of area and its measurement through activities that use figures with checkered surfaces or contained in meshes. In this way, they favoured the assimilation of area calculation with the counting of small squares, thus taking the perception of the area calculation of the rectangular region to the multiplicative process that results in the number of squares that make up the surface of the figure. The transition from area calculation by counting squares to using formulas occurs in the volumes of the 6th grade of elementary school. From that moment on, a teaching that reinforces the use of the area formulas of the main plane figures is emphasised. From this perspective, the authors introduce each formula through reconfigurations, decomposing the figures into rectangular or triangular regions to arrive at the formulas and justify them. The checkered grid is resumed at the end of the study of areas to introduce the discussion that can later lead to the general definition of the area through numerical approximations or by exhaustion.

Figure 9

Area measure of a polygonal region whose vertices are points in the Cartesian plane. (Author 1, 9th grade, 2019, p. 242)

Agora vamos calcular a medida de perímetro e a medida de área desta região triangular *ABC* usando a unidade de medida de comprimento u indicada.



O $\triangle ABC$ é retângulo: $[d(A, B)]^2 = 2^2 + 5^2 = 4 + 25 = 29 \Rightarrow d(A, B) = \sqrt{29}$

O $\triangle BDC$ é retângulo: $[d(B, C)]^2 = 3^2 + 5^2 = 9 + 25 = 34 \Rightarrow d(B, C) = \sqrt{34}$

Medida de comprimento da base do $\triangle ABC$: $d(A, C) = 3 - (-2) = 3 + 2 = 5$

Medida de comprimento da altura do $\triangle ABC$: $d(B, D) = 5 - 0 = 5$

Medida de perímetro do $\triangle ABC$: $d(A, C) + d(A, B) + d(D, C) = 5 + \sqrt{29} + \sqrt{34} \simeq 5 + 4{,}9 + 5{,}8 = 15{,}7$

Medida de área do $\triangle ABC$: $\frac{5 \cdot 5}{2} = \frac{25}{2} = 12{,}5$

Portanto, a medida de perímetro do $\triangle ABC$ é de, aproximadamente, 15,7 u, e a medida de área é de 12,5 u^2.

FINAL CONSIDERATIONS

We found that the analysed collections introduce the concept of area of flat figures at the end of the early years of elementary school, with an approach that aims to use the checkered grid to identify the unit of measurement and the calculation by counting squares. The change in approach actually occurs in the volumes of the 6th grade, in which both authors present the transition from this tool to calculating the area through the application of formulas, proposing sequences of activities unrelated to the grid so that there can be an exploration of the use of formulas.

The introduction of each formula is presented through decompositions and compositions in rectangular or triangular regions. However, few activities help the students develop these possible reconfigurations. We return to the checkered grid in the examples of approximate calculation of the area of non-polygonal figures, an initiative to construct the area function from a more general point of view.

The mobilisation of perceptual apprehension is found throughout all activities, as we identified that recognising the shape of the figures and the unit of measurement to be used is immediate in obtaining a solution. The operative apprehension is also found in the volumes of the early years of elementary school, in activities that require mereological and/or positional modifications of the non-standard measurement unit, reconfiguring halves of squares to compose a unit. In the final years of elementary school, this apprehension is present in the deductions of each formula presented, as well as the discursive apprehension, which is also enhanced at this level by articulating the existing mathematical properties with the developed reconfigurations. The sequential apprehension can be more mobilised in the activities proposed in the volumes of the initial years of elementary school, in the reproductions or constructions of figures that use the checkered grid as a construction tool.

Like Duval (2004), we understand that the use of figural representations can favour true learning if each of the four apprehensions is considered in the development of teaching a concept. We believe that for teaching the surface area of flat figures, perceptive, discursive, and sequential apprehensions are fundamental for the concept understanding, as well as the operative apprehension that enables a rich understanding of the area calculation, consolidating the apprehension of the area equivalence of figures with different formats. Therefore, the authors should propose more activities that help students develop these apprehensions.

Throughout our research, given the analysis by Imafuku (2019), which considered collections prior to the BNCC (Brasil, 2018), we found that the ranking of topics related to surface area teaching was reorganised according to the BNCC indications (Brasil, 2018). However, even organised in different volumes, there are no significant changes in the approach because the contents remain practically unchanged. An observed modification is an increased possibility of mobilising sequential apprehension in the new versions for the initial years, as the authors began to propose discussions based on the construction of new figures in the checkered grid. We found that, in some reformulations aimed at the final years of elementary school, they attempted to give greater prominence to reconfiguring by introducing topics that address the calculation of the area of plane figures by decomposing the figures. Such emphasis is proposed since the use of the checkered grid favours the work with important aspects of teaching the area of flat figures throughout all the grades, leading to the understanding of the calculation of the area of the rectangular region, helping the reconfiguration process, building the idea of equivalence and the understanding of area calculation by approximation.

Based on our analysis, we believe that the teaching of the surface area of plane figures proposed in the studied texts for elementary school requires courses that value the understanding of area calculation by counting squares. They must lead the students to perceive the structure used in determining the area through numerical calculation and make it possible, in the transition from the use of the checkered grid to the formulas, to use and become familiar with the reconfiguration processes. With this, the equivalence between the figures is highlighted and preserved and, consequently, an approach that leads students to understand the formulas and their uses is possible.

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AUTHORSHIP CONTRIBUTION STATEMENT

DBSI actively participated in developing the text for the design, analysis, and interpretation of the data, and authors MEELG and AFGS contributed to the development of the text for the design, analysis, and interpretation of the data for the final review.

DATA AVAILABILITY STATEMENT

The data that support the results of this study are available in textbook collections approved by the National Textbook Programme (PNLD). Copies can be accessed at:

https://www.edocente.com.br/pnld/2020,
https://www.edocente.com.br/pnld/2019,
https://pnldf1.moderna.com.br/matematica and
https://pnld.moderna.com.br/fundamental-2/matematica/

REFERENCES

Bardin, L. (1977). *Análise de conteúdo*. (L. A. Reto & A. Pinheiro, Trad.). Edições 70.

Brasil. Ministério da Educação. (2018). *Base Nacional Comum Curricular – BNCC: Educação é a Base.* http://basenacionalcomum.mec.gov.br/images/BNCC_EI_EF_110518_versaofinal_site.pdf

Brasil. Ministério da Educação (2015). *Guia de livros didáticos: PNLD 2016: Alfabetização Matemática e Matemática: Ensino Fundamental anos iniciais.* http://www.fnde.gov.br/component/k2/item/7027-escolha-pnld-2016

Brasil. Ministério da Educação. (2016). *PNLD 2017: Matemática – Ensino Fundamental anos finais.* http://www.fnde.gov.br/component/k2/item/8813-guia-pnld-2017

Brasil. . Ministério da Educação. (2018). *Guia de livros didáticos: PNLD 2019: Ensino Fundamental Anos Iniciais.* https://pnld.nees.ufal.br/pnld_2019/componente-curricular/matematica

Brasil. Ministério da Educação. (2019). *Guia de livros didáticos: PNLD 2020: Ensino Fundamental Anos Finais*. https://pnld.nees.ufal.br/pnld_2020/componente-curricular/pnld2020-matematica

Cavanagh, M. (2008). Area Measurement in Year 7. *Reflections*, *33*(1), (pp. 55-58).

Clements, D. H., & Stephan, M. (2004). Measuremente in pre-K to grade 2 mathematics. In: D. H. Clements, J. Sarama, & A. M. DiBiase (Eds.), *Engaging young children in mathematics.* (pp.299-317). Lawrence Erlbaum.

Duval, R. (1994). Les différents fonctionnements d'une figure dans une démarche geometric. *Repères - IREM*, 17, 121-138.

Facco, S. R. (2003). *Conceito de área: uma proposta de ensino aprendizagem.* (Dissertação Mestrado em Educação Matemática). Pontifícia Universidade Católica de São Paulo, São Paulo, SP, Brasil.

Gil, A. C. (2002). *Como elaborar um projeto de pesquisa (*4a ed.). Atlas.

Imafuku, D. B. S. (2019). *O Ensino de área de figuras planas nos livros didáticos na transição dos anos iniciais para os anos finais do ensino fundamental.* (Dissertação Mestrado em Educação Matemática). Universidade Anhanguera de São Paulo, São Paulo, SP, Brasil.