Teaching Geometry Using an Adapted Narrative: A Case Study with a Visually Impaired Student

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ABSTRACT

Context: Inclusive education for disabled people is an ongoing debate within schools, making it necessary for teachers and institutions to create methodological pathways that ensure an equitable and effective learning process for all pupils, regardless of their limitations. **Objectives:** To investigate the contributions of a tactile adapted version of the historical narrative "Eratosthenes and the Circumference of the Earth" in the formation of geometry concepts in visually impaired students and the challenges associated with the teaching process, this proposal is based on the application of an adapted didactic structure through Eratosthenes' historical narrative, focusing on teaching the similarity of triangles and geometry. The study was based on Vygotsky's cultural-historical theory and Galperin's theory of the gradual formation of mental actions. **Design:** The methods are qualitative in nature via a case-study strategy. **Environment and participants:** The research was conducted with a class of 46 students, of which one is visually impaired, attending the first grade of high school at a public institution in the countryside of Paraná-Brazil. **Data collection and analysis:** Data were collected through activities performed in class and an evaluation questionnaire. Data were analysed through content analysis. **Results:** The results show that the intervention contributed to the assimilation of mathematical concepts by the visually impaired individual and other students, showing changes in the perceptions of key topics within plane geometry, previously demonstrated by the students as partial and non-scientific concepts. **Conclusions:** We can emphasise the need for research and

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studies targeting the construction of proposals and materials that enable an equitable and inclusive teaching process for all students.

Keywords: Mathematics teaching; geometry teaching; historical narrative; visually impaired student; inclusive education.

Narrativa adaptada para o ensino de geometria: um estudo com aluna com deficiência visual

RESUMO

Contexto: O ensino e a inclusão de pessoas com deficiência é uma discussão que é tema presente no contexto escolar e se faz necessário professor e instituição trilhar caminhos metodológicos que visem a garantia de um processo de aprendizagem equitativo e efetivo para todos os alunos, independentemente de suas limitações. **Objetivos:** Com o objetivo investigar as contribuições da adaptação tátil da narrativa histórica "Eratóstenes e a Circunferência da Terra" na formação de conceitos em geometria para alunos com deficiência visual e os desafios no processo de ensino, esta proposta se baseia na aplicação de uma estrutura didática adaptada via narrativa histórica de Eratóstenes, com enfoque no ensino de semelhança de triângulos e geometria. O estudo se fundamentou na Teoria Histórico-cultural de Vigotski, e na teoria de Assimilação por Etapas de Galperin. **Design:** A estrutura metodológica é de vertente qualitativa e como estratégia o estudo de caso. **Ambiente e participantes:** A pesquisa foi realizada em uma turma de primeira série do ensino médio regular, com 46 alunos, de uma instituição pública do interior do Paraná que conta com uma aluna com deficiência visual. **Coleta e análise dos dados:** Os dados foram coletados por meio das atividades realizadas e de questionários de avaliação, sendo analisados por meio da análise de conteúdo. **Resultados:** Os resultados mostram que a intervenção contribuiu na apropriação de conceitos matemáticos por parte da aluna com deficiência visual e demais alunos, com mudanças conceituais em tópicos elementares da geometria plana, antes apresentados pelos alunos em uma estrutura de experiências conceituais incompletas e não científicas. **Conclusões:** A partir do estudo podemos enfatizar a necessidade de pesquisas e estudos que visem a construção de propostas e materiais que viabilizem um processo de ensino equitativo e inclusivo a todos alunos.

Palavras-chave: Ensino de matemática; ensino de geometria; narrativa histórica; aluno com deficiência visual; inclusão escolar.

INTRODUCTION

Education is a process that contributes to the development of an individual and enables the construction of knowledge that provides them with tools for the exercise of citizenship within society. It drifts from common sense

insofar as it impacts systematised knowledge and the appropriation of scientific concepts (Silva, 2000; Saviani, 1999).

In this sense, this article presents a pedagogical proposal that aims to contribute to the teaching and learning process of students with and without visual impairment who share the same physical space. It is important to mention that for effective inclusive education, a transformation in culture, politics, and practices in educational environments (whether formal or informal) is required, aiming to eliminate educational barriers. We argue that the educational system must be strengthened so that everyone who attends it obtains knowledge, which, added to any prior learning, allows for full and effective participation in accessibility and care for all students, especially those who, due to different circumstances, are excluded or belong to marginalised groups (UN, 2016).

In this regard, Viginheski (2013) debates that visually impaired students, despite having access and permanence in regular education, have a compromised knowledge appropriation of scientific concepts due to inaccessible methodological practices. The results showed several conceptual difficulties connected to an education that mainly uses orality as a teaching method (VIGINHESKI et al., 2016).

Mendes (2017) refers to this phenomenon as "exclusionary inclusion", as inclusion policies determine that students in the same school are taught the same content but, at the same time, within a system where forms/norms generate homogenising practices. Nonetheless, the responsibility for homogeneous practices lies beyond teaching because, just like disabled pupils, teachers are excluded for not having the necessary knowledge and tools to meet their students' educational needs.

Based on these reflections, this work results from a pedagogical proposal included in a master's research. The study aimed to investigate the contributions of a tactile adapted version of the historical narrative "Eratóstenes e a Circunferência da Terra" [Eratosthenes and the Circumference of the Earth] in the formation of geometry concepts by visually impaired students and the challenges associated with the teaching process The study was based on the cultural-historical theory which, in the informal knowledge acquired by students, considers traits and cultural experiences of their daily lives fomented by situations, facts, phenomena, and social relations that later, in school, become the foundation for the development of formal and scientific concepts with the aid of the teacher and peers (Vygotsky, 1991).

The cultural-historical development of individuals, consequently, leads to their psychic transformation and the development of higher mental functions (HMFs) as proposed by Vygotsky (1991). How we represent, perceive, explain, and act on the environment develops HMFs, which are constituted from social relationships and contribute to the formation of concepts, just as these concepts also contribute to the development of the HMFs. The concept results from social intelligence, mediated by signs or words and instruments, which contribute to communication, understanding, and problem solving (Vygotsky, 1991).

Núñez and Pacheco (1998) discuss that non-scientific knowledge is characterised by the conscious absence of perception in existing relationships, whereas scientific knowledge is acquired in an intentional, hierarchical, and systematised manner, characterised by mental operations of abstraction and generalisation. For the authors, scientific knowledge develops from a more generalised and abstract structure towards a more specific and concrete one, while informal, non-scientific knowledge takes the opposite path, starting from concrete sensory experiences towards more generalised and abstract ones.

Regarding appropriation of concepts, Vygotsky (1998) highlights the mediating role of signs. Higher mental functions result from the mediation process, and signs are essential to dominate and guide these processes. The mediator sign is the central part of the process, being indispensable to its structure. Thus, learning processes are generated from the relationship between teacher, sign, student, and object.

Galperin's (2009a) theory on the formation by mental stages and concepts details the formation of internal actions through external objects and describes the role of each functional moment of the guidance-activity, execution, and control of the transformations that occur during the process. The theory shows the assimilation of knowledge, such as the transformation from the social stage (external plane) to the individual level (internal plane). External activity is internalised and becomes ideal internal activity, assuming a psychic and independent form. The process of consciousness and external activity are not distinct but forms of a single process (Núñez and Pacheco, 1998).

According to Galperin (2009a), this internalisation process is conceived as a cognitive cycle that cannot be organised linearly; however, it can be separated methodologically for analysis. These stages are: Stage 1- Motivational; Stage 2- Elaboration of the activity guidance base of action (BOA in the Portuguese acronym, which we will call AGB hereafter); Stage 3-

Formation of the action at the material/materialised plane; Stage 4- Formation of the action at the external language plane; Stage 5- Action at the mental plane.

The motivational stage is the ground zero of the learning process. When no motivation is created in an activity, students develop resistance, disinterest, and apathy, added to other stances that compromise the quality of the development of the activity. This stage prepares students for the assimilation of new concepts. The AGB establishes a connection between the subject and the object of the action, performing the mediation between the action and the solution of the problem, which is given through guidelines, such as a teaching plan, and dictates the set of requirements for the action to be concluded. In carrying out an activity, for example, the AGB is essential for the student to build a system of knowledge and organise models of actions to solve the problem (Rezende and Valdes, 2006; Galperin, 2009b).

In the material or materialised stage, the difference between both must be understood. Galperin (2009b; 2009c) considers that the material form is in the object of study itself, and the materialised form uses its substitute, the representational model. At this stage, Sindeaux (2015) emphasises using real objects or simulated representations, in which the learner performs practical actions with the teacher's support but still in a relationship of dependence for their development.

The formation of the action on the external language plane is the moment in which the knowledge assimilated through external factors and verbal or written language is expressed, establishing communicative relationships with teachers and colleagues (Galperin, 2009b).

The transformation from external to internal language constitutes the final stage of the learning process, the mental stage. The external language is an instrument of communication and codification of external agents, which, in this last stage, becomes internal, providing new forms of thinking, already through abstract cognitive actions (Galperin, 2009b).

These stages constitute the learning process, the formation of new knowledge and skills in those who execute it, or the acquisition of new structures to existing habits (Galperin, 2009b). Thus, the learning process that ensures the quality of knowledge is determined by the type of activity used for its assimilation. Hence, it becomes evident how important it is that teachers educate themselves, the was a knowledge can be assimilated in the most satisfactory way, reorienting the student from being passive learners to actively

taking part in the process of construction, assimilation, and internalisation of knowledge.

METHODOLOGICAL PROCESSES

The proposed methodology was developed with 46 high-school first graders, one of whom was visually impaired (blind). The didactic proposal presents a narrative developed and adapted about the historical experiment of the Greek mathematician Eratosthenes, responsible for calculating the circumference of the Earth around 2,100 years ago. The narrative was constructed based on historical elements present in the bibliography, particularly in the work of Lasky (2001). The adapted narrative version included visual, tactile, and Braille writing elements to meet all the students' needs. This is a novel approach, and a patent application was filed with the National Institute of Industrial Property (Instituto Nacional da Propriedade Industrial - INPI).

All students' results were analysed, with an emphasis on the development of the visually impaired student's skills. The study case is considered an applied research strategy justified by "[...] the interest of the researcher in studying a unique, particular situation" (Oliveira, 2008, p. 4).

For data analysis, the five essential elements for qualitative analysis as defined by Ludke and Andre (2013) were followed: i) the data are obtained in the natural environment of the research; ii) the predominant descriptive form allows a more meticulous analysis and an interpretation of the problem studied; iii) attention with the process is greater than with the results; iv) the centre of the investigations is established in the meaning attributed by people to the objects; and v) the inductive process guides the data analysis.

The research was conducted in a public school located in the state of Paraná, Brazil. Forty-six students took part in the research, including a visually impaired student (identified here as J3). The other students were also identified with a letter of the alphabet followed by a number.

Student J3 has been blind since the first months of life due to retinoblastoma, an intraocular malignant tumour that develops in the retina, and derives from a mutation that occurs in the chromosome gene. The disease can be congenital or develop in the first years of life and can affect one or both eyes (Rodrigues, Latorre, & Camargo, 2004). J3 has been enrolled in regular education since the early years of elementary school and, concomitantly,

attends specialised educational services (SES). The student adopts Braille language for written records, uses soroban to perform mathematical calculations and, in some disciplines, uses specific tools for visual impairment, such as DosVox and Mecdaisy.

The research project was submitted and approved by the Ethics and Research Committee of the Federal Technological University of Paraná, process No 3.147.815 - February 14, 2019.

The tools and procedures for data collection were mainly descriptive to ensure a meticulous analysis of the data subsequently. Table 1 below describes the intervention development phases.

Table 1

Activities' structure

Phases	Description of Activities
Phase I	**Initial or diagnostic evaluation:** Identification of specific concepts on topics of plane geometry, such as angles, plane figures, congruent and similar figures, ratio and proportion, and similar triangles that may have been acquired by the students in previous years.
Phase II	**Didactic intervention development:** initial motivation, reading of the narrative, handling of the didactic structure, development of problems and construction of learning proposals for the appropriation of concepts and gaps evidenced in the initial evaluation from elements of the narrative.
Phase III	**Final Evaluation**: Using specific and targeted questions, the final evaluation presented the problems in contextualised situations.

The compilation and interpretation of the data followed the parameters of the content analysis approach. Based on the data collected, two levels of analysis are structured: the first focused explicitly on the general content of the text, with the purpose of filtering, compiling and organising the data,

emphasising facts and concepts related to the students' learning process. The second level, the centre of the analysis, is explicitly composed of text content, being read and interpreted extensively to understand the terms and behaviour of data with structures that dialogue with other categories and the literature, exploring conclusions and concepts related to the analysed content.

The following section reports the results and discussion. Phase I includes the main analysis obtained from the initial assessment of the students. In phase II (development and application of didactic materials), the methods used for the appropriation of concepts and strategies utilised to fill the gaps evidenced in phase I are discussed. Lastly, in phase III (analysis of the results of the final evaluation), data is cross-referenced with the data from the initial evaluation to verify the conceptual contributions of this research.

RESULTS AND DISCUSSION

Phase I - Initial evaluation

The initial evaluation was conducted with twenty-nine students to map consolidated concepts on plane geometry. For the visually impaired student, the assessment was adapted to Braille.

In the initial evaluation, the concepts about angles were presented in a formal and scientific structure in 12% of the students, most likely due to an incomplete consolidation of knowledge during the learning process of the concepts. Further gaps in geometry concepts, such as similarity and congruence, appeared in the student's responses through fragmented definitions and concepts, suggesting an incomplete learning process and that the students have yet to appropriate concepts of plane geometry. For example, student K2 relates angle measurements only to Greek letters. This concept was taught in previous years but was not effectively consolidated by the student.

Vygotsky (1998) states that for the consolidation of concepts, it is necessary a mediation process between the teacher, the student, and the object of knowledge that enables it. During the observations, articulation between the teacher, the students, and the knowledge was limited as was focused on orality, which may impair the learners' consolidation of concepts. Talizina (2009) states that the mechanical reproduction of knowledge may be one of the reasons students do not consolidate a concept, besides difficulties employing these concepts to solve everyday problems.

In the concepts of angles, the students did not present a scientific and conceptual understanding of these terms, rather, they came from spontaneous knowledge acquired in informal experiences, as evidenced in J3's answer:

> *[...] acute angle is a thin angle; hence, it is called acute. I have no idea what an obtuse angle is, and a right angle is an angle that always has a shape that suits it.*

Other answers given by other students are acute angle: something sharp (B3); something that contains a tip (I1); an acute angle has a tip (I3); the one that is similar to the figure (A3).

The student's speech aligns with Castro's (2019) research, which underscores the fragility of spontaneous, informal concepts in relation to the reasoning ability to incorporate them into an operation, gaining incorrect validity and hindering learning levels.

Students were expected to have already appropriated these concepts scientifically, as they are part of the content taught in previous years. Most likely, these concepts were not consolidated due to learning sessions without material or materialised stage and where teaching was limited to orality and demonstration. As stated by Vygotsky (1998), even though a word is more easily captured by memory than a thought, orality does not enable the assimilation of concepts.

The visually impaired student, just like most of her peers, did not express a scientific understanding of the concepts (except and only partially regarding the definition of a triangle), nor even from spontaneous knowledge in a verbalised form. Thus, the absence of concepts is a learning deficiency for the visually impaired student and certainly, the condition is aggravated due to visual limitations and the lack of resources in the learning processes. These characteristics demonstrate how the learning process occurs, and the difficulties teachers face while mediating the appropriation of knowledge for this class.

During the observational period, orality with expository classes was the most used method for teaching this group. The reality of a classroom with almost 50 students with conceptual gaps makes it difficult to provide individualised attention. Moreover, the situation is further aggravated by the need to comply with the curriculum contents, pedagogical diligences, and adapt materials, in addition to the lack of training to meet the processes and mechanisms of inclusion and learning that suit the needs of a visually impaired student.

Phase II: Narrative Reading

The narrative, as defined by Talizina (2009), is an important element in the learning process, characterised by having a motivating and challenging potential. In association with the history of science, in this study, the narrative presents itself as a literary instrument within a contextualised approach (Matthews, 1995; Santos, 2015).

Figure 1 illustrates the students reading the narrative where the elements and plot are presented. In figure 2, J3 manipulates the tactile structure adapted in reliefs and textures to depict the conceptual aspects of the historical experiment of Eratosthenes.

Figure 1

Students reading the narrative.

The tactile structure of the narrative had conceptual elements about angles that were not demonstrated by J3 at the time of reading, despite the attempt to map spontaneous concepts that could have been acquired in previous experiences. However, we must consider that "[...] to operate spontaneous concepts, the child is not aware of them, because its attention is always centred on the object to which the concept refers, never in its own thoughts" (Vygotsky, 1191, p.198).

Figure 2:

Student J3 manipulating tactile structure.

Phase II - Elementary concepts in geometry

For spontaneous knowledge of words to be transformed into scientific knowledge, it is necessary to develop a sequence of functions with attention to memory, abstraction, comparison, and discrimination (Vygotsky, 1998).

Figure 3

Representation method for angles classification used with student J3

Representação ângulo reto.   Representação ângulo agudo.   Representação ângulo obtuso.

Figure 3 illustrates the method used with student J3 for representation and understanding of concepts related to angles, and examples of acute, right, and obtuse angles.

The activities related to the classification of angles were guided through questions and situations exploring angles, without written records, using the base scheme of Figure 3.

> *A 40-degree angle is considered straight or obtuse? And the 100-degree angle, is it is considered straight, sharp, or obtuse? (teacher).*

In the first attempts, the answers given by student J3 were only *"less than 90°'"* or *"greater than 90°"*. However, when using the representation of the angles (Figure 3), the answers included terms such as "*this is acute (J3)", "this is obtuse (J3)".*

The congruent figures, two figures with equal dimensions, were adapted in relief; each side numerically represented in Braille writing (Figure 4).

Figure 4

Braille adaptation of congruent figures

Student J3 recognised conventional geometric figures such as a square, a rectangle, and a triangle but did not recognise the rhombus, the parallelogram, or the trapezoid figures because they were *"not seen in school (J3)"*. Most likely, those have been previously taught without tactile experiences as part of the learning process. Sá, Campos, and Silva (2009) state that activities with visual resources must be adapted through tools that represent the configuration of the desired landscape, facilitating knowledge acquisition.

Phase II: Concepts of reason and proportionality

The concept of reason is defined following Silva (2016). For the author, the ratio between two numbers, *x* and *y*, with *y* ≠0, is a relation of the x/y type. We understand that the ratio is a fraction, or a division, to analyse quantities. On the other hand, the equality of two ratios is known as a proportion (Silva, 2016).

Student J3 recognised that the face of the three-dimensional pieces (Figure 5) forms a triangle *"because it has three sides (J3)"*. Although the student has demonstrated comprehension concerning the shape of the geometric figure, the answer does not include all the elements and concepts that define this polygon. While manipulating the piece and verifying its protrusions, the angles identified in the three-dimensional pieces, J3 could not demonstrate the concepts of angles, vertices, and sides.

Figure 5

Three-dimensional triangular pieces
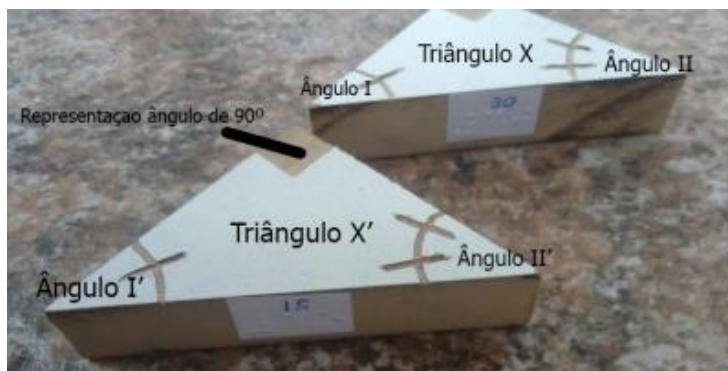
The triangles represented on the faces of the three-dimensional pieces had their units engraved in Braille on the lateral edges of each piece. Each angle was identified as I, II and III on the notched lines located in the formation of each angle. In pairs of triangles, the notches are analogous, demonstrating the congruence of the angles. In the case of faces with right triangles, the representation of ninety degrees (90º) had the engraving in the form of a

quadrilateral at the vertex of the angle formation; this representation enabled tactile recognition of the faces formed by right triangles by the student J3.

In the tactile inferences for the learning of the concept of a ratio by J3, the measurements on each side of the face of a triangle X and their respective measurements of the face of a triangle X' were identified; the distance between angles I and II on the face of triangle X was equal to thirty (30) units of measurement; the distance between angles I' and II' on the face of the triangle X' was equal to fifteen (15) units of measurement.

The concept of ratio was then investigated by the question: *What is the ratio between the measurements expressed on the sides of the faces of the X and X' triangles?* The ratio is represented in the form of a fraction between X and X' given by:

$$\frac{dist\^{a}ncia\ \^{a}ngulo\ I\ e\ II\ do\ tri\^{a}ngulo\ X}{dist\^{a}ncia\ \^{a}ngulo\ I'e\ II'do\ tri\^{a}ngulo\ X\prime} = \frac{30}{15}.$$

Figure 6

Student "J3" identifying measurements between the angles using the triangular faces

When asked about the ratio between triangle X and triangle X',
respectively, J3 was presented with the three-dimensional piece with the face
of the triangle X and then with that of the X', thus being able to verify the
measurements of the corresponding angles and sides (Figure 6).

In the case of the triangular faces from the three-dimensional pieces
shown in Figure 7, in the construction of a non-specific ratio, student J3
presented the following resolution:

> "The distance between the 90° angle and angle I is equal to
> five (5) units of measurement. The distance between the 90°
> angle and the I equals three (3) units of measurement. 5÷3 is a
> ratio (J3).

Figure 7

*Three-dimensional pieces with triangular faces used in the
intervention*

Traditionally, in mathematics, the ratio is represented in the form of a
fraction and not as the algebraic representation of division, although, as
demonstrated by the student, in Braille writing, the signs that represent division
and fraction are the same, while the proportion is the equality between two
ratios, as shown in Figure 8.

Figure 8

Configuration of three-dimensional pieces

J3 wrote down the following when performing the ratio calculation (Figure 8):

> *The distance (triangle X) between the 90° angle and the II angle is nine (9) units of measurement, and the distance between the I and II angle is fifteen (5) units of measurement (J3).*

Regarding triangle X':

> *The distance between angle II and the 90° angle is three (3) units of measurement and between angle II and I is five (5) units of measurement.*

She demonstrated the correct structure of ratios: "*9÷3*" and "*15÷5*", however, she presented difficulty trying to perform the division mentally. For J3 to operate, it was necessary to create contextualised situations by dividing objects.

Gómez and Granell (1983) state that the concept of an algorithm is firstly taught, and then a contextualised problem is used to evaluate knowledge acquisition. However, for student J3, the reverse process was used, as the initial contextualisation of a situation facilitates understanding compared to the initial exposure to the division operation algorithm. Lautert (2005) asserts that the learner, when trying to develop a division, associates these situations with the operative invariants of the division, such as partition division, in which the parts must be distributed equally. In this case, contextualised problems help to understand the operative invariants better.

**Phase II: Concept of triangle similarity**

Due to the visual limitations and the difficulty in understanding a group of information without a hierarchy, a scheme was established to verify the concept of similarity with the student J3:

> 1st: Are the angles of the triangles congruent? That is, if both have equal representation in the angles of type I and I, II and II and finally, III and III. If yes, proceed to the next step;
>
> 2: Assemble and compare the ratios between the sides;
>
> 3: Are the ratios equal? If so, they are similar triangles (researcher)"

The scheme enabled contextualisation and diversification in the activities, which are instruments in the learning process. One of the approaches used to solve exploratory and diversified activities is using concepts appropriated from previous experiences that can be added to new knowledge, as in the problem proposed in Figure 9.

Figure 9

Problem involving variables used to teach triangle similarity.
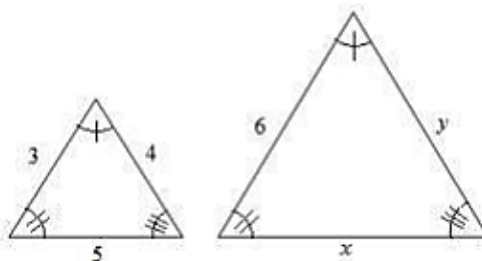
In contextualised situations that do not rely exclusively on calculations but also need interpretation, students with or without disabilities may present difficulties to start solving the problem, especially considering situations where previous appropriated knowledge is needed.

For example, in Figure 9, to start the resolution, there is a need to use the concept of proportions: the product of the means equals the product of the

extremes. Applying this property and taking two ratios, we have: 6÷3= y÷4 where 3 and y, according to the definition, are the means, and 6 and 4 are the extremes.

Assis (2013, pg. 69) evidenced in its research that participants presented several signs to differentiate the denominator from the numerator, such as "part-whole", "whole-part", and "part-part". In "part-whole", the author suggests that "part" represents the numerator while "whole" is the denominator; "part-part" is the "consumed part", signalised as the numerator, whereas the "remaining part" is the denominator; in "whole-part", the numerator is signalled as the "whole", and "part" as the denominator.

Most of the participating students (approx. 90%) chose to solve the problem (Figure 9) using the equivalent fraction method (cross multiplication) instead of the product of proportions (products of means by products of extremes) because they were already familiar with the former as it has been previously used in curricular situations.

In solving this problem (figure 9), the equivalent fraction was not accepted by student J3. It is important to reflect upon this situation since the formation of action in the external language is one of the processes for internalising a concept (Galperin, 2009a); therefore, a constant dialogue between teacher and learner may help identify preferences that meet their needs in an attempt to improve their learning process.

The reason the student cannot accept the concept of equivalent fractions is that it is not possible to create a visual representation of the structure of equal fractions or cross multiplication. This is also justifiable by the characteristics of the Braille language, which is presented linearly and in which the sign for fraction and division has the same writing configuration (Brasil, 2018). The lack of differentiation between division and fraction made it impossible for the student to mentally conceive the structure of a fraction, not being able to identify the numerator and the denominator.

Since there is no specific and accurate form in Braille writing to identify these signs, there are variations in the forms of representing the numerator and the denominator, and specific groups will adopt different signs to build the concept of fractions (Assis, 2013). In the case of student J3, it is possible that during her school life, she has learnt the division sign but without further knowledge of the structure of a fraction, its properties and operations.

It is important to mention that a good part of the students end basic education without mastering notions of fractions, which becomes an even

greater problem when they need to use this knowledge in other content (Pereira, 2009). In the case of students with visual impairments, the situation is worse: there is a greater difficulty in understanding the content due to the absence of vision, and it is through tactile experience that they capture and process the information of the objects (Fernandes and Healy, 2007). Student J3 did not show any concept of fractions. A learning that met her educational needs may not have been offered to her, generating a conceptual gap on the topic.

When faced with evidence of learning deficiency, the teacher must articulate paths that meet the conceptual need. In this case, another way of approaching this property is through the algebraic structure, i.e., using the fundamental definition of the properties of proportions, "product of extremes = product of means". For example, we take as a model the previous ratios from Figure 9, 6÷3= y÷4, now applying the definition "products of the extreme by the means", we have 6.4 = 3.Y.

In the process of mediation through oral language, student J3 registered on the Braille machine,

> [...] first, you build the ratios as a proportion (6÷3= y÷4), [...] now, multiply the two numbers that are close to the equal sign (3.Y), and then numbers on each end (6.4). After this process, 24=3y was obtained, and lastly, now, divide the number that sides with a letter by the one without a letter (J3),

determining the value of the variable *y* using the first-degree equation.

Further difficulties arose from using this second approach in finalising the first-degree equation. As student J3 recorded her progress in Braille and the researcher was not familiar with this form of writing, it was not possible to follow the process used to develop the equation. The student, her peers, and the teacher all had difficulty understanding where in the different steps taken to solve the problem was the error or doubt the student had. This fact highlights the importance of training regular education teachers on specific knowledge, such as knowing Braille writing and how it is registered, in addition to other situations that occur in the school routine, not to meet this need but also other needs.

The transcription of the student Braille registers was made by a specialist teacher who supports the student in the SAS, which, however, was done after the timetabled session. Thus, the demands of both the teacher and the student, which were supposed to be fulfilled at the same time as the other colleagues, were not met until later and, sometimes, not met at all due to the

work dynamics of the regular education teacher and the SAS teacher. This weakened the learning process, culminating in another conceptual gap. Considering this, developing another methodological structure in real time in the classroom is essential.

Given the defining the value of a variable through a first-degree equation (similar to Figure 9), another path was outlined with the student J3 (Figure 10), using the difference in proportions between two triangles, to identify that the two triangles have different measures and one of them contains larger measures. It was possible to build a bridge through arguments that explore the student's tactile experience:
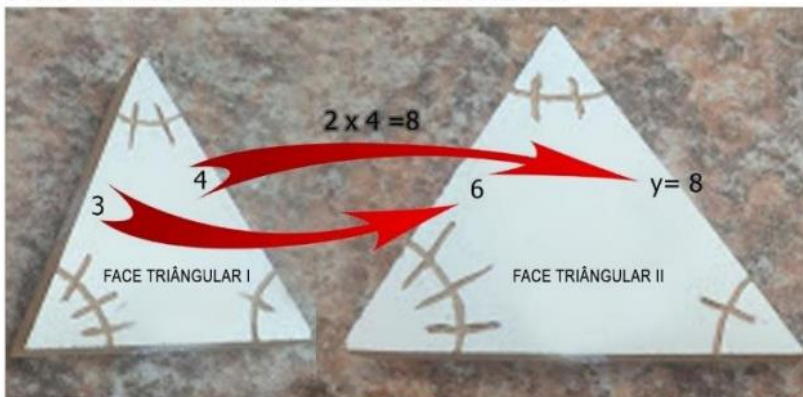
> [...] the triangles have different measures, there was a smaller one that expanded/grew and increased in measure. The proportion is how many times this triangle has grown (teacher dialogues with student J3).

Figure 10

Scheme used with student J3 to demonstrate concepts of reason and proportion.

When stating that the triangle grew or increased "y" times, this "y" represents the value of the proportion or the proportionality ratio. Using as an example the triangular faces of the three-dimensional pieces from Figure 10, the proportionality ratio between the sides is two, since 6÷3= 2.

Thus, the sides of triangular face II expanded twice (2x) the measure of triangular face I, representing for the student the concept of proportion and the proportionality ratio (as shown in figure 10). Therefore, if the distance between the angle I and II of the face I equals four units of measurement (4 u.m.), the corresponding side on the triangular face II (measure y) will have twice the size, 2x4=8, that is, the value of the variable y represents eight units of measurement (8 u.m.). This form of reasoning enabled a higher level of understanding and acceptance by the student "J3", as she had a more concrete approach and a less abstract concept.

The need for a new approach in the teaching process may be common for teachers who work in classrooms which have or do not have disabled students, since visually impaired learners may present different learning conditions from others due to their limitations. Alongside the importance of an accessible curriculum as an educational response to the system, benefiting all students but especially those with disabilities. Besides, these events expose what types of strategies, apart from the usual, are needed to fully enable students, with or without disabilities, to take part in educational opportunities which produce favourable results for their learning.

Phase III - Final Evaluation

In the final evaluation, the proposed exercises were contextualised to some specific situations and were not limited by definitions.

Figure 11 below shows the results of the final evaluation. The percentages of correct answers (fully answered), wrong answers (blank answers and those that did not include any definitions of concepts while completing the exercise) and partial answers (which had errors in calculation or definitions at some point during resolution) are shown.

Figure 11

Final Evaluation results
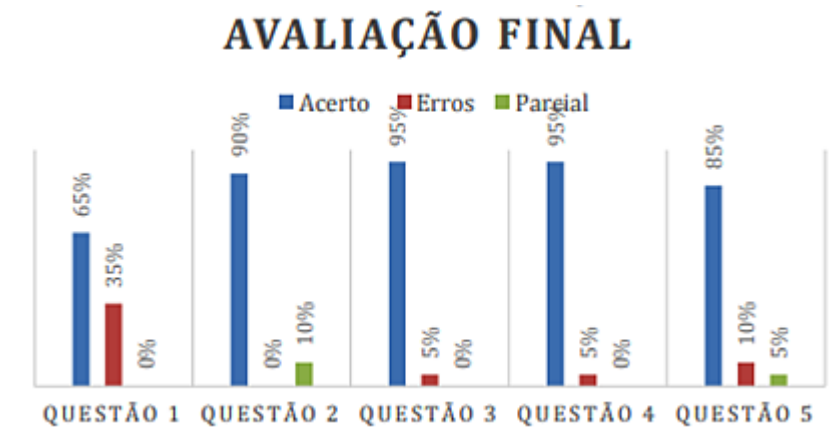
The questions were organised into conceptual categories:

Question 1: Plane geometry figures;

Question 2: Angle classification;

Question 3: Congruent figures and similar figures;

Question 4: Ratio and proportionality;

Question 5: Contextualised problems exploring ratio, proportionality, and similar figures.

The visually impaired student correctly indicated the geometric shapes (question 1) and the classification of angles (question 2) (Figure 12).

Partial errors in the concepts of angle classification; 35% of students mixed the concepts of an acute and obtuse angle. It appears students understood that acute and obtuse angles are types of angles; however (35%) did not consciously employ the concept while solving the problem.

During the verification of learning outcomes for similar figures and congruent figures (question 3), the visually impaired student correctly defined the concept of congruent figures:

> *They are figures that have equal measure outlines; and partially, the concept of similar figures [...] are those that have the same outline but have different measurements (J3).*

Figure 12

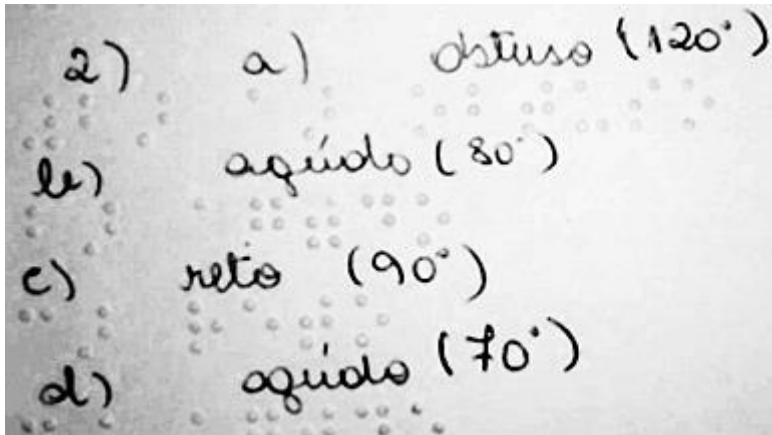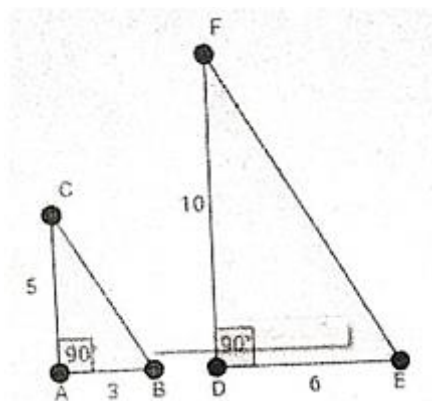*Resolution of question 2 by student J3.*


Figure 13

Question 4

In question 4 (Figure 13), a problem exploring the concept of ratio and proportionality, the visually impaired student gave the following answer: *"it (the triangle) grew twice".*

By saying that the larger triangle *"grew"* twice, J3 refers to the concept of ratio proportion. Thus, it is possible to express that the alternative approach (defining the value of the variables, figure 10) was adequate for the student's needs and contributed to the internalisation of the concept of proportionality in the learning process because the student presented difficulties in solving the fraction, division, and first-degree equations.

In question 5 (Figure 14), student J3 found it difficult to work with diversified concepts (ratio, proportion, angles, multiplication, and division operations) in a single problem.

Figure 14

Question 5: Define the value of the variable x
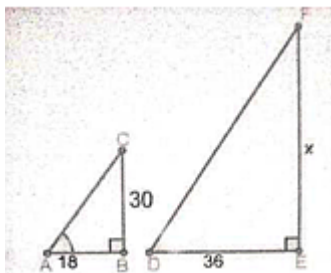
Initially, J3 identifies the larger triangular face noting down it has 36 u.m. From the smaller face, J3 registers as being 18 u.m., which equals the corresponding side of the larger triangular face. Student J3 explains:

> *The larger triangle grew 16 times because I did 36 ÷ 18 (J3). (...) The distance in the larger triangle is X and in the smaller one is 16x30 = I don't know how to do this multiplication (J3).*

First, the student calculates the proportion between the figures when stating that the "larger triangle grew 16 times" and justifies demonstrating the calculation of the ratio *"because I did 36÷ 18 (J3)"*, determining the proportion.

The resolution method presented by J3 is correct but has an error in the division operation to calculate the proportion, as $36 \div 18$ equals 2 and not 16. The difficulties of the student with visual impairment and other students with elementary concepts of mathematics, such as basic operations, are highlighted, especially regarding operations with divisions, concepts that have not been internalised, but that add up and form gaps in the learning process (Lautert, 2005).

To define the value of the variable, J3 then identified the corresponding side of the variable x in the smaller triangle and performed the multiplication represented by *"16 times 30(J3)"*. This multiplication originated from the perception of the student that 16 is the number of times the larger triangle "grew", i.e. the proportion was 16, so the measure *x* in the larger triangle would be equal to 16 times the measure 30 shown in the corresponding side of the smaller triangle. Thus, it is understood that the student's resolution is correct and consistent with the procedures adopted during the intervention.

It was also found that the difficulties in solving question 5, (figure 14), specifically the multiplication and division operations shown as fractions, were also presented by other students. In the case of the visually impaired student, the difficulties were accentuated when performing the mental calculations because she did not bring the soroban to class, which compromised the resolution of the exercises.

CONCLUSIONS

This article discusses the contributions of an intervention proposed for teaching geometry to students with or without visual impairment using an adapted historical narrative. The narrative, built and adapted as a tactile structure, portrays the history of Eratosthenes and the calculation of the circumference of the Earth. The option for an adapted narrative shows the motivational stage, defined by Galperin's (2009b) assimilation theory, as the ground zero and introductory milestone of a learning process. The discussion focused on the articulation between the actions and results presented by the visually impaired student during the intervention and the cultural-historical theory.

Students still presented conceptual errors in the evaluation following the interventions. These errors occur throughout the evaluation process, as students are yet to internalise concepts from the guiding basis of the action. However, it is possible to observe an advance in conceptual knowledge between

the concepts presented in the initial evaluation, scientific concepts not appropriated or partially appropriated in previous learning situations, and concepts presented in a scientific structure in the final evaluation.

It should be noted that the words gained a new meaning and concept, as seen for the definitions of acute, right, obtuse angle, and similar and congruent figures, when compared to the concepts shown in the initial evaluation by students, including student J3, through spontaneous knowledge. In the final evaluation, it is possible to observe new concepts employed in a mathematical and conceptual scientific structure.

This research shows that a concept expressed by a word is an act of generalisation, and that the meaning of words evolves alongside the development of learning and mental operations. A concept can only come under deliberate control and awareness when it becomes part of a system (Vygotsky, 1981). It is the absence of such a system in spontaneous concepts that constitutes the main difference between them and scientific concepts. As the intellect develops, words are replaced by increasingly higher generalisations, a process that leads to the formation of true concepts, which, in turn, presuppose the development of intellectual functions, such as abstractions.

As for the visually impaired student, the need for support and constant training for the teacher who teaches this student is emphatically expressed. Teachers must understand disabilities and be familiar with existing tools and how to use them to meet the paths that enable a quality learning process that overcomes conceptual gaps and challenges imposed not by the disability but by the lack of instruments to manage a meaningful learning process for students.

AUTHORS' CONTRIBUTIONS STATEMENTS

F. and L. conceived the project and developed the experimental design. F. was responsible for data collection. F., L. S., and E. equally contributed to the review of methods. F. L. and S. contributed equally to data analysis. All authors contributed to the draft of the manuscript. E. revised the submission version and made relevant amendments to the Portuguese language.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author, FB, upon reasonable request.

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