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Validation of a Scale to Assess the Reversibility of Thought in Verbal Arithmetic Problems

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SUMMARY

Background: Reversibility is a key concept for the understanding and development of mathematical thinking. There is an agreement regarding problem-solving as a fundamental part of mathematical competence, and some authors regard reversible thinking as a requirement for it. **Objectives**: We want to validate an instrument that assesses the reversibility of thought when solving verbal arithmetic problems (word problems) involving various operations, semantic-mathematical structures and proximity of situational information. **Design**: A qualitative study was carried out from the data obtained by experts, and a quantitative study was carried out to determine the validity and reliability of the instrument. **Setting and Participants**: 318 students from different Spanish schools attending primary education (6 to 12 years) participated. **Data collection and analysis:** Participants performed 180 mathematical tasks distributed over three theoretical scales, two operations, and four semantic configurations. **Results:** To determine the consistency of the data, a reliability analysis was performed globally and on each of the scales, all values being greater than 0.90. Exploratory factor analysis resulted in three factors that explained more than 70%. To analyse the validity of the instrument, confirmatory factor analysis was performed, and its indices showed an adjustment of the models. **Conclusions**: We consider that the designed instrument is sufficiently robust to assess the reversibility of the basic addition and subtraction operations and, in addition, to analyse the discrimination of word problems according to the semantic-mathematical structure and their situational context.

Keywords: Reversibility; Mathematical thinking; Factor analysis; Construct validity; Primary education.

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Validación de una escala para evaluar la reversibilidad del pensamiento en problemas aritmético-verbales

RESUMEN

Contexto: La reversibilidad es un concepto clave para la comprensión y desarrollo del pensamiento matemático. Existe acuerdo en que resolver problemas es una parte fundamental de la competencia matemática, y algunos autores consideran el pensamiento reversible como un requisito exigible para ello. **Objetivos**: Se quiere validar un instrumento que valore la reversibilidad del pensamiento cuando se resuelven problemas aritmético-verbales que involucran diversas operaciones, estructuras semánticas y proximidad de la información situacional. **Diseño**: Se realizó un estudio cualitativo a partir de los datos obtenidos por jueces expertos y un estudio cuantitativo para determinar la validez y fiabilidad del instrumento. **Contexto y Participantes**: Participaron 318 estudiantes de diferentes colegios españoles de toda la etapa de Educación Primaria (6 a 12 años). **Recolección y análisis de los datos:** los participantes realizaron 180 tareas matemáticas distribuidas en tres escalas teóricas, dos operaciones y cuatro configuraciones semánticas. **Resultados:** Para determinar la consistencia de los datos se realizó un análisis de fiabilidad de forma global y en cada una de las escalas, siendo todos los valores superiores a .90. Se realizó un Análisis Factorial Exploratorio que dio como resultado tres factores y que explicaban más del 70%. Para analizar la validez del instrumento, se realizó un Análisis Factorial Confirmatorio cuyos índices mostraron un ajuste de los modelos. **Conclusiones**: Se considera que el instrumento diseñado es suficientemente robusto para valorar la reversibilidad de las operaciones básicas de adición y sustracción y, además, para analizar la discriminación de los problemas aritmético-verbales según la estructura semántica y el contexto situacional de estos.

Palabras clave: reversibilidad; pensamiento matemático; análisis factorial; validez de constructo; educación primaria.

Validação de uma escala para avaliar a reversibilidade do pensamento em problemas aritmético-verbais

RESUMO

Contexto: A reversibilidade é um conceito chave para a compreensão e desenvolvimento do pensamento matemático. Há consenso de que a resolução de problemas é parte fundamental da competência matemática e alguns autores consideram o pensamento reversível como um dos seus requisitos. **Objetivos**: Pretende-se validar um instrumento que avalie a reversibilidade do pensamento na resolução de problemas aritmético-verbais envolvendo diversas operações, estruturas semânticas e proximidade de informações situacionais. **Desenho:** Foi realizado um estudo qualitativo com base nos dados obtidos por especialistas e um estudo quantitativo para determinar a validade e confiabilidade do instrumento. **Ambiente e participantes**: participaram 318 alunos de diferentes escolas espanholas do Ensino Fundamental (6 a 12 anos). **Coleta e análise de dados**: os participantes realizaram 180 tarefas matemáticas distribuídas em três escalas teóricas, duas operações e quatro configurações semânticas. **Resultados**: Para determinar a consistência dos dados, foi realizada uma análise de confiabilidade globalmente e em cada uma das escalas, sendo todos os valores superiores a 0,90. Foi realizada uma Análise Fatorial Exploratória, resultando em três fatores que explicaram mais de 70%. Para analisar a validade do instrumento, foi realizada uma Análise Fatorial Confirmatória, cujos índices mostraram um ajuste dos modelos. **Conclusões**: Considera-se que o instrumento projetado é suficientemente robusto para avaliar a reversibilidade das operações básicas de adição e subtração e, ainda, para analisar a discriminação de problemas aritmético-verbais de acordo com sua estrutura semântica-matemática e contexto situacional.

Palavras-chave: reversibilidade; pensamento matemático; resolução de problemas; análise fatorial; validade do constructo; educação primária.

INTRODUCTION

Reversibility is a mental process that consists of constructing a two-way correlation between an initial condition and a result, and between the result and the initial condition. This concept is a key aspect in the development of mathematical competence. Some authors consider reversible thinking the primary requirement for solving mathematical problems; however, reversibility is often a problem for students (Fitmawati et al., 2019).

Research on mathematical problem solving has been constant for several decades. Authors such as Kilpatrick (1978) or Kulm (1979) studied dependent and independent variables that should be considered in teaching and learning problem solving. They classified the independent variables into subject variables, which describe or measure qualities specific to those who are solving the problem, task variables, which refer to the characteristics of the problems, and situational variables, which specify the peculiarities of the physical, psychological or social environment in which the resolution of the problem occurs. In turn, the task variables can be classified into: structural variables, related to the mathematical structure of the problem; formatting variables, linked to the syntactic structure of the statement; and contextual variables, referring to the situational information offered, its semantic-mathematical structure and the knowledge of the semantic field (Puig & Cerdán, 1988).

One of the most used types of problems in the teaching of mathematics is the word problem (WP) understood as the "verbal description of problematic situations from which one or more questions arise whose answer can be obtained by applying mathematical operations to the numerical data present in the statement" (Verschaffel et al., 2014, p. 641). In this work, we want to validate an instrument that evaluates the reversibility of thought according to the resolution operation, the semantic-mathematical structure of the WP and the proximity of the situational information for the person who solves it. The reasons that lead to the validation of this instrument are the importance of knowledge about the development of the reversibility of thought and the influence that the word-problem task variables may have in said development.

The study of the reversibility of thought begins with Piaget's study of children's cognitive development, which establishes four stages: sensorimotor, preoperational, concrete operational, and formal operational stage. He places the concrete operational stage between the ages of 7 and 11 and suggests that children begin logical operational thinking during this period. At this stage, the principles of conservation and reversibility are developed. From the age of 8, they begin to solve complex problems (Babakr et al., 2019). Piaget (1970) defined mathematical objects as products of coordinated mental actions. In particular, logical-mathematical actions (operations) are characterised by their modularity and reversibility. Modularity enhances mathematical reasoning with the possibility of combining chains of mental actions, for example, students familiar with operations can combine results obtained in one operation with others to obtain new results. Related to these actions, the reversibility of thought was initially broadly defined by Piaget as the structural understanding of operations that allows reversible actions (Norton & Boyce, 2015).

One of the simplest forms of reversibility of thought is to understand how subtraction relates inversely to addition. In fact, researchers in mathematics education have studied reversibility as a critical aspect of students' mathematical reasoning development (Greer, 2011; Hackenberg & Lee, 2015; Simon et al., 2016). By reversing mental actions, such as those involving addition and subtraction operations, students can check the outcome. However, the reversibility of thought can be studied both in the mathematical structures of additive operations and in the WPs that use such mathematical structures. Mathematical skills related to success in problem solving are reversibility and flexibility. Reversibility can be explained based on the mathematical structure, such as the approach, process, and outcome of the operations involved (Rohimah & Prabawanto, 2019).

The WPs are classified according to their mathematical structure in WPs of additive or multiplicative structure. Specifically, additive-structure

WPs are resolved through the formulation and resolution of an addition or subtraction operation. In simple additive problems, depending on the syntactic structure used in their drafting, we can find six open sentences for addition and another six for subtraction, depending on where the unknown term is located (Table 1).

Table 1

Types and configurations of open addition and subtraction sentences (Castro et al., 1995)

Configurations	For addition	For subtraction
Direct	$a + b = ?$	$a - b = ?$
Semidirect 1	$a + ? = c$ $c = a + ?$	$a - ? = c$ $c = a - ?$
Semidirect 2	$? + b = c$ $c = ? + b$	$? - b = c$ $? = a - b$
Indirect	$? = a + b$	$c = ? - b$

Spanish textbooks abound with additive-structure WPs. These problems can be considered authentic discursive entities (Orrantia et al., 2005), which has allowed in-depth studies of their semantic-mathematical structures. Many authors have been interested in this aspect, and different classification schemes have been established (Carpenter & Moser, 1982; Riley & Greeno, 1988; Vergnaud, 1982). One-step WPs are those expressed by simple semantic-mathematical structures and solved with a single arithmetic operation. They describe situations that may refer to the increase or decrease of an amount (Change), the combination of two amounts (Combination) or the comparison of two amounts (Comparison). The Combination and Comparison problems describe static situations, while the Change problems represent actions in dynamic situations. Matching problems combine Change and Comparison characteristics and are less common. Other WPs use compound semantic-mathematical structures, which are combinations of simple structures with a certain number of stages set to reach their solution (Rodríguez et al., 2019).

One-step WPs involve three datasets, one of which is unknown. For Comparison problems, they are indicated as reference, difference, and comparison sets (Stern, 1993). Their equivalents in dynamic situations are the beginning, the change, and the results (Gabler & Ufer, 2021). Depending on the unknown set, the problem responds to different mathematical structures that do not depend on the semantic-mathematical structure of the problem. That is, different forms of problem writing can determine the same mathematical structure. Table 2 shows examples of problems with different semantic-mathematical structure that give rise to the same mathematical structure "4 + 3 = ?".

Table 2

Problems with different semantic-mathematical structures and the same mathematical structure

Semantic-mathematical structure	Problem
Change	Santiago has 4 silkworms and Ana gives him another 3. How many worms does Santiago have now?
Combination	Santiago has 4 silkworms and Ana has 3. How many worms do they have between them?
Comparison	Santiago has 4 silkworms and Ana has 3. How many worms does Ana have?
Equalising	Santiago has 4 silkworms. If he had 3 more worms, he'd have as many as Ana. How many worms does Ana have?

Concerning the semantic-mathematical structure of a one-step WP, Fuson et al. (1988) distinguished between additive and subtractive wording. So, some variations in the wording of a problem can also connect with the reversibility of thought by leading to the same additive or subtractive mathematical structure. Linguistically, relationships in Comparison problems can be expressed by relational terms such as "more" or "larger" (additive wording), or "less" or "smaller" (subtractive wording). For example, "Ana has 3 more silkworms than Santiago" can also be expressed in subtractive words: "Santiago has 3 silkworms less than Ana." Similarly, dynamic word problems can be expressed with action verbs that refer to adding (additive wording, e.g., ‘get’, ‘buy’) or removing an amount (subtractive wording, e.g., ‘give away’, ‘sell’).

The resolution of one-step WPs consists of relating two known amounts by an arithmetic operation to obtain an unknown amount. According to Verschaffel et al. (2000), this can be done genuinely or superficially. A genuine resolution requires understanding both the proposed situation and the semantic-mathematical structure. Instead, the superficial form consists of an ordered translation of the statement into mathematical language regardless of situational information. Only problems that address situational contexts that require a low level of understanding can be solved superficially (Vicente et al., 2018, 2020). The proximity (proximity or remoteness) of situational information to the daily reality of those who try to solve the problem influences the level of abstraction required and, therefore, its understanding (Conejo & Ortega, 2013, Vicente & Manchado, 2017).

Previous studies establish that the complexity of the problems depends as much on the mathematical structure that must be faced as on the proximity of the situational information. For example, Castro et al. (1995) state that mathematical sentences of direct configuration are those that present the least difficulty. Moreover, subtraction sentences usually generate more difficulties than addition sentences, where “? - b = c” is significantly more difficult than the other five. However, there are no differences between the difficulty of semidirect sentences “a + ? = c", "? + b = c” and “a - ? = c". In general, the sentences with the operation on the right side of the equal sign are significantly more difficult than the others. Works such as those of Daroczy et al. (2015) or Stern (1993) show that the WPs with an unknown reference/start set (corresponding to indirect mathematical structures) are more difficult than those with an unknown comparison/result set (corresponding to indirect mathematical structures). On the other hand, Conejo and Ortega (2013) state that the situational information and vocabulary used in their presentation also determine the complexity in the resolution, so that the problems that involve concepts that are distant because they have an unusual use imply greater difficulty than those that use close concepts and concepts known for their habitual use.

Recent research questions various aspects of Piaget’s theory (1970). It shows evidence that children at the concrete operational stage cannot understand the relationship between things that do not exist in the physical world, such as the relationship between numbers; that not all children reach the stage of formal operations or perceive the influence of social contexts on cognitive development (Babakr et al., 2019). The validation of this instrument aims to advance the understanding of the development of reversibility in children 6 to 12 years old.

METHODOLOGY

Population and sample

The population of this work are students of public, private, and concerted primary education schools in Spain. A sample of 318 pupils[1] (150 men and 168 women) from two autonomous communities (Castilla y León and Extremadura), ranging in age from 5 to 12 years, covering the entire stage of primary education (1^{st} to 6^{th} grades of elementary school), was obtained by the availability of educational establishments. All subjects participating in the tests know and practice solving the addition and subtraction algorithms.

Regarding the size of the sample, on the one hand, classic recommendations consider that a sample of between 200 and 300 subjects is sufficient; on the other hand, current recommendations suggest that the size of the sample interacts with other factors such as the commonalities of the items, considering a sample of between 200 and 250 as sufficient (Beavers et al., 2013; Fernando & Anguiano-Carrasco, 2010). Therefore, this sample was considered convenient and sufficient to validate with this sample.

Instrument

The instrument consists of 180 mathematical tasks (Annex 1), grouped into three theoretical scales based on the aspects that can influence the complexity of a verbal arithmetic problem (Adónis, 2006):

- A first scale consisting of 60 mathematical addition and subtraction operations with small numbers, which respond to the various mathematical structures and are called *Structures* (*ST*). Depending

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[1]The students were informed of their participation in the research and gave their acceptance implicitly, by filling in the questionnaire, safeguarding their identity. The authors assume all responsibility and release Acta Scientiae from any action that may arise, including full assistance and possible compensation for any damage that results for any of the participants in the research, in accordance with Resolution 510, of April 7, 2016, of the National Health Council of Brazil.

on the place of the unknown term in the open statement, these mathematical structures were presented in four modalities, according to the configurations explained above: *Direct* (*D*), *Semidirect 1* (*S1*), *Semidirect 2* (*S2*) and *Indirect* (*I*).

- The other two scales are formed by problems in which the proximity of the situational information changes, one with 60 problems where the situational information uses concepts that are considered close to the child because they are frequently used, called *Near Problems* (*NP*) and, the other, 60 problems where concepts are presented that are supposed to be distant from the child because they are of occasional use, called *Distant Problems* (*DP*). The tasks proposed in these scales are related to those proposed in the *ST* scale, both in the configurations (*D, S1, S2,* and *I*) as seen in Table 3 and with the order of appearance in relation to the configuration of the corresponding mathematical structure (Table 4).

Table 3

Examples of tasks in the various questionnaires according to their configuration

Configurations	Structures Questionnaires (ST)	Near Problems Questionnaires (NP)	Distant Problems Questionnaires (DP)
Direct (D)	$4 + 3 = \square$	Santiago has 4 silkworms, and Ana has 3. How many worms do they have between them?	Santiago has 4 breams, and Ana has 3. How many breams do they have between them?
Semidirect 1 (S1)	$4 + \square = 7$	Santiago has 4 silkworms. How many does Ana have, if, between the two of them, they have 7?	Santiago has 4 breams. How many does Ana have, if, between the two of them, they have 7?

Semidirect 2 (S2)	$\square + 3 = 7$	How many silkworms does Santiago have, if Ana has 3 and, between the two, they have 7 worms?	How many breams does Santiago have, if Ana has 3 and, between the two, they have 7 breams?
Indirect (I)	$\square = 4 + 3$	How many silkworms do they have between Santiago and Ana, if Santiago has 4 and Ana has 3?	How many breams do they have between Santiago and Ana, if Santiago has 4 and Ana has 3?

Table 4 shows how *ST*, *NP* and *DP* were related, configurations *D*, *S1*, *S2,* and *I* and the type of operation (addition or subtraction) in the three scales used (expand on the information in Appendix 1).

Table 4

Scheme of tasks of the three scales of the instrument

Operation	Configurations	Scales ST, NP, DP Item Number
Addition	Direct	7, 8, 11, 13, 25, 37, 51, 52
	Semidirect 1	38, 39, 42, 45, 47, 50, 54, 55
	Semidirect 2	6, 9, 23, 28, 30, 35, 40, 41
	Indirect	3, 15, 17, 19, 21, 24, 31, 32
Subtraction	Direct	1, 2, 5, 22, 36, 49, 57
	Semidirect 1	44, 46, 48, 53, 56, 58, 59
	Semidirect 2	4, 12, 18, 20, 33, 34, 43
	Indirect	10, 14, 16, 26, 27, 29, 60

Five experts in mathematical education performed an analysis of the items of each of the scales; for each of them, four characteristics were considered: clarity, appropriateness, relevance and sufficiency (Escobar-Pérez & Cuervo-Martínez, 2008, cited in Marbán & Fernández-Gago, 2021) and they were given a rating of 1 (does not meet the criterion) to 4 (high level of compliance). Initially, there were eight items in each mathematical structure (*D, S1, S2,* and I) of each of the scales (*ST, NP* and *DP*) for each operation (addition and subtraction). After that, experts agreed to eliminate one item from each structure and each scale in the subtraction operation due to duplication of information so that the final instrument was composed of the items listed in Table 4.

Data collection and analysis procedure

All the students who participated in the study performed the tasks proposed in the three theoretical scales. Since the number of tasks was high, we decided to carry out the resolution on three different days, one for each scale, so that the fatigue factor did not distort the results.

The tasks were scored based on the presence or absence of error so that they were assigned number 1 if the answer was correct and number 0 if incorrect. For the totals of the configurations studied (*D*, *S1*, *S2,* and *I*), all the scores of each were added up on each of the scales. *Total Direct Structures* (*TSTD*), *Total Semidirect Structures 1* (*TSTS1*), *Total Semidirect Structures 2* (*TSTS2*), *Total Indirect Structures* (*TSTI*), *Total Direct Near Problems* (*TNPD*), *Total Semidirect Near Problems 1* (*TNPS1*), *Total Semidirect Near Problems 2* (*TNPS2*), *Total Indirect Near Problems* (*TNPI*), *Total Direct Distant Problems* (*TDPD*), *Total Semidirect Distant Problems 1* (*TDPS1*), *Total Semidirect Distant Problems 2* (*TDPS2*) and *Total Indirect Distant Problems* (*TDPI*) were considered.

The IBM SPSS version 26 program was used to analyse the internal consistency of the instrument. Both the reliability of the data and the validation of the construct were studied. To study the reliability of the data, Cronbach's alpha coefficient was calculated for each scale (*ST*, *NP,* and *DP*) with all its items. For construct validation, we calculated the totals of each of the scales in each of the configurations (*TSTD, TSTS1*, *TSTS2*, *TSTI*, *TNPD*, *TNPS1, TNPS2, TNPI*, *TDPD, TDPS1, TDPS2,* and *TDPI*):

- A principal component analysis (PCA) to reduce data and an exploratory factor analysis (EFA) were performed with the IBM SPSS version 26 program. For its application and interpretation, we considered the value of the Kaiser-Meyer-Olkin measure of sampling adequacy (KMO) greater than .70, indicating an adequate interrelation between items and the Barlett's sphericity test, whose significance indicates that the variables can be compared to each other (Pérez & Medrano, 2010).

- A Confirmatory Factor Analysis (CFA) was performed with the IBM SPSS Amos version 26 program. The maximum likelihood adjustment procedure was used, and the CMIN/DF indicators between 2 and 5 were considered; NFI, NNFI/TLI, IFI and CFI > .90 were taken into account; RMSEA < .08; PRATIO, PCFI, PNFI > .70 (Byrne, 2010; Escobedo et al., 2016; Hair et al., 2014; Kline, 2010).

RESULTS

Reliability and analysis of the items

For the reliability analysis, we considered Cronbach's alpha coefficient, which assumes that for values higher than .70, the reliability of the instrument is considered acceptable. The coefficient value for the instrument was .97 for 180 items; and for each of the *ST*, *NP* and *DP* scales, the values were .89, .94, and .96, respectively, so we verified that not only are the scales acceptable but reliable, stable, and with a very high internal consistency, qualifying, therefore, with an excellent level of reliability (Cronbach, 1951).

Principal Component Analysis (PCA) and Exploratory Factor Analysis (EFA)

For the analysis of the variability of the data and possible reduction of their dimensionality, we carried out a PCA with Varimax rotation on each of the scales used in the study (*ST*, *NP* and *DP*).

Table 5

KMO values and Barlett's sphericity test on all three scales

	KMO	Barlett's sphericity test ($\chi 2$)	Sig.
TST	.84	7057.76	.00
TNP	.89	9311.46	.00
TDP	.92	10461.70	.00

The results indicated the possibility of a correct interpretation of the EFA, with KMO values very close to 1 (Table 5). Also, since the Barlett's sphericity test was significant in all the scales, the model can be considered suitable, for it allows comparing all the variables with each other (Pérez & Medrano, 2010).

The EFA was carried out on the totals of each configuration and on each of the scales used to check whether the factors were positioned according to the proposed theoretical model from which we started. This analysis yielded values that indicated the possibility of an interpretation of the factor analysis (KMO = .82; $\chi 2$ = 2585.87; p = .00; Marín-Díaz et al., 2016). The variance explained was 70.77%, and through a Varimax rotation, we obtained three factors that corresponded to the configurations *D*, *S1*, *S2,* and *I*, taking into account the three *ST*, *NP* and *DP* scales used (Table 6).

Table 6

Matrix of factors rotated over totals

	Factors		
	1	**2**	**3**
TNPS2	.91		
TNPS1	.86		
TDPS2	.82		
TDPS1	.76		
TDPI		.88	
TDPD		.88	
TNPI		.67	
TNPD		.61	
TSTS1			.82
TSTD			.74
TSTI			.66
TSTS2			.61

Therefore, the scales and configurations that correspond to each of the rotated factors do not correspond to the theoretical model initially proposed, so we would have an alternative model to the initial theoretical model, leaving the distribution of these new factors as follows:

- To the first factor: *Near Problems* and *Distant Problems* with *Semidirect 1 and Semidirect 2 configurations*, which we will call *Semidirect.*
- To the second factor: *Near Problems* and *Distant Problems* with *Direct* and *Indirect configurations,* a factor that we will call *Direct – Indirect.*
- To the third factor: *Structures* with *Direct, Semidirect 1, Semidirect 2* and *Indirect* configurations, a factor that we will call *Structures*.

Validity of the construct (Confirmatory Factor Analysis)

Since the arrangement of EFA factors did not coincide with what was initially proposed at the theoretical level and, given the configurations of the three factors that resulted from the factor extraction method through the PCA with Varimax rotation (Table 6), the CFAC was performed using structural equation models. This is a robust multivariate technique that combines with multiple regression aspects (Cerón et al., 2020) to analyse the existing relationships between the variables and the two models: the theoretical model initially proposed and the alternative model that the EFA includes (León & Fernández-Díaz, 2019). This checks whether the construct is valid for both models.

CFA, according to the initial theoretical proposal

To check whether the initial theoretical proposal fits correctly, we performed a CFA.

Table 7

Adjustment rates of the initial and adjusted theoretical model

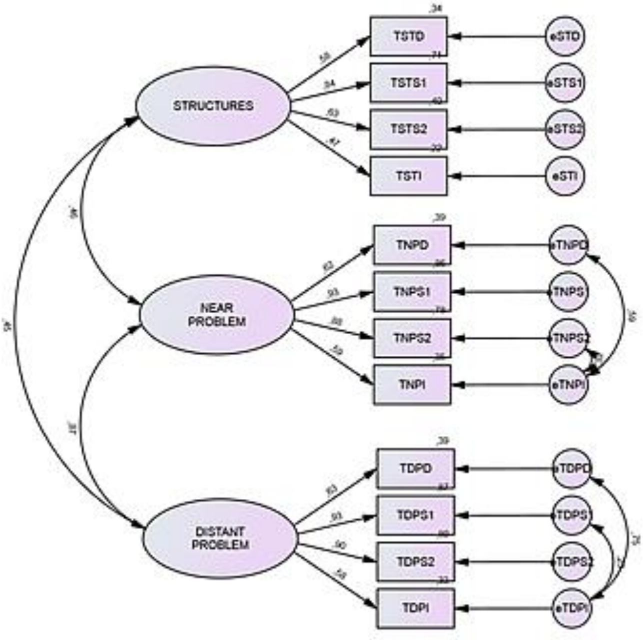
Measure	Recommended Adjustment Level	Initial theoretical model	Adjusted initial theoretical model
Absolute adjustment measures			
Chi-squared	p > .05	p = .00	p = .00
CMIN/DF	2-5	14.10	5.12

RMSEA	<.080	.20	0.11
Incremental adjustment measures			
CFI	>.90	.74	.92
TLI	>.90	.60	.88*
NFI	>.90	.73	.91
IFI	>.90	.74	.93
Parsimony adjustment measures			
PRATIO	>.70	.65	.61*
PCFI	a>value>parsimony	.48	.56
PNFI	a<value>adjustment	.48	.55

This initial model consists of three latent variables (*Structure*, *Near Problems* and *Distant Problems*), 12 observed variables (from *TSTD* to *TDPI*), 12 error terms (from *eSTD* to *eDPI*), 12 factor loads between the factors and the corresponding observed variables, 12 regression loads between the errors and the associated observed variables, and three correlations between the theoretical latent factors.

Figure 1

Adjusted initial theoretical model



The results obtained from the initial theoretical model did not fit correctly (see Table 7), so we proceeded to make the appropriate modifications, obtaining an adjusted initial theoretical model that adds four correlations between the errors to the initial model, according to the modification indices, which causes a reduction in the value of the statistic (Medrano & Muñoz-Navarro, 2017). We observe that the saturations oscillate between .47 and .93 on all three scales (Figure 1).

The absolute adjustment and incremental measures of this adjusted initial theoretical model verify the considerations of the authors cited above, so the model adjusts. Therefore, we can say that this measuring instrument is suitable and quite solid.

CFA, according to the resulting EFA

Since when performing the EFA, the model did not coincide with the initial theoretical model, for it only maintained the configuration in one of its factors of the latent variables *Structure*, being another factor formed by the *Direct* and *Indirect* configurations of the two types of problems, and the third and last factor, formed by the *Semidirect* configuration of both types of problem, we carried out a CFA to check whether the resulting proposal of the EFA adjusted correctly. The initial model, according to the EFA, is made up of three latent variables (*Structure, Direct – Indirect* and *Semidirect*); 12 observed variables (from *TSTD* to *TDPS1*); 12 error terms (from *eSTD* to *eSTI*); 12 factor loadings between the factors and the corresponding observed variables; 12 regression weights between the errors and the associated observed variables and three correlations between the latent factors according to the EFA.

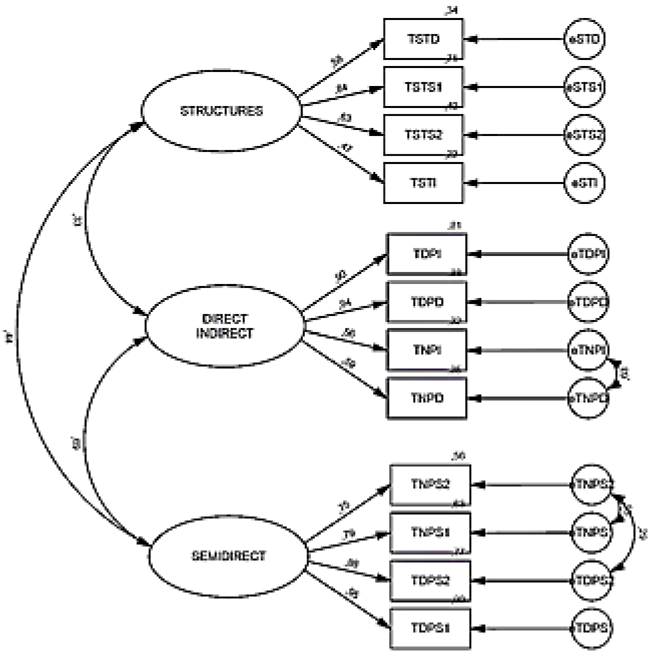
Table 8

Adjustment rates of the model according to initial and adjusted EFA

Measure	Recommended Adjustment Level	Model according to the initial EFA	Model according to the adjusted EFA
Absolute adjustment measures			
Chi squared	p > .05	p = .00	p = .00
CMIN/DF	2-5	10.80	6.07
RMSEA	<.08	.18	.13
Incremental adjustment measures			
CFI	>.90	.81	.91
TLI	>.90	.70	.85*
NFI	>.90	.79	.89*
IFI	>.90	.81	.91
Parsimony adjustment measures			
PRATIO	>.70	.65	.62*
PCFI	a>value>parsimony	.53	.56
PNFI	a<value>adjustment	.52	.55

Figure 2

Model according to the adjusted EFA



Since the initial model, according to EFA, did not fit correctly (see Table 8), we made the appropriate modifications, obtaining a model according to the adjusted EFA, which adds to the initial model, according to EFA, three correlations between the errors, according to the modification indices (Figure 2). We can observe that the saturations oscillate between .47 and .95 on all three scales. According to the adjusted EFA, the adjustment measures are valid in this model, so, again, we can state that the model is suitable.

The fact that both models (adjusted initial theory and adjusted EFA) are valid suggests that perhaps some external factor intervenes in the cognitive functioning of the subjects when facing the tasks. However, this also indicates that, regardless of whether the instrument is used taking into account theoretical factors or taking into account factors according to the EFA, the instrument is sufficiently robust and valid to measure reversibility development when solving one-step WPs and to discriminate between the task variables of the various problems.

DISCUSSION AND CONCLUSIONS

Interest in the variables to be considered in teaching and learning problem-solving is indisputable. It has been shown that there are different factors that can alter the relationship between the subject and a correct resolution of these problems (Kilpatrick, 1978; Kulm, 1979).

The literature review allows us to affirm that a main feature of basic operations is reversibility (Piaget, 1970). There are different configurations for addition and subtraction operations (*Direct, Semidirect 1, Semidirect 2,* and *Inverse*) that are reflected in the WPs at the semantic level (Castro et al., 1995). Moreover, the proximity of the situational information of a verbal arithmetic problem with respect to the daily experiences of people who solve the problem influences the acquisition of a complete mathematical abstraction (Conejo & Ortega, 2013).

Therefore, the validated instrument was elaborated to measure the discrimination with respect to the reversibility of the addition and subtraction algorithms, as well as the WPs based on the semantic-mathematical structure used and the proximity of the situational information.

Each of the scales used (*ST, NP* and *DP*) contained 60 tasks of a dichotomous nature (presence of error – absence of error) that the participants in the experimentation solved. In addition, on the one hand, the items of the *ST* scale contained different open statement configurations of addition or subtraction, *D*, *SD1*, *SD2,* and *I*, depending on where the unknown to be calculated was located in the operation; on the other hand, the items of the *NP* and *DP* scales contained different WPs with information about the student's near (*NP*) and distant (*DP*) situation.

With indices of .90 onwards, the scales showed very high internal consistency with an excellent level of reliability. The KMO values were very close to 1, and Barlett's sphericity was significant, allowing us to perform an EFA that showed the existence of three factors that explain 70.77%, which, although not corresponding with the initial theoretical model, had theoretical logic. The first factor, *Semidirect*, brings together the *Near Problems* and *Distant Problems* with *Semidirect* configurations (*Semidirect 1* and *Semidirect 2*); the second factor, *Direct – Indirect*, brings together the *Near Problems* and

Distant Problems with *Direct* and *Indirect* configurations; and the third factor, *Structure*, brings together the *Structures* with all the *Direct, Semidirect 1, Semidirect 2* and *Indirect* configurations.

To verify the factorial structures, regarding the initial theoretical model and the model according to the adjusted EFA, a CAF was carried out using the maximum likelihood method, since it allows obtaining a category estimator and with high efficiency (Correa & Carmona, 2015), taking into account that the sample size complied with both classic and current recommendations (Anguiano-Carrasco, 2010; Beavers et al., 2013). In both models, both the initial theoretical and the EFA-adjusted models, the incremental adjustment indices exhibited values above .90; likewise, the parsimony adjustment indices showed a correct adjustment; on the other hand, although the absolute adjustment indices did not meet the suitability, they approximated and would be interpretable since, although a fully satisfactory adjustment cannot be ensured, we can affirm that those values would not disrupt the resulting model. The fact that both models adjust suggests the presence of transversal factors extraneous to the instrument itself that can alter the subjects' cognitive functioning in the resolution of the tasks.

Therefore, the results reveal that the designed instrument is sufficiently solid and allows distinguishing the reversibility of the addition and subtraction operations and the WPs according to the semantic-mathematical structure used and the proximity of the situational information to the person who solves it.

The concentration of subjects only in two autonomous communities may be a limitation of this study, and this variable should be considered for the analysis of the results. The number of tasks by scales may seem excessive, but all of them were necessary for us to validate the instrument correctly. We expect that the instrument is used correctly so that, in future research, the errors can be correctly localised, which is a relevant aspect of the correct approach to teacher education through, for example, the request for the creation of mathematical tasks suitable for the development of reversibility.

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AUTHORSHIP CONTRIBUTION STATEMENT

BSB conceived the idea of the research presented and collected the data. BSB and MJC analysed the data. All authors actively participated in the discussion of the results and the formulation of conclusions. They all reviewed and approved the final version of the article.

DATA AVAILABILITY DECLARATION

Data supporting the results of this investigation will be available by the correspondent, MJC, upon reasonable request.

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ANNEX 1

Structures

ST1. $8 - 3 = \square$	ST2. $9 - 2 = \square$	ST3. $\square = 2 + 5$
ST4. $5 - \square = 2$	ST5. $3 - 2 = \square$	ST6. $4 + \square = 7$
ST7. $3 + 2 = \square$	ST8. $4 + 3 = \square$	ST9. $8 + \square = 11$
ST10. $\square = 4 - 3$	ST11. $9 + 2 = \square$	ST12. $8 - \square = 5$
ST13. $8 + 3 = \square$	ST14. $\square = 8 - 3$	ST15. $\square = 4 + 3$
ST16. $\square = 6 - 3$	ST17. $\square = 3 + 2$	ST18. $3 - \square = 1$
ST19. $\square = 6 + 3$	ST20. $7 - \square = 4$	ST21. $\square = 9 + 2$
ST22. $6 - 3 = \square$	ST23. $6 + \square = 9$	ST24. $\square = 8 + 3$
ST25. $6 + 3 = \square$	ST26. $\square = 7 - 3$	ST27. $\square = 3 - 2$
ST28. $9 + \square = 11$	ST29. $\square = 9 - 2$	ST30. $2 + \square = 7$
ST31. $\square = 5 + 3$	ST32. $\square = 7 + 3$	ST33. $6 - \square = 3$
ST34. $9 - \square = 7$	ST35. $7 + \square = 10$	ST36. $7 - 3 = \square$
ST37. $5 + 3 = \square$	ST38. $\square + 3 = 8$	ST39. $\square + 2 = 5$
ST40. $3 + \square = 5$	ST41. $5 + \square = 8$	ST42. $\square + 3 = 9$
ST43. $4 - \square = 1$	ST44. $\square - 3 = 1$	ST45. $\square + 2 = 11$
ST46. $\square - 3 = 5$	ST47. $\square + 3 = 11$	ST48. $\square - 2 = 7$
ST49. $4 - 3 = \square$	ST50. $\square + 3 = 7$	ST51. $2 + 5 = \square$
ST52. $7 + 3 = \square$	ST53. $\square - 2 = 1$	ST54. $\square + 5 = 7$
ST55. $\square + 3 = 10$	ST56. $\square - 3 = 3$	ST57. $5 - 3 = \square$
ST58. $\square - 3 = 4$	ST59. $\square - 3 = 2$	ST60. $\square = 5 - 3$

Near problems

NP1. I had eight candies this morning, but I ate three. How many candies did I have left?

NP2. I had nine marbles, but I lost two. How many marbles do I have left?

NP3. How many kilos of oranges do we have at home, if my mother bought 2 kilos yesterday and 5 today, and nobody ate oranges?

NP4. In my garden this morning there were 5 flowers. How many flowers did the gardener cut this afternoon, if now there are only 2?

NP5. I had 3 dolls, but I lost 2. How many dolls do I have?

NP6. Santiago has 4 silkworms. How many does Ana have, if between the two of them they have 7?

NP7. In one closet there are 3 books and in another there are 2. How many books are in the two closets?

NP8. Santiago has 4 silkworms and Ana has 3. How many animals do they have?

NP9. If in one house 8 people live, how many people live in another house, knowing that between the two together 11 people live?

NP10. How many candies do I have left, if I had 4 and I gave away 3?

NP11. Juan caught 9 fish and Marco 2. How many fish did they fish between the two of them?

NP12. I had eight candies this morning. How many did I eat if I have 5 now?

NP13. 8 people live in one house and 3 live in another. How many people live in the two houses?

NP14. How many candies do I have left, if I had 4 and I ate 3?

NP15. How many silkworms do they have between Santiago and Ana, if Santiago has 4 and Ana has 3?

NP16. How many liters of wine were left in a carafe, if there were 6 liters and they removed 3?

NP17. How many books are in two cabinets, if in one there are 3 books and in the other there are 2?

NP18. If I had 3 dolls, how many did I lose if now I only have 1 doll?

NP19. How many pencils do we have in the classroom, if yesterday we had 6 and today the teacher brought 3 more?

NP20. If Filipa had 7 glasses, how many did she break if she only has 4 now?

NP21. How many fish did Juan and Marco catch together, if Juan caught 9 and Marco 2?

NP22. There were 6 liters of wine in a jug, but we removed 3 liters. How many liters were left in the jug?

NP23. If we had 6 pencils in the classroom, how many pencils did the teacher bring if we now have 9?
NP24. How many people live between the two houses together, if 8 live in one house and 3 in the other?
NP25. In the classroom we had 6 pencils and today the teacher brought 3 more. How many pencils do we have in the classroom now?
NP26. How many glasses does Filipa have, if she had 7 and broke 3?
NP27. How many dolls did I stay with, if I had 3 and lost 2?
NP28. If Juan caught 9 fish, how many did Marco catch if 11 fish were caught between the two?
NP29. How many marbles did I stay with if I had 9 and lost 2?
NP30. My mother bought 2 kilos of oranges yesterday. How many kilos of oranges did she buy today, if we now have 7 kilos?
NP31. How many arrows does an Indian have in total, if he has 5 red and 3 blue arrows?
NP32. How many cows does a cowboy have, if he has 7 black and 3 white cows?
NP33. If there were 6 liters of wine in a carafe, how many liters have we removed from the carafe if now we only have 3 liters?
NP34. If I had 9 marbles, how many did I lose if now I only have 7?
NP35. A cowboy has 7 black cows; how many white cows does he have if in total he has 10 cows?
NP36. Filipa had 7 glasses, broke 3, how many glasses does she have now?
NP37. An Indian has 5 red arrows and 3 blue arrows. How many arrows does he have in total?
NP38. How many red arrows does an Indian have, if he has 3 blue arrows and in total he has 8 arrows?
NP39. How many books are in one cabinet, if there are 2 books in another, and between the two cabinets there are 5 books?
NP40. If there are 3 books in one cabinet, how many books are in another cabinet, knowing that there are 5 books between the two cabinets?
NP41. If an Indian has 5 red arrows, how many blue arrows does he have, if between the two colors he has 8 arrows?
NP42. How many pencils did we have in the classroom, if today the teacher brought 3 pencils and now we have 9?
NP43. If I had 4 candies, how many did I give away if now I only have one?
NP44. How many candies did I have if I gave away 3 and now I only have 1 left?
NP45. How many fish did Juan catch, if Marco caught 2 and between the two they caught 11 fish?

NP46. How many candies did I have, if I ate 3 and now I have 5 candies?
NP47. How many people live in one house, if in another house 3 people live and between the two houses there are 11 people?
NP48. How many marbles did I have if I lost 2 and now I have 7 marbles?
NP49. If I had 4 candies and I gave away 3, how many candies do I have now?
NP50. How many silkworms does Santiago have, if Ana has 3 and between the two they have 7 worms?
NP51. My mother bought 2 kilos of oranges and I bought 5 kilos. How many kilos of orange did we buy between the two of us?
NP52. A cowboy has 7 black and 3 white cows, how many cows does he have in total?
NP53. How many marbles did I have if I lost 2 and now I only have 1 marble?
NP54. How many kilos of oranges did we have if my mother bought 5 kilos today and now we have 7 kilos of oranges?
NP55. How many black cows does a cowboy have if he has 3 white cows and in total he has 10 cows?
NP56. How many liters of wine were in a carafe, if we threw 3 liters away and now there are 3 liters in the carafe?
NP 57. In my mother's garden there were 5 flowers, and the gardener cut 3. How many flowers are there in the garden?
NP58. How many glasses was Filipa carrying, if she broke 3 and now has 4 glasses?
NP59. How many flowers were there in my mother's garden this morning, if the gardener cut 3 flowers and now there are 2 flowers left in the garden?
NP60. How many flowers are left in my mother's garden if there were 5 flowers and the gardener cut 3?

Distant problems

DP1. I had 8 kiwis this morning, but I ate 3. How many kiwis do I have left?
DP2. I had 9 clips, but I lost 2. How many did I stay with?
DP3. How many kilos of oranges do we have at home, if my mother bought 2 kilos yesterday and 5 today, and nobody ate oranges?
DP4. In my garden this morning there were 5 gerberas. How many gerberas did the gardener cut this afternoon, if now there are only 2?
DP5. I had 3 parallelepipeds, but I lost 2. How many did I stay with?
DP6. Santiago has four breams. How many breams does Ana have, if between the two of them they have 7?
DP7. In one cabinet there are 3 trains and in another there are 2. How many trains are in the two cabinets?

DP8. Santiago has 4 breams and Ana has 3. How many breams do they have between them?
DP9. If in one house 8 people live, how many people live in another house, knowing that between the two together 11 people live?
DP10. How many rings did I have left, if I had 4 and I gave away 3?
DP11. Juan caught 9 turbot and Marco 2. How many turbot did they catch between the two of them?
DP12. I had 8 kiwis this morning. How many did I eat if I have 5 now?
DP13. 8 people live in one house and 3 live in another. How many people live in the two houses?
DP14. How many candies do I have left, if I had 8 and I ate 3?
DP15. How many breams do they have between Santiago and Ana, if Santiago has 4 and Ana has 3?
DP16. How many cubic decimeters of wine were left in a carafe, if there were 6 cubic decimeters and they removed 3?
DP17. How many trains are there in two cabinets, if in one there are 3 trains and in the other there are 2?
DP18. If I had 3 parallelepipeds, how many did I lose if I only have 1 now?
DP19. How many slabs do we have in the classroom, if yesterday we had 6 and today the teacher brought 3 more?
DP20. If Filipa was carrying 7 pitchers, how many did she break if she is now carrying only 4?
DP21. How many fish did Juan and Marco catch together, if Juan caught 9 and Marco 2?
DP22. In a jug there were 6 cubic decimeters of wine, but we removed 3. How many cubic decimeters were left in the jug?
DP23. If we had 6 slabs in the classroom, how many slabs did the teacher bring if we now have 9?
DP24. How many people live between the two houses together, if 8 live in one house and 3 in the other?
DP25. In the classroom we had 6 slabs and today the teacher brought 3 more. How many slabs do we have in the classroom now?
DP26. How many pitchers does Filipa have, if she was carrying 7 and broke 3?
DP27. How many parallelepipeds did I stay with, if I had 3 and lost 2?
DP28. If Juan caught 9 turbot, how many did Marco catch, if the two of them caught 11 turbot?
DP29. How many clips did I keep, if I had 9 and lost 2?
DP30. My mother bought 2 kilograms of oranges yesterday. How many kilograms of oranges did she buy today, if we now have 7 kilograms?

DP31. How many feathers does an Indian’s crown have in total, if it has 5 red feathers and 3 blue feathers?
DP32. How many ruminants does a cowboy have, if he has 7 black ruminants and 3 white ones?
DP33. If there were 6 cubic decimeters of wine in a carafe, how many cubic decimeters did we remove if we only have 3 now?
DP34. If I had 9 clips, how many did I lose if now I only have 7?
DP35. If a cowboy has 7 black ruminants, how many white ruminants does he have, if in total he has 10?
DP36. Filipa was carrying 7 pitchers, and she broke 3, how many pitchers did she stay with?
DP37. An Indian has a crown with 5 red feathers and 3 blue ones. How many feathers does he have in total?
DP38. How many red feathers does an Indian’s crown have, if it has 3 blue feathers and in total it has 8?
DP39. How many trains are there in one cabinet, if in another there are 2 and in the two cabinets there are 5 trains?
DP40. If there are 3 trains in one cabinet, how many trains are there in the other cabinet, knowing that there are 5 trains in both?
DP41. If an Indian’s crown has 5 red feathers, how many blue feathers are there if it has a total of 8 feathers?
DP42. How many slabs did we have in the classroom, if today the teacher brought 3 and now we have 9 slabs?
DP43. If I had 4 rings, how many did I give away if I only have 1 now?
DP44. How many rings did I have, if I gave away 3 and now I only have 1?
DP45. How many turbot did Juan catch, if Marco caught 2 and the two caught 11 turbot?
DP46. How many kiwis did I have, if I ate 3 and I have 5 left?
DP47. How many people live in one house, if in another live 3 and in the two together 11 live?
DP48. How many clips did I have, if I lost 2 and I have 7 left?
DP49. If I had 4 rings and I gave away 3, how many rings did I stay with?
DP50. How many breams does Santiago have, if Ana has 3 and between the two they have 7 breams?
DP51. My mother bought 2 kilograms of oranges yesterday and bought 5 kilograms today. How many kilograms of oranges did she buy in the two days?
DP52. A cowboy has 7 black and 3 white ruminants, how many ruminants does he have in total?
DP53. How many parallelepipeds did I have, if I lost 2 and now I only have 1?

DP54. How many kilograms of oranges did my mother buy yesterday, if she bought 5 today and now we have 7 kilograms?

DP55. How many black ruminants does a cowboy have, if he has 3 white and in total he has 10 ruminants?

DP56. How many cubic decimeters of wine were there in a carafe, if we removed 3 cubic decimeters and now we only have 3?

DP57. In my garden this morning there were 5 gerberas, but in the afternoon the gardener cut 3. How many gerberas were left in my garden?

DP58. How many pitchers was Filipa carrying, if she broke 3 and now she has only 4 pitchers?

DP59. How many gerberas did the gardener cut this morning in my garden, if this afternoon he cut 3 and now there are only 2 gerberas left?

DP60. How many gerberas are left in the garden if this morning there were 5 gerberas and this afternoon the gardener cut 3?