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# TPACK Model: Teachers' Perceptions of Their Technological Competence When Conducting an Experimental Virtual Lesson in the Context of Covid-19

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## ABSTRACT

**Background**: One of the current concerns in the face of the changes and challenges imposed by the COVID-19 pandemic is the quality of the education received in non-traditional environments such as virtual or hybrid teaching. Elements associated with this problem include the knowledge and skills that mathematics teachers use to work in these environments and to integrate them into mathematics education. **Objectives**: This investigation aimed to characterise the levels of technological competence self-perceived by mathematics teaching staff when planning and executing a virtual class. **Design**: A qualitative framework was used in an exploitative-descriptive approach. **Setting and participants:** This study is part of a doctoral research in which we sought to identify the knowledge demonstrated by three mathematics teachers when incorporating technology into a virtual class with 24 students. The TPACK model (domains and subdomains linked to technology) was used to achieve this. **Data collection and analysis:** The data was collected through an open-ended interview linked to the video recording of the class, and the analysis used was content analysis. **Results**: The main conclusion was that the teachers perceived the levels of their technological competence to be very high when implementing an experimental virtual class. **Conclusions**: It is suggested that their continuous professional development and, especially, having worked together in a team for several years is a possible factor that makes them feel more able to integrate technology in mathematics education.

**Keywords**: Mathematics Education; synchronous virtual teaching; virtual lesson; TPACK; COVID-19.

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**Modelo TPACK: La percepción docente sobre su competencia tecnológica al ejecutar una lección virtual experimental en el contexto del COVID-19**


## RESUMEN

**Contexto**: Una de las preocupaciones actuales ante los cambios y retos que ha impuesto la pandemia COVID-19 es la calidad de la educación que se está recibiendo en ambientes no tradicionales como los virtuales o híbridos; así, parte de los elementos vinculados a esta problemática son los conocimientos y las competencias que profesores de matemática tienen para trabajar en esos ambientes e integrarlos a la educación matemática. **Objetivo**: El objetivo de esta investigación fue caracterizar la competencia tecnológica que auto perciben el personal docente de matemáticas cuando planifica y ejecuta una clase virtual. **Diseño**: Para esto, se utilizó un marco cualitativo desde un enfoque explotario-descriptivo. **Contexto y participantes**: Este es parte de un estudio doctoral en el cual se buscó identificar los conocimientos evidenciados por tres docentes de matemáticas al incorporar tecnología en una clase virtual con 24 estudiantes; para lograrlo, se utilizó el modelo TPACK (dominios y subdominios vinculados con la tecnología). **Recolección de datos y análisis**: La información fue recolectada a través de una entrevista abierta vinculada a la videograbación de la clase y el análisis empleado fue el análisis de contenido. **Resultados**: La principal conclusión es que los grupos docentes se auto perciben con niveles muy altos en su competencia tecnológica en el momento de ejecutar una clase virtual experimental. **Conclusión**: Se sugiere que su desarrollo profesional continuo y, en especial, haber trabajado en equipo durante varios años es un posible factor que hace que sientan mayor habilidad en integrar las tecnológicas en la educación matemática.



**Palabras clave**: Educación matemática; enseñanza virtual sincrónica; lección virtual; TPACK; COVID-19.


## INTRODUCTION

In March 2020, the World Health Organization declared a global pandemic due to COVID-19. For this reason, it was necessary to impose social distancing, and most countries suspended face-to-face classes. Consequently, lessons have changed to synchronous activities with videoconferencing systems and asynchronous actions with videos and virtual classrooms, among other techniques; this is a new normal. Digital literacy has accelerated (at least in terms of using and managing specific resources) in the child, youth and adult populations (Sehoole, 2020).

Different strategies have emerged in the teaching and learning processes during this global emergency. However, the early adoption of educational models involving the use of new technologies has generated concerns about losing some benefits of teaching carried out in classroom settings and falling into the simple transmission of content (Bakker and Wagner, 2020; Bozkurt & Sharma, 2020; Castro, Pino-Fan, Lugo-Armenta, Toro & Retamal, 2020; Chirinda, Ndlovu, & Spangenberg, 2021; Font, & Sala, 2020; Peña, Pino-Fan & Asis, 2021; Videla, Rossel, Muñoz, & Aguayo, 2022).

In this context, it is crucial to evaluate the efficiency of lessons that integrate the use of information and communication technologies (ICTs), and the Technological, Pedagogical And Content Knowledge (TPACK) model can be used to do so. Among other things, this model allows the promotion of proper use of ICTs, taking into account pedagogical and disciplinary knowledge in the case of mathematics (Abbitt, 2011; Bowers and Stephens, 2011; Cabero, Marin and Castaño, 2015; Lee and Kim, 2014; Morales-López, 2019, Morales-López et al., 2021; Saudelli and Ciampa, 2016).

In this investigation, the TPACK model is used to characterise how teachers self-perceive their competencies in using ICTs in a virtual mathematics lesson. This synchronous lesson was carried out with 16-year-old students from different regions of Costa Rica; the main topic covered was the introduction to functions, and four technological tools were used: a videoconferencing system, an interactive web tool, mathematical software, and an online whiteboard. The primary material for this virtual lesson was derived from the free math resources developed by the Mathematics Education Reform Project in Costa Rica (whose acronym in Spanish is PREMCR).

This research is part of a more general investigation of technological tools for virtual education and the didactics of mathematics, integrating the model of competencies and mathematical didactic knowledge of mathematics teachers (Godino, Batanero, Font and Giacomene, 2016; Godino, Giacomone, Batanero and Font, 2017), based on the ontosemiotic approach (Godino, Batanero and Font, 2007) and the TPACK model (Koehler and Mishra, 2008).

## THEORETICAL BACKGROUND

### TPACK Model

When technology is incorporated into a teacher's professional practice, there may be different scenarios, ranging from its incorporation without considering pedagogical issues to its incorporation with a clear didactic objective. Simply using technology does not necessarily generate significant learning in the student body.

To assist in understanding, investigating, and developing practices that correctly address disciplinary skills and knowledge related to the use of technology, the Technological, Pedagogical, and Content Knowledge (TPACK) model (Koehler and Mishra, 2008) was used in our research. The TPACK is based on the pedagogical content knowledge model of Shulman (1986), who stated that each professional in education must know the content, general pedagogical knowledge and pedagogical knowledge of the content. In 2008, Koehler and Mishra added the technological element to the Shulman model and formalised the TPACK model (formerly known as TPCK), which refers to the knowledge teachers need to incorporate ICTs in pedagogical mediation effectively. The domains of this model are briefly described below.

Content Knowledge (CK). This refers to the knowledge of the subject that the teaching staff must present in the classroom. In the particular case of mathematics, teachers must know the fundamentals, representations, possible errors, processes, definitions and the relationships of that knowledge with other topics outside and inside mathematics. The mathematics curricula of each country typically indicate the topics to be developed.

Pedagogy Knowledge (PK). This domain is associated with knowledge about methodologies and strategies for teaching in general, as well as topics including learning theories and assessment forms.

Technological Knowledge (TK). This domain refers to knowledge about technological resources both in hardware (such as computers, electronic tablets, and motion sensors), software (including spreadsheets, text templates, presentations, dynamic geometry software, and software for statistical analysis) and the use of web resources (including videoconferencing systems, specialised websites, and applications).

The intersection of these three domains gives rise to subdomains, including those discussed below.

Pedagogical Content Knowledge (PCK). This subdomain is what the Shulman model proposes, i.e., the specialised knowledge teachers use when teaching a particular body of content, using and merging the characteristics of knowledge with the particularities of the learners and their context.

Technological Content Knowledge (TCK). As Morales-López (2019) puts it, TCK “refers to the relationship between the technology used and the content studied. It is based on different representations of objects using technological resources” (p. 80). For example, in the case of mathematics, it refers to knowing the differences between dynamic geometry software and symbolic calculation software.

Technological Pedagogical Knowledge (TPK). This subdomain refers to or describes the knowledge about how technology can support the teaching and learning processes, using the advantages and overcoming the disadvantages of technological resources. This refers to general knowledge and is not linked to any particular content.

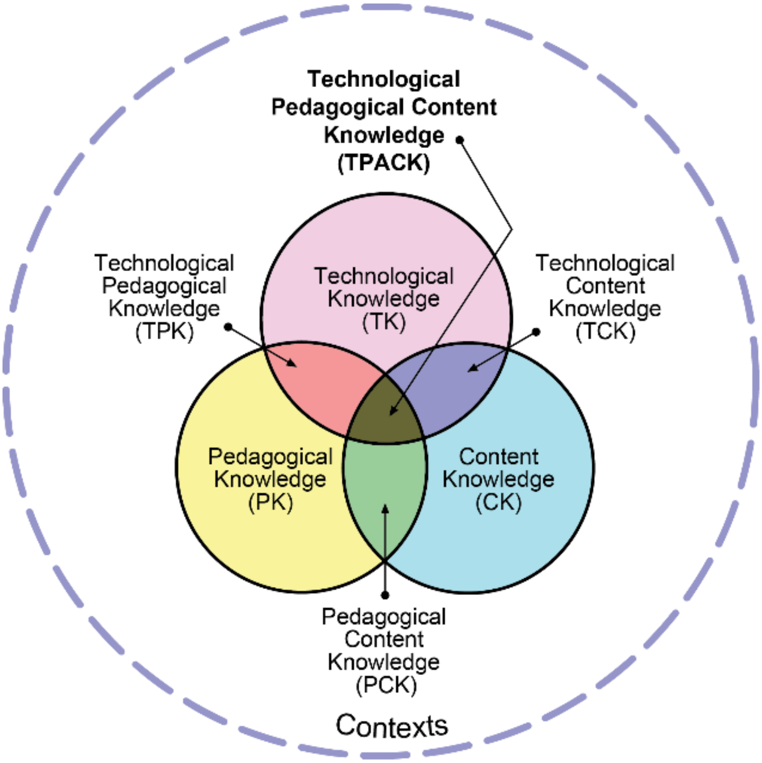
Technological Pedagogical Content Knowledge (TPACK). This subdomain is the intersection of the three principal domains and "represents the heart of the knowledge of the teacher who interprets the content and integrates technology into educational processes" (Morales-López, 2019, p. 81). This implies using technology with a clearly established pedagogical purpose, placing technological tools in the teaching and learning process, particularly in mathematics.

For Cabero, Roig-Vila and Mengual-Andrés (2017), the TPACK model "precisely delimits the consideration of instrumental, disciplinary and methodological knowledge in a context of ICT integration" (p. 75). And although "The discourse about TPACK may be seen by practitioners as a purely academic debate. However, this debate has an impact in the practical use of TPACK, particularly in how a (student-) teacher's TPACK development is determined" (Voogt, Fisser, Pareja Roblin, Tondeur, & van Braak, 2013, p. 119).

On the other hand, experiences with the use of this model in different investigations related to mathematics education have allowed the definition of various units of analysis for each of the domains and subdomains of the model (e.g., Arévalo, García and Hernández, 2019; Cavanagh and Koehler, 2013; Kirikçilar and Yildiz, 2018; Önal, 2016; Schmidt, Baran, Thompson, Mishra, Koehler and Shin, 2009). According to Lee, Chung, and Wei (2022), "TPACK's core themes from the highly cited articles have been surrounding PCK, teacher education, skill, and pedagogy" (p. 11). In particular, in the present investigation, the units of analysis of Önal (2016) are used. Figure 1 describes the relationships of the domains and subdomains of this model.

**Figure 1**

*Representation of the TPACK model and the types of knowledge that result from the intersection of each kind of knowledge.* (from http://www.tpack.org/).




## Use of technology and the Costa Rican mathematics curriculum

In Costa Rica, a new mathematics curriculum was approved for Primary and Secondary Education in 2012 (Ministry of Public Education, 2012). These study programs propose a complete reform of mathematics education in the country, complying with international standards but adjusting to the national reality. For Ruiz (2015), some of the essential elements of this reform are (a) a pragmatic vision that emphasises the ability to use mathematics to solve problems; (b) the development of mathematical skills through tasks of different levels of complexity; (c) problem solving as a methodological strategy for organising lessons; (d) reflections on and contributions to the study of mathematical areas based on international experiences, with an emphasis on promoting mathematical thinking; and (e) within the disciplinary axes of the curriculum, the use of technology is proposed in a precise manner that supports the development of mathematical learning.

This curriculum proposes two stages for mathematics lessons, the first called "Learning knowledge" and the second "Mobilisation and application of knowledge" (MEP, 2012, p. 41). In addition, the first stage is made up of four moments. It should be noted that the purposes of the LVE were associated with Stage I.

The initial moment is posing a problem and consists of proposing an initial challenge to the students. The second moment is the space for student work, but independently, i.e., with minimal teacher intervention, which is limited to supervising individual or subgroup work and posing generative questions that allow the student to face an obstacle.

In a third moment, an interactive and communicative discussion is encouraged so that the students, together with the teacher, exchange solution strategies, reflections on the problem, and even errors or difficulties faced. For the fourth and last moment, called closure, the teacher leads the systematisation of the knowledge learnt. Still, the raw material for this formalisation must come from the process of solving the problem posed in the initial moment.

In the conception of the LVE, a premise was established: the design of the lesson had to be coherent with the approach for the organisation of Mathematics lessons, according to MEP (2012), and from there, make the necessary adjustments to execute it through a synchronous lesson.

Concerning this way of organising the classroom, Ruiz (2015) indicates that,

> It might be thought that this four-step scheme is too rigid, that it was not necessary to establish a scheme. In another educational scenario, providing a model might not have been necessary. Still, in the Costa Rican scenario, it was essential to be able to strongly support Mathematics teaching at a higher level and of higher quality in a few years. A curriculum is always temporary and must correspond to a historical moment; in the future, it may no longer be necessary to propose a specific lesson outline. But now, it was indispensable (p. 20).

In this sense, the programs promote the use of technology with pedagogical utility, which "must be done strictly based on the contribution it offers to the achievement of the stated learning purposes" and whose use "should not be adopted for the intrinsic value of the technology, whatever it may be." (MEP, 2012, p. 61).

The use of different technological resources occurs at all educational levels. These resources include (a) calculators, which allow concentrating efforts on the processes of reasoning and application; (b) Computers and specialised software, which permit modelling of everyday life situations and taking advantage of the use of different representations of mathematical objects; (c) the Internet, with an enormous number of resources aimed at learning mathematics, and making lessons more interactive; (d) mobile devices, whose applications can help to demonstrate the applicability of mathematics through solving problems in everyday life.

## COVID-19 and technology

Information and communication technologies have been vital to achieving educational spaces during the global COVID-19 pandemic. Carrying out synchronous and asynchronous teaching and learning processes, remotely or online, as well as in bimodal spaces (face-to-face and remote), have become a "new normal" in education, and, according to Han (2020), this is likely to continue. However, there are also fears that the obligatory incorporation of ICTs due to the pandemic will cause them to be used without careful consideration of their usefulness in education (Engelbrecht, Llinares and Borba, 2020).

> After being faced with the reality of COVID-19, following measures introduced by governments all over the world, and the various arrangements made by schools and universities in compliance with these measures, students and teachers have had to make drastic changes to the traditional teaching and learning approach, working and learning from home. The world of teaching and learning has changed dramatically, and we find ourselves relying on technology to conduct lectures and other teaching and learning activities. Our students are in remote locations away from campus, and we connect with them using technology (Engelbrecht, Llinares and Borba, 2020, p. 821).

There thus exists ambiguity about the use of technology in education because, on the one hand, the need for communication and developing lessons supported by technology is evident. Still, on the other hand, there has been no real training (initial and continuous) about what it means to incorporate technological resources into the classroom.

DeRosa, in Kamanetz (2020), warns that while teaching groups are managing to care for students and are trying to build community in the face of the global emergency, they are making little progress in adjusting the pedagogical and didactic elements of the activities that a class entails. Faced with this situation, Engelbrecht et al. (2020) state that there is a wide range of technological means to create hybrid forms of teaching that can attract students and transform classes into active learning spaces. Among the tools that could be used, these authors highlight social networks, videos, the Internet, software for creating and editing videos, and those tools that allow personalised learning.

As noted by Videla, Rossel, Muñoz, and Aguayo (2022),

> The impact of the pandemic on the transformation of distance education processes compared to face-to-face teaching not only translates into sudden changes and the adaptation of new virtual teaching and learning methods but also generates mental health problems for students. In addition, primary school students show reluctance and little concern for homework. This becomes another demand for educators who must deal not only with changes in their teaching but, in many cases, also be involved with the educational processes of their own families. Mathematics education has been one of the disciplines that most need new didactic teaching strategies, considering its symbolic nature and emphasis on paper rather than technology (Videla et al. 2022).

This phenomenon goes far beyond access to materials and videos, exposes major health challenges, and relates to community members' economic, social and affective-emotional stability (Morales-López, Gavarrete-Villaverde y Alpizar-Vargas, 2021).

## METHODOLOGY

This research was carried out using a qualitative framework and an exploitative-descriptive approach (Hernández, Fernández and Baptista, 2010) and uses the TPACK model to structure the investigation of how teaching staff perceive themselves when using technological resources in a virtual lesson. In the experimental virtual lesson (EVL), the topic covered was an introduction to functions, based on the free mathematics resources designed by the PREMCR. Four technological tools were used to develop the lesson: 1. GeoGebra (mathematical software); 2. Zoom (videoconferencing software); 3. Nearpod (an interactive classroom web application), and 4. Awwapp.com (a digital web whiteboard).

### Subjects

The research was carried out in a mathematics lesson developed non-face-to-face and in a synchronous way through the Zoom platform. The subjects who participated in the EVL[1] were:

a. **Teachers 1, 2 and 3**: These were the teachers who participated at different moments in the planning and execution of the EVL. They have an average of 20 years of teaching experience in high school, 15 years in university teaching, and ten years of work at the PREMCR.
b. **Non-participant observers**: Among the observers were three PREMCR participants (Obs1, Obs2, Obs3), three regional mathematics pedagogical advisors residing in the country (Obs4, Obs5 and Obs6), and four secondary school teachers (Obs7, Obs8, Obs9 and Obs10).
c. **Students**: 24 young persons from different types of institutions in five provinces of Costa Rica.

This investigation will consider statements made by teachers 1, 2 and 3. Aspects of interest included contact activities that occurred during lesson times, as well as the components necessary to deliver the lesson, particularly those related to the technological tools used within the lesson and the pedagogical strategies that articulated the cognitive materials and the tools. Technical difficulties encountered while teaching in the virtual modality were also considered (both in terms of the platform, the tools used, and the way the teaching team handled the situation).

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[1] The subjects signed an Informed Consent Form (ICF) but there was no approval by the Ethics Committee. Therefore, the authors assume and exempt Acta Scientiae from any consequences arising, including full assistance and possible compensation for any damage to any research participants, per Resolution No. 510, of April 7, 2016, of the National Health Council of Brazil.

## Pilot testing

The planning of the Experimental Virtual Lesson included the development of a pilot lesson plan, the execution of the pilot, the application of differentiated observations (Mathematics Education specialists and in-service teachers), the redesign of the original lesson plan and finally, the execution of the VLE.

The pilot plan was developed with tenth-grade students from a private educational institution in the province of Heredia. Six students, two secondary school teachers, a pedagogical advisor, and the members of PREMCR participated in this plan.

## Information collection and class design

**Table 1**

*Resources used in the virtual class at different times in the lesson*

| Block | Minutes | Activities | Resources |
|---|---|---|---|
| 1 | 5 | Introductory activity | *Zoom* |
| 2 | 10 | Greeting and welcome | *Zoom* |
| 3 | 10 | Presentation of the problem in the free mathematics resources (FMR) of the Mathematics Education Reform Project in Costa Rica. | *Zoom* |
| 4 | 20 | Independent work and interactive discussion | *Zoom, Nearpod, Awwapp.com* |
| 5 | 5 | FMR Video Presentation | *Zoom, Nearpod* |
| 6 | 15 | Closure or explanation of concepts | *Zoom, GeoGebra* |
| 7 | 5 | Evaluation of the first part of the class | *Zoom* |
| 8 | 5 | Break | *Zoom* |
| 9 | 10 | Practice 1 | *Zoom, Nearpod, Awwapp.com* |
| 10 | 15 | Practice 2 | *Zoom, Nearpod, Awwapp.com* |
| 11 | 5 | Metaclosure of the problem | *Zoom, Awwapp.com* |

| Block | Minutes | Activities | Resources |
| --- | --- | --- | --- |
| 12 | 5 | Evaluation of the second part of the class | *Zoom* |
| 13 | 5 | Metaclosure of the activity | *Zoom* |

The EVL in which the research was carried out lasted 120 minutes. An unstructured interview (Díaz-Bravo et al., 2013) was conducted in which teachers 1, 2 and 3 shared their reflections on the lesson being taught. Although the class was recorded, the teachers shared their impressions without thoroughly revising the video recording. The activities' timing and the resources used are described in Table 1.

### Categories for data analysis

The categories of analysis of the data obtained, established a priori, are carried out based on the TPACK model of Koehler and Mishra (2008). Only the domains and subdomains related to technology (TK, TPK, TCK and TPCK) were considered since this research is limited to the study of self-perceived characteristics of teachers in their use of technological tools. The indicators for each domain and subdomain are based on Önal (2016). Blocks 1, 2 and 8 are not considered for the analysis of results since they are moments with a general pedagogical orientation rather than a technological one.

## RESULTS AND ANALYSES

As a basis for characterisation, an assessment of the opinions of teachers 1, 2 and 3 was carried out. The analysis is organised according to domains and subdomains: TK, TPK, TCK and TPCK.

### Technological knowledge

According to the teachers, they are competent in the domain of technological knowledge associated with the EVL. Although there were no technical problems in the lesson, a contingency plan (TK1) was detailed in the lesson plan. Likewise, the technological tools in the class were previously analysed in terms of their advantages and disadvantages and their complementarity. For example, the teachers considered that the Nearpod and Awwapp.com tools are web platforms whose use may or may not be educational; however, in the case of the EVL, they were used for educational purposes (TK2, TK3).

It was necessary to install the GeoGebra tool on the teachers' devices to carry out the virtual lesson; according to the teachers, this did not generate a problem. They already used the Zoom tool in their different academic activities; however, it was used for educational purposes in the virtual lesson (TK5 and TK6).

The teachers were alert to any need to solve problems related to using technological tools, mainly in the case of Nearpod, whose use required another device. In addition, a code had to be entered to be able to observe and carry out the different activities; these issues were resolved using Zoom chat or audio (TK7). Observing the TK4 analysis unit was impossible because the virtual lesson was executed without hardware problems. Table 2 summarises the level of competence teachers expressed about themselves concerning TK.

**Table 2**

*Proficiency level in the technological knowledge (TK) domain in an experimental virtual mathematics lesson, according to teachers*

| Unit of analysis | NC | SC | MC | C | HC | NA |
|---|---|---|---|---|---|---|
| TK1. Ability to overcome technical problems that may be experienced in self-instruction. | | | | | ✓ | |
| TK2. Ability to choose appropriate technologies to be used for educational purposes. | | | | | ✓ | |
| TK3. Ability to use new, developing technologies for educational purposes. | | | | | ✓ | |
| TK4. Ability to solve hardware problems with technological devices in the educational environment. | | | | | | ✓ |
| TK5. Ability to install the necessary software on technological devices in the process of instruction. | | | | | ✓ | |
| TK6. Ability to use software already installed on technological devices in instruction. | | | | | ✓ | |
| TK7. Ability to help students with problems they may experience with computers. | | | | | ✓ | |

Note: NC - not competent; SC - somewhat competent; MC - moderately competent; C - competent; HC - highly competent; NA - not applicable.

### Pedagogical technological knowledge

According to the teachers, the detailed planning of the lesson that included the contents to be presented in the class and which tools to use in each block shows an ability to introduce ICTs at appropriate moments in the lesson (see Table 1). For teachers, this planning involves a detailed selection of platforms, depending on what they need to do. For example, in independent work, the teachers believed that a virtual space was required for the students to solve the problems presented and provide an answer; to do so, the teachers used Nearpod in Block 4. They then discussed using the Zoom tool based on the students' responses and procedures. The Awwapp.com digital whiteboard was used to express the students' ideas. These actions and justifications can be classified in indicators TPK1, TPK2 and TPK5.

In the case of the indicator TPK3, in Blocks 9 and 10, these same three tools were used to assess students in terms of their formation; according to the teachers, they were used appropriately in the Nearpod Time to Climb and Draw It activities.

At the end of the lesson, in Block 13 (see Table 1), the teachers carried out a meta-closure, explaining to students how they can use these technological resources in other cases and mathematical activities. This can be classified as a high competence in the TPK4 and TPK6 indicators (see Table 3).

**Table 3**

*Level of competence in the technological pedagogical knowledge (TPK) subdomain in a practical virtual mathematics lesson, according to teachers.*

| Unit of analysis | NC | SC | MC | C | HC | NA |
|---|---|---|---|---|---|---|
| TPK1. Ability to plan the use of technology for educational purposes. | | | | | ✓ | |
| TPK2. Ability to predict how technology may affect the learning and instruction process. | | | | | ✓ | |
| TPK3. Ability to assess students in a class in which technology is used effectively. | | | | | ✓ | |
| TPK4. Ability to provide students with online environments that contribute to their knowledge and skills. | | | | | ✓ | |
| TPK5. Ability to use various methods and approaches during online instruction. | | | | | ✓ | |
| TPK6. Ability to promote online learning among students. | | | | | ✓ | |

Note: NC - not competent; SC - somewhat competent; MC - moderately competent; C - competent; HC - highly competent; NA - not applicable.

### Technological knowledge of the content

According to the teachers, the use of GeoGebra was intended to show the behaviour of a function and elements such as the domain, the range, preimages, images and monotonicity intervals (TCK1). This identification process was carried out with the help of sliders to provide a dynamic visualisation of the concepts involved. The graphical representation of a function was also used, where it is observed that the level of competence of the TCK2 criterion also appears. However, the teachers stated that more detailed presentations could be carried out with the design of more complex animations. Throughout the virtual lesson, different multimedia elements of free mathematics resources were used, including the presentation of problems on the website (Block 3), explanatory video (Block 5) and practice (Block 9), which demonstrates competence in the TCK3 and TCK4 units.

**Table 4**

*Proficiency level in the technological content knowledge (TCK) subdomain in an experimental virtual math lesson, according to teachers*

| Unit of analysis | NC | SC | MC | C | HC | NA |
|---|---|---|---|---|---|---|
| TCK1. Ability to use software already installed on computers (MS Office, calculator, GeoGebra, etc.) for mathematics. | | | | | ✓ | |
| TCK2. Ability to use flash animations and graphic drawings to enrich math classes. | | | | ✓ | | |
| TCK3. Ability to make presentations or multimedia to teach mathematics. | | | | | ✓ | |
| TCK4. Ability to search the Web for topics and concepts related to math classes. | | | | | ✓ | |

Note: NC - not competent; SC - somewhat competent; MC - moderately competent; C - competent; HC - highly competent; NA - not applicable.

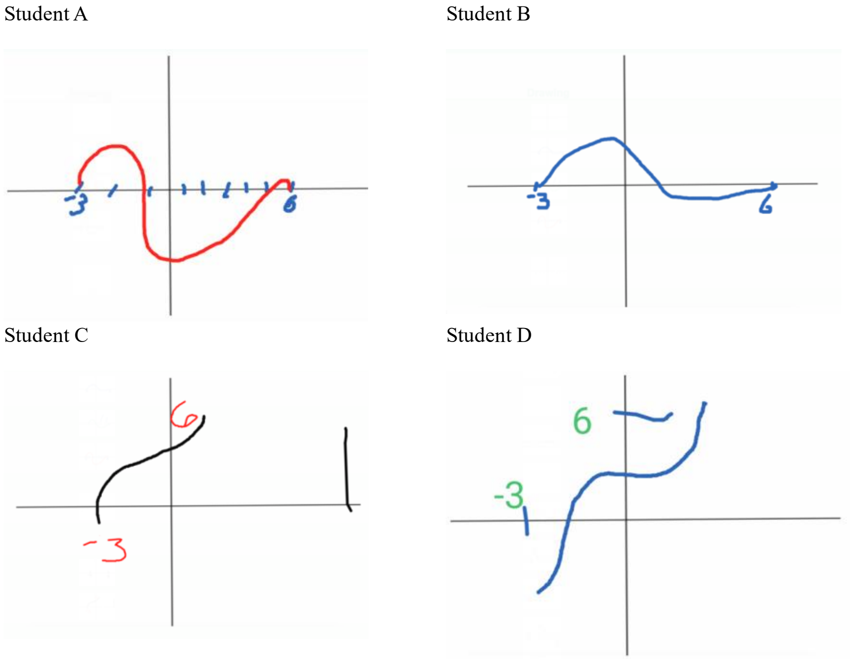
### Technological and pedagogical knowledge of the content

According to the teachers, there was integration between pedagogy-technology and knowledge-technology; they could also observe a relationship between pedagogy-technology-knowledge since they believe that all the elements presented in the different blocks of the lesson are related.

For example, in Block 10 (see Table 1), the teachers indicated that they foresaw the use of the Nearpod Draw It activity so that students could use their cell phones or electronic tablets to draw the graphical representation of a function whose domain was [-3.6]. It was clear to the teachers that this type of activity is a good use of technology from the pedagogical point of view, combining it with a mathematical question intended to determine whether or not the students managed to understand the concept of the domain of a function. Figure 2 shows some of the representations selected by the teachers, where different responses by the students in real-time can be observed, both correct and incorrect.

**Figure 2**

*Answers of some students of the virtual lesson on the graphical representation of a function whose domain is [-3,6]*




During the virtual lesson, the teachers in charge took advantage of one of the Nearpod features to share some of the student graphs with all the participants (who could see the graphs on their devices) while discussing why these graphs were either correct or incorrect. For example, the students made the same mistake in graphs C and D; therefore, in the lesson, the teachers used the space to explain and reinforce the concepts of domains and ranges of real numbers. The teachers stated that this episode showed high proficiency in the TPCK1, TPCK 2, TPCK4, TPCK5, and TPCK8 units of analysis.

According to the teachers, Block 4 of independent work and interactive discussion displayed: 1) the use of the problem-solving strategy for mediation with the help of ICT and 2) consideration of the students' prior knowledge, both of which are relevant to the TPCK3 criterion.

Lastly, according to the teachers, since different means were used at all times during the class to clarify doubts from the students, such as Zoom chat or the awwapp.com whiteboard, a high level of competence was also shown in the TPCK7 criterion.

Since the experimental virtual lesson did not follow any textbook but relied on free math resources, the TPCK6 criterion does not directly apply and cannot be evaluated. In addition, the TPCK9 criterion is not observable in a virtual lesson, only when students are present. Table 5 shows the level of competence of the TPACK subdomain.

**Table 5**

*Proficiency level in the technological and pedagogical content knowledge (TPCK) subdomain in a practical virtual math lesson, according to teachers.*

| Unit of analysis | NC | SC | MC | C | HC | NA |
|---|---|---|---|---|---|---|
| TPCK1. Ability to consider mathematical content, learning-teaching strategies and new technologies during lesson planning. | | | | | ✓ | |
| TPCK 2. Ability to use technology-assisted assessment tools while evaluating the learning-teaching process. | | | | | ✓ | |
| TPCK3. Ability to make use of technological devices to measure preliminary results of students. | | | | | ✓ | |
| TPCK4. Ability to make use of technological devices to identify student misconceptions. | | | | | ✓ | |
| TPCK5. Ability to use technology to reinforce mathematical skills. | | | | | ✓ | |
| TPCK6. Ability to use technology to provide effective examples parallel to the mathematics textbook. | | | | | | ✓ |
| TPCK7. Ability to clarify student doubts during the teaching of mathematics online. | | | | | ✓ | |
| TPCK8. Ability to integrate technology with math classes appropriately and effectively to make topics easier and more understandable. | | | | | ✓ | |
| TPCK9. Ability to assist others in the school in coordinating math, technology, and teaching strategies. | | | | | | ✓ |

Note: NC - not competent; SC - somewhat competent; MC - moderately competent; C - competent; HC - highly competent; NA - not applicable.

## CONCLUSIONS

The global pandemic caused by COVID-19 has brought about many problems in all sectors, and education is one of those most affected. Teachers must use technologies that they do not necessarily know how to use (in technical terms) nor precisely how to use them as teaching tools. Resources such as the experimental virtual lesson are intended to include necessary elements to provide teachers with more support in integrating technology, mathematics and its teaching.

In the cases of practically all indicators, the teachers who participated in the design and implementation of the EVL expressed a belief in their high level of technological competence. Their self-perception is that, at least in the technological component, they have sufficient capabilities to plan and execute a virtual class of this type. It should be emphasised that this investigation was carried out in the case of a virtual class with a specific design planned by an entire work team. This does not limit the scope of this research but instead gives a clear signal that planning offers the teacher a certain degree of confidence that the use of ICT can be integrated into all their work.

Although this investigation focused on characterising the teachers' self-perceptions of their technological competence in an EVL, there are possible hypotheses about why teachers believe they have this professional profile. The central hypothesis is that teachers have been linked to teamwork activities with other professionals in secondary and higher education systems, so they have been more exposed to research and recent texts on the didactics of mathematics and material designs. In other words, the three have been continuously involved in the professional development process, whose results could be classified as adequate. Although the results of this investigation do not provide direct evidence to support this hypothesis, it is clearly essential to investigate further the relationship between continuous professional training and the ability to intelligently use technological resources in virtual classes (Goos and Bennison, 2008). This hypothesis agrees with the one considered by Bakar et al. (2020) but disagrees with the fact that professional experience was not a critical factor in their study. In their case, they proposed the inclusion of academic qualifications and the courses the teachers have received as factors to be included in further investigations.

As indicated by Borba (2021),

> COVID-19 has pushed forward the agenda of the digital technology trend in mathematics education. With the need for

> social isolation, it became necessary to offer education to children and undergraduates at home. In most of the world, the first semester of teaching in 2020 was suspended or went online. Many are discussing different kinds of hybrid education as health conditions allow students and teachers to return to school and universities (p. 388).

Finally, the evidence produced by this research is relevant not only for the professional development of in-service teachers but is also crucial at a more general level, offering strategies to define and sustain initial training programs that promote the intelligent use of technological tools in mathematics education. And even more broadly, some questions whose answers are still open are: what are we learning from our experience of the pandemic that can improve mathematics education right now and in the post-pandemic future? How do we balance possible shifts from traditional, fully face-to-face models to more flexible modalities?

## AUTHORS' CONTRIBUTIONS STATEMENTS

YML and RPV conceived the presented idea, developed the theory, adapted the methodology to this context, created the models, performed the activities, and collected the data. YML and RPV analysed the data. All authors actively discussed the results and reviewed and approved the final version of the work.

## DATA AVAILABILITY STATEMENT

Data supporting the results of this study will be made available by the corresponding author, YML, upon reasonable request.

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## REFERENCES

Abbitt, J. T. (2011). An investigation of the relationship between self- efficacy beliefs about technology integration and Technological Pedagogical Content Knowledge (TPACK) among preservice teachers. *Journal of Digital Learning in Teacher Education, 27*(4), 134–143. https://doi.org/10.1080/21532974.2011.10784670

Arévalo, M. A., García, M. Á. & Hernández, C. A. (2019). Competencias TIC de los docentes de matemáticas en el marco del modelo TPACK *[ICT competences of mathematics teachers in the framework of the TPACK model]*. *Civilizar, 19*(36), 1-27. https://doi.org/10.22518/usergioa/jour/ccsh/2019.1/a07

Bakar, N. S. A., Maat, S. M. & Rosli, R. (2020). Mathematics teacher's self-efficacy of technology integration and technological pedagogical content knowledge. *Journal on Mathematics Education, 11*(2), 256-276. https://doi.org/10.22342/jme.11.2.10818.259-276

Bakker, A. & Wagner, D. (2020). Pandemic: lessons for today and tomorrow? *Educational Studies in Mathematics, 104*, 1–4. https://doi.org/10.1007/s10649-020-09946-3

Bowers, J. S. & Stephens, B. (2011). Using technology to explore mathematical relationships: A framework for orienting mathematics courses for prospective teachers. *Journal Mathematics Teacher Education*, *14*, 285–304. https://doi.org/10.1007/s10857-011-9168-x

Bozkurt, A, & Sharma, R. C. (2020). Emergency remote teaching in a time of global crisis due to CoronaVirus pandemic. *Asian Journal of Distance Education, 15*(1), i–vi. https://doi.org/10.5281/zenodo.3778083

Cabero, J., Marín, V. & Castaño, C. (2015). Validación de la aplicación del modelo TPACK para la formación del profesorado en TIC *[Validation of the application of the TPACK model for ICT teacher training]*.

*Revista d'innovació educativa, 14*, 13-22. https://ojs.uv.es/index.php/attic/article/view/4001

Cabero, J., Roig-Vila, R. & Mengual-Andrés, S. (2017). Conocimientos tecnológicos, pedagógicos y disciplinares de los futuros docentes según el modelo TPACK *[Technological, pedagogical and disciplinary knowledge of future teachers according to the TPACK model]*. *Revista d'innovació educativa* (32)*,* 73-84.

Castro, W. F., Pino-Fan, L. R., Lugo-Armenta, J. G., Toro, J. A., & Retamal, S. (2020). A Mathematics Education Research Agenda in Latin America Motivated by Coronavirus Pandemic. *Eurasia Journal of Mathematics, Science and Technology Education, 16*(12), em1919. https://doi.org/10.29333/ejmste/9277

Cavanagh, R. F. & Koehler, M. J. (2013). A turn toward specifying validity criteria in the measurement of Technological Pedagogical Content Knowledge (TPACK). *Journal of Research on Technology in Education, 46*(2), 129–148. https://doi.org/10.1080/15391523.2013.10782616

Chirinda, B., Ndlovu, M., & Spangenberg, E. (2021). Teaching Mathematics during the COVID-19 Lockdown in a Context of Historical Disadvantage. *Education Sciences*, *11*(4), 177. https://doi.org/10.3390/educsci11040177

Díaz-Bravo, L., Torruco-García, U., Martínez-Hernández, M. & Varela-Ruiz, M. (2013). La entrevista, recurso flexible y dinámico *[The interview, a flexible and dynamic resource]*. *Investigación en educación médica, 2*(7), 162-167. https://doi.org/10.1016/s2007-5057(13)72706-6

Engelbrecht, J., Llinares, S. & Borba, M. C. (2020). Transformation of the mathematics classroom with the Internet. *ZDM Mathematics Education, 52*, 825–841. https://doi.org/10.1007/s11858-020-01176-4

Font, V., & Sala, G. (2020). 2021. Un año de incertidumbres para la Educación Matemática [*A year of uncertainties for Mathematics Education*]. *Bolema: Boletim de Educação Matemática*, 34, i-v. https://doi.org/10.1590/1980-4415v34n68e01

Godino, J. D. Batanero, C. & Font, V. (2007). The onto-semiotic approach to research in mathematics education. *ZDM. The International Journal on Mathematics Education*, *39* (1-2), 127-135. https://doi.org/10.1007/s11858-006-0004-1

Godino, J. D., Batanero, C., Font, V. & Giacomone, B. (2016). Articulando conocimientos y competencias del profesor de matemáticas: El modelo CCDM *[Articulating knowledge and skills of the mathematics teacher: The CCDM model]*. In J. A. Macías, A. Jiménez, J. L. González, M. T. Sánchez, P. Hernández, C. Fernández, F. J. Ruiz, T. Fernández & A. Berciano (Eds.), *Investigación en Educación Matemática XX* (pp. 285-294). SEIEM. http://funes.uniandes.edu.co/8859/1/Batanero2016Articulando.pdf

Godino, J. D., Giacomone, B., Batanero, C. & Font, V. (2017). Enfoque ontosemiótico de los conocimientos y competencias del profesor de matemáticas *[The onto-semiotic approach to the knowledge and skills of the mathematics teacher]*. *Bolema, 31*(57), 90-113. https://doi.org/10.1590/1980-4415v31n57a05

Goos, M. & Bennison, A. (2008). Surveying the technology landscape: Teachers' use of technology in secondary mathematics classrooms. *Mathematics Education Research Journal, 20*(3), 102–130. https://doi.org/10.1007/BF03217532

Han, H. (2020). *How to make the switch to online teaching more effective*. University World News. https://www.universityworldnews.com/post.php?story=20200317134745462

Hernández, R., Fernández, C. & Baptista, P. (2010). *Metodología de la investigación [Research methodology]*. Mc Graw-Hill.

Kamanetz, A. (2020). *'Panic-gogy': Teaching online classes during the coronavirus pandemic*. NPR. https://www.npr.org/2020/03/19/817885991/panic-gogy-teaching-online-classes-during-the-coronavirus-pandemic

Kirikçilar, R. & Yildiz, A. (2018). Technological Pedagogical Content Knowledge (TPACK) Craft: Utilisation of the TPACK When Designing the GeoGebra Activities. *Acta Didactica Napocensia, 11*(1), 101-116. https://doi.org/10.24193/adn.11.1.8

Koehler, M. & Mishra, P. (2008). Introducing Technological pedagogical Knowledge. In AACTE (eds). *The handbook of technological pedagogical content knowledge for educators*. Routledge/Taylor and Francis Group for the American Association of Colleges of Teacher Education.

Lee, C. J. & Kim, C. (2014). An implementation study of a TPACK-based instructional design model in a technology integration course. *Educational Technology Research and Development, 62*(4), 437-460. https://doi.org/10.1007/s11423-014-9335-8

Lee, H.-Y., Chung, C.-Y., & Wei, G. (2022). Research on TPACK: A Bibliometric Analysis from 2011 to 2020. *Frontiers in Education*, 7, 1-14. https://doi.org/10.3389/feduc.2022.765233

Ministerio de Educación Pública de Costa Rica. (2012). *Programas de Estudio Matemáticas. Educación General Básica y Ciclo Diversificado [Mathematics Study Programs. Basic General Education and Diversified Cycle]*. Author.

Morales-López, Y. (2019). Knowledge evidenced by prospective mathematics teachers when performing a task involving geometry, teaching and the use of technology. *Acta Scientiae, 21*(2), 75-92. https://doi.org/10.17648/acta.scientiae.v21iss2id5081

Morales-López, Y., Chacón-Camacho, Y. & Vargas-Delgado, W. (2021). TPACK of Prospective Mathematics Teachers at an Early Stage of Training. *Mathematics, 9(*15), 1741. http://dx.doi.org/10.3390/math9151741

Morales-López, Y., Gavarrete-Villaverde, M. E., & Alpizar-Vargas, M. (2021). Acciones y desafíos en la formación de docentes de matemáticas en el contexto de la pandemia en la Universidad Nacional - Costa Rica. *[Actions and challenges in the training of mathematics teachers in the context of the pandemic at the Universidad Nacional - Costa Rica]. Cuadernos de Investigación y Formación en Educación Matemática, 20,* 216-228.

Önal, N. (2016). Development, Validity and Reliability of TPACK Scale with Pre-Service Mathematics Teachers. *International Online Journal of Educational Sciences*, *8*(2).

Peña, C., Pino-Fan, L., & Assis, A. (2021). Norms that regulate the management of virtual mathematics classes in the COVID-19 context. *Uniciencia, 35*(2), 1-20. https://doi.org/10.15359/ru.35-2.21

Ruiz, A. (2015). Balance y perspectivas de la Reforma de la Educación Matemática en Costa Rica *[Balance and perspectives on Mathematics Education Reform in Costa Rica]*. *Cuadernos de investigación y formación en educación matemática, 13,* 15-33.

Saudelli, M. G. & Ciampa, K. (2016). Exploring the role of TPACK and teacher self-efficacy: an ethnographic case study of three iPad language arts classes. *Technology, Pedagogy and Education, 25*(2), 227-247. https://doi.org/10.1080/1475939X.2014.979865

Schmidt, D. A., Baran, E., Thompson, A. D., Mishra, P., Koehler, M. J. & Shin, T. S. (2009). Technological pedagogical content knowledge (TPACK) the development and validation of an assessment instrument for preservice teachers. *Journal of research on Technology in Education, 42*(2), 123-149. https://doi.org/10.1080/15391523.2009.10782544

Sehoole, C. (2020). Marching on to a new way of learning and working. *Intuition, 14*(2), 2

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14. https://doi.org/10.3102/0013189x015002004

Videla, R., Rossel, S., Muñoz, C., & Aguayo, C. (2022). Online Mathematics Education during the COVID-19 Pandemic: Didactic Strategies, Educational Resources, and Educational Contexts. *Education Sciences, 12*(7), 492. https://doi.org/10.3390/educsci12070492

Voogt, J., Fisser, P., Pareja Roblin, N., Tondeur, J. and van Braak, J. (2013). Technological pedagogical content knowledge – a review of the literature. *Journal of Computer Assisted Learning, 29*, 109-121. https://doi.org/10.1111/j.1365-2729.2012.00487.x