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Ethnomathematical and Mathematical Connections Activated by a Teacher in Mathematical Problems Posing and Solving

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ABSTRACT

Background: Connections are essential for understanding concepts, but difficulties have been evidenced in connecting representations and meanings of concepts and creating contextualised mathematical problems by teachers and students. **Objective**: Therefore, ethnomathematical and mathematical connections were analysed in a teacher's mathematical activity when posing and solving mathematical problems. **Design**: The methodology was qualitative-ethnographic, developed in a workshop done in stages. **Setting and participants**: An indigenous Mokaná teacher from Sibarco was selected. **Data collection and analysis**: Semi-structured interviews were conducted in the workshop, and the data were analysed based on the connections; the workshop was initially designed considering previous literature on the issue, and the researchers were familiarised with the teacher. **Results**: For the analysis of the mathematics used by the teacher in the classroom, we considered his sociocultural context, where he set problems about the area and perimeter of lots of land and enclosures. Then, the researchers presented the ethnomathematical connections that emerged in the elaboration and commercialisation of the pigeon peas sancocho, which was the basis for the teacher to pose and solve problems involving conversions between units of measurement, volume of the totumas (ellipsoid), etc. Simultaneously, mathematical connections of different representations, procedural, meaning, and modelling were identified. Finally, the researchers gave feedback by assessing the

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potential of the mathematics known and explained by the teacher. **Conclusion**: This research provides input for teachers to pose and solve problems contextualised through connections.

Keywords: Ethnomathematical and mathematical connections; Problem posing and solving; Teacher; Mathematics education.

Conexiones etnomatemáticas y matemáticas activadas por un profesor en la creación y resolución de problemas matemáticos

RESUMEN

Contexto: Las conexiones se consideran importantes para la comprensión de conceptos, pero se han evidenciado dificultades para conectar representaciones, significados de conceptos y crear problemas matemáticos contextualizados por parte de profesores y estudiantes. **Objetivo**: por lo tanto, se analizaron las conexiones etnomatemáticas y matemáticas en la actividad matemática de un profesor cuando crea y resuelve problemas matemáticos. **Diseño**: La metodología fue cualitativa-etnográfica desarrollada en el contexto de un taller desarrollado en etapas. **Contexto y participantes**: se seleccionó un profesor indígena Mokaná de Sibarco. **Recolección y análisis de datos**: En el taller se realizaron entrevistas semiestructuradas y los datos se analizaron con base en las conexiones, donde inicialmente se diseñó el taller considerando la literatura previa y se hizo la familiarización de los investigadores con el profesor. **Resultados**: se reportan las matemáticas usadas por el profesor en el aula de clase considerando su contexto sociocultural, donde planteó problemas sobre área y perímetro de terrenos y cercados. Luego, los investigadores presentaron las conexiones etnomatemáticas que emergen en la elaboración y comercialización del sancocho de guandú, lo cual fue la base para que el profesor creara y resolviera problemas que involucran conversiones entre unidades de medida, volumen de las totumas (elipsoide), etc. Simultáneamente, se identificaron conexiones matemáticas de representaciones diferentes, procedimental, significado y modelado. Finalmente, se realizó la retroalimentación entre los investigadores valorando el potencial de las matemáticas conocidas y explicadas por el profesor. **Conclusión**: Esta investigación ofrece un insumo para que los profesores creen y resuelvan problemas contextualizados a través de conexiones.

Palabras clave: Conexiones etnomatemáticas y matemáticas; creación y resolución de problemas; profesor; Educación Matemática.

Conexões etnomatemáticas e matemáticas ativadas por um professor na criação e resolução de problemas matemáticos

RESUMO

Contexto: As conexões são consideradas importantes para a compreensão dos conceitos, mas foram evidenciadas dificuldades em conectar representações, significados de conceitos e criar problemas matemáticos contextualizados por professores e alunos. **Objetivo**: portanto, foram analisadas as conexões etnomatemáticas e matemáticas na atividade matemática de um professor ao criar e resolver problemas matemáticos. **Desenho**: A metodologia foi qualitativa-etnográfica desenvolvida no contexto de uma oficina desenvolvida em etapas. **Contexto e participantes**: foi selecionada uma professora indígena Mokaná do Sibarco. **Coleta e análise dos dados:** Na oficina foram realizadas entrevistas semiestruturadas e os dados analisados a partir das conexões, onde a oficina foi inicialmente concebida considerando a literatura anterior e os pesquisadores foram familiarizados com a professora. **Resultados**: a matemática utilizada pelo professor em sala de aula é relatada considerando seu contexto sociocultural, onde ele levantou problemas de área e perímetro de terrenos e cercas. Em seguida, os pesquisadores apresentaram as conexões etnomatemáticas que surgem na elaboração e comercialização do sancocho de guandu, que serviu de base para o professor criar e resolver problemas que envolviam conversões entre unidades de medida, volume dos totumas (elipsóide), etc. Simultaneamente, foram identificadas conexões matemáticas de diferentes representações, procedimentais, significados e modelagem. Por fim, foi realizada a retroalimentação entre os pesquisadores, avaliando o potencial da matemática conhecida e explicada pelo professor. **Conclusão**: Esta pesquisa oferece subsídios para que professores criem e resolvam problemas contextualizados por meio de conexões.

Palavras-chave: Conexões etnomatemáticas e matemáticas; criação e resolução de problemas; professor; Educação matemática.

INTRODUCTION

The literature on mathematics education recognises that establishing mathematical connections is essential for problem solving and understanding real-life concepts and phenomena (Ministry of National Education [MEN], 2006; Rodríguez-Nieto et al., 2021). In fact, posing contextualised problems is fundamental to favour the understanding and application of mathematics (Miranda & Mamede, 2022; Silber & Cai, 2016). In turn, ethnomathematics focuses on assessing the several kinds of mathematics practised by cultural groups and connecting them with school environments (D'Ambrosio, 2001; Rosa & Orey, 2021a, 2021b).

Ethnomathematics have explored, recognised, valued, and evidenced the knowledge and mathematical ideas developed daily by all peoples (D'Ambrosio, 2001; Gerdes, 2013), highlighting the relationships between the mathematical knowledge of people who use it in different cultural environments (e.g., carpenters, farmers, and cooks, among other craftsmen) and formal mathematics. Rosa and Orey (2018) believe that ethnomathematics connects cultural practices developed and used locally with school mathematics. In this sense, ethnomathematics "is the area of research that studies the multifaceted relationships and interconnections between mathematical ideas and other elements that compose culture, such as language, art, crafts, construction, education" (Gerdes, 2013, p. 150).

Notably, research on ethnomathematical connections shows that there are studies focused on the production of tortillas, yucca buns, kites and flying crates, agriculture, fishing, drums, and cheeses, among others. Ethnomathematical connections seek to relate the mathematics developed in daily practices with the universally accepted institutionalised mathematics found in curriculum resources (Rodríguez-Nieto, 2021). In this sense, we agree with what is suggested by different curricular bodies and institutions of mathematics education, connecting mathematics with real life; for example, the National Council of Teachers of Mathematics [NCTM] (2000) affirms that by connecting mathematics with each other and mathematics with real life, students understand mathematical concepts better.

Rodríguez-Nieto (2020) explored the connections between measurement systems used in daily practices such as bun making, kites and flying crates, and agriculture, among others, and proposed a model of internal and external connections for the study of measurement systems related to the universal activities proposed by Bishop (1999). Also, Rodríguez-Nieto (2021) analysed the ethnomathematical connections between geometric concepts (e.g., circumferences, the circle, and the cylinder) evidenced in the elaboration of the tortilla in Mexico. These connections allowed him to make a teaching proposal for the treatment of the circumference, circle, and cylinder in GeoGebra, connecting the sociocultural environment of students and teachers. In addition, Mansilla-Scholer et al. (2022) investigated the mathematical knowledge in the practice of Chilean fishermen who use non-conventional measures such as the fathom. Rodríguez-Nieto et al. (2022) investigated the ethnomathematical connections between the shapes of cheeses and musical drums in Mexico, based on the relationships between geometric concepts such as the cylinder, circumference, circle, and the truncated cone, among others, counting processes and arithmetic operations in the marketing of products. Therefore, these results are an input for the teacher to approach with his students the theme of the cylinder using the cheese or the drum in a similar way, i.e., making external connections based on internal connections.

Rodríguez-Nieto and Alsina (2022) articulated ethnomathematics, STEAM education, and the globalised approach to analyse an empirical phenomenon about the connections in the daily practice of a group of artisans dedicated to the elaboration of papalotes, cabinets, masks and crates, agriculture, and masonry. In addition, they found that intradisciplinary connections allow the introduction of mathematics as an integrated whole and not as a set of isolated knowledge: the interdisciplinary connections between knowledge of different disciplines (STEAM) that give mutual feedback and the connections between the mathematics practised by cultural groups with institutionalised mathematics that are, at the same time, globalised, because they relate mathematics with the sociocultural context (ethnomathematics). Rodríguez-Nieto et al. (2022) explored the ethnomathematical connections and ethnomodelling processes in the elaboration of Mexican spinning tops and tacos, where they remarked that the definition of ethnomathematical connection shares the features of ethnomodelling, for example, the emic is local mathematics or mathematics practiced by a certain cultural group and the etic refers to institutional mathematics. Additionally, they reported that the trope of meat has a paraboloid shape, and the tortilla has a disc or cylinder shape, both of which have their volume, area, and applications to be considered in math classes at school and university.

Mania and Alam (2021) reported that there is little evidence of research about teachers and the ethnomathematical approach. However, in their study, they found that the teachers positively perceived the ethnomathematical approach, where they applied several buginese and makassar traditional foods and games to teach mathematics and decided to include the results in the curriculum. In addition, they acknowledged that there are high chances for students to learn mathematical content easily and to recognise their own culture, according to Indonesia’s national curriculum. Another study emphasised raising the awareness of mathematics teachers about ethnomathematics and, in their results, showed that teachers in Zimbabwe defined ethnomathematical approaches differently and were aware of live ethnogeometry in cultural experiences that could be integrated into the teaching and learning of geometry (Sunzuma & Maharaj, 2020).

Some investigations have problematised the results obtained through ethnographic and ethnomathematical studies with in-service teachers. For example, Oliveras (1996) and Oliveras and Gavarrete (2012) created a training model for indigenous teachers with an ethnomathematical approach. Likewise, Knijnik and Meregalli-Schreiber (2012) revealed that teacher education courses allow students to inquire and reflect on their school maths experiences and their appropriation of the language game, taking culture into account. Gavarrete and Albanese (2015) explored ethnomathematics and cultural signs in teacher education, stating that "working with ethnomathematics promotes the teacher's creativity for the development of a mathematics curriculum in connection with the sociocultural environment" (p. 299).

Blanco-Álvarez et al. (2017) proposed a structured model to analyse the sociocultural aspects used by the mathematics teacher, emphasising: What features should the school curriculum have based on an ethnomathematical perspective? What should be the didactic-mathematical knowledge of the teacher in order to observe this curriculum? What are the features of initial and continuing education courses from an ethnomathematical perspective? What theoretical frameworks and methodologies did the investigations use?

Sunzuma and Maharaj (2020) identified ethnogeometry and definitions based on teachers' ethnomathematics. However, Mania and Alam (2021) reported that there is little evidence of investigations about teachers and mastery of the ethnomathematical approach. This fact motivates further research on what mathematics people use in daily life and how tasks can be designed to help students at different educational levels and pre-service and in-service teachers understand mathematical issues.

The literature reviewed revealed work on ethnomathematics, ethnomathematical connections, and mathematics; however, we did not identify work on ethnomathematical connections linked to the development of a training workshop for teachers of indigenous mathematics. In this sense, it is pertinent to investigate this topic because, without understanding mathematical concepts and their functionality, teachers may lack adequate education to involve their students in mathematical connections, reasoning, and problem solving as basic skills (Eli et al., 2013). *For this reason, this research aims to analyse the ethnomathematical and mathematical connections established by a teacher when posing and solving mathematical problems in a daily context.*

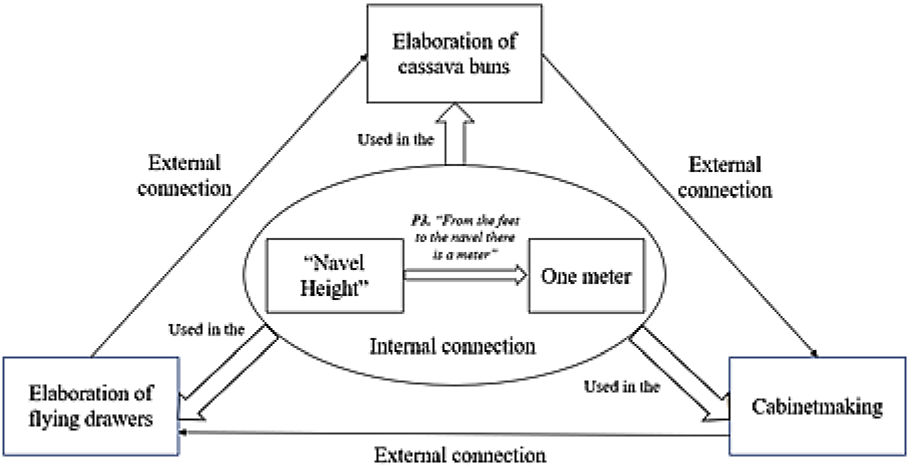
THEORETICAL FOUNDATION

Ethnomathematical connections

For this research, we define ethnomathematical connection as the relationship between the mathematical knowledge developed by people in daily practices and the institutional or public mathematics proposed in the curricular resources (Rodríguez-Nieto, 2021).

Figure 1

Example of internal and external connections



Rodríguez-Nieto (2020) has characterised ethnomathematical connections as internal, external, and of ethnomathematical meaning. Internal connections are "the relationships that a subject makes between units of measurement (conventional or non-conventional) of the same measurement system used in an everyday practice, considering equivalences and conversions" (Rodríguez-Nieto, 2020, p. 12), and an external connection "is promoted when a unit of measure (conventional or nonconventional) is used in a similar way in different measurements systems of different daily practices" (Rodríguez-Nieto, 2020, p. 26). The connection of ethnomathematical meaning "is identified when a person attributes a meaning to a mathematical concept or object establishing an expression-content relationship, emitting what a cultural object or artifact, a measure, a design, among other universal activities, means to him according to daily practice" (Rodríguez-Nieto et al., 2022). For example, three people use the "navel height" similarly in different daily practices (bun making, crate making, and cabinetmaking) and attribute to it a meaning such as the non-conventional, recursive, and cultural measure that equals one metre (Figure 1).

Extended Theory of Mathematical Connections

In this research, we understand a mathematical connection as "a cognitive process through which a person relates two or more ideas, concepts, definitions, theorems, procedures, representations and meanings with each other, with other disciplines or with real life" (García-García & Dolores-Flores, 2018, p. 229). The literature on mathematical connections reports two groups: intra-mathematical and extra-mathematical (Table 1). Intra-mathematical connections "are established between concepts, procedures, theorems, arguments and mathematical representations among themselves" (Dolores-Flores & García-García, 2017, p. 160), and extra-mathematical connections "establish a relationship of a mathematical concept or model with a problem in a (not mathematical) context or vice versa" (Dolores-Flores & García-García, 2017, p. 161).

Table 1

Mathematical Connection Categories

Categories	Mathematical Connections
Oriented towards instruction	It refers to a concept C based on two or more previous concepts, A and B, that a student should understand. Moreover, these connections can be identified in two ways: 1) the association of a new topic with prior knowledge, and 2) the mathematical concepts and procedures connected are considered prerequisites or skills that students must master before developing a new concept (Businskas, 2008).
Modelling	They are relationships between mathematics and real life and are evidenced when the individual solves non-mathematical or

	application problems where they must propose a mathematical model or expression (Evitts, 2004).
Procedural	They appear when the individual uses rules, algorithms, or formulas to complete or solve a mathematical task. These mathematical connections are formal, A is a procedure used to work with concept B (García-García & Dolores-Flores, 2019). Likewise, "it includes the explanations or arguments that a student offers to use those formulas and how he applies them to achieve a result" (García-García & Dolores-Flores, 2019, p. 5).
Different representations	Identified when the individual represents a mathematical concept using alternate or equivalent representations (Businskas, 2008; García-García & Dolores-Flores, 2019). The equivalents are transformations of representations made in the same representation or register (algebraic-algebraic). Alternate representations refer to representations where the register in which they were formed is modified.
Feature	Identified when the individual manifests features of mathematical concepts or descriptions of their properties in terms of other concepts that make them different or similar to other concepts (Eli *et al*., 2011; García-García, 2019; García-García & Dolores-Flores, 2019).
Reversibility	It appears when an individual starts from concept A to arrive at concept B and inverts the process, starting from B to return to A (García-García & Dolores-Flores, 2019).
Part-whole	These connections manifest when the individual establishes logical relationships in two ways (general-particular and inclusion). The generalisation relationship is of form A is a generalisation of B, and B is a

	particular case of A (Businskas, 2008; García-García & Dolores-Flores, 2019). The inclusion relationship occurs when one mathematical concept is contained in another (García-García, 2019).
Meaning	They are identified in two ways. 1) When the mathematical connection refers to the moment the individual assigns meaning to a concept, distinguishes it from another concept and what it represents by finding it as a definition constructed for a mathematical concept. 2) The mathematical connection between meanings is evidenced when the individual connects meanings attributed to a concept to solve a problem (García-García, 2019).
Implication	Identified when a concept P leads to another concept Q through a logical relationship ($P \rightarrow Q$) (Businskas, 2008).
Metaphorical	Rodríguez-Nieto et al. (2020) report that metaphorical connections are understood as the projection of the properties, features, etc., of a known domain, in order to structure another lesser-known domain.

METHODOLOGY

This research was carried out with a qualitative-ethnographic methodology (Cohen et al., 2018; Restrepo, 2016) in seven stages linked to the implementation of a workshop (Figure 2).

Participants and context

The participant in this research was an indigenous teacher from the district of Sibarco, Colombia, noted for his work in helping students of different school levels to carry out their homework and/or school activities related to mathematics. Table 2 presents the teacher's personal information collected in stage 2 of the operation of the workshop.

Figure 2

Process to follow in the investigation

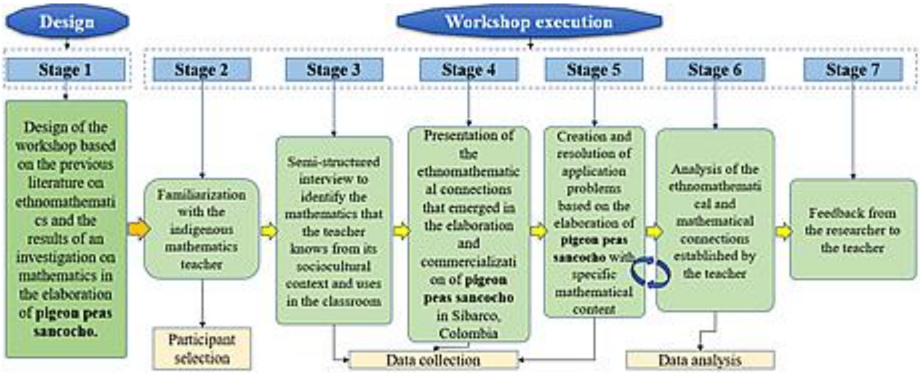


Table 2

Information regarding the participant

Name	Age:	Profession	Years of experience	Schooling	Racial group
Francisco	46 years old	Math teacher	20 years	Indigenous teacher specialising in indigenous educational systems (SEIP); senior teacher, human training; technician in psychological services; knowledge of environmental care and public administration.	Mokaná

Data collection

To collect the data, a semi-structured interview based on a workshop was applied, as outlined in Figure 2. The researchers designed the workshop to explore the mathematics used by an indigenous teacher and promote in him other mathematics immersed in his same sociocultural context as the mathematics used by storekeepers in the elaboration of the pigeon peas sancocho (Rodríguez-Nieto & Escobar-Ramírez, 2022), see Figure 3. In the middle of the interview, the researchers spoke with the indigenous teacher not only to remark on the mathematics he teaches but also on his way of living, acting, and his work experience; this was done from the ethnography (ethnos (people, peoples) and graph (writing, description)), which allows appropriate people's culture and report events as they develop in reality (Restrepo, 2016).

Figure 3

Phases of preparation of pigeon peas sancocho

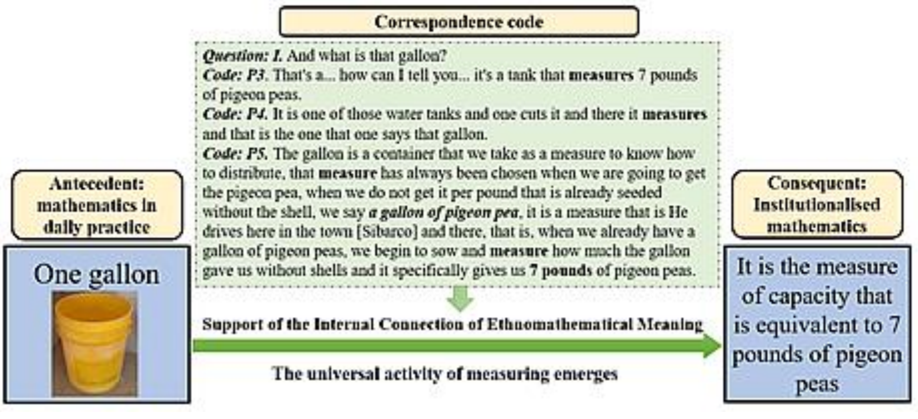
The information presented in Figure 3 helped activate the mathematical knowledge of the indigenous teacher about the elaboration of the pigeon peas sancocho and, in turn, served as an input for posing new mathematical problems from an ethnomathematical connections perspective. It should be mentioned that the collection of information was obtained through video recorders, field notes, and a laptop where the performance of the mathematics immersed in this daily practice was shown.

Data analysis

The data were analysed following the detailed qualitative analysis method proposed by Hernández et al. (2014) with adaptations by Rodríguez-Nieto (2020) and activating the theoretical foundation of mathematical and ethnomathematical connections. In this way, different moments of analysis are presented: 1) the interviews carried out in the workshop were transcribed and kept as texts. 2) some codes or keywords were identified from which ethnomathematical or mathematical connections are inferred. 3) groups of ethnomathematical and mathematical connections were recognised that refer to topics (definite appointment of connections) based on universal activities (counting, designing, measuring, playing, explaining, and locating). Finally, 4) a report of connections was made, which is included in the results section. By way of example, a structured ethnomathematical connection is shown based on an interview applied to some cooks (Rodríguez-Nieto & Escobar-Ramírez, 2022) who stated that the gallon is a measure equivalent to seven pounds of pigeon peas (Figure 4).

Figure 4

Example of the ethnomathematical connection (Rodríguez-Nieto & Escobar-Ramírez, 2022)



It should be noted that, for the cooks and farmers of the district of Sibarco, the gallon is an non-conventional measure of capacity considering the pigeon peas bean in its shell, equivalent to seven pounds of pigeon peas (pigeon peas bean without its shell), and is different from the conventional or universal unit of measure "gallon" which is equivalent to 3,785 litres (Rodríguez-Nieto & Escobar-Ramírez, 2022).

Simultaneously, we identified some mathematical connections (intra-mathematical and extra-mathematical) that differ from the ethnomathematical connections because they do not originate in the teacher's or the cook's daily practice but are generated in the resolution of the tasks that he proposes in the interviews. For example, connections of the modelling type were identified when the teacher modelled the volume of the totuma with the expression $v = \frac{4}{3}\pi(a * b * c)$, and then a connection of the procedural type was identified in order to find the volume starting with the substitution of the values of a, b, and c. Other mathematical connections evidenced are of the alternate representations type when he relates the graphical representation of the totuma with an ellipsoid and its symbolic representation.

RESULTS AND ANALYSIS

The methodological section shows the design and implementation of a workshop developed into seven stages, which guide this results section highlighting a special element, the mathematics teacher's contribution to the practice. For the passages of the transcript, we took into account the following tags: Researcher (R) and, Mathematics Teacher (MT).

First stage

The design principles of the workshop were: 1) researchers' review of the literature to obtain information for implementing a workshop based on the results of an ethnographic and ethnomathematical study to contribute to teaching practice, 2) promote training environments for a mathematics teacher based on ethnomathematical and mathematical connections where the mathematics used in the daily practices of his environment and the mathematics proposed in the curricular or institutionalised materials are linked, and that 3) in the future, the teacher implements the tasks proposed in the workshop with his students without disengaging from the school grade.

Second stage

After the teacher agreed to participate in the workshop, we got acquainted with him. He was told about the research, its main objective, and how it was being conducted (Figure 5).

Figure 5

Presentation of the research project to the indigenous teacher

Third stage

We interviewed the teacher and asked him about the type of mathematics he had in the context of the Sibarco district and which one he used with his students. In this way, we confirmed that the indigenous teacher knows the mathematics present in daily practices frequently carried out in the village, and he told us about each of them during the workshop. In the interview, he was asked to design problem situations with the mathematics that he knows, and the problems he posed were related to the calculation of areas and perimeters that the activity of fencing land involves (Figure 6).

Figure 6

The teacher explains the mathematics he knows in the context of Sibarco.

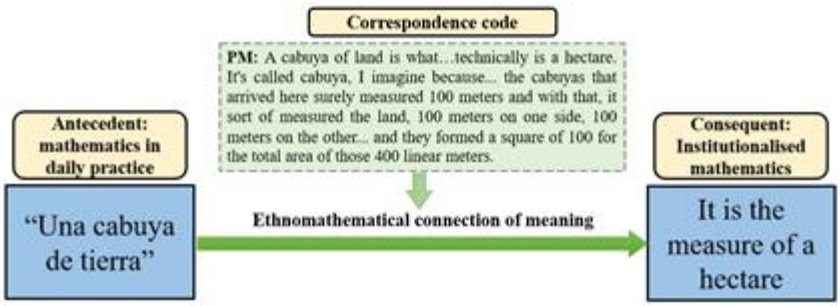
In the interview, the teacher establishes a connection of ethnomathematical meaning; for example, he states that a "*cabuya de tierra*/cabuya of land" is equivalent to one hectare and supports it with the concept of perimeter and area (see the excerpt from the transcript and Figure 7).

I: *For example, when you say a cabuya, what do you mean?*

PM: *A cabuya is what... technically, it is a hectare (...)*

Figure 7

Connection of meaning between cabuya de tierra and hectare

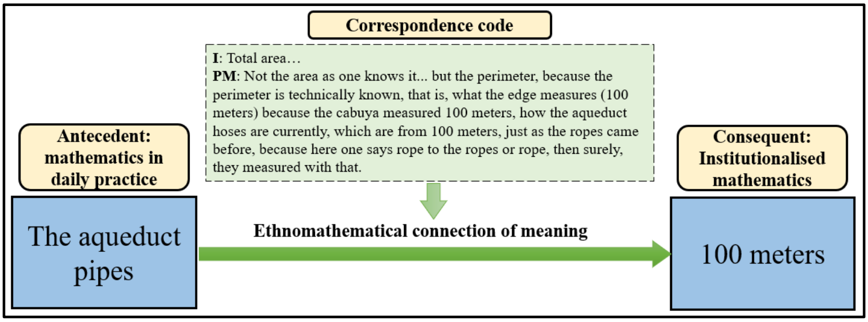


The professor relates the cabuya with the pipes of the aqueduct; he mentions that these are equivalent to the measure of the cabuya, i.e., 100 metres (see the excerpt from the transcript in the correlation code and Figure 8).

In his explanation, he introduced new terms or popular non-conventional measures, such as the fathom, and stated that this was equal to 2 metres. The teacher asked, "Why the fathom?" He answered by stating that this measure avoided dividing by two an irregular piece of land (see extracts from the transcript and Figure 9). The teacher states that the fathom is important when measuring the areas of irregular pieces of land.

Figure 8

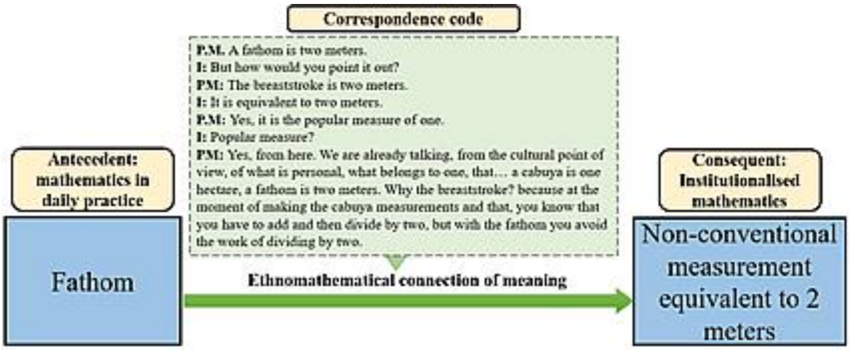
Ethnomathematical connection between aqueduct pipes and metres



PM: *So, when an area is irregular, the two parts are added together, this irregular part (points to an object) does not have the same measure as that, so it is added. For example, if there are 5 metres here and 2 metres there, you add what these two parts measure (points to the object), and it must be divided by two, if it is in metres, but if you use the fathom, you perform the addition and multiplication, and you get the extension of the land right away.*

Figure 9

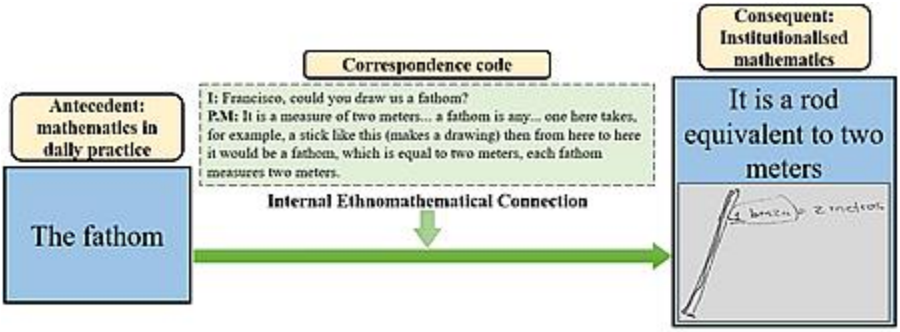
Ethnomathematical connection between fathom and metres



When the teacher designed a problem situation that involved calculating the area of a plot in terms of fathoms (Figure 10), the following excerpt from the interview was taken into account.

Figure 10

Internal ethnomathematical connection and of the different representations type



It should be noted that, in the correlation code of the ethnomathematical connection of Figure 10, a mathematical connection of the type of different alternate representations can be inferred simultaneously since the term fathom (verbal) is related to its graphic representation and is described as equivalent to a two-metre rod.

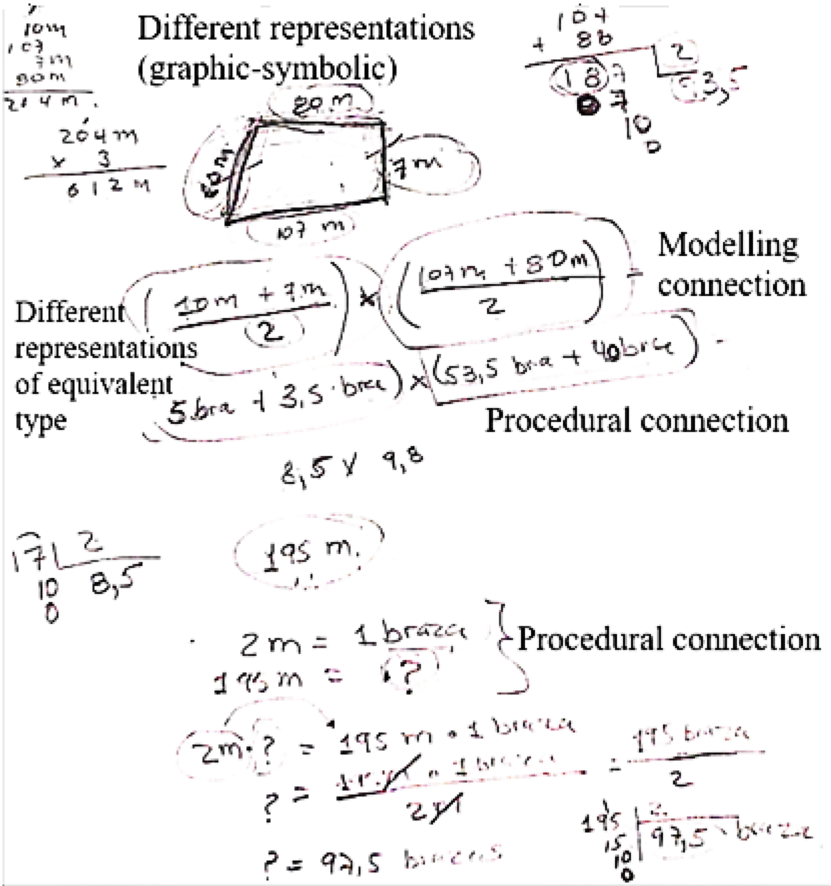
On the other hand, the teacher created a problem situation to calculate the area of an irregular plot of land. He began by drawing a connection of the type of different representations; then, he established a connection of the modelling type where he mathematically represented the verbal information as a mathematical model or formula to calculate the area. Finally, based on the above, the teacher established connections of the procedural type involving arithmetic operations, equivalences and conversions (see the excerpt from the transcript and Figure 11).

> PM: *As for the irregular measurements, I was saying this: suppose you have here... for example, we are going to measure 10 metres on this side, 7 metres (emphasis on the "m" of metres) on this other side, here you can assume that there are 107 metres and over here 80 metres, they are all assumptions*

(the teacher is drawing as he speaks). So, what happens? To be able to calculate the area, you add the 10 metres plus the 7 metres, all this divided by two, then you add the 107 metres plus the 80 metres, all this divided by two. And here we find what we want, and what about the fathoms? The fathoms... according to them, you do not need to divide by two because you would have... 5 fathoms plus 3.5 fathoms here, so that would be...

Figure 11

Problem situation with the calculation of an area in metres and fathoms

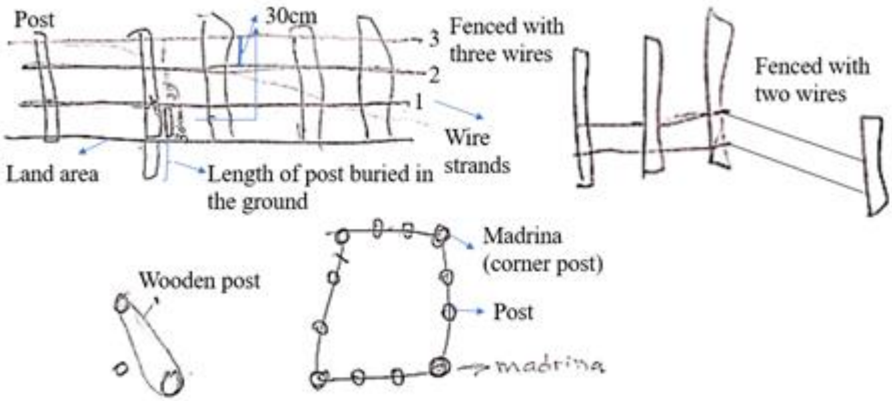


On the other hand, the professor stated that if one knew the perimeter of a piece of land, then one knew the amount of wire necessary to make a fence. Then, the teacher posed a new problem situation and drew a fence (Figure 12 and the excerpt from the transcript), evidencing the design of a problem situation involving the wire used in the fence and the perimeter:

> I: *What mathematical problem would you formulate for the student?*
>
> PM: *For example, if we take into account the previous problem, then, first of all, we would have to know the perimeter, and how do we calculate the perimeter? I have already told you that by adding all the existing distances (he points them out in the drawing), you add the 10 metres plus the 107 metres plus 7 metres plus 80 metres, we add them up, and the result is 204 metres of wire that would be needed to do this (points out in the drawing) but for a single thread of wire. If we put three wires in the fence, it means that we are going to multiply these 204 metres by 3, resulting in that 612 metres of wire would be needed to make this fence around the perimeter of this piece of land with three threads of wire.*

Figure 12

Fencing a piece of land



However, emphasising the ethnomathematical connection, it is evident that the teacher, when referring to the measurements of land and its respective fence, used geometric concepts such as the perimeter, which he put into use to calculate the amount of wire needed to fence the piece of land. Therefore, the ethnomathematical connection had a precedent in considering the perimeter as "the sum of all the distances of the four sides of a plot of land" and a consequent in the Derechos Básicos de Aprendizaje (DBA) (MEN, 2016) for the third grade. In statement 4, they affirmed that the possible relationships between the amounts of the area and the perimeter of flat figures such as quadrilaterals should be described and argued, for example, 1) make decisions about the magnitudes to be measured (area or length), 2) cover surfaces of different flat figures, 3) measure and calculate the area and perimeter of a rectangle, 4) explain how figures of an equal perimeter can have different areas).

Fourth stage

Figure 13

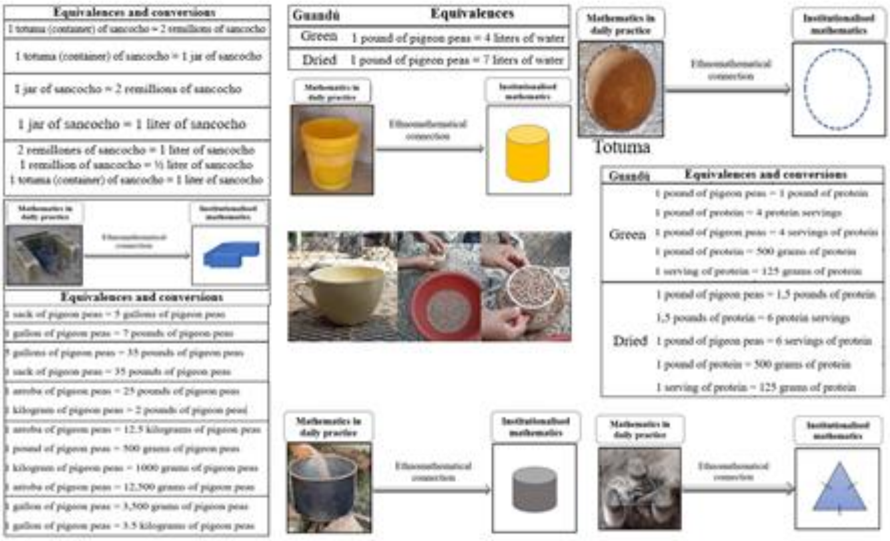
Presentation of the ethnographic results to the teacher

We give the indigenous teacher the results from an ethnographic study of the preparation of the pigeon peas sancocho and associated handicrafts (Rodríguez-Nieto & Escobar-Ramírez, 2022) via a PowerPoint presentation (Figure 13), which showed the mathematics used by traditional cooks.

Then, the teacher verbally assessed the information provided by the researchers in the PowerPoint presentation, highlighting the importance of this mathematics investigation in the preparation of the pigeon peas sancocho (Figure 14) and acknowledging that the results can be turned into math problems and brought into the classroom to improve mathematics teaching and learning.

Figure 14

Some mathematics revealed in the elaboration of the pigeon peas sancocho (Rodríguez-Nieto & Escobar-Ramírez, 2022).



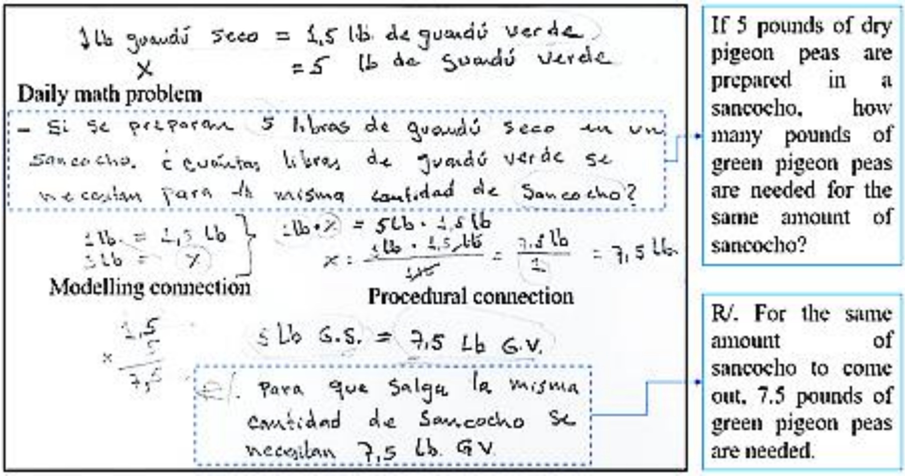
Fifth stage

At this stage, the indigenous teacher was asked to set problems in the context of the preparation of pigeon peas sancocho. Faced with this situation, the teacher created mathematical problems where conversions were made between conventional and non-conventional units of measure, proportions, ratios, and area volume, among others. In this context, the teacher proposed a first conversion problem involving the number of pounds of green or dry pigeon peas required to make a sancocho for a certain number of people (see excerpts from the transcript and Figure 15). In addition, to solve this problem, the teacher used modelling and procedural connections, for he constructed and used the rule of three, initially indicating whether it was direct or inverse, and then expressed the quantities of each magnitude in the same unit. Finally, he calculated the value of *x*.

> I: *You can set a problem in the context of the pigeon peas sancocho.*
>
> PM: *A conversion problem would be… with five pounds of green pigeon peas, I prepare a sancocho; how many pounds of dry pigeon peas would I need for the same amount of sancocho?*

Figure 15

Proportionality problem for the same amount of sancocho

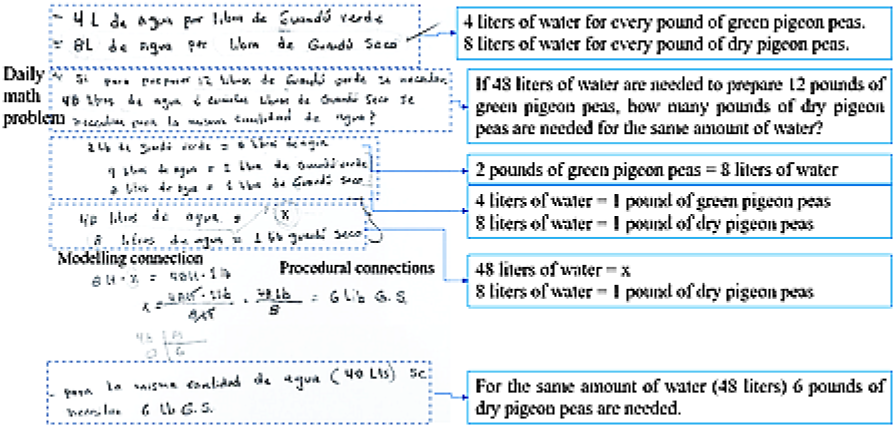


For a second problem in the context of pigeon peas sancocho, the indigenous teacher considered the amount of water per pound of pigeon peas and set a problem, as shown in Figure 16 and the transcript excerpt. It should be noted that the teacher solved the problem using the rule of three, following modelling and procedural connections. The modelling connection was useful for constructing the mathematical expression that helped him find the ratio between the data and the procedural connection for performing the operations in the established ratio.

> I: *What other problem emerges at this time in the context of the elaboration of the pigeon peas sancocho?*
>
> PM: *The amount of water per sancocho.*
>
> I: *Well, the amount of water per pound of pigeon peas.*

Figure 16

Proportionality problem with litres of water per pound of pigeon peas



In the case of the ethnomathematical connection, this type of problem is related to the curricular content encompassed in statement 5, which refers to estimate and measure the volume, capacity, length, area, and weight of objects or the duration of events as part of the process to solve different problems (MEN, 2016). Here, we highlight the comparison of objects according to their length, area, capacity, volume, etc.; the estimation and measurement of length, area, volume, weight and time according to the situation's needs with conventional or non-conventional measuring instruments; problem situations focused on packing objects in boxes and varied containers and calculus of the amount that could fit in. To this end, he took into account the shape and volume of the objects to be packed and the capacity of the container that should be used.

In addition, the MEN (2006) states that students must model, solve, and formulate problems on direct and inverse proportionality and the product of measures.

Another problem posed by the teacher deals with a situation for mathematics addressed in the early years of primary education that involves pigeon peas to perform counting and arithmetic operations (see the transcript extract and Figure 17).

> I: *For children, could you use pigeon peas in maths?*
>
> PM: *Well, let's say that children are going to conceptualise it from an early age: that pigeon peas is something that is used for nourishment.*
>
> I: *But how would you, as a teacher, use it?...*
>
> PM: *Thus… for example, this symbol (indicates it in the sheet of paper) means this amount of pigeon peas, and adding this amount here. This is a symbol that is used universally*.
>
> I: *There you have it in a numerical, graphic or pictorial way, and how would you formulate it to tell it to someone? That you should have to tell it to someone.*
>
> PM: *I mean, like this... for example, bring me five beans of pigeon peas, he should bring the five beans of pigeon peas, for example, if you ask the child how much is five plus three, I tell him to bring me five beans of pigeon peas, he brings them; I tell him to count them, he counts them, well, this is number five; plus three, bring me three beans of pigeon peas, he brings them, tell me how many beans of pigeon peas there are, he counts them, now I tell him to mix them and I ask him: how many do you have? It should be a simple problem for him to learn in order to solve this "conflict" that it sometimes appears to be, regarding the abstract symbolism. And that he understands that this is something that is written down on paper, but that it actually exists*.
>
> I: *How would you do it in the cultural context?*
>
> PM: *If Pedrito has 48 beans of pigeon peas and his mother gives him 16 beans of pigeon peas. How many beans of pigeon peas does Pedrito have?*

Figure 17

Addition problem with beans of pigeon peas

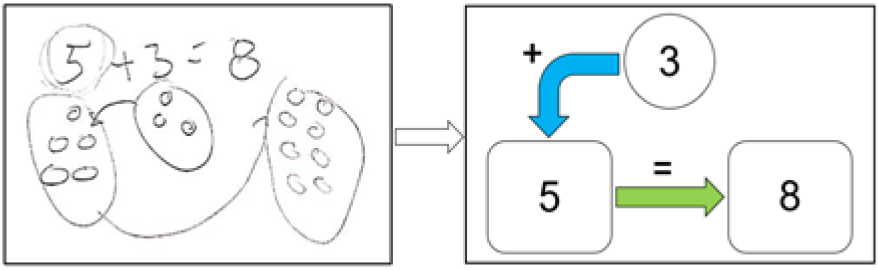


Figure 17 shows that the teacher proposed an addition problem stated verbally, connected in its resolution with pictorial representations. This type of problem is of a stage and destructures the semantic of change 1 with the unknown in the final quantity, as evidenced in the work of Rodríguez-Nieto et al. (2019). Moreover, an outline of the problem situation that allows organising the information or data of the problem and the action that must be done is presented. These types of problems are relevant because they coincide with those suggested in the DBAs (MEN, 2016), where students use arithmetic operations and solve problems regarding collecting, removing, and completing that involve the number of items in a collection or the measurement of quantities. Likewise, primary school pupils must count and solve varied problem situations with addition and subtraction operations in problems whose structure can be a + b = ?, a + ? = c, or ? + b = c. Similarly, the MEN (2006) suggests that students solve and formulate problems in additive situations of composition, transformation (change), comparison, and equalisation.

Lastly, the professor proposed a problem related to the calculus of the totuma volume, stating that the pumpkin is not completely spherical and the totuma is not completely round, given the characteristics of the totumo (Figure 18).

Figure 18

Representation of the pumpkin and the totuma

Then, the teacher said that in the pumpkin, the representation of an ellipsoid is observed, and he set a problem that required calculating the volume of the pumpkin (Figure 19). To solve the problem, he began by representing the ellipse in a drawing, and he measured the length of the axes (major and minor). Then, the teacher established mathematical connections between the different representations type in his drawing process.

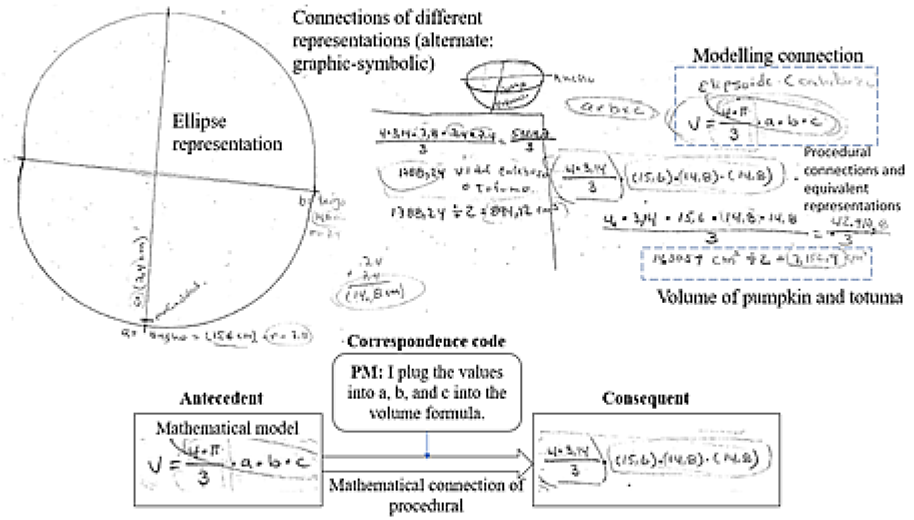
Figure 19

Construction of the ellipse using the edge of the totuma

Now, in order to find the volume of the totuma, the teacher first established modelling connections when considering the formula that allowed him to calculate the volume of an ellipsoid $\left(v = \frac{4}{3}\pi(a * b * c)\right)$, and then the procedural-type connection was revealed when he started substituting the values of a, b, and c in the algebraic-symbolic expression of the volume. Other mathematical connections revealed are those of the alternate-representations type, when he relates the graphical representation of the totuma with an ellipsoid and its symbolic representation. And the professor made connections of the alternate-representations type between the graph of the ellipsoid and equivalents when he wrote: $\frac{4*3{,}14}{3} * (15{,}6)(14{,}8)(14{,}8) = \frac{4*3{,}14*15{,}6*14{,}8*14{,}8}{3}$. He performed the multiplications and obtained the volume of the pumpkin $(14.305{,}9cm^3)$. Finally, he divided the volume of the pumpkin by 2 and found the volume of a totuma $(7.152{,}9cm^3)$ used to pack the pigeon peas sancocho (Figure 20).

Figure 20

Approximate calculation of the volume of a totuma



In terms of the ethnomathematical connections, it is evidenced that the mathematics used by the teacher based on the elaboration of the pigeon peas sancocho are related to the DBA (MEN, 2016) of the fourth grade of primary school, especially in statement 4, where it is suggested to characterise and compare measurable attributes (e.g., capacity of the containers, temperature, and mass) vis à vis procedures and measuring instruments. This means that the student must measure the capacity and the mass from comparisons with the capacity of containers of different sizes and with packages of different masses, respectively (litres, centilitres, gallons, and bottles, etc., for capacity; grams, kilograms, pounds, *arrobas*[1], etc., for mass). Statement 5 specifies that the student must choose standardised and non-standardised instruments and units to estimate and measure length, area, volume, capacity, weight and mass, duration, speed, and temperature, make calculations to solve problems, and use direct and inverse proportionality relationships.

In the case of the DBAs for the fifth grade, the statements referred to establishing relationships between surface and volume, where students must find their measurements of area and volume with direct and indirect measurements. In the sixth grade, the teacher is encouraged to propose situations where the student represents and constructs 2D and 3D shapes based on measuring instruments and that he calculates areas and volumes. Finally, the DBAs for the eighth grade of secondary education DBAs (MEN, 2016) point out different strategies to find the volume of regular and irregular objects to solve problems in mathematics and other sciences (e.g., relating capacity units with volume units (litres, dm^3, etc.) in the resolution of a problem and estimating volume quantities with standardised and non-standardised units).

Sixth stage

At this stage, the ethnomathematical connections established by the teacher before he learnt about the ethnographic input of the workshop were summarised. For example, he made ethnomathematical connections of meaning and internal type that involved representations of the fathom and drawings of the plots of land to calculate the area and the perimeter to obtain the amount of wire necessary for a fence. After providing evidence that supports the ethnomathematical connection, the teacher performed the essential procedures where he used the mathematical connections of different representations,

1 T.N.: a Spanish and Portuguese unit of weight of varying value, equal to 25.37 pounds avoirdupois (9.5 kilograms) in Mexico and 32.38 pounds avoirdupois (12 kilograms) in Brazil.

meaning, procedural, and especially that of the modelling type as a starting point for the resolution of the problems he proposed. It should be noted that, in this article, the connections were shown simultaneously with the setting and resolution of problems; otherwise, information would have had to be repeated.

Seventh stage

To conclude the workshop, we carried out a feedback process assessing the mathematics known by the indigenous teacher in the context of Sibarco, especially the mathematics explored in the elaboration of the pigeon peas sancocho. In this sense, we acknowledged that the results obtained in the ethnographic phase contributed to the teacher's practice, the appropriation of geometric notions, and the existence of a measurement system of their own for the preparation of traditional Sibarco food. The workshop ended after the feedback (see excerpt from the transcript where the teacher, after posing and solving the problems, reflected on the importance of contextualised problems from an ethnomathematical connections approach).

> I: *What was the contribution of this to you? What learning, what knowledge...? And/or is this actually beneficial for students in elementary and high school.*
>
> PM: *I say that it is beneficial because they are learning both practices, the daily one, and at the same time, they are using rules, techniques that are not that empirical to solve certain problems, certain everyday situations...*

CONCLUSIONS

In this research, we identified the ethnomathematical and mathematical connections recognised in the mathematical activity of a teacher in the context of a workshop. In this context, the teacher proposed problems related to the amount of wire needed for a fence around a plot of land involving area and perimeter measurements using non-conventional units of measure such as the fathom. After having explained to the teacher the mathematics involved in the pigeon peas sancocho, he posed and solved problems of proportionality, conversions, and pumpkin volume, where mathematical connections were identified as of the modelling and procedural types. These results are innovative and different from other studies carried out related to ethnomathematics and ethnomathematical connections (Mosquera et al., 2015; Rey & Aroca, 2011;

Rodríguez-Nieto et al., 2019; Rodríguez-Nieto, 2021; Rodríguez-Nieto et al., 2022) given that it directly contributed to the practice of the teacher who, although having lived in the county for more than forty years, had not noticed the persuasive power of the daily practice of the elaboration of the pigeon peas sancocho to improve mathematics teaching and learning processes.

Some research has problematised ethnographic and ethnomathematical results with in-service teachers; for example, Sunzuma and Maharaj (2020), who identified ethnogeometry and definitions based on the ethnomathematics that teachers have. Oliveras (1996) and Oliveras and Gavarrete (2012) investigated ethnomathematics and teacher education and created a model for applying ethnomathematics in teacher education for indigenous contexts in Costa Rica. Likewise, Knijnik and Meregalli-Schreiber (2012) revealed that teacher education courses allow students to inquire and reflect on their school math experiences and their appropriation of the language game, taking culture into account. Gavarrete and Albanese (2015) explored ethnomathematics and cultural signs in teacher education, expressing the importance of the connections between culture and the mathematics taught in the classroom. Blanco-Álvarez et al. (2017) proposed a structured model to analyse the sociocultural aspects used by the mathematics teacher, emphasising the characteristics of initial and continuing education courses from an ethnomathematical perspective. However, Mania and Alam (2021) found little evidence of investigations about teachers and mastery of the ethnomathematical approach, which motivates further research on what mathematics people use in daily life and how tasks can be designed so that students at different educational levels and pre-service and in-service teachers can understand mathematical issues.

We acknowledge that in the literature, there are several models to study the teachers' mathematical and ethnomathematical knowledge. However, this work highlights the extended theory of mathematical and ethnomathematical connections to explore connections made by the teacher, which contributes to the students' problem-solving processes, where permanent relationships are established between sociocultural aspects, teachers, students, and the mathematical curriculum. We are confident that this work can be applied to the design of assignments and promote connections in the classroom to ensure mathematics learning.

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AUTHORS' STATEMENTS OF CONTRIBUTION

CARN and YCER conceived the idea presented and developed the theoretical and methodological section. CARN and YCER collected the data and analysed them. VFM and AAA assisted in the organisation of the article and validated the information and results of the mathematical and ethnomathematical connections. All authors actively participated in the discussion of the results, reviewed, and approved the final version of the article.

DATA AVAILABILITY STATEMENT

The data supporting the results of this research will be made available by the appropriate author [CARN] upon reasonable request.

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