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Knowledge for Teaching Geometry in Brazilian Primary Education: A Study Over Two Scholar Manuals by Theobaldo Miranda Santos

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ABSTRACT

Background: Studies about teacher education from a historical perspective have been outstanding in the field, above all, those that investigate teacher's knowledge. In the data collection, we identified bibliographic research on knowledge disclosed by Theobaldo Miranda Santos, but most of them do it with a focus on mathematical fields of arithmetic. Therefore, we are going to study the geometry-related knowledge. **Objective:** To analyse from a historical perspective the possible knowledge for teaching geometry in primary education present in two school manuals by Santos: *Noções de Didática Especial* (1960) and *Metodologia do Ensino Primário* (1952). **Design:** This is a qualitative and documentary study. The analysis of the materials was based on cultural history and considered the knowledge for teaching. **Setting and Participants:** On the Repositório Institucional da Universidade Federal de Santa Catarina, we accessed the two books analysed in this work. Under the influence of the new school movement, subjects such as Didactics and Teaching Methodology started to be introduced in teacher education courses, which justifies the selection of these books. **Data collection and analysis:** We seek to identify knowledge primary school teachers must incorporate into their teaching practices to consolidate geometry learning. **Results:** The investigated handbooks have features oscillating between the intuitive and the new school perspectives. **Conclusions:** Santos introduced knowledge for teaching geometry in primary education with influences from the traditional and the new school that could influence and guide the practice of teachers who taught geometry in primary education.

Keywords: knowledge for teaching geometry; Theobaldo Miranda

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Santos; scholar manuals; Brazilian primary education; cultural history.

Saberes para ensinar geometria no ensino primário brasileiro: Um estudo sobre dois manuais pedagógicos de Theobaldo Miranda Santos

RESUMO

Contexto: Pesquisas que estudam a formação de professores sob a perspectiva histórica têm ganhado espaço em eventos da área, sobretudo, aquelas que investigam os saberes associados à constituição desses profissionais. No levantamento bibliográfico, foram identificadas pesquisas voltadas aos saberes docentes divulgados por Theobaldo Miranda Santos. Foi observado considerável foco sobre a aritmética desse autor, por isso, estamos interessados nos saberes relacionados à geometria. **Objetivos**: Analisar, sob a perspectiva histórica, os possíveis saberes para ensinar geometria no ensino primário presentes em dois manuais escolares de Santos: Noções de Didática Especial (1960) e Metodologia do Ensino Primário (1952). **Design**: Este é um estudo qualitativo e documental. A análise dos materiais foi feita com base na história cultural e considerando o saber para ensinar segundo Hofstetter e Schneuwly (2017). **Ambiente e participantes**: No Repositório Institucional da Universidade Federal de Santa Catarina, temos acesso aos dois livros analisados neste trabalho. Sob influência do movimento escolanovista, disciplinas como Didática e Metodologia do Ensino começam a ser introduzidas nos cursos de formação, o que justifica a seleção desses livros. **Coleta e análise de dados**: Buscamos identificar saberes que o professor do ensino primário deve incorporar às suas práticas para que a aprendizagem de geometria se consolide. **Resultados**: Os manuais investigados possuem características que oscilam entre a vaga intuitiva e a escola nova. **Conclusões**: Santos introduziu saberes para ensinar geometria no ensino primário com influências da educação tradicional e da educação nova que puderam influenciar e orientar a prática de professores que ensinavam geometria no ensino primário.

Palavras-chave: saberes para ensinar geometria; Theobaldo Miranda Santos; manuais pedagógicos; ensino primário brasileiro; história cultural.

FIRST CONSIDERATIONS

This article discusses possible knowledge *for* teaching geometry in the early years of elementary education presented in two works produced by the carioca education teacher Theobaldo Miranda Santos: *Metodologia do Ensino Primário* (1952) [Theobaldo Miranda

Santos: Primary Education Methodology] and *Noções de Didática Especial* (1960) [Notions of Special Didactics]. This study is the result of the master's research entitled "Saberes *para* ensinar geometria no ensino primário: Um estudo sobre manuais escolares produzidos por Theobaldo Miranda Santos" [Knowledge for teaching geometry in primary education: A study on school manuals produced by Theobaldo Miranda Santos], defended in 2021, which also worked with some textbooks produced by this author.

We contextualise this research according to Chartier's (1990) cultural history, which studies the construction of a social reality according to the different places and moments that locate it, in addition to the history of books and textbook editions by Choppin (2002, 2004), which points to the importance of sources such as textbooks in research on the history of education. We also admit Julia's (2001) concept of school culture, according to which school knowledge and practices vary according to society and its time, and knowledge *for* teaching by Hofstetter and Schneuwly (2017), a concept that will be covered in greater depth in the following sections.

Theobaldo Miranda Santos's influence on the Brazilian educational scene justifies this work. Throughout the 1930s, Santos wrote dozens of articles for magazines and newspapers in Rio with themes related to education (Almeida Filho, 2008), and many of his materials for teacher education, inspired by a Catholic teaching model in response to the diffusion of the New School movement, were a reference for the establishment of models of school knowledge in the country.

According to research and documents analysed in our bibliographic survey (Marquez, 2020; Monteiro, 2011; Pardim, 2013), some of these works circulated in the former state of Mato Grosso, the region of inquiry of the analysis. In addition, we found several textbooks from his collection *Vamos Estudar? [Let's study?]* produced in special editions for several Brazilian regions, which indicates this writer's considerable influence on the education of teachers in the country. We will see, below, what some researchers have found about Theobaldo Miranda Santos and his works.

BIBLIOGRAPHIC SURVEY

The first searches that guided the writing of this work occurred through four databases: the CAPES Catalogue of Theses and Dissertations, the proceedings of the National Meeting of Mathematics Education (Encontro Nacional de Educação Matemática - ENEM) and the National Meeting of Research in History in Mathematics Education (Encontro Nacional de Pesquisa em História em Educação Matemática - ENAPHEM) and the UFSC Institutional Repository. The CAPES Catalogue and the two proceedings were a basis for searching for dissertations, theses, and scientific articles exploring Santos' works.

Of the works investigated, we identified considerable focus on the philosophy of education manuals (Silva, 2014; Calixto, 2016) or the arithmetic in the books of Theobaldo Miranda Santos, as observed in the articles by Schneider and Costa (2019), Lacava and Costa (2016), and Costa (2015), who focused, respectively, on the use of games in teaching arithmetic, the test of nine and the methodology for teaching arithmetic in some of Santos' works.

We focus here on teaching knowledge focused on primary school geometry since this field of mathematics in Santos' works had not been explored as much as arithmetic. As we are interested in the knowledge for teaching that can be identified in these books, we sought the definition of knowledge given by Hofstetter and Schneuwly (2017), which does not limit the knowledge for teaching to methodologies and didactics but has a basis for understanding this concept.

Articles about Theobaldo Miranda Santos and digitalised versions of some of his productions were extracted from the repository and the search for "Theobaldo Miranda Santos" resulted in the following books: *Aritmética Prática* [Practical arithmetic] (Santos, 1952), *Noções de Didática Especial* [Notions of special didactics] (Santos, 1960), *Metodologia do Ensino Primário* [Methodology of primary education] (Santos, 1952), *Noções de Metodologia do Ensino Primário* [Notions of the methodology of primary education] (Santos, 1962), *Vamos Estudar? 3ª série primária* [Let's study? 3rd grade of elementary school] (Santos, 1966) and *Vamos Estudar? 4ª série primária* [Let's study? 4th grade of elementary school (Santos, 1960). Two of them will be analysed more carefully in the following sections.

KNOWLEDGE *TO* TEACH AND KNOWLEDGE *FOR* TEACHING: KNOWLEDGE THAT CONSTITUTES THE TEACHING PROFESSION

This research adopts cultural history as its theoretical-methodological basis, which, according to Chartier (2002, p. 16-17), "has as its main objective to identify how in different places and moments a particular social reality is constructed, is thought, given to read." Furthermore,

> The representations of the social world thus constituted, although they aspire to the universality of a diagnosis based on reason, are always determined by the group interests that forge them. Hence, for each case, the necessary relationship between the speeches given and the position of those who use them (Chartier, 2002, p. 17).

Thus, in this work, we only have a representation (Chartier, 2002) of a geometry mobilised by Santos, who could guide primary school teachers in carrying out their work. Still, we cannot ignore the school culture established at the time. Considering the period between the 16th and 19th centuries, Julia (2001, p. 10) describes school culture as

> [...] a set of standards that define knowledge to be taught and behaviours to be inculcated, and a set of practices that allow the transmission of this knowledge and the incorporation of these behaviours; norms and practices coordinated to purposes that may vary according to the times (religious, sociopolitical, or simply socialisation purposes.

These norms and practices result from pedagogical devices employed by the professional education body (Julia, 2001): norms, pedagogical projects, and purposes attributed to the school, its teacher's education, school practices, and the contents taught.

Such purposes can be propagated by school manuals, important sources for the historian of education, as they are one of the means of disseminating pedagogy at the time of study; they are cultural products (Choppin, 2002) that carry traces of the past, signs of an education and

a society of a particular time. We take the concepts of knowledge *to* teach and knowledge *for* teaching by Hofstetter and Schneuwly (2017) as we turn to geometry to be taught (knowledge to teach) and knowledge mobilised by primary school teachers (knowledge for teaching) to deal with this work topic.

Since textbooks and pedagogical manuals are one of the ways through which "knowledge to teach and behaviours to inculcate" were defined (Julia, 2001), especially regarding knowledge of school mathematics, we seek to identify which knowledge and behaviours can be extracted when we look at a teaching manual, i.e., what mathematical knowledge and teaching practices those publications shared with their readers, teachers, and prospective teachers. To this end, we reflect on the teacher's relationship with knowledge.

It is not new that researchers deal with this topic; after all, Tardif, Lessard, and Lahaye (1991) bring us, in the article "Professores face ao saber" [Teachers in face of knowledge], discussions about teaching knowledge including knowledge from experience, knowledge produced by the teacher from their practice and linked to subjects' context and subjectivity.

In this research, however, we will follow the perspective in which the knowledge produced by the teacher is related to the knowledge *objectified*, knowledge not associated with a subject or its context but systematised and ready to circulate (Valente, 2019). This knowledge is also the result of the articulation between two types of knowledge: *knowledge to teach*, the object of the teacher's work, and *knowledge for teaching*, a teacher's work tool (Hofstetter & Schneuwly, 2017).

Knowledge *to* teach corresponds to the scientific subjects present in the teacher's education, to the knowledge that constitutes their professional field. For example, knowledge in the mathematical field that composes mathematics teacher education courses is part of the mathematics teacher's knowledge *to* teach. However, the domain of knowledge *to* teach does not characterise professional teaching knowledge, nor does it differ from other professions. Knowing how to add natural numbers does not make a person capable of teaching another person how to add natural numbers. The teacher needs a working tool that gives them conditions to carry out the task of educating an individual, specific knowledge of the teaching profession

that has as reference the fields of educational sciences: this is the knowledge *for* teaching,

> Knowledge about the "object" of teaching and training work (about the knowledge *to* teach and about the student, the adult, their knowledge, their development, ways of learning, etc.), about teaching practices (methods, procedures, devices, choice of knowledge to teach, organizational and management modalities) and about the institution that defines its field of professional activity (study plans, instructions, purposes, administrative, and political structures, etc.). (Hofstetter & Schneuwly, 2017, p. 134)

Given the above, we see that knowledge *for* teaching is not limited to methodologies that guide the teacher in carrying out their profession or in didactics to deal with their object of work –knowledge *to* teach– but it extends to the educational system and the entire universe that involves the teaching profession. But how would this knowledge have been introduced into teacher education courses?

According to Borer (2017), at the end of the 19th century, the need to train primary and secondary teachers who met social demands began to emerge in governments. Primary education should allow future citizens access to basic knowledge, and secondary education would be responsible for producing qualified labour and educating future elites. To adapt to the new needs of society, training courses began to require increasingly specific qualifications.

From the 20th century onwards, those who intended to work in public education abandoned self-taught preparation and began to receive "institutionalised education under the power of public authorities that offer formalised courses within which the teaching of academic knowledge and then of knowledge *for* teaching is little by little systematised" (Borer, 2017, p. 187, emphasis added).

Historically, the systematisation of primary school teachers' specific knowledge for teaching occurred through two models of training schools: the normal education model and the higher education model. The so-called normal schools are responsible for education at the secondary level and qualifying future teachers. They offer general education (knowledge of school subjects at the secondary level) and

professional education (knowledge of education sciences), the latter only gaining more space at the end of the studies and over the years (Borer, 2017).

On the other hand, in the higher education model, training takes place in two distinct spaces. First comes general education in secondary-level institutions: once the qualification certificate has been obtained, one moves on to professional education in higher education institutions, where they are taught pedagogy, psychology, and education sciences. Professional education is more relevant than normal schools and covers pedagogical studies responsible for the knowledge *for* teaching (Borer, 2017).

This means that knowledge *for* teaching has not always been relevant in teacher education. The epoch when it was enough for a teacher to know how to read, write, and master the four operations was soon replaced by one that requires more qualified teachers. This qualification grew as pedagogical studies gained more and more space in teachers' curricula in the normal and higher education models.

We will see below that those changes in the structure of teacher education courses echoed the historical context of their society, the educational movements on the rise, and their respective perceptions about the teacher's role and the purposes of schools. These changes came, first, with the advancement of intuitive teaching and later, with the introduction of psychology, methodology, and New School practicum studies.

THEOBALDO MIRANDA SANTOS: HISTORICAL CONTEXT AND PROFESSIONAL TRAJECTORY

Although we adopt in this research the concepts of knowledge from Hofstetter and Schneuwly (2017), we cannot disagree with Tardif, Lessard, and Lahaye (1991) that dominant pedagogical doctrines of a given time, presented by the authors as pedagogical knowledge, guide the teacher's educational activity, and are incorporated into their professional education. Therefore, we will historically locate the educational movements that supported teacher education courses during the publication period of the analysed books.

A movement of pedagogical renewal was spreading in Europe

at the beginning of the 19th century within a social context that sought transformations for primary schools, until then, marked by teaching based on memorisation, abstraction, and repetition (Valdemarin, 2004). Contrasting with what was considered ancient pedagogy came modern pedagogy, defined by two main aspects: the traditional, emphasising theorists such as Pestalozzi and Herbart, and the progressive, represented by Dewey (Zanatta, 2012).

Pestalozzi's ideas marked the so-called traditional pedagogy based on data obtained through the senses, perception, and observation. The educator "proposed a teaching method based on the following principles: from the known to the unknown, from the concrete to the abstract, from intuitive vision to general understanding. His teaching method is described as a 'lesson of things'" (Zanatta, 2012, p. 106) and is part of the logic of the analytical process, in which teaching moves from experiences of meaning and intuition towards definition and abstract thought.

The intuitive method remained strong in Brazilian primary education throughout the 19th century until the beginning of the 20th century, mainly due to the dissemination of foreign pedagogical manuals by educator Rui Barbosa. During that period, in Europe and the United States, a movement known as "new education," "active school," "active pedagogy," "work school." or "new school" appeared. Later,

> A set of ideas from post-war Europe that preached the renewal of teaching methods and processes, still dominated by the coercive regime of the old Jesuit pedagogy, began to reach us. This school renewal movement, which came to be known as the "New School" or "Active School," was based on the most recent advances in child psychology, which called for greater freedom for the children and respect for each one's personality characteristics in the various phases of their development, placing "interest" as the main driver of learning. (Lemme, 2005, p. 167)

The document that determined the establishment of the new school in Brazil was the "Manifesto dos Pioneiros da Educação Nova" [Pioneers of New Education Manifesto], released in 1932 and signed by 26 Brazilian intellectuals. Among the characteristics of the new school,

we can highlight secularism, compulsory and free-of-charge education, and co-education, with the State's duty to disseminate public education in the country and guarantee its access to citizens of all social classes. This social reconstruction movement also argues that the child and their interests must be at the centre of the school, and the proposed activities must be real and similar to their practical life (Monarcha, 2009).

Active Catholic thinkers fought the New School movement and mainly criticised the support given to the secularism of the State. To spread its ideals, the Church took some initiatives: it created associations of Catholic teachers and published books and magazines, such as the *Revista Brasileira de Pedagogia* [Brazilian Journal of Pedagogy] and the periodical *A Ordem* [The Order] (Da Cunha; Da Costa, 2002).

Later, a set of pedagogical manuals began to be produced that appropriated the ideas of the new school, associating them with Catholic priorities (Vidal, 2006), indicating that the understanding of the education of each of the two groups was not radically opposed. In this period of transition of movements in the Brazilian educational scene, Theobaldo Miranda dos Santos was one of the authors who collaborated in producing these pedagogical manuals with Catholic content but with New School ideas.

Theobaldo Miranda Santos (1904 – 1971) was born in Campos dos Goytacazes, Rio de Janeiro, on June 22, 1904. After completing the Dentistry and Pharmacy course at Colégio Metodista Grambery in Minas Gerais, he began his teaching career, and from then on, he followed several segments in the education field: Santos was a teacher, director, and secretary of various departments of education and culture and wrote dozens of books aimed at educating teachers and students at all levels of education.

During the 1930s, he published articles in magazines and newspapers in Rio with themes related to education (Almeida Filho, 2008). At the same time, he approached Catholic Education, which, unlike the New Schools, demanded a religious, conservative, and traditional education, and wrote several articles from this educational perspective. In this context, there was an extensive circulation of teacher education materials inspired by a Catholic teaching model in response to the spread of the New School movement in the country.

> Santos' collections, aimed at teacher education, systematised this field [of educational sciences], building models of school knowledge from the perspective of Catholic pedagogy. In this sense, the collections were model, as they organised the educational field, shaping school culture. (Almeida Filho, 2008, p. 5)

At 66 years old, Theobaldo Miranda Santos left five children and ten grandchildren, in addition to a legacy of more than 150 textbooks intended mainly for primary and normal courses. Theobaldo Miranda Santos (1904-1971) lived in a context of transformations: educational reforms shaped teaching at various levels, and conflicts arose between different groups, which had their particular ideals of school and education. Throughout this research, we will see that Theobaldo Miranda Santos presents characteristics that oscillate between intuitive teaching and new school in the books investigated here.

GEOMETRY AND KNOWLEDGE IN SANTOS' WORKS

Based on the definition of knowledge by Hofstetter and Schneuwly (2017) and considering the historical and social context of Theobaldo Miranda Santos and his role and influence in the education of primary teachers of his time, we read two pedagogical manuals written by this author.

In their pages, we sought mathematical and pedagogical knowledge that should be mobilised by primary school teachers regarding geometry teaching. We also studied the possible meanings that the concepts of "didactics," "special didactics," and "methodology" could assume in the educational context in question (between the intuitive and the new school movement) based on authors such as Gualtieri (2020) and Pinto (2020), besides Santos himself, the author of the two handbooks.

The two pedagogical manuals investigated, *Noções de Didática Especial* (1960) [Notions of Special Didactics] and *Metodologia do Ensino Primário* (1952) [Primary Education Methodology] are part of the *Curso de Psicologia e Pedagogia* [Psychology and Pedagogy

Course] collection. The collection is composed of volumes authored by Santos and "aims to explain, summarise, and systematise the historical evolution, philosophical foundations, scientific bases, and methods and techniques of contemporary pedagogy for Brazilian students" (Santos, 1960, p. 7). The incorporation of didactics and methodology subjects in Brazilian primary teacher education courses -under the influence of the institutionalisation and specialisation of human and social sciences that appeared at the end of the 19th century in Europe- justifies our choice.

In this process, new subjects emerged to guarantee teachers a training base that could legitimise the teaching profession. "In Brazil, historiography shows that, partly due to the influence of New School ideas, the professionalisation of primary education teachers was consolidated in the 1930s, with the reduction of knowledge *to* teach and expansion of pedagogical studies" (Gualtieri, 2020, p. 86). Consequently, professional knowledge was standardised in subjects in teacher education courses, such as Didactics and Teaching Methodology.

According to Pinto (2020, p. 152), Didactics, which had teaching as its object, was no longer idealised as a teaching practice "conceived as ways of organising and giving shape to the knowledge to be taught in a subject." Furthermore, the growing specialisation of professional knowledge meant that General Didactics, responsible for dealing with general issues about teaching and learning, also specialised in terms of different subjects, hence the so-called Special Didactics: Didactics of Mathematics, Geography, and Natural Sciences, among others.

According to Pinto (2020), we understand that general didactics establish theories, standards, and techniques for all types of teaching and students, while special didactics target specific subjects and different audiences and modes of teaching. The teaching methodology is responsible for establishing the different procedures in the teaching-learning process, methods that indicate a path to achieving a specific objective. Hence, both didactics and methodology have teaching methods in their studies.

Since Teaching Methodology and Special Didactics emerged in teacher education courses to meet teachers' specialisation needs in each school subject, we can verify that both pedagogical handbooks supported these professionals' education. Let us see below what

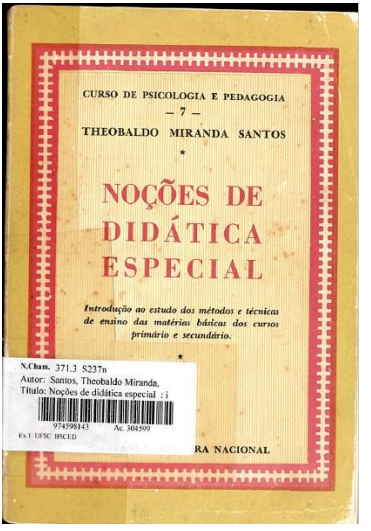
knowledge for teaching geometry in primary education could be identified in each of the volumes under study.

Noções de Didática Especial (1960)

The book *Noções de Didática Especial* (Figure 1) is volume 7 of the *Curso de Psicologia e Pedagogia* collection.

Figure 1

Cover of the book Noções de Didática Especial. (Santos, 1960)



The single edition was published in 1960 by Editora Companhia Nacional in São Paulo and consists of 300 pages. The author tells us that the volume:

> [...] represents a small introduction to the study of teaching methods and processes of basic primary and secondary subjects. [...] our sole purpose was to help young students who, in normal schools and philosophy colleges, prepare for the most difficult, beautiful, and worthy of all tasks –educating the new generations.

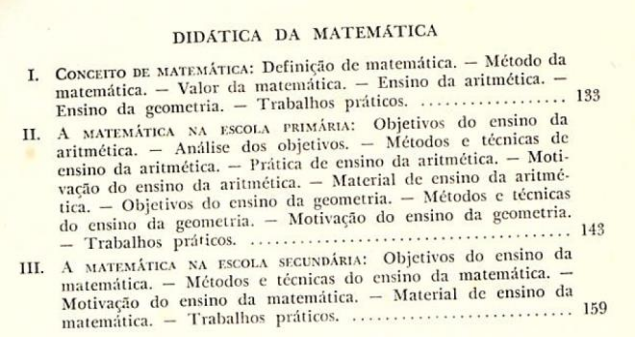
(Santos, 1960, p. 7)

As we saw in the previous section, Special Didactics were guidelines developed specifically for each of the school's curriculum subjects, having the knowledge gathered by General Didactics as a common denominator. The summary brings the structure of the work, which is divided into five parts: Didactics of Geography, Didactics of History, Didactics of Natural Sciences, Didactics of Mathematics, and Didactics of the Portuguese Language.

Each of these chapters is divided into three identical sections: the first deals with the concept of the subject, the second deals with its use in primary school, and the third, in secondary school. In this work, we will focus our study on the Didactics of Mathematics (Figure 2) paying attention to the items (I) Concept of Mathematics and (II) Mathematics in Primary School, and more specifically, the teaching of geometry at this level of education.

Figure 2

Summary with the structure of the chapter dedicated to the Didactics of Mathematics. (Santos, 1960)



DIDÁTICA DA MATEMÁTICA

When approaching the definition of mathematics, Theobaldo Santos treats mathematical notions from a philosophical perspective, according to which "mathematical objects are constructed by the spirit with data resulting from experience. [...] The formation of mathematical notions is therefore based on experience but is only achieved thanks to the human being's capacity for abstraction" (Santos,

1960, p. 134). Here, we can perceive the philosophical and religious character of the author when approaching the construction of mathematical objects by the spirit. We also identify remnants of the new school movement when the author characterises the construction of the child's knowledge through experience, with the teacher being just the learning mediator.

Santos also attributes three educational values the study of mathematics provides. The first are practical values, after all,

> the handling of basic arithmetic operations, the understanding of algebraic language, the interpretation of graphic representations, and familiarity with geometric shapes constitute highly useful resources for modern man's social, economic, and professional life. (Santos, 1960, p. 136)

These values make the author's position very clear regarding mathematics study as a tool for human life. There are also mentally developed academic values (clarity, precision, mental discipline, ability to create relationships) and cultural values (appreciation of the beauty of geometric shapes and the human being's ability to create), characteristic of the intuitive teaching movement. Values of that nature can be identified when Santos (1960) talks about the geometry of primary education, which has multiple values: utilitarian and instrumental (for applications in practical and professional life), and educational and formal (for the development of creativity and manual and mental skills). Furthermore, we see the names of famous intuitive teaching educators, such as Pestalozzi and Herbart, which highlights Santos' tendency to value the intuitive character of geometry and its practical application in solving daily problems.

Of the fundamental objectives of teaching geometry in primary school, Santos (1960, p. 154) underscores four fundamental ones:

> 1) Provide the child with an instrument to resolve life situations related to issues of shape, extension, and position;
>
> 2) Provide the child with knowledge of lines, surfaces, and volumes as resources for solving practical problems of everyday existence;

> 3) Get the child used to analysing and solving these problems;
>
> 4) Form fundamental habits of thought and action in the child's spirit through studying the subject.

For the geometry teaching, the author explains two methods: the analytical and the synthetic method:

> In the first case, we start from the bodies to reach the lines. In the second case, we start with the lines to get to the bodies. The analytical process is the only one that should be used in elementary school, even though it is not the specific method of studying geometry. However, when it comes to recognising, describing, and classifying geometric shapes, it is justified to start with solid bodies, which, being concrete and material, even young children can understand. From there, we move on to surfaces and, from there, to lines, and thus, we will have developed the entire program of the subject in primary school." (Santos, 1960, p.155)

Following the thoughts of Florentino Rodrigues, Santos considers that geometry teaching should occur through intuition and discovery, starting from what the child sees and knows and not from relationships and principles presented as ready-made ideas for the child. Once again, geometry teaching reveals characteristics of the intuitive wave because, in the analytical method, the child begins studying geometry from what they perceive with their eyes and hands, and only then, they follow a path formed by increasingly abstract concepts, but never far from associations about their reality.

To motivate the teaching of geometry, Santos cites other didactic proposals based on foreign authors, such as the Spanish Margarita Comas, an intellectual from the New School movement in her country. In addition to reinforcing that learning geometry must always start from real objects and never from abstract models, we must associate it with drawing, games, manual work, the environment, and movement for students to find it interesting and enjoyable. The notion of line, for example, must come from the idea of a trajectory made by a point, and the concept of surface must be given as the trajectory followed by a line; the notion of angle should not be restricted only to

the inclination of two straight lines starting from the same point, but should also be associated with the measure of rotation of a body around an axis.

We see that the author dedicates this volume to the didactics of each primary course subject, which would justify the "special" adjective in the title. Analysing the Didactics of Mathematics, we see that Santos brings the definition of mathematics, the method of mathematics, the value of mathematics, and the teaching of arithmetic and geometry as essential themes to address the mathematics concept. The objectives, methods, techniques, and motivation for teaching arithmetic and geometry are subjects that characterise mathematics in primary school.

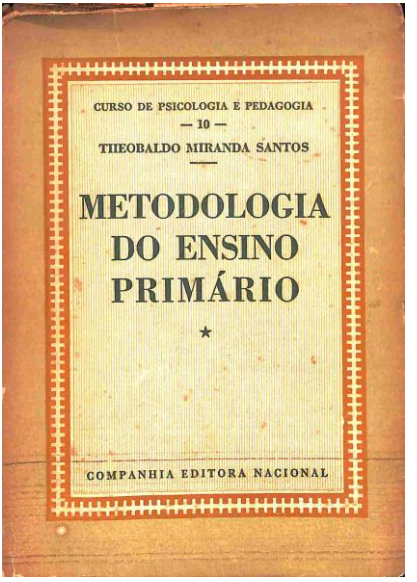
Thus, we have mathematics, knowledge *to* teach, one of the elements associated with didactics, which is also related to knowledge *about* teaching. The objectives and motivation for teaching geometry equip the teacher with arguments that justify teaching this subject in primary school and situations that inspire and help them "make learning geometric shapes interesting and enjoyable" (Santos, 1960, p. 156). Finally, the methods and techniques presented constitute elements of mathematics teaching, and, as we will see later, together with the other items, they are also part of the so-called geometry methodology.

Metodologia do Ensino Primário (1952)

Volume 10, *Metodologia do Ensino Primário* (Figure 3) of the collection *Curso de Psicologia e Pedagogia* was published in 1952 (third edition) with 246 pages, by Companhia Editora Nacional, in São Paulo.

Figure 3

Cover of the book Metodologia do Ensino Primário *(Santos, 1952)*



The version analysed is the third of 11 editions published between 1948 and 1967 (Almeida Filho, 2008). The book was intended for students in normal schools and educational institutes who wanted to apply for teaching in the country. This compendium was created to summarise briefly methodological knowledge that met the requirements of the organic law of normal education (Santos, 1952). The summary in Table 1 shows that the book is divided into two parts.

Table 1

Overview of the summary of the book Metodologia do Ensino Primário *(Santos, 1952)*

Name of the part	Content
First part **General Methodology**	Methodology Pedagogical methods Evolution of pedagogical methods Classification of pedagogical

	methods
	Didactic processes
	Didactic forms
	Didactic modes
	Teaching material
	The lesson
	Active methods and new schools
	Reading methodology
	Writing methodology
	Oral language methodology
	Arithmetic methodology
Second part	Geometry methodology
Special Methodology	Geography methodology
	History methodology
	Natural sciences methodology
	Manual work methodology
	Drawing methodology

In the first, General Methodology, Santos explains definitions and concepts associated with pedagogical methods, and in the second part, Special Methodology, he brings general characteristics and teaching techniques for each of the subject groups in Brazilian primary education programs. Each subject is finished with paragraphs of exercises, notes, and a bibliography. The exercises are a sequence of questions about the main concepts covered in the respective chapter; the notes contain quotes from other authors with additional information that complement the text, and the bibliography contains a list of authors who served as references for the development of the text.

To facilitate the study of the method, in the General Methodology part, the main classifications known and adopted by authors of contemporary methodology are presented, and then a general classification of the methods adopted by most of them and by Santos (1952, p. 44):

> We can divide pedagogical methods into two large groups: 1) general methods, composed of logical processes used by all methods rather than pedagogical methods themselves; 2) special methods, which are

> specifically pedagogical methods, that is, specially designed to achieve educational purposes. These two groups of methods are, therefore, intimately, and organically related.

Santos (1952) states that the method is a principle created by the teacher to carry out his educational work and fulfil the purposes of education based on psychological knowledge. "The function of the method must, therefore, consist of provoking, stimulating, and directing the manifestation of this [learning] activity. Hence, the need for the method to adapt to the nature of the student and the purposes of education" (Santos, 1952, p. 24). However, he reiterates that the method is not limited to rigid and mechanical formulas; it must be flexible, adjusting to the child's freedom and spontaneity. We see here that, for the author, the pedagogical method, a professional knowledge (Hofstetter & Schneuwly, 2017) of the primary school teacher, is closely linked to the purposes of education, purposes that, during the new school movement, were based on psychology and the child's needs.

Besides the method, Santos also presents his reader with concepts associated with the teaching process, forms, modes, and materials. The concept of Didactic Process is defined by the practical resources used by the teacher in applying the methods, such as the inductive processes of analysis, intuition, observation, exemplification, and the deductive processes of synthesis, synopsis, diagram, scheme, demonstration, and repetition (Santos, 1952). We see that those processes are resources that the teacher can adopt when conducting their classes, they are knowledge specific to the education professional.

To develop these didactic processes, the teacher can adopt a Didactic Form that adapts to their teaching style (Santos, 1952). This Form is the way in which the teacher presents the material to the students: the oral exposition, the interrogative form, and the conversation and discussion or a combination of the previous forms, which is best defended by the author of the book. At the end of this unit, Santos includes a quote from Adolfo Rude in his notes. The passage contrasts the interrogative didactic form of the "old school," in which only the master asked questions and the student answered, and the "new school," which understood questioning as a natural need for children, and part of their education. This passage may indicate that,

although Santos belonged to groups of Catholic intellectuals, he still shared educational objectives with the so-called pioneers of new education.

The next item, Didactic Modes, refers to room organisation for the development of school activities: in the individual mode, teaching occurs between the teacher and one student at a time; in simultaneous mode, teaching occurs collectively; in mutual mode, the teacher takes their most advanced students to help them with school tasks, and in mixed mode, best defended by Santos, there is a combination of the three previous modes. At the end of this unit, the author presents quotes that contrast characteristics of the traditional school and the so-called active school: using Aguayo, he considers the process of transmitting knowledge as a "pedagogical error," as the role of the teacher is to provoke and direct the learning process. Quoting the Brazilian educator Lourenço Filho (1897-1970), one of the signatories of the "Manifesto dos Pioneiros da Educação Nova" [Manifesto of the Pioneers of New Education], he differentiates the student (from traditional schools), who learns everything when taught, who prepares for exams and must be disciplined, from the child (from the active school), the latter being a developing being with specific needs for whom one cannot transmit knowledge (Santos, 1952).

Didactic Materials, according to Santos, are any objects that help the teacher exercise their educational role, and the tendency for primary education at the time was to replace illustrative materials (manipulated by the teacher and observed by students) with work materials (observed and manipulated by the students), implying, once again, that the centre of learning is the child and that they learn through their senses and their perception of things.

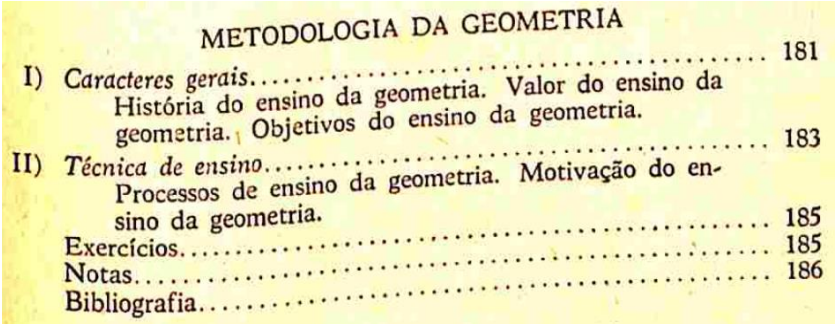
In this first part of the book, General Methodology, we see that Theobaldo Miranda Santos provides his reader, teacher, or prospective teacher with basic tools that can help them carry out his work: knowledge about student development and their ways of learning and knowledge about teaching practices (pedagogical methods and processes, forms, modes, and teaching materials). While in Notions of Special Didactics, Santos looks at the general teaching guidelines – objectives, methods, techniques, and motivation for teaching– in Primary Education Methodology, the author turns to the individual character of the method, linked much more to the relationship between

the teacher and their teaching tools than to the teaching itself.

Finally, the second part of the book, entitled Special Methodology, presents methodologies for each subject that compose the primary education course. To address the Geometry Methodology, the author adopts the same texts used in the Didactics of Mathematics chapter in the book *Noções de Didática Especial* (Santos, 1960). The few differences correspond to the organisation of the texts and the order in which they are presented, in addition to the fact that, if previously the texts were divided into three sections –(1) Objectives of teaching geometry, (2) Methods and techniques of teaching geometry, and (3) Motivation for teaching geometry– in *Metodologia do ensino primário* (Santos, 1952) they unfolded into more items, as can be seen in Figure 4:

Figure 4

Summary with the structure of the chapter dedicated to geometry methodology (Santos, 1952)



METODOLOGIA DA GEOMETRIA

Methodology of geometry presents what has already been explained previously, when we studied the volume of *Noções de Didática Especial*: notions of the history of geometry teaching; values of teaching geometry, which, in addition to being valuable for practical life, is considered one of the few subjects that put into action "so many faculties, which exercise so many organs and are, therefore, so educational" (Santos, 1952, p. 182). The objectives of its teaching in primary school are giving the child an instrument to solve life situations, in addition to their teaching processes (using intuition and

analytical or synthetic methods) and motivations (geometry must be interesting and based on things from the environment).

SOME RESULTS

Through a literature review and search for newspaper reports and documents with a list of textbooks suggested for use in the country, we identified widespread use of works by Theobaldo Miranda Santos for Brazilian primary school teacher education. However, we did not find many works discussing Brazilian primary school geometry, so we focus here on the knowledge *for* teaching geometry in two of his pedagogical manuals. This type of material, however, only gained visibility in Brazil from the 1940s onwards, when movements that sought to give the old normal schools a less general and more technical and professional character emerged. Therefore, specific courses for teacher education were created, including didactics and methodology subjects. During this period Theobaldo Miranda Santos began to write his first pedagogical manuals, publishing, in 1952, the third edition *of Metodologia do Ensino Primário*, and in 1960, the single volume of *Noções de Didática Especial*.

Regarding the Didactics of Mathematics, we identified two important subjects to be studied by primary school teachers: (1) the concept of mathematics, which addresses the definition of mathematics, the method of mathematics (invention and mathematical demonstration), the value mathematics (practical, academic, and cultural) and the teaching of arithmetic and geometry (their definitions, values, and studies throughout history); (2) mathematics in primary school, which addresses the objectives of teaching arithmetic and geometry in primary school and the motivations, methods, and techniques for their teaching.

In *Metodologia do Ensino Primário*, Santos (1952) understands that the method is a principle created by the teacher to provoke, stimulate, and direct learning and fulfill the purposes of education. Furthermore, the method must follow the principles governed by psychology and adjust to the nature of the student and the educator and the characteristics of the environment in which they both find themselves. Among the knowledge that can help teachers in carrying out their work, there is knowledge of student development and their

ways of learning and knowledge of teaching practices (pedagogical methods and processes, forms, modes, and teaching materials).

To teach geometry in primary education, teachers must have knowledge of the value, objectives, and motivation of geometry for primary education; after all, the child is not interested in what does not seem valuable to them in practical life, and it is up to the educator to mobilise these values in class. The teacher must also know about the historical development that geometry teaching has undergone in different civilisations and eras and of the methods and techniques adopted for teaching geometry. We noticed, however, that the treatment given to geometry is reduced, compared to arithmetic, which has many more texts and approach suggestions than geometry content for primary education.

School handbooks investigated oscillate between the intuitive wave and the new school but were shaped according to the norms and purposes attributed to the school and the teacher education. Santos associates the intuitive value of geometry with names such as Pestalozzi and Herbart, arguing that geometry teaching must occur through intuition and discovery, starting from what the child sees and knows, from concrete problems and manual activities that they can perform. On the other hand, it cites several Brazilian and foreign authors who were pioneers in disseminating the new school movement in their countries and uses concepts from child psychology and the child's developmental stages to guide teachers in conducting their classes. It reinforces that primary school children's learning and construction of knowledge take place when they are the centre of interest in learning, and contrasts the "old school" model, in which the teacher asks questions and the student answers, with the model of the "new school," where questioning is part of the child's learning process.

FINAL CONSIDERATIONS

This research arose with the guiding question: What knowledge for teaching geometry in primary education can be identified in Theobaldo Miranda Santos's school handbooks? To answer this question, we chose to analyse the pedagogical manuals *Noções de Didática Especial* (1960) and *Metodologia do Ensino Primário* (1952).

Besides presenting typical elements of these two educational

movements, in some passages, Santos exposes characteristics of a Catholic pedagogy when involving themes linked to human formation, such as moral and spiritual values. This line of thought is reinforced mainly when the educator relates the method adopted by the teacher to their philosophical conception of life. We then see that this educator shared educational objectives with the so-called pioneers of new education, a situation that was reflected in his manuals, which attempted to adapt the Catholic model to New School precepts.

In this study, we outline some of the knowledge for teaching geometry mobilised in a sample of books developed by Theobaldo Miranda Santos, an educator from Rio, with influences from traditional education and new education, who introduced knowledge for teaching into his pedagogical manuals, professional knowledge that could guide primary teachers their work with geometry.

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AUTHORSHIP CONTRIBUTION STATEMENT

ASM and ESCS conceived the idea presented and mobilised the theoretical methodological framework. ASM collected and analysed the documentary data. ESCS supervised the research and reviewed the theoretical framework. Both authors revised the final version of the text.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding authors, ASM and ESCS, upon reasonable request.

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