

Reflections on the Documentational Work of Two Teachers in a Remote Collaborative Environment for Compound Interest Teaching

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ABSTRACT

Background: Documentational Approach to Didactics was adopted, this way considers a continuous process in which resources, associated with the utilisation schemes, are transformed into documents. **Objectives:** Characterize the Documentational Genesis of two High School Mathematics teachers in remote collaborative work for teaching Compound Interest articulated with the GeoGebra software. **Design:** As a methodological framework, the reflective investigation methodology was used, in which the teachers contribute to data collection and analysis. The paper was structured in four professional situations: reflection situation, training situation, implementation situation, and confrontational situation. **Settings and participants:** Teachers worked from March to June 2021, and resources were developed through the Google Docs and Google Meet virtual platforms and later made available on Classroom. **Data Collection and Analysis:** Data obtained through the recordings of the meetings and the proposals of the participants, analysed according to documentational trajectory concept, the instrumentation and instrumentalisation dialectics, productive and constructive activity dialectics, and the self-efficacy concept. **Results:** Based on our analyses, it was possible to characterise the teachers' Documentational Genesis. **Conclusions:** The Documentational Approach to Didactics was relevant to the study because it was possibly identified the resources in individual and collective utilisation schemes, mobilised in the documentational work and in class implementation by the teachers and the instrumentation and instrumentalisation dimensions in document construction for teaching Compound Interest, i.e., the Documentational Genesis process. In addition, we have identified the teachers' self-efficacy during the documentational work and implementation of classes on Compound Interest.

Keywords: Documentational Genesis; Collaborative Work; Compound Interest; GeoGebra; Reflective Investigation.

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Reflexões sobre o Trabalho Documental de Dois Professores em um Ambiente Colaborativo Remoto para o Ensino de Juros Compostos

RESUMO

Contexto: Adotou-se a Abordagem Documental do Didático, um processo contínuo no qual os recursos, associados aos esquemas de utilização, são transformados em documentos. **Objetivos:** Caracterizar a Gênese Documental de dois professores de Matemática do Ensino Médio, em trabalho colaborativo remoto, para o ensino de Juros Compostos com o uso do GeoGebra. **Design:** Utilizou-se a metodologia de investigação reflexiva na qual os professores contribuem para a coleta e análise dos dados, estruturado em quatro situações profissionais: de reflexão, de treinamento, de implementação e de confronto. **Ambiente e participantes:** Os professores trabalharam de março a junho de 2021 e foram desenvolvidos recursos, por meio das plataformas virtuais *Google Docs* e *Google Meet*, e posteriormente disponibilizados no *Google Sala de Aula*. **Coleta e análise de dados:** Obtidos por meio das gravações dos encontros e das propostas dos participantes, analisados segundo o conceito de trajetória documental, a dialética de instrumentação e instrumentalização, a dialética da atividade produtiva e construtiva e o conceito de autoeficácia. **Resultados:** Com base nas análises, foi possível caracterizar a Gênese Documental dos professores. **Conclusões:** Por meio da Abordagem Documental do Didático foi possível identificar os recursos em esquemas de utilização individual e coletiva mobilizados no trabalho documental e na implementação em sala de aula pelos professores, assim como as dimensões de instrumentação e instrumentalização na construção de documentos para o ensino de Juros Compostos, ou seja, o processo de Gênese Documental reconhecendo a autoeficácia dos professores durante o trabalho documental e na implementação das aulas sobre Juros Compostos.

Palavras-chave: Gênese Documental; Trabalho Colaborativo; Juros Compostos; GeoGebra; Investigação Reflexiva.

INTRODUCTION

In this paper, we present results of a research whose general purpose was to characterize the Documentational Genesis of two High School Mathematics teachers in remote collaborative work for teaching Compound Interest articulated with the GeoGebra software, through the Documentational Approach to Didactic - DAD (Gueudet & Trouche, 2015).

Based on the DAD, we focused on the process of building documents for teaching Compound Interest, through an association of resources adopted by the teachers and utilization schemes, and the impact caused by documentational work on the teachers' professional development.

Accordingly, in our research, we used reflective research as a methodological framework (Gueudet and Trouche, 2010), in a context of

remote collaborative work of teachers, which was structured in four professional situations: reflection situation, training situation, implementation situation, and confrontational situation. Based on this methodological framework, we analyzed the teachers' professional action and improvement, checking the topics of stability and advances in documentational work in the period considered.

The question that guided this investigation was: How to characterize the Documentational Genesis of two High School Mathematics teachers in remote collaborative work, for the teaching of Compound Interest, articulated with the GeoGebra software?

It should be noted that formulation of the research question had an important contribution from the discussions developed in the ten-seminar cycle around the Documentational Approach to Didactics (Trouche, 2021), organized by the Postgraduate Studies Program in Mathematics Education at the Pontifical Catholic University of São Paulo (PUC-SP), in the which the basic notions related to the documentational genesis were presented: notion of resource, document, resource systems and schemes.

In this article, we focus on the concept of documentational trajectory (Rocha, 2019), the instrumentation and instrumentalization dialectic, “mother” and “child” resource, productive activity and constructive activity (Assis and Trouche, 2021), and the concept of self-efficacy (Bandura, 1997). Based on our analyses, we were able to characterize the teachers' Documentational Genesis.

THEORETICAL FRAMEWORK

The theoretical construction of the DAD had an important contribution from the Instrumental Approach (Rabardel, 2002), which presents the concept of Instrumental Genesis. According to Rabardel (2002), Instrumental Genesis involves two processes: instrumentation and instrumentalization, defined as follows:

Instrumentalization processes concern the emergence and evolution of artifact components of the instrument: selection, regrouping, production and institution of functions, deviations and catachresis, attribution of properties, transformation of the artifact (structure, functioning etc.) that prolong creations and realizations of artifacts whose limits are thus difficult to determine.

Instrumentation processes are relative to the emergence and evolution of utilization schemes and instrument-mediated

action: their constitution, their functioning, their evolution by adaptation, combination coordination, inclusion and reciprocal assimilation, the assimilation of new artifacts to already constituted schemes, etc. (Rabardel, 2002, p.103, translation by the authors).

According to Abar and Alencar (2013),

Rabardel's (1995) focuses on the transformation of the artifact utilization into an instrument, then proposing a model of situations in which an instrument is used, comprising: *subject*: user, operator, worker, etc. It is he who directs the psychic action on the object; *instrument*: tool, machine, product, etc. It is the mediator between the subject and the object; *object*: material, real, activity object, work object or other subjects. It is at the object that the action is directed (Abar & Alencar, 2013, p. 353).

Instrumental Genesis therefore seeks to clarify how a subject acts to obtain an instrument from an artifact. In this sense, Bittar (2011) argues:

Let us consider a teacher for whom the software is unknown. When first coming into contact with this unknown material, he does not know how to handle even its basic tools. This software is, for this teacher, an artifact. As he begins to unravel the material, discover how it works and elaborate situations for using the software, the teacher is developing and adding utilization schemes to the artifact, and then, the artifact is transformed, for this teacher, into an instrument. The more he uses this instrument, the more schemes can be built, added to the software and the teacher will then have a new instrument (Bittar, 2011, p. 161).

Thus, when the teacher begins to explore the material, linking utilization schemes to the artifact, we have the instrumentation process; as new schemes are elaborated, linked to the instrument, we have the instrumentalization process.

It is worth highlighting the Theory of Instrumental Orchestration (TOI) (Trouche, 2004; Drijvers *et al.*, 2010), which is based on the Instrumental Approach. According to Trouche (2005):

Instrumental orchestration is the systematic and intentional arrangement of elements (artifacts and human beings) in an environment, carried out by an agent (teacher) in order to effect

a given situation and, in general, to guide learners in the instrumental genesis, and in the evolution and balance of their instrument systems. It is systematic because, as a method, it is developed in a defined order and with a determined focus and can be understood as an arrangement integrated into a system; it is intentional because an orchestration does not describe an existing arrangement (there is always one), but points to the need for a priori thinking of this arrangement (Trouche, 2005, p. 126, translation by the authors).

Thus, under this theory, through a metaphor, a classroom is compared with an orchestra. According to Lucena, Gitirana and Trouche (2016),

The TOI instrumental approach studies the development of subjects in the utilization of artifacts, transforming them into instruments, through the instrumentalization and instrumentation processes. The Orchestration Theory seeks to model the teaching action in an environment rich in digital technologies that favors individuals' instrumental genesis, based on three phases: the didactic configuration, the mode of execution and the didactic performance. The first two were characterized by Trouche (2004) and the last one by Drijvers *et al.* (2010) (Lucena, Gitirana & Trouche, 2016, p. 3, translation by the authors).

Instrumental orchestration presents a time element that is associated with didactic performance. Didactic performance encompasses the choices made during the teaching process, as decided in the didactic configuration and mode of execution: what question to formulate now, how to validate a particular student intervention, how to deal with an unforeseen element of the mathematical task or technological resource, or other purposes that may arise.

Regarding the scheme concept, in his studies on the Theory of Conceptual Fields, Vergnaud (1990) highlights that scheme is the invariant organization of conduct for a given class of situations, through which one must investigate the subject's knowledge in action, i.e., the cognitive elements that allow this action to be operative.

According to Vergnaud (1998), a conceptual field is, at the same time, a set of situations and a set of concepts, all interconnected. Therefore, the meaning of a concept cannot be analyzed by a single situation and, reciprocally, a situation cannot be analyzed through a single concept, but by many of them, creating systems. Thus, a teacher's professional knowledge and their evolution

are closely associated with the situations in which the teacher matures this knowledge.

Vergnaud (1990) considers that the subject observes meaning for a concept in situations and problems, not limiting themselves to a definition. In addition, he points out that rational knowledge is necessarily operational. About situations, he considers that there are those in which the subject has the necessary skills for a relatively instantaneous approach to the situation, and those in which the subject does not have all the necessary skills. In the first situation, the subject's actions will be mechanized and will be structured by only one scheme. In the second situation, the subject is encouraged to ponder, explore, experiment, in order to delineate different schemes that will be adapted and recombined. Accordingly, the subject will have access to new discoveries.

Still on the concept of utilization schemes, Bellemain and Trouche (2019) point out that:

A scheme is defined by Vergnaud (2009) as an entity composed of four components: objectives, rules of action, taking of information and control, operational invariants, and inferences in situation. The operational invariants are the epistemic components of the schemes, composed of knowledge, often implicit, that results from the activity and, at the same time, guides the action. This means that the scheme develops through repeated performance of a given task. (Bellemain & Trouche, 2019, p. 108).

According to Pepin and Guedet (2020), rules of action generate behavior according to the situation characteristics; operational invariants involve concepts-in-action, which are concepts considered relevant (e.g. "teaching differentiation"), and theorems-in-action, which are propositions considered to be true (e.g. "if students perform poorly, then they need more help from the teacher"); as possibilities of inferences, the example provided was "in this class, I need to adapt my differentiation scheme, because there are some students with very high performance."

The scheme components simultaneously record the balance of activity structuring and its capacity for variation. Inferences can lead subjects to adjust their rules of action to situation particularities; adjustments can cause other operational invariants, or even other schemes, to appear. It should be noted that, for the same situation, a teacher can elaborate certain schemes, whilst another teacher can elaborate other different schemes. In the same way, a teacher can elaborate certain schemes in a joint work with other teachers and elaborate other schemes when working with the students for the same activity.

We will now address significant aspects of the theoretical framework considered, the Documentational Approach to Didactics. Gueudet and Trouche (2015) highlight this approach and examples of its use related to Mathematics teachers. According to these authors:

The theoretical approach we propose and call the documentational approach to didactics feeds on various theoretical frameworks, some of which are common in mathematics education (the theory of situations – Brousseau, 1998 - the Anthropological Theory of Didactics – Chevallard, 2002 - or conceptual theory - Vergnaud, 1996), others already well established in studies on the inclusion of instruments in Didactics (Rabardel ergonomic theory - 1995), other approaches, finally, more directly related to our purpose, in the field of documentational engineering (Pedauque, 2006, 2007) (Gueudet & Trouche, 2015, p. 6, translation by the authors).

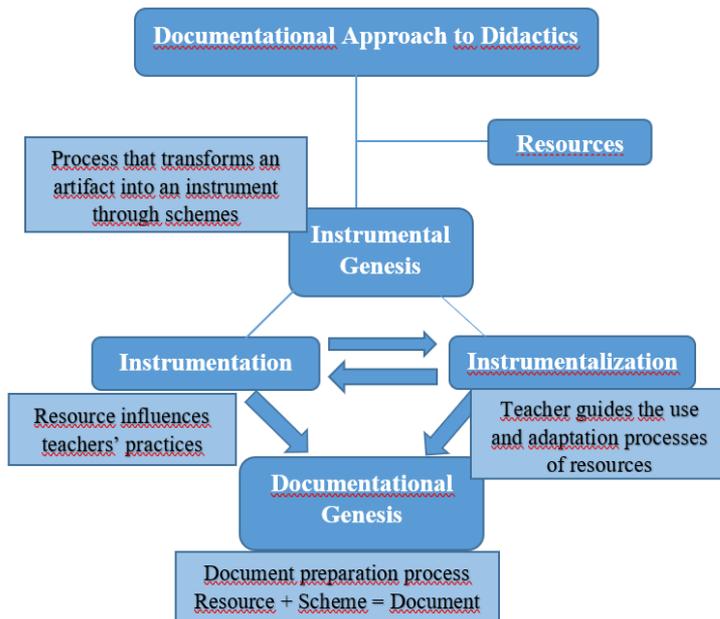
In the Documentational Approach to Didactics, the documentational work conducted by the teacher when preparing their classes is considered. This work is the essence of pedagogical actions implemented by the teacher and their professional development. This approach involves a process called Documentational Genesis, in which, according to Gueudet and Trouche (2015), the teacher's resources are constantly updated, transformed, and combined in association with utilization schemes, generating a document.

According to Assis and Trouche (2021), there are three documentational genesis dialects: the instrumentation/instrumentation dialectic takes into account the relationship between teachers and their teaching resources; the “mother”/“child” resource dialect occurs between all the available resources acquired by the teacher for teaching (the mother resources) and the resources obtained from their transformation (the child resources); the productive activity/constructive activity dialectic refers to the relationship between productive activity (the teacher develops resources for teaching) and constructive activity (the teacher produces new knowledge).

In Documentational Genesis, resources guide teachers' practices (instrumentation process) and, reciprocally, teachers begin to command them, adapting and modifying them as they are used (instrumentalization process). Figure 1 presents a scheme of this theoretical framework.

Figure 1

Documentational Approach to Didactics Scheme (Dias, Almeida & Abar, 2021, p. 8)



Just as the instrumental approach differentiates what is within reach for the activity (the artifacts) and what is elaborated by the subjects (the instruments), the documentational approach differentiates what is within reach for teachers' practices (the resources) and what they elaborate to help their teaching practice (the documents).

METHODOLOGICAL FRAMEWORK

The methodological framework used was the reflective research methodology (Gueudet & Trouche, 2010), through which teachers' action and professional improvement are researched, with perception of stability and advances in documentational work within a considered timeframe. It is important that the researcher take into account, as much as possible, the teacher's work, both in classroom and in other different situations (at the teacher's home, in meetings with other teachers, in the library, etc.). There is a deep relationship between the teacher and data collection, in the sense that data

is extensively collected and the teacher is monitored in different situations. Also, there is a dynamic commitment by the teacher, which favors a reflective attitude, as the teacher is encouraged to detail their own activities, sharing it with others.

In this sense, the methodological contract (Sabra, 2016) was clearly explained to teachers. From the outset, the asymmetry of positions was discussed, in which each of the parties would have a well-defined role, but each would work collaboratively within their roles. From the start, they were aware that the researcher would analyze work development, class preparation and implementation. As the methodology is reflective, teachers contributed to data collection and analysis, so that the researcher sought to relate what they explained from their points of view, inferring that there was professional improvement of the teachers for the teaching of Compound Interest.

Regarding the collaborative work definition, it is considered

[...] joint work (two or more subjects) with benefit for the professional development of those involved, aiming at achieving the following common objective: the integral formation of students, in which learning and the expansion of success are the goals. The concept assumes mutual support, interaction that produces knowledge and expertise, and the implementation of joint actions between school actors (Pinto & Leite, 2014, p. 148, translation by the authors).

In this regard, we highlight the importance of using research paths that enrich connections between school and university, bringing benefits to teachers' practices, providing opportunities for elaboration of new understandings about actions at school, establishing an engagement of the university with the pursuit of answers to the challenges observed in the school context. Thus, there is involvement between an external researcher and a teacher or a group of teachers, so that all investigation is conducted *with* the teacher rather than *for the teacher*, allowing joint formulation of knowledge and intervention in the focused context. The collaborative work presented in this research is framed in this proposal.

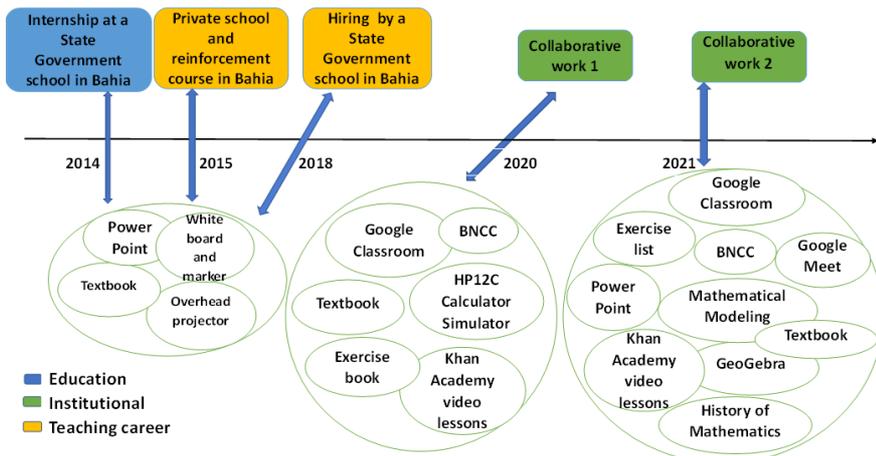
The teachers followed a trajectory to obtain a document for teaching Compound Interest, called a documentational trajectory, which, according to Rocha (2019, p. 75, translation by the authors), is "like a path (which expresses continuities and changes) that articulates professional events (individual and/or collective) experienced by teachers and the transformations in their documentational work over time."

Thus, in the course of their documentational trajectory, the teachers have participated in some events that impacted their documentational work for teaching Compound Interest, increasing their resources, including new rules of action and developing different schemes.

Rocha (2019) proposed reflective mapping of the documentational trajectory (MRTD), in which teacher are asked to remember events they deem most important in their professional lives and the influence of these events on documentational work. The representation of Ms. Carla's (teacher) MRTD for teaching Compound Interest is shown in Figure 2.

Figure 2

Reflective map of Ms. Carla's documentational trajectory Documentational (Reported by Ms. Carla)



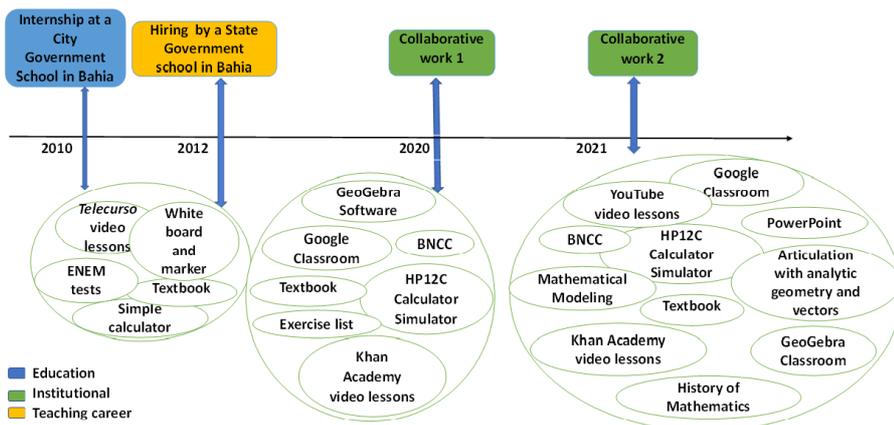
Ms. Carla (teacher) has a degree in Mathematics Teaching and six years of experience as a teacher in Elementary School. We observed 5 reflective events considered important by the teacher and 13 resources linked to them, with her involvement in two collaborative works. The map summarizes the evolution of her documentational work towards teaching content. Next, Ms. Genilda's (teacher) MRTD representation for teaching Compound Interest is shown.

Ms. Genilda (teacher) has a degree in Mathematics Teaching and nine years of experience as a teacher in High School. We observed 4 reflective

events considered important by the teacher and 16 resources linked to them, with her involvement in two collaborative works. The map (Figure 3) summarizes the evolution of her documentational work towards teaching content.

Figure 3

Reflective map of Ms. Genilda's documentational trajectory Documentational
(Reported by Ms. Genilda)



We should point out that, among the data collection techniques, we used, in addition to the teachers' MRTD, online questionnaires and interviews, and video recordings of professional collaborative work situations.

DESCRIPTION AND ANALYSIS OF THE COLLABORATIVE WORK

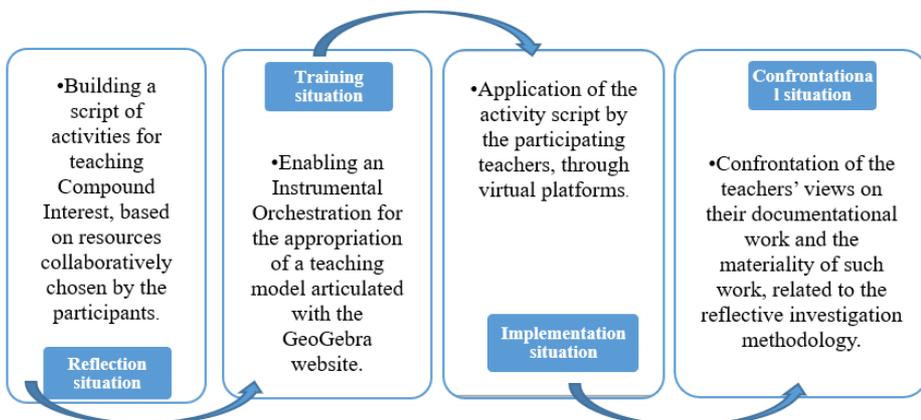
Work took place from March to June 2021 and was developed through virtual platforms Google Docs (an online word processor that makes it possible to create and edit documents and share them with others) and Google Meet (a pattern-based videoconferencing application that uses proprietary protocols for transcoding video, audio, and data) in which resources for teaching Compound Interest were developed. These resources would later be made available in the Google Classroom environment and/or implemented with students via Google Meet, for each of the classes under the responsibility of each teacher.

The reflection situation experiment was designed in order to enable teachers to critically reflect on their decisions for teaching Compound Interest in the 3rd (senior) year of High School. This professional situation took place in March 2021 with the purpose of proposing to research subjects a collaborative construction of an activity script for teaching Compound Interest through free choice of tasks and resources. During its development, teachers were expected to articulate schemes and classes of situations to meet the demands caused by the situation.

The work was structured in four professional situations, as shown in Figure 4.

Figure 4

Collaborative Work



In accordance with the intersection of both logics “practical questioning” and “formal research” discussed by Desgagné *et al.* (2002), the following guiding question for the collaborative work was defined: “What resources can be developed, collaboratively and remotely, in order to enable the understanding of the compound capitalization regime concept?”

Thus, the following approaches were considered: modeling of problem situations, use of Digital Technologies (particularly the GeoGebra software), History of Mathematics, games (digital or non-digital), articulation with other areas of Mathematics. That collaborative work would be based on specific competence 2 for Mathematics and its Technologies in High School at BNCC was also discussed, specifically on skill (EM13MAT203): “Planning and

performing actions involving the creation and use of applications, games (digital or non-digital), spreadsheets for family budget control, compound interest calculation simulators, among others, to apply mathematical concepts and make decisions” (Brasil, 2017, p. 526).

Each teacher posted her resources and strategies on Google Docs, so that the other teacher could comment thereon and suggest adjustments to the resources and strategies proposed by her colleague, also counting on the researcher’s participation, who shared reflections.

The training situation experiment aimed at making subjects use a teaching model articulated with the GeoGebra software. To achieve this purpose, an Instrumental Orchestration was conceived (Trouche, 2004; Drijvers *et al.*, 2010), whose didactic configuration and mode of execution were planned in order to allow subjects to face a new teaching situation. This situation purposely suggested teaching articulated with the GeoGebra software and was adopted based on its potential to contribute to the class planning and implementation processes by teachers.

Thus, on April 9, 2021, we held a meeting through Google Meet, when the researcher provided guidance to the teachers regarding the contributions of GeoGebra software, available at www.geogebra.org, in order to favor reflections of these teachers on the possibilities of its use in the construction of teaching documents. Through said guidance, the teachers and the researcher interacted about the creation of activities with GeoGebra, the integration of didactic materials from GeoGebra with Google Classroom and the use of GeoGebra Classroom (virtual platform linked to the website, through which teachers can assign interactive assignments and view student progress, synchronously and asynchronously).

The implementation situation took place between May and June 2021 and was planned to execute the script of activities developed in the reflection situation, verify teachers’ documentation process evolution in the context of remote classes on Compound Interest, including the possible use of the teaching model articulated with GeoGebra.

Ms. Carla implemented her classes synchronously, using the Google Meet platform. The implementation involved three classes of the 3rd year of high school in the morning shift, each class with approximately 35 students. However, due to low attendance of students, classes were combined into a single class on class implementation. Given the short time available to complete the unit, the teacher claimed that she selected some resources developed in the collaborative work that she judged to be more relevant, leaving the others for another opportunity.

In order to implement a teaching situation that favored the instrumental genesis of students in relation to GeoGebra, Ms. Carla conceived an Instrumental Orchestration (Trouche, 2004; Drijvers *et al.*, 2010), whose didactic configuration and whose mode of execution were planned in order to allow students to face this new teaching situation. This situation purposely suggested the construction of graphs of functions through GeoGebra and was adopted based on the potential of this software in this direction. In this context, the teacher also guided students on how to access the GeoGebra website to use the software interfaces.

Ms. Genilda implemented her classes asynchronously, integrating Google Classroom and GeoGebra Classroom platforms. The implementation involved a class of the 3rd year of high school students in the evening shift, with only five students. In this class, in particular, teachers of all disciplines decided to teach their classes only asynchronously, due to the difficulties presented by students in attending classes synchronously.

Ms. Genilda made interactive tasks available in the GeoGebra Classroom, which were accessed by the students. In addition, teacher and students interacted through Google Classroom, in order to discuss, dispel doubts and make corrections to the tasks answered by students in GeoGebra Classroom.

It was found that the resources and knowledge associated with teachers' making the implementation situation feasible were understood as factors for observing the evolution of the documentation developed by them for teaching Compound Interest. Data collection was enhanced and the ability to monitor its documentation processes was strengthened. Relying on this monitoring, we can also conclude that the training and implementation situation experiments contributed to the use of the teaching model articulated with the GeoGebra software by both teachers, each one in a different way.

The confrontation situation took place in June 2021 and aimed to enable an evaluative experiment on the implementation of classes by the teachers. This situation was planned in such a way that the researcher had prior access to the video of class implementation by the teachers and could prepare some comments and questions about the resources and schemes mobilized by them in this process.

These comments and questions were considered helpful in making the reflective follow-up process viable and confronting the teachers' documentation work, based on the principle of confronting the teachers' views about their documentation work and the materiality of this work in relation to the reflective research methodology (Trouche, Gueudet & Pepin, 2018).

The confrontation situation experiment was also considered to allow teachers to enrich the activity script, through critical evaluations on resources adopted from construction to implementation, when they would continue to articulate schemes and adapt the produced document.

DOCUMENTATIONAL WORK ANALYSIS

An important concept associated with documentational trajectory is that of documentational experience: the relationship of teachers with resources, over time, allows “the accumulation and appropriation of their documentational work history” (Rocha, 2019, p. 74, translation by the authors). According to the author, this experience provides the teachers’ documentational work with particularities identifiable by them that will guide the development of the documentational trajectory, which conceptualizes the documentational experience trait. The author also highlights the documentational dominant (orientation of documentational work towards a certain family of activities in a certain period) and the symbolic transition events, which change the documentational dominant.

In Ms. Carla’s MRTD for teaching Compound Interest (Figure 2), we observed that the first reflective event indicated is her supervised internship, which took place in 2014, the teacher’s initial training stage in which she had her first contact with a classroom. According to the teacher, she was somewhat insecure about performing her didactic-pedagogical actions, so she began to recall classes delivered by her Financial Mathematics professor in the teaching degree course, who would use PowerPoint and the overhead projector.

Based on Rocha (2019), we may say that the documentational experience experienced by Ms. Carla in her initial training (insecurity in didactic-pedagogical actions) left a trait of this experience (use of PowerPoint and overhead projector) which accompanied her in her actions in the second reflective event (hiring by a private school and reinforcement course), which took place in 2015; in the third reflective event (hiring by a state government school in Bahia), which took place in 2018; and in the fifth reflective event (collaborative work 2), which took place in 2021.

Also based on Rocha (2019), we may say that Ms. Carla’s documentational dominant, prior to the fourth reflective event (collaborative work 1), which took place in 2020, was the creation of resources for her teaching. It should be noted that the third reflective event (hiring by a state government school in Bahia) produced another event (attendance of an online course promoted by the State Department of Education), in 2018, which

addressed the use of some digital resources for teaching, such as tools from Google, YouTube and Khan Academy. However, this event did not have an immediate impact on the teacher's documentational work, as her documentational experience left another trait: resistance to non-traditional teaching methods.

Ms. Carla's documentational dominant after the fourth reflexive event (collaborative work 1) was the discovery of new resources. Accordingly, this is the symbolic event of transition. It should be highlighted that this event took place in the context of the Covid-19 pandemic, which caused the need to use platforms and models for online learning at home, whilst teachers sought to adapt their own models and support systems based on local needs and availability of online teaching. Thus, the course previously promoted by the Department of Education had an impact on Ms. Carla's documentational work during this period, as she was already familiar with the functionality of some resources such as Google and Khan Academy tools covered in the course.

During the fifth reflective event (collaborative work 2), in the context of this research and still in the context of the pandemic, the documentational experience (sharing new modalities, resources and teaching strategies) experienced by Ms. Carla in the previous reflective event left some traits of this experience: adoption of different digital and non-digital resources to design and implement classes, continued adoption of the textbook as base reference, and continued use of PowerPoint, but less dependent on it.

Ms. Carla, at the beginning of her career, for teaching Compound Interest, only mobilized schemes related to oral exposition of the concept and resolution of some exercises in the textbook. According to Ms. Carla, the interaction with Ms. Genilda and the researcher in this work gave her the opportunity to get in touch with some digital technological resources, whose functionalities could support her in adopting differentiated teaching strategies, especially in this context of social distancing that demands such adoption.

Ms. Carla highlights the opportunity she had, during the course of her work, to become familiar with the GeoGebra software and its website (which she had never used before with her students) and to be able to experience some of its geometric and algebraic possibilities, in order to be able to prepare and teach Compound Interest classes that are increasingly enriching. The teacher only used squared paper for graphic constructions.

Ms. Carla also considers that using the teaching approaches highlighted in this work, together with the functionalities of digital technological resources, can favor adequate teacher intervention in situations

where students have greater difficulties in understanding, which contributes to the teaching qualification.

In Ms. Genilda's MRTD for teaching Compound Interest (Figure 3), we observed that the first reflective event indicated is here supervised internship, which took place in 2010, the teacher's initial training stage in which she had her first contact with a classroom. According to the teacher, since she graduated high school, she had difficulties understanding some questions about compound interest addressed in the Brazilian High School Exam (ENEM) tests. This concern she took to her undergraduate course, where she was supported by colleagues and professors to answer certain questions, including for the classes she taught in her internship. In addition, she exchanged ideas with classmates from the video lessons on the content and applied them in the classroom.

Based on Rocha (2019), we may say that the documentational experience experienced by Ms. Genilda in her initial training (difficulties with questions about compound interest on the ENEM tests) left a trait of this experience (resolving questions on the ENEM tests) that accompanied her in her actions in the second reflective event (performing in a state government school in Bahia), which took place in 2012, and another trait (application of video lessons in the classroom), in all subsequent periods.

Also based on Rocha (2019), we may say that Ms. Genilda's documentational dominant, prior to the third reflective event (collaborative work 1), which took place in 2020, was the creation of resources for her teaching. It should be noted that the second reflective event (hiring by a state government school in Bahia) produced another event (attendance of an online course promoted by the State Department of Education) which, similarly to Ms. Carla's event, had no immediate impact on Ms. Genilda's documentational work, as her documentational experience left another trait: resistance to non-traditional teaching methods.

Ms. Genilda's documentational dominant after the third reflective event (collaborative work 1) was the discovery of new resources. Thus, this is the symbolic event of transition, with the same information about the pandemic context and the impact of the course delivered by the Department of Education on documentational work, related to Ms. Carla.

During the fourth reflective event (collaborative work 2), in the context of this research and still in the context of the pandemic, the documentational experience (sharing new modalities, resources and teaching strategies) experienced by Ms. Genilda in the previous reflective event left some traits of this experience: adoption of different digital and non-digital resources to design

and implement classes, continued adoption of the textbook as base reference, and continued use of video lessons and GeoGebra.

Just like Ms. Carla, Ms. Genilda, at the beginning of her career, for teaching Compound Interest, also mobilized schemes related to oral exposition of the concept and resolution of some exercises in the textbook. However, unlike Ms. Carla, Ms. Genilda also mobilized other schemes related to the use of video lessons, ENEM questions and simple calculator that could favor the assimilation of the compound capitalization concept by her students.

According to Ms. Genilda, her participation in this collaborative work allowed her to improve some knowledge she had about the GeoGebra software, which can favor her performance in the classroom, since greater exploration of the potential of this software with her students may allow them to build compound capitalization-related knowledge more dynamically and interactively.

According to Ms. Genilda, the opportunity she had to get to know and adopt GeoGebra to design activities on compound capitalization, and GeoGebra Classroom to make classes asynchronously possible was very fruitful, both considering the context in which her students were unable to attend online classes, and envisioning the use of these resources, in the future, synchronously, through association with videoconferencing platforms and automatic feedback. Accordingly, it is possible to expand the channels of interaction with students, make them more motivated, and contribute to the improvement of teaching.

Ms. Genilda also points out that the use of GeoGebra associated with other collaborative work resourced for teaching compound capitalization could make the process of mediating students' knowledge construction more motivating and profitable, thus contributing to teachers' continuing education.

CONSIDERATIONS ON THEORETICAL ASPECTS

Regarding the first dialectic, in the instrumentation dimension, textbooks served as a resource for teachers to study and also for the implementation of classes on compound interest. Textbooks were one of the resources that guided the activity script organization. However, reflecting on how to teach made teachers consider possible obstacles for students with the textbook approach.

For teachers, for example, the study of compound capitalization without understanding the historical development of the concept and without the articulation of numerical, algebraic, and graphic representation records

would be a limitation to students' learning.

In the instrumentalization dimension, we highlight the adaptations made to textbook questions to elaborate the script of activities. In fact, the adjustments to the questions involved access to video lessons, the financial calculator simulator, modeling through an exponential function, integration with other areas of mathematics, and the use of GeoGebra (in order to promote the articulation of numerical, algebraic and graphical representation records).

Regarding the second dialectic, in the case of this research, we considered "mother" resources: the textbook, the text on the history of mathematics, the scientific paper on modeling, the video lessons, the financial calculator simulator, the board game, the BNCC (2017) and GeoGebra. As presented, these resources were articulated in the documentational genesis. Otherwise, as a product of the teachers' relationship with these resources, we highlight the "child" resources. As an example, we have the problem situations elaborated from textbook questions and the recommendations for using GeoGebra for the articulation of algebraic and graphic representation records. In the analysis of this dialectic, we also highlight the emergence of a system of teachers' resources designed to teach compound interest with GeoGebra.

Regarding the third dialectic, resources produced comprise the productive dimension of the documentational genesis. In this research, the script of activities on compound interest with GeoGebra and everything that composes it form a product of the teachers' activity in the situation. However, it is in the dynamics of building the document that we see the teachers articulate knowledge.

Considering the activity script production scheme for the professional situation of preparing a lesson plan on compound interest for 3rd-year High School students, we found some related knowledge, through interviews with the teachers:

- a) the teachers see textbooks as an important resource for teachers and students, as they address mathematical content in an organized manner, present solved and proposed exercises, self-assessment and supplementary activities,
- b) the teachers understand the text on the history of mathematics as an important resource, as it highlights the development of the concept of interest, favoring its assimilation by the students,
- c) the teachers consider video lessons as important resources for approaching mathematical content, as they complement textbook information,

- d) the teachers understand that obtaining a mathematical model (a mathematical formula or pattern) through the situation problems approach can improve students' interpretation skills, awakening a critical attitude and an exploratory spirit in them when trying to solve those problems, realizing that there may be several alternatives to obtain the solution,
- e) the teachers believe that GeoGebra functionalities contribute to the mathematical content approach, as they allow the exploration of algebraic and graphic representation records, obtaining the answers to questions through the CAS window and the relationship with other mathematical contents, such as analytic geometry and vectors.

The procedures with GeoGebra were the product of learning about software possibilities, which took place through individual research of the subjects and the interaction between them and the researcher in collaborative work, and which did not make up the teachers' resource system. We understand that the transition from the pencil-and-paper context to the GeoGebra environment increased their professional training. Explorations were increasing in the dynamics of reflecting on how to teach. In this study, we considered the assimilation of the mathematical content itself about compound interest in textbook and GeoGebra resources, their geometric and algebraic records, and the preparation of a script of activities with the use of the software.

CONSIDERATIONS ABOUT THE TEACHERS' SELF-EFFICACY

According to Bandura (1997 apud Iaochite *et al.*, 2016, p. 46), self-efficacy is defined as a judgment by the individual about their current skills and abilities to act in a specific domain, and constitutes the basis for motivation, well-being and future personal achievements. In the field of teaching, self-efficacy refers to the evaluation made by the teacher about their abilities to teach, including for discouraged students. In this aspect, it is related to how teachers see their roles, organize their classes and the perception that all students can evolve in the construction of new knowledge, even those who have considerable difficulties in understanding contents.

In their paper which addresses self-efficacy in computer use, Guimarães and Abbad (2015) highlight four sources of information that can influence the degree of self-efficacy observed by the individual: personal experience, vicarious observation, verbal persuasion and emotional focus. The

first one is associated with the individual's personal experience and reveals that a subject's perception of their abilities to perform a task tends to evolve if their preliminary experiences provide them with favorable data on similar abilities.

The second source, called vicarious observation, is associated with the opportunity for the subject to observe other similar individuals experiencing successful or unsuccessful situations. In this regard, observation of close subjects being successful (or unsuccessful) through personal dedication impacts the observers' belief regarding their ability to also be successful (or unsuccessful) in these situations.

The third source, verbal persuasion, indicates that a subject can impact the degree of self-efficacy of another, through verbal information related to the activity and the individual's ability to perform it. In this sense, subjects who are verbally persuaded to believe they have the potential to perform tasks properly are prone to greater and continuous dedication, rather than being dominated by doubts and personal difficulties in the face of challenges. The fourth source of influence is related to emotional focus, through which subjects are more likely to believe that they will be successful if they do not have a high level of anxiety related to a certain situation.

As an example, Ms. Carla, using her personal experience as a source of information, when adopting a text on the history of compound interest as a resource, observed an impact on the degree of self-efficacy related to this approach. This text was a new resource for the teacher until the moment of the collaborative work, when the teacher started to use it, guided by it and associating it with some common utilization schemes (referring to reading it and writing a short, related text). In this case, we have the instrumentation process. Reciprocally, the teacher started to associate new utilization schemes with the resource (related to discussions based on students' comments), which characterized the instrumentalization process.

In relation to Ms. Genilda, a similar situation took place, but the source of information that influenced the degree of self-efficacy observed by the teacher was verbal persuasion. In this sense, Ms. Carla impacted Ms. Genilda's degree of self-efficacy through verbal information related to the resource and the individual's ability to use it.

Regarding the adoption of GeoGebra (www.geogebra.org) as a resource for teaching Compound Interest, Ms. Carla and Ms. Genilda had vicarious observation as a source of information, as they had the opportunity to observe the researcher experiencing successful situations and observed an impact on the degree of self-efficacy related to this strategy. The teachers started to use it, guided by it and associating it with some common utilization

schemes (referring to the recognition of its functionalities). In this case, we have the instrumentation process.

Reciprocally, the teachers began to associate new utilization schemes with the resource related to accessing the various GeoGebra interfaces (in the case of Ms. Carla), to the creation and monitoring of activities through the GeoGebra Classroom (in the case of Ms. Genilda), which characterized the instrumentalization process.

FINAL REMARKS

Our concern to characterize the Documentational Genesis of two High School Mathematics teachers in remote collaborative work for the teaching of Compound Interest, articulated with the GeoGebra software, motivated this study. The Documentational Approach to Didactics and the Theory of Conceptual Fields contributed to research on this topic. We identified the resources that, added to the individual and collective utilization schemes constructed in the teachers' work, became documents.

Thus, we believe that the Documentational Approach to Didactics was relevant to the study, because it enabled us to analyze the collaborative work of teachers when they used resources in the construction of documents for teaching Compound Interest, i.e., we analyzed these teachers' Documentational Genesis process for teaching this mathematical item, the preparation of documents in the work developed, in both orientations: instrumentation and instrumentalization.

We understand that the teachers' documentational work has progressed, through their documentational trajectory (Rocha, 2019) for the teaching of Compound Interest, considering the concepts of documentational experience, documentational experience trait, symbolic transition events, and documentational dominant.

We referred to the Theory of Instrumental Orchestration, which contributed to the expansion of our reflections on the entire process and underlying knowledge that teachers reveal in the action of thinking or rethinking a resource, something immediately related to the operational invariants discussed by Vergnaud (1990).

We also referred to the self-efficacy concept, which contributed to the expansion of our reflections when we discussed how collaborative work development influenced the degree of self-efficacy of Ms. Carla and Ms. Genilda (teachers), analyzing it in association with the instrumentation and

instrumentalization processes related to the Teachers' Documentational Genesis for teaching Compound Interest.

Given the limitations imposed by the situation of social distancing caused by the Covid-19 pandemic, we took into account, as much as possible, the remote work of teachers, both in the construction of documents and in the implementation of synchronous or asynchronous classes.

We made methodological choices that, in our perception, would cooperate with the objectives of our study, leading us to answer the research question and reveal new contributions to the field of Mathematics Education. As the methodology is reflective, the teachers contributed to data collection and analysis, so the researcher tried to relate what they explained with their points of view, inferring that there was professional improvement of the teachers for the teaching of Compound Interest.

At this point, we return to the research question: how to characterize the Documentational Genesis of two High School Mathematics teachers in remote collaborative work, for the teaching of Compound Interest, articulated with the GeoGebra software?

Based on the reflections presented in the analyses and at this point in our work, we believe we have answered the research question, as we have identified the resources and the individual and collective utilization schemes mobilized in the documentational work and in class implementation by the teachers. We have presented the instrumentation and instrumentalization dimensions in document construction by teachers for teaching Compound Interest, i.e., the Documentational Genesis process. In addition, we have identified the teachers' self-efficacy during the documentational work and implementation of classes on Compound Interest.

In this research literature review, we did not find any work with the Documentational Approach to Didactics with a focus on the remote collaborative aspect for teaching Compound Interest. Thus, we believe that this investigation contributed to the advancement of this theoretical approach and, therefore, to Mathematics Education.

Lastly, we believe that, for collaborative work involving the teaching of mathematical content, this research presents the specificity of approaching the Compound Interest content, adding to several others based on the Documentational Approach to Didactics. Continuing this research, we can think of other works that fill gaps related to other mathematical contents, at different levels of education.

AUTHORS' CONTRIBUTIONS STATEMENTS

CBA participated in all phases of research development: preparation, participation in remote meetings and data collection.

CAAP and CBA actively participated in the discussions and reviewed and approved the final version of this work.

DATA AVAILABILITY STATEMENT

Data supporting the results of this study will be made available by the authors for correspondence, upon reasonable request.

REFERENCES

- Abar, C. A. A. P. & Alencar, S. V. (2013) A gênese instrumental na interação com o GeoGebra: uma proposta para a formação continuada de professores de matemática. *Bolema*, 27(46), 349-365.
- Assis, C. F. C. & Trouche, L. (2021) Unindo as peças de um quebra-cabeça: perspectivas dialéticas e a gênese documental na formação inicial de professores de matemática. *Educ. Matem. Pesq.*, São Paulo, 23(3), 399-427.
- Bandura, A. (1997) *Self-efficacy: the exercise of control*. Freeman.
- Bellemain, F. & Trouche, L. (2019) Compreender o trabalho do professor com os recursos de seu ensino, um questionamento didático e informático. *Caminhos da Educação Matemática em Revista*. 9(1), 105-144.
- Bittar, M. (2011) A abordagem instrumental para o estudo da integração da tecnologia na prática pedagógica do professor de matemática. *Educar em Revista*, Curitiba, 157- 171.
- Brasil. Ministério da Educação. (2017) *Base Nacional Comum Curricular - BNCC - Ensino Médio*.
- Desgagné, S., Bednarz, N., Lebuis, P., Poirier, L. & Couture, C. (2002) L'approche collaborative de recherche en éducation: un rapport nouveau à établir entre recherche et formation. *Consortium Erudit*, 27(1), 33-64. <http://dx.doi.org/10.7202/000305ar> .
- Dias, A. O., Almeida, C. B. & Abar, C. A. A. P. (2021) Estado de conhecimento sobre a abordagem documental do didático em

pesquisas na língua portuguesa. *Revista de Ensino de Ciências e Matemática*, 12(1), 1-25

- Drijvers, P., Doorman, M., Boon, P., Reed, H. & Gravmeijer, K. (2010) The teacher and the tool: instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics*, 75(2), 213-234.
- Gueudet, G. & Trouche, L. (2010) Des ressources aux documents, travail du professeur et genèses documentaires. In: G. Gueudet; L. Trouche (dir.), *Ressources Vives. La documentation des Professeurs en Mathématiques*, (pp; 57-74).
- Gueudet, G. & Trouche, L. (2015) Do trabalho documental dos professores: gêneses, coletivos, comunidades: o caso da matemática. *Em Teia*, 6(3).
- Guimarães, V. F. & Abbad, G. S. (2015) Autoeficácia no uso do computador em situações de aprendizagem: uma análise da literatura internacional. *Revista Psicologia: Organizações e Trabalho*, 15(2), 170-187.
- Iaochite, R. T., Costa Filho, R. A., Matos, M. M. & Sachimbombo, K. M. C. (2016) Autoeficácia no Campo Educacional: Revisão das Publicações em Periódicos Brasileiros. *Psicologia Escolar e Educacional*, 20(1), 45-54.
- Lucena, R, Gitirana, V. & Trouche, L. (2016) Teoria da orquestração instrumental: um olhar para a formação docente. In: *Anais do I Simpósio Latino-Americano de Didática da Matemática*, Bonito.
- Pepin, B. & Gueudet, G. (2020) Studying teacher collaboration with the documentational approach: from shared resource to common schemes? Teachers of Mathematics Working and Learning in Collaborative Groups. *Proceedings of ICMI Study 25*, (pp.158-165).
- Pinto, C. L. L. & Leite, C. (2014) Trabalho colaborativo: um conceito polissêmico. *Conjectura: Filos. Educ.*, Caxias do Sul, 19(3), 143-170
- Rabardel, P. (2002) *People and technology: a cognitive approach to contemporary instruments*. Paris: Université Paris 8.
<https://hal.science/hal-01020705> .
- Rocha, K. M. (2019) *Une étude des effets du travail documentaire et collectif sur le développement professionnel des enseignants de mathématiques: apport des concepts d'expérience et de trajectoire*

documentaires. Doctoral Thesis Didactique des Mathématiques, Ecole Normale Supérieure de Lyon. <https://theses.hal.science/tel-02399664> .

- Sabra, H. (2016) L'étude des rapports entre documentations individuelle et collective: incidents, connaissances et ressources mathématiques. *Recherches en didactique des mathématiques*, 36(1), 49- 95.
- Trouche, L. (2004) Environnements informatisés et mathématiques: quels usages pour quels apprentissages? *Educational studies in mathematics*, 55, 181-197.
- Trouche, L. (2005) Construction et conduite des instruments dans les apprentissages mathématiques: nécessité des orchestrations. *Recherches en didactique des mathématiques*. 25, 91-138.
- Trouche, L., Gueudet, G. & Pepin, B. (2018) The documentational approach to didactics. In: S. Lerman (ed.), *Encyclopedia of Mathematics Education*. Springer.
- Trouche, L. (2021) *Ciclo de 10 seminários em torno da Abordagem Documental do Didático*. PUC-SP, Projeto Piprint- PG 9302/2020- PUC-SP, São Paulo.
- Vergnaud, G. (1990) La teoría de los campos conceptuales. [traducción de Juan D. Godino. Espanha]. *Recherches em Didactique dès Mathématiques*, 10, 133-170.
- Vergnaud, G. (1998) Toward a cognitive theory of practice. In: Sierpinska, A.; Kilpatrick, J. (eds.), *Mathematics Education as a Research Domain: A Search for Identity*, (pp. 227-241). Kluwer.