


Interactions Promoted by Teacher Educators Based on an Erroneous Approach to the Subject of Fractions

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Received for publication 15 Mar. 2023. Accepted after review 15 May 2023

Designated editor: Claudia Lisete Oliveira Groenwald

ABSTRACT

Background: An episode developed within the framework of a training process for primary school teachers is presented, based on a previous diagnosis where, among other shortcomings, those related to the subject of fractions and their teaching were detected. In this context, a set of interactions between the participants and two mathematics teacher educators take place, based on an error related to this theme.

Objectives: To analyse the types of interactions promoted by two mathematics teacher educators from the appearance of an error in the context of a continuous training program for primary school teachers.

Design: Naturalistic research of a qualitative-ethnographic nature.

Scope and participants: Two mathematics teacher trainers and teachers of the second stage of primary school (9-11 years old) belonging to an educational network that offers its services in disadvantaged sectors.

Data collection and analysis: Sixteen hours recorded on video were observed and analysed from two categories with their respective properties; each category characterises a type of interaction.

Results: The information collected shows a different type of interaction on the part of each of the two mathematics teacher educators when an error appears about the fraction and its teaching.

Conclusions: The two types of interactions that are evident in a different way in each one of the trainers, show that the characteristics proposed by an initial, continuous or professional development training program are marked by the types of interactions promoted by the trainer.

Keywords: Teacher training; mathematics teacher trainer; mistake; teaching fractions

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Interacciones promovidas por formadores de profesores a partir de un planteamiento erróneo acerca del tema de las fracciones

RESUMEN

Contexto: Se presenta un episodio desarrollado en el marco de un proceso de formación para profesores de primaria sustentado en un diagnóstico previo donde, entre otras falencias, se detectaron aquellas relacionadas con el tema de las fracciones y su enseñanza. En ese contexto se producen un conjunto de interacciones entre los participantes y dos formadores de profesores de matemáticas, a partir de un error relacionado con esa temática. **Objetivos:** Analizar los tipos de interacciones promovidas por dos formadores de profesores de matemáticas a partir de la aparición de un error en el contexto de un programa de formación continua para profesores de primaria. **Diseño:** Investigación naturalista de carácter cualitativo-etnográfico. **Ámbito y participantes:** Dos formadores de profesores de matemáticas y profesores de la segunda etapa de primaria (9-11 años) pertenecientes a una red educativa que ofrece sus servicios en sectores desfavorecidos. **Recopilación y análisis de datos:** Se observaron dieciséis horas registradas en vídeo y se analizaron a partir de dos categorías con sus respectivas propiedades; cada categoría caracteriza un tipo de interacción. **Resultados:** La información recopilada muestra de parte de cada uno de los dos formadores de profesores de matemáticas un tipo de interacción diferente cuando aparece un error sobre la fracción y su enseñanza. **Conclusiones:** Los dos tipos de interacciones que se evidencian de manera diferente en cada uno de los formadores, muestran que las características que se proponga un programa de formación inicial, continua o de desarrollo profesional, está marcado por los tipos de interacciones que promueva el formador.

Palabras claves: Formación de profesores; formador de profesor de matemáticas; error; enseñanza de las fracciones

Interações promovidas por formadores de professores a partir de uma abordagem errônea do tema das frações

RESUMO

Contexto: Apresenta-se um episódio desenvolvido no âmbito de um processo de formação de professores do ensino básico, com base num diagnóstico prévio onde foram detectadas, entre outras deficiências, as relacionadas com o tema das frações e o seu ensino. Nesse contexto, ocorre um conjunto de interações entre os participantes e dois formadores de professores de matemática, a partir de um erro relacionado a esse tema. **Objetivos:** Analisar os tipos de interações promovidas por dois formadores de professores de matemática a partir do aparecimento de um erro no contexto de um programa de formação contínua de professores do ensino básico. **Desenho:** Pesquisa naturalística de natureza qualitativo-etnográfica. **Âmbito e participantes:** Dois formadores de professores de matemática e professores do 2.º ciclo do ensino básico

(9-11 anos) pertencentes a uma rede educativa que oferece os seus serviços em setores desfavorecidos. **Coleta e análise dos dados:** Foram observadas e analisadas dezesseis horas gravadas em vídeo a partir de duas categorias com suas respectivas propriedades; cada categoria caracteriza um tipo de interação. **Resultados:** A informação recolhida mostra um tipo diferente de interação por parte de cada um dos dois formadores de professores de matemática quando surge um erro sobre a fração e o seu ensino. **Conclusões:** Os dois tipos de interações que se evidenciam de forma diferente em cada um dos formadores, mostram que as características propostas por um programa de formação inicial, contínua ou de desenvolvimento profissional são marcadas pelos tipos de interações promovidas pelo formador.

Palavras-chave: treinamento de professor; formador de professores de matemática; erro; ensinando frações

INTRODUCTION

We present an episode of a training session for primary school teachers, in which a set of interactions take place between the participants and two trainers, based on an error related to the teaching of the part-whole notion of fractions. The episode is part of a research project that studies the interactions led by teacher educators in a training program for primary school teachers.

In the educational task it is unavoidable to establish different relationships within the classroom that generate, in turn, various interactions of the agents or elements that intervene in that context. In this sense, Coll and Sánchez. (2008), point out that the interaction is the articulation of the actions carried out by the teacher and the students, which revolve around an activity and a specific content within the educational process, giving rise to the emergence of knowledge or joint construction. of meanings from interactions (Chico, 2018). These interactions involve procedures, conjectures, forms of validation and counterexamples, hence the importance of analyzing the dialogue to study the interactions. Among the theories that deal with interactions, there is symbolic interactionism that has great influences from the interactionist approach in Mathematics Education (Godino and Llinares, 2000), since sociological concepts are adapted to those of teaching and learning mathematics.

The importance of studying interactions in the school context lies in the fact that a large part of the knowledge that the student achieves comes from interactions in classroom contexts between teachers and students (Sánchez-Barbero, Calatayud and Chamoso, 2019; Mendoza - von right Borch , 2018). In this sense, Donoso, Valdés and Cisternas (2020) indicate that the information about the interaction between teachers and students in the classes is still

incipient, however, it is of interest to analyze the interactions that occur in the actions aimed at communicating and negotiate mathematical meanings in the classroom (Chico, 2018)

On the other hand, in every interaction in the classroom, errors on the part of those who participate in it come to light. This occurs because there is a close relationship between classroom interactions, errors, and learning processes (Lannin, Barker, & Townsend, 2007; National Council of teachers of Mathematics (NCTM), 2000). Any learning situation is subject to the possibility of assuming errors and these should be used in favor of learning

Over the years, interest in studying errors has increased and initially this type of research was more focused on students than on teachers (Tulis, 2013, Heinze and Reiss, 2007, Santagata, 2005). Today this reality has progressively changed; Research that addresses errors in teachers being trained has increased, however, those that study errors and other aspects related to practicing teachers have not been as abundant (Aguirrea, Solís and Huincahue, 2022; González; Gómez and Restrepo, 2015; Heinze and Reiss, 2007, Santagata, 2005 and Tulis, 2013; Rhoads, Radu and Webber, 2011) and much less, in the case of teacher educators, key actors that have been little studied (Contreras, 2021; Parra-Sandoval, 2020; Chick and Beswick, 2018). Therefore, any research on error that focuses on these two populations - practicing teachers and teacher educators - can provide significant data for learning processes and this is the case that concerns us in this paper.

Consequently, we present an episode in which a practicing primary school teacher manifests an error when issuing his opinion on the subject of the fraction as part-whole; this situation generates an abundant number of interactions between this participant, his peers and the two trainers. This episode is part of the ongoing training program that we already mentioned.

Taking into consideration that the erroneous approach is about the teaching of the fraction in the context of a continuous training program for primary school teachers and that this generated a series of exchanges of opinions and knowledge between the trainers and participants, we set out to analyze the types of interactions that are promoted by two teacher educators based on an erroneous approach by one of the participants on the subject of teaching fractions, in the context of a continuous training session.

THEORETICAL FRAMEWORK

To achieve the stated objective, we address a bibliographic review that covers the following three aspects: classroom interactions, errors in the field of mathematics education, and the teaching of fractions.

Classroom Interactions

According to Godino and Llinares (2000), interactionism in mathematics education contributes to studying the interactions between individuals within a culture, instead of focusing on the individual in isolation. To characterize it, one must describe its position in relation to meaning, the nature of mathematical knowledge, the processes of getting to know and understand the mathematical object, and the role of language.

Leguizamón (2017) states that different approaches can be used to characterize interactions in the classroom in a natural way; This is done by analyzing the possible pairs involved in this process, that is, teacher-student interaction, teacher-group, teacher-class, student-student, student-group, student-class, group-class and in all, it must be taken Keep in mind that they are asymmetrical power relations.

Another approach to interactions is to analyze them within the framework of a mathematical activity; for example, studying the processes that are promoted and the degree of participation of teachers and students during the interactions (Sánchez-Barbero et al. 2019). Among the processes are cognitive, referring to the reasoning that occurs in the interaction (Smart and Marshall, 2013) and metacognitive (Vélez and Ruis , 2021), which include the reflective capacity that allows one to become aware of one's own cognition. Regarding the degree of participation of teachers and students, it is the participation they have in the construction of knowledge for learning (Sánchez-Barbero et al., 2019).

The study of interactions also revolves around their characteristics. For example, Ingram and Riser (2019) address the interactions that are generated in mathematics classes from problem solving, showing that students participate in a very limited range of problem-solving actions, generally controlled by the teacher.

Another perspective is through the study of patterns. Voigt (1985) points out that the identification and study of patterns in the context of interactions facilitates the observation of regularities that occur there, helping

to analyze the work in the classroom. According to the author, the study of patterns avoids the diversity of interpretations that could be given from the first perceptions. For their part, Pagés, Olave and Lezama (2018), point out that an interaction pattern is an interaction structure that occurs face to face between two or more subjects, such that:

- “It serves to reconstruct a specific regularity of interaction focused on a theme.
- It refers to concerted actions, interpretations and mutual perceptions of at least two participants, and is not the sum of their individual actions.
- The structure is explainable by means of a set of rules. (pp. 146-147).

Based on the study of patterns, we find that Hermkes , Mach and Minnameier (2018) are committed to the identification of significant units of what is observed and from there, they identify patterns; the researchers highlight how useful this way of studying interactions in the classroom is for them.

In that study by Hermkes et al. (2018) highlight that the predominant interaction pattern shows a low level of cognitive efficiency in the teacher's interventions with their students. Leguizamón (2017) for his part, examines the interaction patterns that arise when observing future teachers in different mathematics classes, highlighting among the most common interactions those in which the teacher asks short questions, to which brief answers correspond on the part of the teacher. of the students. In the same way, in that same study, the self-response on the part of the teacher was reported in the teacher trainer.

In relation to learning, Donoso et al. (2020) reveal that most of the interactions promoted by teachers require low cognitive demand from their students and little feedback from the teacher. The opposite is the case reported by Borssoi , Silva and Ferruzzi (2021), who observe a teacher promote interactions characterized by guiding their students to reflect and share their knowledge, thus achieving learning that requires a complex cognitive demand such as mathematical modelling.

Pages et al. (2018) report a study carried out with three future teachers and propose two types of patterns derived from interactions: the *extractive-funnel pattern* and the *discussion-focus pattern* . In the first, students do not need to be cognitively involved in the activity to adequately answer the

questions that are posed or arise, but it is the teacher who does it, and knows what answer he wants to arrive at. In the second, the student works actively and has the responsibility of developing an explanation and justification of what is done, the teacher seeks to develop cognitive and metacognitive processes in students and achieve the purpose of the tasks. The results of this study show a predominance of the extractive-funnel pattern in future teachers.

Given the practical nature of studying interactions by classifying according to certain repeating patterns, we decided to assume the interaction patterns established by Pagés et al. (2018). The patterns make it easier for us to focus on the actions that are carried out in the training session and allow us to detect the characteristics of the interactions and the role that the trainer plays in them.

The error

The interactions in the classroom give rise to the appearance of different types of errors by those who participate. In this sense, there is unanimity in assuming that errors are inherent to learning processes (Lannin et al., 2007; NCTM, 2000), so any learning situation is subject to the possibility of their assumption and these should be taken advantage of.

One of the main characteristics of the error is its persistence over time; Thus, McAllister and Beaver (2012) analyze the type of errors that future teachers make when creating problems with fraction operations. Among the most outstanding of the study is the weak conceptual understanding of operations with fractions. This presence of errors in populations that are supposed to have already studied this and other topics is reported by other studies such as those by Aguerrea et al. (2022); Plaza, González and Vasyunkina (2020); Booth, Barbieri, Eyer, and Paré-Blagojev (2014) and Cangelosi, Madrid, Cooper, Olson, and Hartter (2013). According to Cangelosi et al. (2013), errors persist because they can be stuck at a low level of the associated concept and overcoming them means being addressed in different ways (Aguerrea et al., 2022; Cangelosi et al., 2013).

On the other hand, we found studies that focus on how teachers deal with the appearance of errors in their classes. Schleppenbach, Flevares, Sims and Perry (2007) in a comparative study between teachers from China and the United States, analyze the speeches of these teachers and their reactions to the appearance of errors in class. The results show that Chinese teachers react differently to American ones; Faced with the appearance of errors in their

classes, teachers in China formulate questions that promote reflection on the manifested error; instead, US professors are more inclined to comment on the error. For their part, Pinzón, Gómez and González (2022) in a questionnaire applied to teachers, inquire about their curricular practices in the face of the errors that students manifest. Among the results, it stands out that two thirds proposed activities focused on the teacher himself, leaving only a third that propose activities in which students assume the leading role in the activities under the guidance of the teacher. These results indicate the non-uniformity of reactions to the appearance of errors in their classes.

Fractions

Fractions gather a set of errors that persist over time. Part of this reality is very probably the conceptual fragility referred to by Cangelosi et al. (2013).

In the case of fractions, this conceptual fragility could be partly due to the fact that their study is often reduced to representation as part-whole. At this point it is important to point out that the mobilization between the different interpretations of the fraction is usually limited. In this regard, Tunç-Pekkan (2015), through a test carried out on 656 4th and 5th grade American children, finds low performance in items where more advanced reasoning is required, compared to those where the required interpretation is linked to the fraction as part/whole.

Researchers like Cortina; Zuñiga and Visnovska (2013) state that the conceptual flexibility of the fraction is affected when an interpretation is favored. For example, teachers, by privileging the part-whole conception in the context of surfaces, rely on the assumption that the activities of this interpretation turn out to be more significant for students because they allow, through the counting scheme, to identify the numerator and denominator of a fraction, for the areas already divided and shaded. On the other hand, it also allows the use of representation systems such as verbal (reading fractions) and symbolic (numerical expression associated with a fraction).

Privileging the interpretation of the fraction as part-whole and its figural representation in a continuous context turns out to be a fragile didactic practice in which errors can be generated that will later affect other learning. A clear example of a possible error in students can be seen in the research by Singh, Hoon, Nasir, Han, Rasid and Hoong (2021), where they ask a student to graph $10/9$. The student indicates that he cannot use the graphic representation of the pie (figural representation in a continuous and circular

context) because it is a fraction where the numerator is greater than the denominator (improper fraction).

The researcher now asks him to graphically represent $4/9$, the student after several attempts generates the representation in Figure 1.

Figure 1

Adapted from Singh et al. (2021)



At this point it is important to note that when representing proper fractions using the circle, the student tends to divide the figure using the segment called diameter, allowing the circle to be divided into only an even number of circular sectors. Then he takes one of those circular sectors and divides it in half, thus obtaining a total of 9 parts, but two of them are of a different size from the rest. This action on the part of the student is due to the double counting scheme (the total number of parts into which the figure has been divided is counted and the number of parts that have been shaded is also counted) allowing them to identify fraction symbols without taking into consideration that the parts of the subdivision are of the same size. (Tunç-Pekkan, 2015).

METHODOLOGY

We present an episode framed in a research project that analyzes the interactions led by two teacher educators in a refresher program for primary school teachers. The characteristic of this episode is that it generates a set of interactions from an erroneous approach to the conceptualization of fractions and their teaching by one of the participants. This training program was based on the main deficiencies found in a diagnosis made to primary school teachers, in which one of the shortcomings was the subject of fractions. The participating

teachers belong to a network of schools that serve disadvantaged populations in Venezuela.

As a methodological approach we assume ethnography because it allows the reconstruction of events as they happen (Flick , 2015; Álvarez, 2011). In this sense, we recorded sixteen hours of video, which is the duration of three training sessions, and from them we selected for this writing an episode of about 17 minutes, characterized by the rich repertoire of interactions between two trainers and the participating teachers generated by the occurrence of an error.

To analyze the interaction patterns of each of the trainers, we assume as analysis categories the two interaction patterns proposed by Pagés et al. (2018). The first pattern is the so-called *extractive - funnel*, which is characterized by the fact that the person conducting the activity directs all the interventions towards the desired response , which leads the participants not to get involved in complex cognitive processes to issue their responses. The second pattern is called *discussion – focus* , in this case those who direct the activities are two trainers through questions, examples and counterexamples, making the participants reach their own conclusions resorting to argumentation based on their own reflection; This supposes, on the part of the participants, the activation of complex cognitive processes.

To help us characterize and analyze these two patterns, we watched the video of the episode numerous times and paid attention to the following aspects: first , the intention of the questions posed by the trainers; the second, characteristics of these questions and the respective answers that these questions generate in the participants and finally, the way in which the trainers deal with the error that arises among the answers of the participants. Taking these four aspects into account, we formulated the properties of each of the patterns, adapting to the data that emerged from the observation.

With the two patterns proposed by Pagés et al. (2018) and the properties that define each of the categories, we elaborate Table 1.

Table 1

Analysis Categories

Categories	Properties
Extractive Pattern - funnel	<ul style="list-style-type: none">*The trainer directs the participants towards the expected answer.* The trainer does not take into account errors or issues that may divert attention to where he wants to direct it.*The trainer checks if the participant responds as expected.*Participants try to answer as they suppose the trainer expects them to.*The trainer's questions place little cognitive demand on the participants.
Pattern Discussion – Targeting	<ul style="list-style-type: none">*Participants are guided through questions that allow them to respond correctly.* The trainer considers errors and issues as an opportunity to reflect on the issue.*Through questions, the trainer verifies if the participant responds with arguments resulting from their reflection*participants answer according to what they think is the correct answer*The participants in their answers show complex levels of cognitive effort (argumentation, reflection)

RESULTS

We present the results of the observations made addressing each of the analysis categories with their respective properties (see Table 1) .

Interactions Extractive pattern - funnel

Let us remember that the extractive pattern - funnel is characterized because the leading role is assumed by who directs the activity; In this case, it is the trainer who formulates the questions, characterized by being short and in turn leading to brief answers. In the same way, these types of questions require little cognitive demand to whom they are addressed. In this same pattern, errors or topics that are not of interest to the trainer are left aside.

In the training session in which this episode is located, the trainer previously begins by commenting on the results of a mathematics test applied to the teachers participating in the continuous training program; In particular, the answers to the problems and exercises posed around fractions and percentages are discussed, which then leads to a dialogue on the subject of teaching the concept of fraction, as we indicate here below:

Trainer 1 (F1): *But how did they do it? What contents? What did you guys use? What content?*

Teacher 1 (D1): *First we analyze the statement of the problem and we also begin to build the rectangle. With the indications we begin to make our layout...*

F1 interrupts the intervention of D1

F1: And in that layout, what concepts are implicit? because you divide that rectangle in equal parts, what concept is implicit there? What concept?

(voices are heard and the teachers try to respond, as if trying to say the answer expected by F1, until one of them succeeds)

D1: At least the function (others continue to affirm the word function as well) . When they are dividing the rectangle into equal parts, the fraction!...

F1: The fraction, right?

The first thing that we distinguish in the dialogue is the effort on the part of the trainer to direct the discussion towards the subject of fractions as part-whole, that is, to direct attention towards the answer expected by him (the correct one, according to his expectations). . Note that from the beginning F1 takes expressions used by D1 such as, “we start to build the rectangle”, “we start to make our layout” and transforms them if necessary, which is the case when F1 expresses “because you divide *that rectangle into equal parts*”. The idea is to bring the discussion to the context of the fraction as part – whole. It does so because these phrases are closely related to the definition of a fraction as part-all in school math culture. This effort on the part of F1 to direct the discussion is consolidated at the moment that he closes his intervention by expressing “*right?*”. *With this question* , you avoid any possibility of diverting attention to other issues and thus achieve the expected response .

Another characteristic of F1 when assuming the extractive - funnel pattern, is leaving aside expressions that are not directed towards the topic of fractions. Thus, when D1 and other participants mention the function, F1 simply does not take such response into consideration, much less investigates why they allude to this term. The interest of F1 in directing all the attention of the participants towards the fractions as part-whole is reiterated, hence it stays with that word or phrase that it assumes as the correct answer.

In relation to the characteristics of the questions and their corresponding answers, we highlight the formulation of brief questions that in turn generate short answers; For example, F1 asks questions like, “ *what concept is implicit there?, what concept?*” , “ *the fraction right?*” . One consequence of this type of questions is that they lead to short answers that require little cognitive effort from the participating teachers, since they only need to guess the answer . In this case D 1 responds to what it considers that F1 wants as a reply.

Finally, regarding the error, we noticed on the part of F1, not reacting as it had been doing, but giving way to F2. Let's look at the next part of the dialogue:

D1 : ... then the children make a mistake that sometimes they want to use a circumference and divide it into five. It cannot be divided into 5 because it does not have 5 equal parts. So that is where the teacher's orientation is in telling them: You can use a rectangle, right, and even the same little squares in the squared notebook are divided into 5 equal parts.

Once D1's intervention has finished, a brief waiting period begins in which F1 does not react to what D1 asserted and it is, at that moment, that F2 intervenes.

This set of interactions shows in F1 a tendency to direct the interventions of the participants towards the subject of their interest, the fraction as part - whole. His questions - most of them short - require brief answers and, consequently, not very complex cognitive processes; In addition, its lack of interest in interventions where another topic is mentioned and its demarcation in the face of an error presented by one of the participants, allow us to affirm that F1 leans more towards the extractive-funnel pattern.

Discussion pattern interactions - targeting

The discussion-focusing pattern is characterized by the fact that the person leading the training activity does so through questions, examples and counterexamples, taking advantage of the error to reflect on and learn about it. As a consequence, the interventions of the participations are characterized by being supported by arguments based on reflection.

We find these types of interactions registered when an error arises related to the figural representation of the fraction $1/5$ through a circle. The person who characterizes it best is trainer 2 (F2). In this sense, F2 does not let errors pass and addresses and guides them through questions, allowing those who express themselves in an erroneous way to respond correctly. Let's see when D1 confuses circumference and circle:

D1 : ... then the children make a mistake that sometimes they want to use a circle and divide it into five (referring to a circle by means of gestures with their hand and a leaf) . It cannot be divided into 5 because it does not have 5 equal parts. So that is where the teacher's orientation is in telling them: You can use a rectangle, right, and even the same little squares in the squared notebook are divided into 5 equal parts.

Faced with this confusion between circumference and circle, F2 intervenes:

F2: I didn't understand the 5 equal parts of the circumference ... (doesn't finish the word), the... Are you talking about the circle or circumference?

D1 : "yes, the circle" ...

Despite the fact that D 1 rectified the misuse of the term "circumference", it continued to be expressed as if it were synonymous with circle and F2 intervenes again . Let's see the continuation of the dialogue :

D1 : "yes, the circle" ... Sometimes I at least have been ... I always had the opportunity to give a third grade three times and then the children wanted to do the division in a circle (sic) of five equal parts, so they sometimes divided the circumference (sic) into 4 and took one of the 4 and divided it, that is, it is supposed that if it is a fraction, it must be divided into equal parts, then, in this aspect, the child should be

suggested to use the appropriate geometric figures, which ones can be divided into equal parts and which ones cannot.

F2: *Are you talking about the circle or the circumference ?*

D1: *The circle*

After once again correcting the misuse of the term "circumference" by D1, the trainer F 2 begins to formulate questions that lead to reflect on what D1 affirmed in terms of "that a circumference (circle) cannot be divided into five equal parts" referring to the fraction $\frac{1}{5}$. This performance of F2 leads to another characteristic of the focus discussion pattern, which is to promote reflection, looking for the participants to respond with arguments. Let's see:

F2: *Can the circle be divided into 5 parts?*

D 1: *No (it is a No with a prolonged voice) (While teacher D 1 is saying no, the voices of the other participants are heard, some say no and others yes.)*

D1 intervenes again

D1: *Not necessarily, because they are equal parts. Generally, (voices of participation are heard), generally, exactly! (confirming the intervention of another teacher), they are 4 equal parts.*

F2: *What do the others think?*

F1: *Who has a different appreciation?*

We observe that F2 when faced with an error, reacts without immediately correcting it, asking for a clarification. In this way, F2 guides the discussion through questions ("Can the circle be divided into 5 equal parts?"). As a result of these questions, a debate is generated between the participants and F2, verifying if the correct answer is reached through the argumentation of their answers.

It is noteworthy that before this attitude of F2, F1 assumes a new position and asks "Who has a different appreciation?". It seems that F1 rectifies its initial attitude in this dialogue.

Continuing the discussion, another teacher, D2, points out:

D2 : *Yes you can, because in any case you would have to measure the circle, right, you divide the diameter (referring to*

the radius with your hands) and divide it, that is, you can divide it into 5 parts, equal parts. They are going to measure the same, you can divide the 5 parts and have them all measure the same, that the arch be the same”,

Although D2's explanation is not entirely correct, it is noted that D2 responds according to what she thinks is the correct answer and supports it. This seems to lead the participants to understand that the situation initially expressed is an error (indicating that a circle could not be divided into 5 equal parts).

After the intervention of D2 who affirms that, if a circle can be divided into five equal parts and, therefore, represent $\frac{1}{5}$ in this way, D1 intervenes again and says:

D1: I say this from the experience that sometimes we form the cake. At least I tell my children: if I have a cake and I divide it into 2, then I divided a cake into 2, then that fraction represents a half, then they begin to schematize that nothing else I am going to use the circle. And if you present a fifth, they sometimes make the circumference (sic) divide it into 4 and one of the fourth, they come and draw the line. (It is interrupted with interventions by others who are little understood)

Again D1 reiterates that a circle cannot be divided into five equal parts; however, there is an attempt to qualify his answer. At times D1 seems to no longer be situated in the field of mathematics, where he initially contextualized his intervention, but in the field of teaching the concept of fractions. For this, D1 resorts to his experience and gives indications that this "cannot" refers to the fact that it is not convenient to divide the circle into five equal parts because from his perspective, it confuses his students. It becomes an argument more of a didactic type of content than a mathematical one. However, it is observed in the dialogue that the confusion exists and that is why F2 insists asking "...but can or can't you divide the circle by five"? to which the teachers respond that it is possible. In this way, F2 seeks to confirm through this question if the error has already been overcome, at least for that moment.

Then F2 delves into the error manifested by D1 and resumes the example expressed by this teacher, posing the following question:

F2: Okay. They do that (divide the circle into four equal parts and take one of them and divide it in half). That is an error. Very good. Why that mistake? What is the cause?

By asking F2 the question “Why this error? What is the cause?”, we noticed questions that require a complex cognitive effort, in this case, it promotes metacognition and argumentation. A position to reflect on the error and thus discover the causes is observed among the participants. This reflective process is manifested in the following interventions:

D3: *I think it is because of not using the measurement instruments and that is a more complicated fact, that is what the protractor is for.*

F2: *Aha, on the one hand, that may be something, but why do they tend to do that, why do you say that they always divide by 4 first?*

D1 intervenes again

D1: *Well, because generally, sometimes from experience, we take the circle and start: a half, a quarter, and that's it, but sometimes we don't give those measures to a child of one fifth and we take that, that care, when it is a fifth we have to do it like this, we take measures, not because as we are starting it in what is the knowledge of fractions. So when they are presented with a fifth, they divide, as they already are. Accustomed to seeing a room, they divide the circle into 4 and one of those rooms they share in 2.*

F2: *But why do they do that? I agree with you ...*

D5: *Because it is easier for us to teach the child to divide into pairs (referring to the denominator): we are going to divide into a fourth, we are going to play the two fourth. We are always teaching them to divide into pairs and we are not making use of the measuring instruments.*

Trainer 2 : *Why the tendency for boys to go to a room, to a middle in a circle? Why will it be? Is it a mental maturity problem or are there other reasons?*

D4: *We schematize them, we teach them that there are only some figures or elements that are distributed in certain quantities and the student always learns, it is recorded that we can always divide the square into 2 parts or 4 parts. We schematized those models and it should not be like that.*

It could seem at this moment that F2 tries to direct attention to the correct answer, seeming to locate itself in the extractive-funnel pattern; however, the type of F2 questions, emphasizing why, indicate that their intention is to promote reflection as a resource to analyze the error made by the students.

From here the dialogue now opens to discuss the reasons why students make this type of error and F2 offers an explanation about it. In this way F2 closes the activity oriented to analyze the causes of the errors in the fractions. The set of these F2 interventions indicate, on their part, an inclination to favor interactions of the focus-discussion type.

DISCUSSION OF RESULTS

The information reports interactions promoted by the two trainers based on an erroneous approach by one of the participants in relation to the subject of fractions, giving rise to distinguishing different tendencies in each of them. On the one hand, F1 favors extractive-funnel type interactions. On the other hand, F2 tends to promote interactions of the discussion-focus type.

To help us, we pay attention to the following aspects derived from the categories and properties proposed: intentionality of the questions on the part of the trainer and their consequent answers in the participants, cognitive demand that the questions require and positioning of the trainers before the appearance of erroneous approaches by the participants.

Regarding intentionality, F1 takes into consideration only those interventions that can help to raise the issue of fractions; Through brief questions, he extracts the answer that he considers to be a guide or a sign to express the subject on which the training session is concerned. This type of positioning of F1 coincides with the results of Pagés et al. (2018), when they analyze the actions of three future teachers and observe a tendency to direct the questions and statements to a certain type of response. Similar results coincide with those of Leguizamón (2017) who reports the same attitude in three professors belonging to the Mathematics Degree at the Pedagogical and Technological University of Colombia (UPTC). It is an attitude that assumes control on the part of the person who directs the activity in order to focus attention on the subject he wants, leaving aside any other type of response that diverts his attention, as stated by Ingram and Riser . (2019) and Donoso et al. (2020) in their studies, when inquiring about the behavior and interactions derived from the performance of teachers in their classes.

For its part, F2 assumes a more orienting than directive position when interrogating. F2 raises open questions that allow diversifying responses, seeking to explore the thinking of participating teachers. This attitude of F2 contrasts with F1 and the same with the results of the aforementioned studies (Leguizamón, 2017; Pagés et al., 2018; Ingram and Riser, 2019 and Donoso et al., 2020). However, the way of asking the F2 questions coincides with what was reported by Borssoi et al. (2021) who analyze the interactions promoted by a teacher at the time of a mathematical modeling class in a differential equations course. For their part, Schleppebach et al. (2007) in a study in which they compare the discourses of teachers from China and the United States, note a tendency in Chinese teachers to formulate open questions such as those of F2, a situation that is less noticeable in American teachers. These divergent positions indicate how much the interactions depend on the intentionality - conscious or not - of the type of questions.

A second aspect is in relation to the type of responses of the participating teachers based on the questions formulated by the trainers. As a logical consequence of the type of questions, it is observed that when faced with the questions formulated by F1, the answers of the teachers in the training session are generally brief, coinciding with what was reported by Leguizamón (2017), Pagés et al. (2018), Ingram and Riser (2019) and Donoso et al. (2020). On the contrary, the questions posed by F2 mostly correspond to broader answers, which are accompanied by arguments, as also reported by the studies by Borssoi et al. (2021) and from Schleppebach et al. (2007), the latter in the case of teachers in China.

As a consequence of the intentionality of the questions formulated by F1 and F2 and their respective types of answers, it is evident that the cognitive demands vary between both cases. In the case of F1, the responses of the participating teachers tend to reflect a low cognitive demand; On the contrary, in the case of F2, the answers of the participating teachers reflect more complex cognitive processes because they invite reflection and their consequent response based on arguments. The appearance of responses that reflect more complex cognitive processes is also reported by Borssoi et al. (2021); They observe that when the mathematics teacher guides his students to model a situation related to the context of his students, they develop more argumentation and reflection due to the type of questions they promote.

Regarding the position of the trainer before the appearance of errors, the attitude of F1 is not to give it importance if this means diverting attention to the subject of the fraction as part-whole; Thus, when D1 mentions the word

"function" in a context of fractions, he avoids it, he does not ask what those teachers mean with that statement. For his part, F2 assumes a different attitude, he reacts to the erroneous comment that D1 manifests in relation to the representation of $1/5$ in a circle. These differences between F1 and F2 indicate different ways of acting when faced with errors, as reported by González et al. (2022) and Schleppenbach et al. (2007); Both studies report two types of positioning in the face of error, one in which the teacher, when errors appear, is the one who clarifies it, and another, in which the teacher is the one who guides his students to analyze the error and reflect on it. until the student or students become aware of their wrong answers. That is, they consider the appearance of the error in class as an opportunity to promote learning in their students.

FINAL CONSIDERATIONS

There is no doubt that the interactions promoted by the trainers tend to make a difference between the two. F1 shows a tendency towards the extractive - funnel pattern and F2 promotes more interactions typical of the discussion - focus pattern, the consequences of each type of interaction being different. For this reason, as a result of what has been observed, we make a balance of these consequences, without pretension to generalize.

In the first place, it is clear that the intention of the questions posed by the trainers generates different interactions between the participants, which is reflected in the type of answers and opinions that arise in the episode. In the extractive-funnel pattern, the answers are generally brief and with little argumentation; In the case of discussion-focus interactions, the answers are usually accompanied by more arguments. By presenting answers supported by argumentation, whether they are wrong or not, the discussion-focusing pattern promotes reflection among the participants, which shows more complex cognitive processes that enrich teacher training processes.

Secondly, the way in which the trainers deal with the error is different in each type of pattern. In the extractive-funnel pattern, the errors in the answers of the participants are ignored or partially used for the convenience of the trainer. Otherwise, it happens in the discussion-focusing pattern; the trainer assumes the presence of the error, confronts it by questioning around it, avoiding from the beginning issuing opinions about it. The trainer asks the participants in order to promote reflection and their corresponding argumentation. In this way, the cognitive processes associated with the

questions are richer since they are the product of reflective processes, thus contributing to improving the training of the participants (Hummes , Font Moll , & Breda, 2019).

Another aspect to highlight from the episode studied is the ratification of what was stated by Pagés et al. (2018) and Hermkes et al. (2018), who report in their studies how useful the establishment of patterns is for the analysis of interactions in the classroom. This way of studying interactions in the classroom not only has a methodological utility for research, it also constitutes a resource with great potential to contribute to the initial training of teachers, their continuous training and their professional development, since they serve to identify the different actions that take place in the classroom and their subsequent analysis. All this allows us to reflect on the implications of our actions in the teaching and learning processes.

Regarding the limitations of this study, given its nature and objectives, there are aspects that could not be deepened, such as the scope in the transformation of the educational practices of this group of teachers in relation to the teaching of the conceptualization of the fractions. We affirm it, because the persistence of errors plays against it, as confirmed by numerous investigations already mentioned here; In this sense, we believe that the analysis of interactions in classes through patterns contributes to the diagnosis of class situations, however, the consolidation of training requires more time and that is why we propose to continue with this type of study in other contexts. and also thinking about the convenience of longitudinal studies.

Another need that arises, given the limitations of this study, is to investigate more about the teacher educator in terms of the interactions that he promotes in his professional work and his way of approaching the error when it arises in his classroom. This study about the trainer deserves to be investigated both in the context of initial training, as well as in continuous training and professional development.

The path is open to investigate the characterization of the different types of interactions, their respective limitations and potentialities, even more so when it comes to studies related to the professional development of teachers and the role of their trainers.

AUTHORS' CONTRIBUTION STATEMENTS .

HPS coordinated the entire process. GP and AP conceived the idea presented. All authors developed the theory. HPS collected the data. HPS and AP adapted the methodology. All authors analyzed the data. All the authors actively participated in the discussion of the results, reviewed and approved the final version of the paper.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author, HPS, upon reasonable request.

REFERENCES

- Aguerrea, Maitere, Solís, María Eugenia y Huincahue, Jaime. (2022). Persistent mathematical errors when entering initial teacher math training: the linearity case. *Uniciencia*, 36(1), 49-65. <https://doi.org/10.15359/ru.36/1.4>
- Álvarez, y Carmen. (2011). El interés de la etnografía escolar en la investigación educativa. *Estudios Pedagógicos*, XXXVII(2), 267–279. <https://doi.org/10.4067/S0718-07052011000200016>
- Booth, Julie L.; Barbieri, Christina; Eyer, Francie & Pare-Blagoev, E. Juliana (2014). Persistent and Pernicious Errors in Algebraic Problem Solving. *The Journal of Problem Solving*, 7(1), 3. <https://doi.org/10.7771/1932-6246.1161>
- Borssoi, A.H., Silva, K.A.P. da e Ferruzzi, E.C. (2021) Aprendizagem Colaborativa no Contexto de uma Atividade de Modelagem Matemática. *Boletim de Educação Matemática*, 35(70). <https://doi.org/10.1590/1980-4415v35n70a17>
- Cangelosi, Richard; Madrid, Silvia; Cooper, Sandra; Olson, Jo & Hartter, Beverly (2013). The negative sign and exponential expressions: Unveiling students' persistent errors and misconceptions, *The Journal of Mathematical Behavior*, 2(1), 69-82. <https://doi.org/10.1016/j.jmathb.2012.10.002>.
- Chick, H. & Beswick, K. (2018) Teaching teachers to teach Boris: a framework for mathematics teacher educator pedagogical content knowledge. *J*

Math Teacher Educ, 21, 475–499. <https://doi.org/10.1007/s10857-016-9362-y>

- Chico, J. (2018). Impacto de la interacción en grupo en la producción de la lengua del álgebra en clase de matemáticas. *Avances de Investigación En Educación Matemática*, 14, 31–47.
- Coll, C., y Sánchez, E. (2008). *Presentación. El análisis de la interacción alumno-profesor: líneas de investigación*. 346, 15–32. <https://www.educacionyfp.gob.es/dam/jcr:9a33c94d-977b-4290-bcaf-3ed775a8195b/re34601-pdf.pdf>
- Contreras, L. C. (2021). Una aproximación a un modelo de conocimiento del formador de profesores de matemáticas. *Revista Venezolana de Investigación en Educación Matemática*, 1(1), e202101. <https://doi.org/10.54541/reviem.v1i1.12>
- Cortina, José Luis; Zuñiga, Cñaudia y Visnovska, Jana (2013) La equipartición como obstáculo didáctico en la enseñanza de las fracciones. *Educación Matemática*, 25(2), 7-29. <https://doi.org/10.24844/EM>
- Donoso Orosio, E., Valdés Morales, R., y Cisternas Núñez, P. (2020). Las interacciones pedagógicas en las clases de resolución de problemas matemáticos. *Páginas de Educación*, 13(1), 82-106. <https://doi.org/10.22235/pe.v13i1.1920>
- Flick, U. (2015). *El diseño de la investigación cualitativa*. Morata.
- Godino, Juan D. & Llinares, Salvador (2000). El interaccionismo simbólico en educación matemática. *Educación Matemática*, 12(01), 70-92.
- Pinzón, Andrés; Gómez, Pedro & González, María José (2022) Mathematics Teachers' Feedback Responses to Students' Errors and Unexpected Strategies. *Australian Journal of Teacher Education*, 47(3), 19-34. <http://doi.org/10.14221/ajte.2022v47n3.2>
- Heinze, A. & Reiss, K. (2007). Mistake-handling Activities in the Mathematics Classroom: Effects of an In-service Teacher Training on Students' Performance in Geometry. In J. H. Woo, H. C. Lew, K. S. Park & D. Y. Seo (Eds.), *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 31, pp. 9-16). Seoul, Corea: PME.

- Hermkes, R., Mach, H., & Minnameier, G. (2018). Interaction-based coding of scaffolding processes. *Learning and Instruction*, 54, 147–155. <https://doi.org/10.1016/j.learninstruc.2017.09.003>
- Hummes, Viviane; Font Moll, Viçent y Breda, Adriana (2019) Uso Combinado del Estudio de Clases y la Idoneidad Didáctica para el Desarrollo de la Reflexión sobre la Propia Práctica en la Formación de Profesores de Matemáticas: *Acta Scientiae* 21(1), 64-82. <https://doi.org/10.17648/acta.scientiae.v21iss1id4968>
- Ingram, J., & Riser, P. A. (2019). Experiences of problem solving in whole class interactions. *Avances de Investigación En Educación Matemática*, 16, 5–21.
- Lannin, J.K., Barker, D.D. & Townsend, B.E. (2007) How students view the general nature of their errors. *Educ Stud Math* 66, 43–59. <https://doi.org/10.1007/s10649-006-9067-8>
- Leguizamón, José Francisco (2017) Patrones de interacción comunicativa del profesor universitario de matemáticas. Un estudio de caso. *Praxis y Saber*. 8(16), 57-82. <https://doi.org/10.19053/22160159.v8.n16.2017.6200>
- McAllister, C.J. & Beaver, C. (2012), Identification of Error Types in Preservice Teachers' Attempts to Create Fraction Story Problems for Specified Operations. *School Science and Mathematics*, 112: 88-98. <https://doi.org/10.1111/j.1949-8594.2011.00122.x>
- Mendoza von der Borch, T. (2018). Aprender del problema y de las formas de interacción. La construcción de conocimientos relativos al porcentaje en clases de secundaria. *Revista Colombiana de Educación*, 74, 133–154. <https://doi.org/10.17227/rce.num74-6901>
- National Council of Teachers of Mathematics (2000) *Principles and Standards*. National Council of Teachers of Mathematics.
- Pagés, Daniela, Olave, Mónica, y Lezama, Javier. (2018). Estudio de interacciones en clase de matemáticas: un caso con futuros profesores de matemáticas. *Educación matemática*, 30(2), 140-170. <https://doi.org/10.24844/em3002.06>
- Parra-Sandoval, Hugo (2020). Problematización y conocimiento especializado del formador de profesores de Matemáticas. *Revista Paradigma*, 41,

251-270. <https://doi.org/10.37618/PARADIGMA.1011-2251.2020.p251-270.id873>

- Plaza, L., González, J., y Vasyunkina, O. (2020). Obstáculos en la enseñanza – aprendizaje de la matemática. Revisión sistemática. *Acta Latinoamericana de Matemática Educativa*, 33(1), 295–304.
- Rhoads, K., Radu, I. & Weber, K. (2011) The Teacher Internship Experiences of Prospective High School Mathematics Teachers. *International Journal of Science and Mathematics Education*. 9, 999–1022. <https://doi.org/10.1007/s10763-010-9267-7>
- Sánchez-Barbero, B., Calatayud, M. y Chamoso, J. M. (2019). Análisis de la interacción de maestros cuando resuelven problemas realistas conjuntamente con sus alumnos en aulas de primaria, teniendo en cuenta su experiencia docente. *Uni-pluriversidad*, 19(2), 40-59. <https://doi.org/10.17533/udea.unipluri.19.2.03>
- Santagata, Rosella (2005) Practices and beliefs in mistake-handling activities: A video study of Italian and US mathematics lessons. *Teaching and Teacher Education*, 21(5), 491-508, <https://doi.org/10.1016/j.tate.2005.03.004>
- Schleppenbach, M.; Flevares, L. M.; Sims, L.M. & Perry, M. (2007) Teachers' Responses to Student Mistakes in Chinese and U.S. Mathematics Classrooms, *The Elementary School Journal*, 108(2), 131-147. <https://doi.org/10.1086/525551>
- Singh, P., Hoon, T. S., Nasir, N. A. M., Han, C. T., Rasid, S. M., & Hoong, J. B. Z. (2021). Obstacles Faced by Students in Making Sense of Fractions. *The European Journal of Social y Behavioural Sciences*, 30(1), 34–51. <https://doi.org/10.15405/ejsbs.287>
- Smart, J. B. & Marshall, J. C. (2013). Interactions between classroom discourse, teacher questioning, and student cognitive engagement in middle school science. *Journal of Science Teacher Education*, 24(2), 249–267. <https://doi.org/10.1007/s10972-012-9297-9>
- Tulis, María (2013) Error management behavior in classrooms: Teachers' responses to student mistakes. *Teaching and Teacher Education*, 33, 56-68. <https://doi.org/10.1016/j.tate.2013.02.003>
- Tunç-Pekkan, Z. (2015). An analysis of elementary school children's fractional knowledge depicted with circle, rectangle, and number line

representations. *Educational Studies in Mathematics*, 89(3), 419–441.
<https://doi.org/10.1007/S10649-015-9606-2>

Vélez Gutiérrez, C. F., y Ruíz Ortega, F. J. (2021). Una revisión sobre metacognición. Algunas implicaciones para los procesos educativos. *Tesis Psicológica*, 16(1), 100-117.
<https://doi.org/10.37511/tesis.v16n1a5>

Voigt, J. (1985). Patterns and routines in classroom interaction. *Recherches en Didactique des Mathématiques*. 6(1)