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Tasks About Impossible Random Events: A Pedagogical Game as a Teaching Tool for the Early Years of Elementary School

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ABSTRACT

Background: The concepts of impossible random events are an integral part of the National Common Curricular Base – BNCC and of the thematic unit "Probability and Statistics", in which 6- to 8-year-old students must classify events involving chance, such as "it is impossible to happen", indicating the need for proposals that help in the teaching and learning process. **Objectives**: A possibility was shown to develop a pedagogical work for the initial years of Brazilian Elementary School based on pedagogical games, creating methodological theoretical subsidy to rethink strategic methods, resizing them in order to minimise the existing gap between ludic activities and everyday tasks carried out by students, spontaneously, and the work triggered in the classroom. **Design**: Cards were prepared for the game "Probability in Action", called Questions (?), which are tasks based on problem situations with the objective of favouring the apprehension of contents and the development of probabilistic knowledge, having as theoretical support the Anthropological Theory of the Didactic - ATD by Yves Chevallard, consisting of two blocks, the practical and the theoretical. **Setting and Participants**: It was a theoretical work that presented a didactic and mathematical praxeology that exposed and explored, in detail, the practice and the probabilistic theory involved in the cards of the game "Probability in Action" focused on impossible random events in which it is associated with the solving tasks or problem situations. **Data collection and analysis**: Tasks associated with impossible random events were created and described using ATD, comprising type of tasks (T); techniques (τ) that solve tasks of this type; technology (θ) that justify the techniques and guarantee their validity, and, finally, the theory (Θ) that justifies the technology. This praxeological quartet is denoted [T, τ, θ, Θ], and the block [T, τ] is called practical-technical (praxis), or know-how block; and the block [θ, Θ] is called the technological-theoretical block (logos) or

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knowledge block. **Results**: In terms of the ATD, its use allowed identifying a set of praxeologies that made it possible to characterize both the probabilistic object and the didactic approach for such an object, and the praxeological organization was composed of four elements: 1) Task (T) and its subtasks (t), which was characterized by the action required by the problem situation proposed for the question cards in the game; 2) Technique (τ), identifying the way to perform the task and its subtasks; 3) Technology (θ), specified by the set of definitions, properties, axioms and theorems that justify the technique, that is, that the random phenomenon refers to experimental realization when its observation is impossible; 4) Theory (Θ), the field in which technology is justified, probability theory. **Conclusions**: Supported by the results of this study, basic probabilistic concepts were thought of that meet the needs of Brazilian fundamental education, so that it is possible to contribute to the growth and development of an autonomous, critical, active society capable of making decisions in front of to the information you encounter.

Keywords: impossible random event; elementary school; pedagogical game; anthropological theory of the didactic; Brazil.

Tarefas sobre eventos aleatórios impossíveis: um jogo pedagógico como ferramenta de ensino para os anos iniciais do ensino fundamental

RESUMO

Contexto: Os conceitos de eventos aleatórios impossíveis são parte integrante da Base Nacional Comum Curricular – BNCC e da unidade temática “Probabilidade e Estatística”, na qual alunos de 6 a 8 anos devem classificar eventos envolvendo o acaso, tal como “é impossível acontecer”, indicando a necessidade de propostas que auxiliem no processo ensino e aprendizagem. **Objetivos**: Mostrou-se uma possibilidade de desenvolver um trabalho pedagógico para os anos iniciais do Ensino Fundamental brasileiro, baseado em jogos pedagógicos, criando subsídio teórico metodológico a um repensar sobre os métodos estratégicos, redimensionando-os a fim de minimizar o hiato existente entre as atividades lúdicas cotidianas realizadas pelos alunos, espontaneamente, e o trabalho desencadeado em sala de aula. **Design**: Foram elaboradas cartas para o jogo “Probabilidade em Ação”, denominadas Perguntas (?), que são tarefas baseadas em situações problemas tendo como objetivo favorecer a apreensão dos conteúdos e o desenvolvimento do conhecimento probabilístico tendo como aporte teórico a Teoria Antropológica do Didático - TAD de Yves Chevallard, composta por dois blocos, o prático e o teórico. **Ambiente e participantes**: Tratou-se de um trabalho teórico que apresentou uma praxeologia didática e matemática que expôs e explorou, de forma detalhada, a prática e a teoria probabilística envolvidas nas cartas do jogo “Probabilidade em Ação” focadas em eventos aleatórios impossíveis na qual associa-se à resolução de tarefas ou situações problemas. **Coleta e análise de dados**: Foram elaboradas tarefas associadas a eventos aleatórios impossíveis e descritas por meio da TAD, composto por: tipo de tarefas T; técnicas (τ) que resolvem as tarefas desse tipo; tecnologia (θ) que justificam as técnicas e garantem sua validade, e, finalmente, a teoria (Θ) que justifica a tecnologia. Esse quarteto praxeológico é denotado [T, τ, θ, Θ], sendo que o bloco [T, τ] é denominado de prático-técnico (*práxis*), ou bloco do saber-fazer; e o bloco [θ, Θ] é denominado bloco tecnológico-teórico (*logos*) ou bloco do saber. **Resultados**: Nos termos da TAD, a sua utilização permitiu identificar um conjunto de praxeologias que possibilitou caracterizar, tanto o objeto probabilístico quanto a abordagem didática para tal objeto, sendo que a organização praxeológica foi composta por quatro elementos: 1) Tarefa (T) e suas subtarefas (t), que se caracterizou pela ação demandada pela situação-problema proposta para as cartas de perguntas do jogo; 2) Técnica (τ), identificando a forma de realização da tarefa e suas subtarefas; 3) Tecnologia (θ), especificada pelo conjunto de definições, propriedades, axiomas e teoremas que justificam a técnica, ou seja, que o fenômeno aleatório se refere a realização experimental quando sua observação é impossível; 4) Teoria (Θ), o campo no qual se justifica a tecnologia, a teoria da probabilidade. **Conclusões**: Apoiados nos resultados desse estudo, pensou-se nos conceitos probabilísticos básicos que vão ao encontro das necessidades do ensino fundamental brasileiro, para que assim seja possível contribuir com o crescimento e o desenvolvimento de uma sociedade autônoma, crítica, ativa e capaz de tomar decisões frente às informações as quais se depara.

Palavras-chave: evento aleatório impossível; ensino fundamental; jogo pedagógico; teoria antropológica do didático; Brasil.

INTRODUCTION

Random thinking, also called probabilistic, helps to make decisions in situations of uncertainty, chance, risk or ambiguity due to lack of reliable information, in which it is not possible to predict with certainty what will happen. In addition, it helps to find reasonable solutions for problems for which there is no clear and safe solution, approaching them with a spirit of exploration and investigation through the construction of models of physical, social or game phenomena and using strategies such as the exploration of data systems, simulation of experiments and performance of counts.

Specifically, about chance, it is considered that this is related to the absence of specific patterns or schemes in the repetition of events or occurrences, and other times with situations in which it is not known what these patterns are, if any, as is the case: of epidemics and diseases; ballot elections; the results of devices such as those used to extract numbered balls for lotteries; techniques for throwing dice or coins or distributing cards or chips in games that are called "chance", etc.

Furthermore, it is considered that in the everyday experiences that students already have about certain events and that can be approached through games, they begin to become aware that their occurrence and that their results are unpredictable, seeking to make intuitive estimates about the possibility of one or another event to occur.

For Cano (2017) the unpredictability of random events composes an initial intuition of chance and allow some numerical assignments to be made to measure the probabilities of events or occurrences, even if initially a little arbitrary, which begin by assigning, for example, probability 0 the impossibility or maximum improbability of occurrence; assign ½ to any two alternatives considered equally likely and 1 (one) to the necessity or maximum probability of occurrence.

For Cano (2017), the situations and processes that allow the systematic counting of the number of possible combinations that can be presumed equally likely, together with the recording of the different results of a random experiment, as well as the attempts to interpret and predict them from the exploration of data systems, they develop in the students the distinction between deterministic and random situations, allowing to refine the measures of probability with numbers between 0 and 1. Later, these situations and processes can be modelled through mathematical systems related to the theory of probability.

Adding to these factors, Alsina (2012) recalls that the probabilistic contents in fundamental education in the curricular guidelines, national and international, propose to work initially on the understanding of typically probabilistic terms, such as "certain", probable" or "impossible", always starting from of situations close to the child.

Franklin et al (2007), Gal (2005) and Jones (2005) state that there are reasons to include probability in schools, related to its usefulness in everyday life, its instrumental role in other disciplines, the need for basic knowledge in many professions, and the important role of probabilistic reasoning in decision-making. Thus, students will encounter randomness not only in the math classroom, but also in biological, economic, meteorological, political, and social (games and sports) environments.

Garfield (2002) reveals that probabilistic reasoning is defined in “how” and in what “sense” people reason, interpret and make inferences with probabilistic information. It involves making interpretations about data, ideas of variability, distribution, chance, uncertainty, randomness, probability, sampling, among other concepts.

For Tsakiridou and Vavyla (2015), teaching probability content has numerous advantages, which other mathematical disciplines lack. When dealing with the mentioned contents, children learn to accept the fact that not all everyday situations are deterministic. To work with this fact, the solution is to make it possible for several possibilities to be critically interpreted and to determine a greater or lesser probability of happening. In this way, children accumulate experiences for real-life situations, in which it is necessary to decide on a daily basis the best option among many.

At the same time, children need to accept the fact that some events are impossible. Thus, it is necessary to act deliberately and solve the problem, so one must use their way of thinking, different from that applied in learning other mathematical disciplines (HodnikCadez & Skrbe, 2011).

Vásquez and Alsina (2014) recall that in the Chilean national curriculum, the teaching of probability began with very simple activities in which chance was present, thus favouring the emergence of intuitions. And considering the importance of children apprehending the concept of chance, it was suggested to play random games, for example, with coins and dice.

Thus, our research problem was to show a possibility of developing a pedagogical work for the initial years of Elementary School, based on pedagogical games, creating methodological theoretical subsidy to rethink strategic methods, resizing them in order to minimize the gap existing between the daily recreational activities carried out by the students, spontaneously, and the work triggered in the classroom.

In view of the theme and the research problem, the objective of this work was to elaborate problem situations (tasks according to the Anthropological Theory of the Didactic - ATD) involving the classification of impossible random events for the game "Probability in Action" as a proposal for a possibility of work involving the teaching of probability in the early years of elementary school, following the principles of ATD by Chevallard (1996) and Chevallard, Bosch and Gascón (2001), in the organization of didactic praxeology and mathematics (probability).

THEORETICAL BACKGROUND

In its relationship with learning, the game is a didactic strategy and a ludic activity in the integral development of the child, as it can act as a mediator between a concrete problem and abstract mathematics depending on the intentionality and type of activity (Aristizábal, Colorado & Álvarez, 2011). It is a very useful teaching strategy that provides meaningful learning results for students, as it highlights the acquisition of numerical sense through rich, varied and meaningful situations that stimulate intelligence and imagination (Aristizábal, Colorado & Gutiérrez, 2016).

Lorenzato (2009), indicates that the student, when faced with ludic situations, learns the logical essence of the game and also how to deal with the present mathematics. The game assumes the purpose of developing problem solving skills, allowing the student an easy method to create action plans to achieve certain goals, and thus evaluate their efficiency in the achieved results.

Based on the way Probability was born, we defend the idea that playing is the best way for children to learn probabilistic concepts. Supported by this idea, Góngora (2011) proposes that, in order to work on the concepts associated with this area, games of chance be used from a playful and pedagogical approach, so that not only do students have a first contact with this field of knowledge in a fun but also meaningful way.

We also consider, supported by Hurtado and Costa (1999), that the use of games in the classroom becomes a very useful tool to familiarize the student with the probabilistic world, since random experiences provide a direct correspondence of teaching with his daily life. Difficulties found in this type of experience can help to overcome obstacles of an epistemological nature, which arise in the construction of this knowledge.

Ribeiro and Goulart (2010), Campos and Novais (2010) and Soukef (2014), point out that the use of games in teaching Probability helps students understand the probabilistic nature of games of chance, developing a more critical attitude towards regarding their actual chances of winning games while understanding probability calculation and the concepts of, for example, event and sample space.

According to the National Common Curricular Base – BNCC (Brasil, 2018) it is important to make a distinction between games as specific content and games as an auxiliary teaching tool. It is not rare that, in the educational field, games and games are invented with the aim of provoking specific social interactions between their participants or to fix certain knowledge.

Introducing the game or ludic tasks in the classroom does not need to be a complex process in teaching, in which several approaches and problems arise from problem solving that can be seen as a prize or a goal to be achieved. Some researchers have already analysed the advantages of introducing games in the classroom through the study of practical cases (Malaspina, 2012; Villarroel & Sgreccia, 2012).

In Brazil, Oliveira Júnior (2013) and Oliveira Júnior et al. (2015, 2017, 2018) aimed to present and apply the game "Playing with Statistics and Probability" for elementary school students, with the intention of facilitating the understanding of the content for teaching and learning the basic concepts of statistics and probability, using the problem solving methodology. After the application of the game, most students declared that they enjoyed the activity and that they learned from it, and it can be seen that it served as a methodological support for Statistics and Probability.

Oliveira Junior et al. (2020), Oliveira Júnior and Datori Barbosa (2020, 2022) and Oliveira Júnior and Kian (2022), presented tasks related to the concept of sample space contained in a pedagogical game aimed at teaching Probability for the early years of Elementary School according to the National Common Curriculum Base - BNCC (Brasil, 2018). Letters of questions were elaborated for the game “Playing with Probability”, which are tasks based on problem situations with the objective of favouring the apprehension of contents and the development of probabilistic knowledge, having as a theoretical contribution the Anthropological Theory of Didactics - ATD by Yves Chevallard, composed divided into two blocks, the practical and the theoretical.

For Jackson and Sirois (2009, 2022), children's expectations about the world around them should be broadly assessed through the violation of the expectation paradigm and related habituation tasks. Looking at impossible events should be considered to reflect the surprise in their occurrence.

THE IMPORTANCE OF KNOWING THE CONCEPT OF IMPOSSIBLE RANDOM EVENTS

Shtulman and Carey (2007) and Shtulman, (2009) show that discrimination between possible and impossible events is not always so easy for young children. It's hard for them when they have to compare and discriminate highly unlikely events from completely impossible ones. Their studies have shown that 4- to 7-year-olds are very good at judging impossible events, like catching a shadow, but tend to include unlikely events, like catching a fly with a pair of chopsticks, in the impossible category as well. Children aged 8 to 9 years, in general, did better in this task, although some of them still confused improbable with impossible events.

In a study by Vásquez and Alsina (2017), terms and expressions were identified (Table 1) that students in the early years of elementary school associate with the word and concept of impossible. We sought to analyse the linguistic elements present in the development of problem situations, whose purpose was for students to start training in the formation of the language of chance and probability, so that later they would be able to predict and conjecture in relation to the possibility of occurrence of successes.

Table 1

Terms and verbal expressions used by students in research when dealing with the word impossible (Vásquez & Alsina, 2017, p. 466)

Unlikely
Accidental
Unexpected
It is not possible for this to happen
Bad luck
Very little chance
Luck
Good luck
Very difficult to happen
Of causality

These words that are associated with probabilistic concepts are linguistic elements whose objective is to introduce students to the use of the language of chance and probability, so that later they are able to predict and conjecture about the possibility of occurrence of events.

Vásquez and Alsina (2017) concluded that the terms and verbal expressions that students use to conceptualize the word "impossible" denote difficulties in understanding the vocabulary associated with chance and probability, because in the different terms presented by students it is possible to identify that they associate, in several cases, an incorrect vocabulary. It is observed that they use these terms to refer to those events or situations that have little possibility of occurring or that are very difficult for them to occur and if they do occur, they attribute it to the fact of bad luck or very good luck, such as this is the case with lottery games.

Bringing elements associated with the teaching of probability, in Brazil, according to the National Common Curricular Base - BNCC (Brasil, 2018), the study of probabilities in the early years of Elementary School (students aged 6 to 9 years) aims to promote the understanding that not all events are deterministic. Therefore, initial work with probability should focus on developing the notion of randomness, so that students understand that there are certain events, impossible events and possible events. In addition, the BNCC warns that:

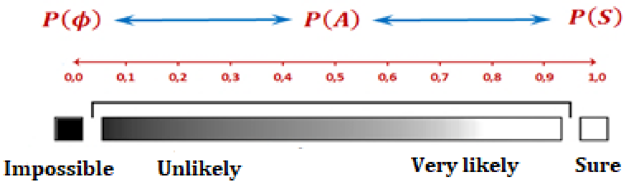
> It is very common for people to judge impossible events that they have never seen happen. At this stage, it is important for students to verbalize, in events involving chance, the results that could have happened as opposed to what actually happened, initiating the construction of the sample space (Brasil, 2018, p. 230, our translation).

Taking MOOC (2020) it is expressed that random events are considered subsets of a sample space, and an event is considered a possible result of an experiment. We can still consider that the probability of the impossible event is zero and the probability of the right event (which represents the entire sample space) is one, so for any other event A it is true that: $0 \leq P\ (A) \leq 1$. considering the scheme (Figure 1) all possibilities between 0 and 1 (exclusive), can be considered possible or probable events and that is indicated by P(A).

In probability theory, it can also be observed in Figure 1, that the impossible event is the one that never occurs, being associated with the empty set, opposite of the safe event that is the sample space itself. In other words, they are all those events or elements that are not contained in the sample space. We can still indicate that it is the result of a random experiment that will never occur, that is, the probability of occurrence of an impossible event is 0%.

Figure 1

Scheme indicating random events (MOOC, 2020, p. 81)



According to Lipschutz (1993, p. 58) a sample space is associated with a random experiment and consequently the associated events, that is,

> The set S of all possible outcomes of an experiment is called the sample space. A particular result, that is, an element of S, is called a sampling point. An event A is a set of outcomes or, in other words, a subset of the sample space S. The event {a} consisting of the unique sample point a $\in$ S is called an elementary event. The empty set ∅ and S are events; ∅ is called an impossible event, and S a certain event.

Subsets of the sample space will be called events. We say that an event occurs when the outcome of the random experiment belongs to the event. We can also say that an event is a particular outcome of a random experiment. In terms of sets, an event is a subset of the sample space.

For Meyer (1982), in the terminology of set theory, an event is a subset of the sample space S, and one of its subsets, the empty set, constitutes an event, called impossible.

For Fonseca and Martins (2011), event is a set of experiment results, and in terms of sets, it is a subset of S, the sample space. Furthermore, it indicates that ∅ is an event, being said an impossible event.

In Magalhães and Lima (2005), the subsets of the sample space are called events and represented by the capital letters A, B, ..., and the empty set, impossible event, is denoted by ∅.

Thus, the probabilistic contents to be addressed, focused on aspects associated with impossible random events, among others, according to the BNCC (BRASIL, 2018) for the early years of Elementary School (1st year to

3rd year), are presented in Table 2 (description of knowledge objects and respective skills).

Table 2

Objects of knowledge *and Skills of the probabilistic contents proposed in the BNCC from the 1st to the 3rd year of Elementary School* (Brasil, 2018, p. 280-281; 284-285; 288-289)

	1st year	2nd year	3rd year
Objects of knowledge	Notion of chance.	Analysis of the idea of randomness in everyday situations.	Analysis of the idea of chance in everyday situations: sample space.
Skills	Classify events involving chance, such as "it will happen for sure", "maybe it will happen" and "it is impossible to happen", in everyday situations.	Classify outcomes of random everyday events as "unlikely", "very likely", "unlikely" and "impossible".	Identify, in random family events, all possible outcomes, estimating those with greater or lesser chances of occurrence.

We understand that the pedagogical systematization with the resolution of problems in the context of games directed to the teaching of probability, if well-structured and guided, allows the approximation and apprehension of probabilistic terms and the meaning of these terms, facilitating the development of the first notions.

Furthermore, in the North American document called Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): a framework for Statistical Education and Data Science (Bargagliotti & Franklin et al., 2020), it is suggested that students should understand that probability is a measure of the chance that something will happen, as it is a measure of the degree of certainty or uncertainty. The probability of events should be seen as founded on a continuum from impossible to certain, with the least likely, equally likely, and most likely being between those limits. Using these probabilistic notions, students can think about crafting answers to a statistical research question, such as: If a student in your class is randomly selected, is he more or less likely to prefer rap, MBP, or rock?

As previously pointed out, there is a large amount of evidence in several studies that, even very young children are able to discriminate impossible events from common events, as in Nóbrega and Spinillo (2015), however, it is not always easy for children to differentiate possible events, but highly improbable, a completely impossible event (Bryant & Nunes, 2012). This fact should be explored in studies to observe what can be done to broaden this understanding.

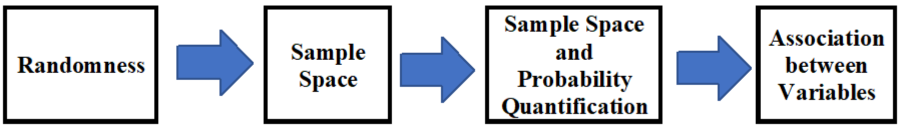
Regarding the teaching program aimed at teaching probability developed by Nunes et al. (2015), these focus on two aspects: (1) on the one hand, it is concerned with promoting children's understanding of the concepts of randomness, evaluating the improvement in their ability to solve mathematical problems in situations involving uncertainty; (2) on the other hand, to promote children's understanding of numerical operations in a context in which they can be sure of the results and, from there, to assess whether the understanding of mathematical ideas that involve certainty can contribute and also improve their knowledge of Probability.

It is worth mentioning that this teaching program includes some of the elements described in the probabilistic literacy model proposed by Gal (2005), related to the treatment of randomness, probabilistic calculations and critical questions.

In Figure 2, we present the scheme of this teaching program that starts with the simplest ideas about randomness, goes through the quantification of probabilities and reaches the understanding of risk (association between variables).

Figure 2

Stages of the teaching program on probability and risk (Nunes et al., 2015)



The focus of our study, in relation to the first unit of the study program, randomness, Nunes et al. (2015) indicate that in probabilistic situations in which a set of possible events can happen, it is presumable to find difficulties in children with these situations. This problem found is explained by the difficulty in identifying, in the set of possible events, which ones will happen or in what order they happen, due to randomness, since it is not possible to determine the way in which the events occur in a sequence or in a random spatial arrangement.

METHODOLOGY

This study is a theoretical work that brings a didactic and mathematical praxeology that exposes and explores, in detail, the practice and mathematical theory (probability) involved in the cards of the game "Probability in Action" focused on random events impossible in the which we associate with the resolution of tasks or problem situations.

We emphasize that the cards for the game "Probability in Action" presented here were created considering the objective (analysis of the idea of randomness in everyday situations) and skills (classifying events involving chance, such as "it is impossible to happen", in everyday situations) of the BNCC curriculum proposal for the early years of Elementary School, Brazil (2018), in order to enable students to understand basic concepts of probability.

When preparing the tasks indicated in the "Question" letters, we highlight that the probabilistic notions were based on the National Common Curricular Base – BNCC (Brasil, 2018), bringing all the objects of knowledge and skills to be developed in the initial years, in the Teaching Program on probability and risk by Nunes et al. (2015), which aims to apprehend probabilistic concepts and in the North American document GAISE II (Bargagliotti, Franklin et al., 2020) for incorporating skills necessary to understand the advances related to the teaching of statistics and probability in recent years, maintaining the spirit of the first version called GAISE I (Franklin et al., 2007).

Thus, we call tasks the various problem situations that will compose the "questions" cards of the game, based on the methodology of problem solving. According to Van de Walle (2009) the game may not look like a problem, but it may be based on a problem. In allowing the game to allow students to reflect on ideas that they have not yet formulated very well, then it fits the definition of a task based on problem solving.

The notion attributed in this work to the task reflects the anthropological meaning of ATD, including only those actions that are human and not derived from nature (Chevallard, 1999), and it is also highlighted that it has its focus on study activities, not being a teaching theory or learning. Chevallard (1996) uses the term "task" in ATD and in this text we use it to represent a problem.

Bosch and Chevallard (1999) restrict the notion of task in mathematics by distinguishing mathematical activity from other human activities, that is, when faced with a task, one must know how to solve it. The "how to solve the task" is the engine that generates a praxeology, that is, it is necessary to have (or build) a technique, which must be justified by a technology, which, in turn, needs to be justified by a theory. The word technique will be used as a structured and methodical process, sometimes algorithmic, which is a very particular case of technique.

In order to achieve these objectives, this research was guided mainly by the BNCC (BRASIL, 2018), which brings the contents and probabilistic skills to be worked on in the early years of Elementary School and by the ATD, which allowed a mathematical praxeological analysis (probabilistic) and didactic about the tasks.

This praxeological notion, in its simplest form, according to Chevallard, Bosch and Gascón (2001), based on mathematical activity or any other activity, refers to two blocks that complement each other, that is, on the one hand, tasks and techniques, and on the other, technologies and theories. In the practical-technical block (praxis) the techniques associated with solving the task will be presented. According to Chevallard (1996), a praxeology related to the task T needs (in principle) a way of performing, that is, a way of executing a certain task. In the block of knowledge (logos), the first component is a rational discourse, called technology (θ) and theory (Θ) which represents a higher level of justification, explanation and production that plays the same role in relation to technology (θ) that it has in relation to the technique (τ).

We can still present, according to Bittar (2017), that the praxeological model proposed to describe any activity, mathematical or not, is composed of: type of tasks T; techniques (τ) that solve tasks of this type; technology (θ) that justify the techniques and guarantee their validity, and, finally, the theory (Θ) that justifies the technology. This praxeological quartet is denoted $[T, \tau, \theta, \Theta]$, and the block $[T, \tau]$ is called practical-technical (praxis), or know-how block; and the block $[\theta, \Theta]$ is called the technological-theoretical block (logos) or knowledge block.

According to Chevallard (1999), a praxeology related to the task T needs (in principle) a way of performing, that is, a way of executing a certain task. Still in Almeida (2018), in his reading of Yves Chevallard, it is expressed that the block of knowledge, is constituted by the organizing and formalizing part of knowledge, in it are: technology and theory. In it appear the tests, descriptions and confirmations that formalize the use of the applied techniques, in a technological discourse, being that the technology justifies the technique. This technological discourse, in turn, is supported by theory, that is, as this knowledge is formalized by the academy in the guiding documents, the theory justifies the technology.

Thus, thinking about this praxeology, the elaborate problem situations that make up the game cards are composed of tasks, consisting of a sequence of subtasks that can be performed using various techniques, justified by technology, which uses theories related to probability as an object of analysis. study. As a theoretical basis, the ATD was used to detail the elaboration of problem situations, which we will call tasks identified by (T), consisting of a sequence of subtasks (t), which can be performed using various techniques (τ) justified by technology (θ) which uses the theory (Θ) of Probability as an object of study.

Thus, the elaboration of the tasks basically obeyed the following steps: (1) present at least one technique to solve the requested tasks; (2) for the described techniques to establish, at least, an outline of a technological discourse; (3) articulate different types of tasks around probabilistic concepts; (4) articulate different types of tasks using the methodology of problem solving.

Therefore, the ATD was used in the preparation of the tasks that make up the game cards due to its mathematical praxeological organization (probabilistic) that allows us to detail the tasks in an organized way, emphasizing both the practical and theoretical aspects, in a complementary way.

RESULTS AND ANALISES

Starting from our object of study and what research carried out in the area reveals to us about the contribution of games associated with problem solving, we seek to create tasks that make up a game based on problem solving that addresses probabilistic content for the early years of Elementary School .

Thus, the curricular activities elaborated by the proposition of problems had their creation process considering the contents of the curricular proposal of the BNCC (Brasil, 2018), in order to allow the students, the initial understanding of basic concepts of probability, such as the analysis of the idea of chance in everyday situations, chances of random events and sample space. In this topic, we present some tasks related to "impossible random events", directed to the objectives to be achieved from the 1st to the 3rd year of Elementary School.

These tasks (problem situations), figure 3, make up the pedagogical game focused on the principles of ATD in the didactic and mathematical (probabilistic) praxeological organization. Each of them is a game card. The intention is for students to recognize how to represent all the possibilities that can be listed from the proposal of a problem aimed at situations that may even be experienced. Thus, some of the game cards involving probabilistic content such as Questions (?), refer to the tasks proposed for the didactic game that consider impossible events (E = ∅), followed by the principles of ATD.

Figure 3

Task 1: Determine the impossible event

1. Choose the option that characterizes the impossible event by drawing one of the cards below:

( ) come out a blue card.
( ) come out a card n° 7.
( ) comes out a red number 7 card.

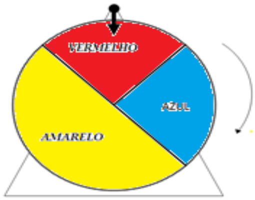
2. Choose the option that characterizes the impossible event by picking one of the fruits from the basket below:

( ) take an orange.
( ) take an avocado.
( ) take a banana.

3. Choose the option that characterizes the impossible event when removing a dice from the bag below:

4. Choose the option that characterizes the impossible event by spinning the wheel below once:

( ) take a blue dice.
( ) take a red dice.
( ) take a yellow dice.



( ) The yellow colour comes out.
( ) The green colour comes out.
( ) The blue colour comes out.

5. Choose the option that characterizes the impossible event when picking up one of the pens below:

( ) pick a black pen.
( ) pick a green pen.
( ) pick a purple pen.

6. Choose the option that characterizes the impossible event when picking up one of the toys below:

( ) get a ball.
( ) get a cart.
( ) get a doll.

Task T_1 consists of determining, based on the proposed situations, when an event should be considered impossible. The subtasks t_1 to t_6 presented in figure 3 propose that the student determine which of the indicated options or events is configured as an impossible event. To analyse the problem situations proposed in Figure 2, Table 3 presents the description of the technique according to the principles of ATD.

Table 3

Description of the technique referring to Task 1, which aims to identify events that are impossible

Technique	Description
τ_1	The proposed subtasks (t_1 to t_6), figure 3, are based on the same principle, therefore, they use the same technique to solve them. According to the proposed situations, the student will have to reflect

	on the problems in question and, based on the observation of the sample space, determine which of the listed options is impossible to occur. To do so, it will be necessary to reflect under what circumstances an event is considered impossible and, based on analyzes of the conditions and context of each one, identify, among the listed options, which one will not have a chance of occurring. We consider as an impossible event when the event does not contain any element of the sample space.

Still returning to the subtasks in Table 3 and detailing technique τ_1, we have this detailed description in Table 4.

Table 4

Detailed description of technique 1 (τ_1).

Technique	Description
τ_1	In the case of subtask t1, the response to the task is the option “draw a red card n° 7”, considering that the random experiment is: remove one of the cards presented in the figure associated with this subtask and the sample space all the cards that can be observed in the same figure. In this way, no red card with the number seven is observed, characterizing it as an impossible event. In the case of subtask t2, the response to the task is the option “take avocado”, considering that the random experiment is: take a fruit (grape, orange, passion fruit and banana) in the basket in the figure associated with this subtask and the sample space all the fruits that can be observed in the basket of the same figure. In this way, no avocado is observed, thus characterizing it as an impossible event. In the case of subtask t3, the response to the task is the option “take a yellow die”, considering that the random experiment is: remove one die from the bag shown in the figure associated with this subtask and the sample space all 15 data, being 5 red and 10 blue that can be seen in the same figure. In this way, no green data is observed in the bag, therefore, characterizing it as an impossible event. In the case of subtask t4, the response to the task is the option “leave the colour green”, considering that the random experiment is: observe the colour after spinning a roulette wheel shown in the figure associated with this subtask and the sample space all colours (yellow, blue and red) that are observed in the same figure. In this way, no region of the roulette wheel that has the green colour is observed, therefore, characterizing it as an impossible event.

	In the case of subtask t5, the response to the task is the option "choose a black pen", considering that the random experiment is: choose one of the pens shown in the figure associated with this subtask and the sample space all pens of colour (blue, red, purple and green) that can be seen in the same figure. In this way, no black pen is observed, therefore, characterizing it as an impossible event. In the case of subtask t6, the response to the task is the option "to catch a ball", considering that the random experiment is: to take one of the toys presented in the figure associated with this subtask and the sample space all the toys that can be observed in the same figure (pets, doll, cart and captain America doll). In this way, no ball is observed, therefore, characterizing it as an impossible event.

The theoretical-technological discourse (θ_1, Θ_1), which allows justifying and explaining the $\tau 1$ technique, can be explained according to MOOC (2017) when it says that an impossible event represents the event that does not have sampling points, that is, the referred event cannot occur, being represented by the empty set "$\varnothing$".

CONCLUSIONS

Our proposal, guided by the BNCC (Brasil, 2018), sought to explore probabilistic concepts from the teaching methodology of problem solving because we believe that this brings important achievements and evolutions to students.

Supported by the results of this study, thinking about the basic concepts of Probability was to develop research that meets the needs of primary school, so that it is possible to contribute to the growth and development of an autonomous, critical, active society capable of taking decisions based on the information they are presented with.

And, in terms of ATD, its use allowed identifying a set of praxeologies that make it possible to characterize both the mathematical object (probabilistic) and the didactic approach for such an object. The praxeological organization was composed of four elements:

1. Task (T) and its subtasks (t), which characterized the action required by the proposed problem situation for the question cards in the game. For example, identify and list all possible results of a random experiment (sample space) as well as its events (subsets),

in this case, those that are considered impossible and associated with the empty set that is a subset of any set or sample space .

2. Technique (τ), identifies the way of carrying out the task and its subtasks. Each task has at least one technique associated with it. For example, determine the impossible events.

3. Technology (θ), was specified by the set of definitions, properties, axioms and theorems that justify the technique. For example, the technology that justifies the technique is the definition of random event, that is, it is the random phenomenon of experimental realization when its observation is impossible.

4. Theory (Θ) is the field in which technology is justified. In the examples presented, the theory is given by the probability that allows us to describe impossible random events, that is, those in which uncertainty is present.

For Chevallard (1999), didactic organizations are the responses to these practices, with their two components: "praxis", which is formed by didactic tasks and techniques, and "logos", which is formed by didactic technologies and theories. In the case highlighted in the mathematical and praxeological organization, addressing the didactic praxeology for teaching probability aimed at identifying experiments or daily situations, specifically impossible events, the didactic objective is that players expand: (1) The idea of event focusing on the estimation of results that are configured as impossible: (2) The determination of all results of a random experiment; (3) Classification of events, subset of the sample space.

Finally, we reinforce, based on Vásquez and Alsina (2017) that the probabilistic language begins to emerge from the experiences of everyday life and that it gradually becomes a probabilistic language, where the concepts of impossible play a fundamental role.

The game "Probability in Action" can be used in different ways, it all depends on the moment and the intention of the teacher. It can be used both to assess children's prior knowledge and to introduce a subject that can later be developed with more time in the classroom, or introduce a subject in the classroom and then reinforce it with the game. In addition, carrying out some experiments in the classroom that are proposed in the game cards would provide an excellent opportunity for intervention in favor of building notions of probabilistic concepts.

AUTHORS' CONTRIBUTIONS STATEMENTS

APOJ and NDB conceived the presented idea, adapted the methodology to this context, carried out the activities and collected the data, as well as analysed the data. All authors actively participated in the discussion of results, revised, and approved the final version of the work.

STATEMENT OF DATA AVAILABILITY

Data sharing is not applicable to this article, as it is theoretical research that brings a didactic and mathematical praxeology that exposes and explores, in detail, the practice and mathematical theory (probability) involved in the cards of the game "Probability in Action" focused on impossible random events that can be associated with the resolution of tasks or problem situations and that are available in the text of the article.

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