



# Meaning of Problem in School Mathematics: From Exercise and Application to the Learning-Triggering Problem

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## ABSTRACT

**Background:** There are, in the school context, especially in mathematics education, different historical and cultural meanings that are being produced and impregnated in the term *Problem*, as well as different understandings regarding the roles of *Problems* in teaching practices, which justifies studies focused on to the analysis of the implications of such conceptual meanings in the training of teachers who teach mathematics. **Objectives:** The aim is to answer the question: What are the meanings of *Problem* present in research and publications that dialogue with training for teaching mathematics? What would be, specifically, the meaning of *Problem* from a cultural-historical perspective? **Design:** The research was carried out with students of the degree course in mathematics of a Brazilian public institution, in which they participated in a training movement that had *Problems* as mediators. In the context of that research, theoretical and bibliographical studies were developed to understand ideas impregnated in didactic-pedagogical proposals that exalt the importance of *Problems* in teaching practices. We recognise these ideas as meanings based on the analysis that considers the theoretical and methodological assumptions of the cultural-historical theory. **Setting and participants:** Publications that circulated in the context of the training course for mathematics teachers were analysed, which contextualised the research that gave rise to this clipping about the understandings of the authors of those texts regarding their understanding of the possible relationships between the *Problem* and the teaching of mathematics. **Data collection and analysis:** The texts were selected from the bibliographies present in the teaching plans of the subjects of that degree course or that were used as a textual reference during the formative experiment carried out to produce research data and were analysed according to the assumptions of the cultural-historical theory to the concept of meaning. **Results:** The

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results point to the constitution of meanings according to which the *Problems* are usually adopted, in teaching practices, as lessons already presented or as opportunities to apply concepts in supposedly everyday situations. There is also a process of meaning from which the *Problems* are understood, from a cultural-historical perspective, as triggers of mathematics learning. **Conclusions:** The main conclusion points to the understanding that the *Problems* play, historically and culturally, some roles in mathematics teaching practices, among which, three stand out: they are opportunities to practice lessons already taught; they are opportunities to apply techniques, algorithms, or definitions in simulated situations (*problem solving*); or they are *learning-triggering problems*.

**Keywords:** problems; problem solving; learning-triggering problem; cultural-historical theory; mathematics teaching.

### **Significação do conceito de Problema na matemática escolar: De exercício e aplicação até o problema desencadeador de aprendizagem**

#### **RESUMO**

**Contexto:** Há, no contexto escolar, em especial na educação matemática, diferentes significados históricos e culturais que vão sendo produzidos e impregnados no termo *Problema*, bem como diversas compreensões a respeito de quais seriam os papéis dos *Problemas* nas práticas de ensino, o que justifica estudos voltados à análise das implicações de tais significados conceituais na formação do professor que ensina matemática. **Objetivos:** Busca-se responder à questão: quais são os significados de *Problema* presentes em pesquisas e publicações que dialogam com a formação para o *ensino de matemática* e qual seria, especificamente, o significado de *Problema* em uma perspectiva histórico-cultural. **Design:** A pesquisa foi desenvolvida com estudantes do curso de licenciatura em matemática de uma instituição pública brasileira, na qual eles participaram de um movimento formativo que tinha os *Problemas* como mediadores. No contexto daquela pesquisa, foram desenvolvidos estudos teóricos e bibliográficos para compreender ideias impregnadas em propostas didático-pedagógicas que enaltecem a importância dos *Problemas* nas práticas de ensino. Essas ideias é que reconhecemos como significados a partir da análise que considera os pressupostos teóricos e metodológicos da *teoria histórico-cultural*. **Ambiente e participantes:** Foram analisadas publicações que circulavam no contexto do curso de formação de professores de matemática que contextualizou a pesquisa que deu origem a este recorte no que diz respeito a compreensões dos autores de tais textos a respeito das possíveis relações entre o *Problema* e o ensino de matemática. **Coleta e análise de dados:** Os textos foram selecionados a partir das bibliografias presentes nos Planos de Ensino das disciplinas daquele curso de licenciatura ou que foram utilizados como referência textual durante o experimento formativo realizado para produção de dados da pesquisa e foram analisados segundo os pressupostos da teoria histórico-cultural, em especial ao conceito de significado. **Resultados:** Os resultados apontam para a constituição de

significados segundo os quais os *Problemas* costumam ser adotados, nas práticas de ensino, como exercícios de lições já apresentadas ou como oportunidades de aplicação de conceitos em situações, supostamente, cotidianas. Verifica-se, ainda, um processo de significação a partir do qual os *Problemas* são compreendidos, em um perspectiva histórico-cultural, como desencadeadores de aprendizagens de matemática. **Conclusões:** A principal conclusão aponta para a compreensão de que os *Problemas* protagonizam, histórica e culturalmente, alguns papéis nas práticas de ensino de matemática, dentre os quais, três se destacam: são oportunidades para exercitar lições já ensinadas; são oportunidades para aplicar técnicas, algoritmos ou definições em situações simuladas (*resolução de problemas*); ou são *problemas desencadeadores de aprendizagem*.

**Palavras-chave:** problemas; resolução de problemas; problema desencadeador de aprendizagem; teoria histórico-cultural; ensino de matemática.

## INTRODUCTION

This article presents part of the results from a doctoral research work dedicated to monitoring mathematics undergraduates' *senses* (Vygotsky, 2001; Leontiev, 1978) regarding the role of Problems<sup>1</sup> in the learning processes of teaching organisation. The excerpt presented here highlights the production process of the *meanings* (Vygotsky, 2001; Leontiev, 1978) of Problems in research circulating in the Brazilian educational context, especially those aimed at mathematics teacher education. This section aims to analyse the understanding of the role attributed to Problems by researchers focused on studying relationships between such Problems and mathematics teaching practices.

The research that originated this text was carried out with mathematics degree students at a Brazilian public higher education institution. We produced and analysed data extracted from seven undergraduates' movements of the *senses*. In the context of the research, we felt the need to know and understand *meanings* steeped in the concept of *Problem* and the *meaning-making* process of such a concept, in particular, the ideas that allowed us to understand

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<sup>1</sup> Considering the semantic variation of possible meanings embedded in the term “problem” and to avoid loss of meaning in translation processes, we adopted the term “Problem” with a capital letter, when referring to the mathematics problem adopted in the classroom as a resource focused on teaching and learning practices. When the spelling adopts “problem” (with a lowercase initial letter), we are referring to a more general notion, as a synonym for “defect” or “difficulty”. It is important to highlight, however, that in direct quotations we maintain the original spelling, with the expectation that the context can differentiate the meanings to be attributed.

Problems as triggers of mathematical learning in the context of the *teaching guiding activity* (TGA) (Moura et al., 2010).

To achieve our objective, in the original research, we resorted to the assumptions of cultural-historical theory (Vygotsky, 2001), especially the concept of *meaning* (Vygotsky, 2001; Leontiev, 1978), from which we present interpretations of the understanding of researchers whose works circulated in the context of the initial teacher education course that contextualised the doctoral research of which this text is an excerpt. The texts were selected from the bibliographies present in the teaching plans of the components of that course or were used as textual references during the training experiment carried out to produce research data.

Specifically, we analysed texts in which we could infer aspects of how the authors understood the role of the Problem in mathematics teaching practices. The articles that supported this understanding in the research and this section are those of Onuchic (1999, 2008), D'Ambrósio (2008), Stanic and Kilpatrick (1989), Pólya (1978/1995), Dante (1998, 2009), Moretti (2007, 2014), and Moura et al. (2011).

The movement presented in the text discusses the processes of dispute in which the *meanings* being impregnated in the term *Problem* are historically and culturally produced. We can currently recognise ideas according to which, in short, “the teaching-learning of mathematics has problem-solving as its starting point” (Brasil, 1998, p. 56). But what *means* saying that the Problem and its resolution processes are *the starting point of* practices that aim at teaching and/or learning mathematics? It is this *meaning*, always in dispute and under construction, that we seek to discuss in this text.

We could verify that Problems play, historically and culturally, some roles in mathematics teaching practices, among which we highlight three: they are opportunities to exercise lessons already taught; to apply techniques, algorithms or definitions in simulated situations; or they are elements that trigger learning.

The article is divided into three parts besides this introduction. The first part brings the theoretical and methodological foundations adopted in our research, in particular the assumptions that guide the analysis in this excerpt, and the second discusses and reveals the results of the analysis of the meanings produced about the Problem, according to research focused on studying their role in mathematics teaching practices. Finally, the third part highlights our conclusions regarding the movement discussed and its relevance in

mathematics education, especially the education of teachers who teach mathematics.

## THEORETICAL AND METHODOLOGICAL CONTEXT

As we indicated in the introduction, the research that gave rise to this text aimed to monitor the movements of the subjects' *senses* about the role of Problems in their future teaching practices, considering their conditions as prospective mathematics teachers in initial education. The methodological precepts of the research were in line with historical dialectical materialism (HDM) proposed by Karl Marx and consolidated as a method that underlies the cultural-historical theory (CHT) proposed by Vygotsky.

By assuming the objective of monitoring the movements of the degree students' *senses*, we must also know the historical and cultural *meanings* of the concept of Problem in mathematics teaching practices. This involves not only recognising the possible definitions of Problems but also understanding their role in teaching practices, according to ideas that circulate in degree students' training context. We understand that the work of researchers focused on understanding the place of the Problem in teaching practices is a good representation of the *meaning-making* movement of Problems and their role in mathematics teaching.

In other words, from the research interest in the movements of undergraduates' *senses* about the role of *Problems* in their future teaching practices emerged a need to understand the *meanings* and the *meaning-making* processes of the concept of Problem. This is because these three constructs – *senses*, *meanings*, and *meaning-making*– form, in the context of research carried out according to the theoretical and methodological precepts of the CHT, an indecomposable theoretical unit, that is, a dialectical unit. Thus, it becomes pivotal to present, albeit in general terms, an understanding of what we mean by *senses*, *meanings* and *meaning-making* from the CHT perspective.

We bring our understanding closer to the *senses* and *meanings* of those initially proposed by Vygotsky (2001) and later consolidated by Leontiev (1978), considering historical and cultural aspects of human development. Like other researchers, such as Asbahr et al. (2017), we identified the process that refers to the idea of a movement of the historical and social constitution of a concept as *meaning-making*.

Vygotsky (2001) argues that the relationships between thought and language materialise in words since social interactions establish, historically and culturally, a semiotic system formed by signs and represented by words that, in turn, develop new social interactions, characterising a continuous dialectical relationship between what is thought and what is said. In other words, the social interactions that relate words and signs and that take place in a historical and cultural process give words their respective *meanings*. According to Vygotsky (2001, p. 398), “a word devoid of meaning is not a word, it is an empty sound”.

Thus, according to Vygotsky (2001), *meaning-making* is a social and collective production; therefore, it is historical and cultural. *Sense*, in turn, results from each one’s personal experiences within the same culture during the *meaning-making* process. Thus, according to Vygotsky (2001), *sense* is “broader” than *meaning*, as it encompasses all the psychological facts to which the word refers us based on our experiences. Vygotsky (2001, p. 465) states that *sense* is “a dynamic, fluid, complex formation with several zones of varying stability. *Meaning* is just one of those zones of *sense* that the word acquires in the context of some discourse; moreover, a more stable, uniform and exact zone” (emphasis added).

Thus, Vygotsky (2001) believes that *senses* and *meanings* are related dialectically so that *meanings*, which are produced collectively in social, historical and cultural contexts, influence the constitution of *senses*, which are personal and have their genesis in individual contexts, expanding them. At the same time, the *senses* influence the production of *meanings*, consolidating or modifying them. This relationship between *personal senses* and *social meanings* is established in a process called *meaning-making*.

To add to the above, Leontiev (1978) argues that the concepts of *sense* and *meaning*, in the process of *meaning-making*, gain context in understanding the processes of awareness, overcoming the bourgeois idea that consciousness can exist independently, and must be understood in the context of the activity that humanises the human being.

Human consciousness is not an immutable thing. Some of its typical features are, in given concrete historical conditions, progressive, with prospects for development, while others are survivals condemned to disappear. Therefore, we must consider consciousness (the psyche) in its becoming and development, in its essential dependence on the way of life, which is determined by existing social relations and the place

the individual occupies in these relations. (Leontiev, 1978, p. 89)

If consciousnesses were always and everywhere identical, configuring such a *human consciousness*, then consciousness studies would always have the same structure and refer to the same things. But Leontiev (1978) argues that the human way of life and the interactions between the subject and the environment, i.e., the subject's *activity*, determine consciousness at the same time as they are determined by it. According to Leontiev's understanding, we can infer that a Problem, for example, is not a Problem because it has a particular systematic structure or because it refers to specific actions but, instead, because it mediates a relationship between what the subject thinks (consciousness) and the environment (objective reality).

Leontiev (1978, p. 94) further defines that "meaning-making is what in an object or phenomenon is objectively discovered in a system of connections, interactions, and objective relationships. Meaning-making is reflected and fixed in language, which gives it stability." The author argues that meaning-making is the historical and social movement that allows us to relate the word and the object.

Meaning-making is the generalisation of reality that is crystallised and fixed in a sensitive vector, ordinarily the word or phrase. It is the ideal, spiritual form of the crystallisation of humanity's social experience and practice. Its sphere of representations of a society, its science, its language, exist as systems of corresponding meaning-making. Meaning-making, therefore, belongs, first and foremost, to the world of objectively historical phenomena (Leontiev, 1978, p. 94).

Dialectically, however, the *meaning-making* also exists as a fact of individual consciousness. Leontiev (1978) understands that man perceives and thinks about the world as a socio-historical being and, therefore, has the representations and knowledge of his time and society as a criterion for analysing reality. However, his consciousness is not reduced to his individual experience since a human's way of life does not start *from scratch* to make their own discoveries. According to the author, during their lives, "humans assimilate the experience of previous generations [and] this process takes place precisely in the form of the acquisition of meaning-making and to the extent of this acquisition" (Leontiev, 1978, p. 94). Therefore, *meaning-making* influences how a person assimilates generalised and reflected human experience.

To understand the idea regarding the process of *meaning-making*, which occurs in a historical and social materialist context, Leontiev (1978, p. 95) makes an analogy from the field of geometric knowledge:

[...] what I think, understand and know about the triangle, may not coincide perfectly with the meaning-making [of] “triangle” admitted in modern geometry. But it is not a fundamental opposition. Meaning-making have no existence outside concrete human brains; there is no independent realm of meaning-making comparable to the Platonic world of ideas. Consequently, we cannot oppose a “geometric”, logical, and, in general, objective meaning-making to this same meaning-making in an individual’s consciousness as a particular psychological meaning-making. The difference is not between the logical and the psychological but between the general and the particular, the individual. A concept does not cease to be a concept when it becomes the concept of an individual. [...] Reality appears to the human being in its meaning-making, but in a particular way. Meaning mediates a human being’s mirroring of the world to the extent that they are aware of it, that is, to the extent that their mirroring of the world is based on the experience of social practice and integrates it. (Leontiev, 1978, p. 95)

Leontiev (1978) states that meaning-making is generally absent from consciousness, but this is not the same as saying that the subject “is never aware” of meaning-making. From meaning-making, we portray what is perceived or thought, but it is not conscious, it is not thought. Leontiev (1978) says this psychological fact is fundamental, as it enables the understanding that meaning-making “is mirroring reality regardless of human beings’ individual or personal relationship to it. Humans find a ready-made system of meaning-making, historically created, and appropriate it just as they appropriate an instrument” (Leontiev, 1978, p. 96).

In the case of the section that we summarise in this text, the premise is that the concept of *meaning-making* and understanding the process of *meaning-making* allow us to carry out an analysis of research that deals with Problems in an attempt to recognise *meanings* of the role of Problems in mathematics teaching practices that can be **determinants** of degree students’ *senses* of the concept and role of the Problem in teaching practices, at the same time that they



can be **determined**, also, by how these degree students understand the role of the Problem and its place in teaching practices.

Therefore, considering the methodological perspective of our research, we sought to materialise formative movements aimed at preparing for teaching so prospective teachers could be in *activity* (Leontiev, 1978) while learning general ways of organising *teaching activity*. For this training, the degree students' *initial senses* regarding the role of Problems in pedagogical practices aimed at teaching mathematics found contradictory movements, in which Problems would have different functions in the context of mathematics teaching practices.

It is important to emphasise that this item so far aims at introducing the theoretical context and aspects of the methodology in which this section is inserted, but it is not our objective, in this text, to uncover aspects of the relationship between the *senses* and the *meanings* prospective teachers assign the role of the Problem in teaching practices<sup>2</sup>. Our scope is to unravel *meanings* impregnated in research and publications that had Problems – or Problem Solving – as an object of study focused on discussing aspects that relate these Problems with their roles in mathematics teaching practices.

Specifically, we carried out theoretical and bibliographical research covering publications with mathematics teaching Problems and practices as the central object of analysis. We also brought some texts that adopted the CHT as a theoretical and methodological perspective and that could contribute to understanding the *meanings* of the Problem from this perspective. We analysed the texts we raised according to the assumptions above so that we could understand the movement of *making meaning* of the concept of Problem in mathematics teaching practices, as discussed below.

## RESULTS AND ANALYSIS

Initially, we saw that Problems have been part of the history of mathematics since immemorial times. Their presence in ancient historical documents points to an understanding of their importance in mathematics teaching practices. Onuchic (2008), for example, indicates that “mathematics problems have occupied a central place in the school mathematics curriculum since ancient times” (p. 1). With this, the author proposes that the Problems had

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<sup>2</sup> For this presentation, we recommend reading the thesis: Virgens (2019) in < <https://teses.usp.br/teses/disponiveis/48/48134/tde-03102019-120541/pt-br.php>>.

fundamental importance in the development of mathematics and that this fact would have been appropriated by school mathematics so that the Problems would have come to occupy a prominent place in pedagogical practices simultaneously with the development of mathematics. According to Onuchic (2008, p. 1), “records of mathematical problems are found in ancient Egyptian, Chinese, Babylonian, and Greek history”. We understand that the author’s considerations seem to bring the presence of Problems in the history of mathematics closer to the relevance of such Problems in mathematics teaching.

Similarly, D’Ambrósio (2008, p. 1, emphasis added) argues that “problem solving *has always been considered an important part of mathematics teaching*” and that concern about how problem solving is taught in schools has always been present in school mathematics. The author also refers to the theory spread in the 19th century, according to which Problems would be exercises for developing the brain in a movement of mental discipline, making an analogy to the muscular benefit of physical exercise. According to D’Ambrósio (2008), this theory would assume that, under the justification of the need to exercise the brain for its development, a Problem would be any situation that presents the student with the opportunity to exercise previously taught lessons. For the author, it is possible to perceive those ideas in teachers’ practices until today.

On the other hand, Stanic and Kilpatrick (1989) point out that the importance of Problems in mathematics contrasts with the little relevance of Problems in school curricula. These authors highlight Pólya’s work, especially the book *How to Solve It*, 1945, as a turning point in attributing importance to Problems in mathematics teaching practices. According to Stanic and Kilpatrick (1989, p. 1), “Mathematics educators have only recently appeared accepting the idea that the development of problem-solving skills deserves special attention”. They state that Problems are found in mathematics textbooks from the 19th and 20th centuries, indicating the presence of Problems in mathematics classes. However, according to their interpretations, they had a *too-narrow* vision of learning to solve problems. For them, teaching Problem Solving referred to presenting Problems and, perhaps, including an example of a specific technical solution.

A *meaning-making* of the concept of Problem inferred from such understandings seems to point, on the one hand, to the constitution of a *meaning* of Problems that brings together situations from the history of mathematics and school Problems, treating both concepts – Problem in mathematics and school Problem – as synonyms. On the other hand, Stanic and Kilpatrick’s (1989)

defence presupposes a distinction between the Problems studied in mathematics and those that have teaching and learning practices in perspective. The latter, however, would be associated with an understanding that the authors classify as *too narrow* and which refers to the idea of models to be reproduced, i.e., it is close to the idea of Problems that would serve, in teaching practices, to exercise what has already been taught previously.

Indeed, as Stanic and Kilpatrick indicate, the contributions of the Hungarian mathematician George Pólya receive great prominence in speeches about the characterisation of Problems and, above all, aspects of their resolution in mathematics education. The book *How to Solve It* by Pólya highlights the need for a teaching process aimed specifically and especially at Problem-solving processes. The importance attributed to Pólya's work was such that, according to D'Ambrósio (2008), there was an unfair reduction of his work to the heuristic process that he presents as a suggestion for organising problem-solving processes.

Pólya's heuristic process (1995, p. 86) "seeks to understand the problem-solving process, particularly the useful mental operations typical of this process". However, this process has reached our days *summed up* in the steps he proposed to solve a Problem: understand the problem, locate what is required (the unknown), draw up a resolution plan, execute the resolution plan and examine the solution (check the answer).

We highlight that Pólya's work is generally interpreted from a perspective according to which there would be a need to teach how to solve any Problem in school mathematics. An interpretation of Pólya's proposals even leads to certain practices that presuppose problem solving as curriculum content, such as functions, equations, etc. There is no doubt that this was not Pólya's objective (Stanic & Kilpatrick, 1989), but we understand that this idea also constitutes a *meaning* which was being constructed and which appears in some proposals aimed at discussing the role of Problems in mathematics teaching (e.g., Dante, 1998, 2009).

Pólya did not specifically target mathematics teachers. He proposed to deal with any person solving any Problem. We believe this to be a primary factor in the distorted interpretations of his work that followed his publications. After all, it is tempting that when we identify this objective (anyone learning to solve any Problem), we look for it in their text *prescriptions* that help us solve any Problem.

The understanding that such propositions would constitute resolution rules led scholars that study the Problems and the aspects of their resolution in the teaching and learning processes to present later their criticisms of Pólya's proposals, attributing an excessively generalist character to the heuristics. In an interview with Guimarães (2011), Jeremy Kilpatrick, who was Pólya's student and assistant, understands that the reason for the wide acceptance of Pólya's heuristics was due to the following:

[...] Because they are good suggestions. They are useful if you, if you... I think they are difficult to learn. I mean, one of the issues that comes up later is [that] ... Alan Schoenfeld argues that these [suggestions] are too general, that they are too difficult to teach, and that they are not specific enough for ... programming a machine to solve problems. But I would argue that if you see enough examples of how these heuristics work, you gradually come to adopt them in your own problem solving. And you can see ... you can see how they would operate, and Pólya gives a lot of examples of that. Actually, Mathematical Discovery includes even more examples — [Pólya's] book Mathematical Discovery — and I learned all about that later. But even with How to Solve It, which has ... fewer examples of the kind of problems [from] secondary mathematics, even How to Solve It gives a lot of good advice on ways of how to think about problems. (Guimarães, 2011, p. 72, our translation)

In any case, the great emphasis attributed to Pólya's works concerning the importance of learning to solve Problems culminated in the dissemination of ideas and, therefore, in the potential formation of *meanings* of the Problem itself in the school context. Even though this was not, as we indicated, Pólya's intention, School Problems were acquiring characteristics that were premised on planning a resolution process that could be developed according to Pólya's heuristics. In this context, publications such as the book *Didática da Resolução de Problemas de matemática* [Mathematics Problem Solving Didactics] (Dante, 1998, p. 9, emphasis added) indicate “the *suitable* way to propose problems and how to involve students in solving them”.

*Meaning-making* then starts to incorporate elements that understand Problems as situations that can be resolved through logical and strategic sequences of steps (*resolution algorithms*), displacing the action of solving the Problem from the subject and depositing it in the algorithm. With this, we can recognise the constitution of another *meaning*, according to which Problems

need to be composed so that they can be solved by applying general techniques; that is, by *efficient resolution algorithms*. Problems permeated with such *meaning* are those that we can summarise in the expression *Standard Problems*.

By *Standard Problems*, we understand those that present all the information for the resolution in the statement and keywords that lead us to identify the strategy, algorithm, property or definition to be *applied* for the resolution. This one *meaning* of Problems is located at the intersection of *meanings* that point to Problems seen as an opportunity to exercise already learned lessons and apply knowledge in so-called *real, useful or interesting* situations.

Problems that comprise movements that aim to provide the opportunity to simulate situations in which the concepts under study can be applied in supposedly everyday contexts, linked to the idea of reality, usefulness or enhancement of students' interest or curiosity, can be understood, in the process of *meaning*, as being in dialogue with premises from the educational field related to *pragmatism in teaching*, which the American philosopher John Dewey championed dearly. Dewey (2010) claims that only those concepts that have direct application in students' daily lives should have a place in the school curriculum, whether through usefulness or their potential to arouse students' interest.

Dewey's ideas echo the propositions of researchers of his time, such as the American psychologist Thorndike (1936, p. 154), who argued that the Problems should:

- (1) talk about situations that are likely to occur many times in real life;
- (2) treat them as they would be in practical life;
- (3) present them in a way that is neither much more difficult nor much easier to understand than they would be if reality itself presented them to the student's senses;
- (4) arouse somehow the same interest that accompanies the resolution of problems that they encounter in the real course of their occupations.

The significance that historically and culturally produced ideas that the role of the Problem in mathematics teaching practices would refer to this Deweyan pragmatism reverberated in ways that had the potential to influence teacher practices. This is the case, for example, of the document *An Agenda for Action – recommendations for School Mathematics of the 1980s*, produced and published by the National Council of Teachers of Mathematics (NCTM, 1980, p. 2), which argued that “Problem solving involves applying mathematics to

the real world, serving the theory and practice of current and emerging sciences, and resolving issues that extend the frontiers of the mathematical sciences themselves”.

Amid influences on the need for applicability of what was learned at school in pragmatic everyday situations, *meanings* according to which the Problems should enhance such application, even in simulated situations, have gained space and appear in much more recent publications, as in Dante (1998). He starts with a definition according to which a Problem would be “any situation that requires the student to think to solve it” (Dante, 1998, p. 9), arguing that Problems should make students think productively, develop students’ reasoning, teach students to face new situations, allow them to get involved with mathematics applications, make mathematics classes more interesting and challenging, equip them with strategies for solving Problems, and give them a good mathematical foundation.

Such objectives presented for using Problems in the classroom point to the consolidation of *meaning* in which the Problem is an instrument used to apply the mathematical content taught, and the ability to solve such Problems is a necessary and sufficient condition for solving any type of problem in the student’s life.

Dante (1998, 2009) classifies Problems according to the processes to be applied in their resolution. Classification is the resource that students generally use when faced with the challenge of solving a Problem. In this situation, the student resorts to a previous knowledge already acquired and checks which procedures inherent to such knowledge (often memorised from definitions, formulas, algorithms or properties) are applied to present the expected answer. Thus, in theory, Problems could serve, at the same time, as resources to exercise previous lessons and to apply the knowledge taught by the teacher in pragmatic everyday situations.

But, if so, how could we tell the difference between the *meanings* of Problems in mathematics teaching practices? The answer to this question raises the discussion about the need, defended by Dante (1998, p. 43), to “make a clear distinction between an exercise and a problem”. For the author, “teaching how to solve problems is more difficult than teaching mathematical concepts, skills, and algorithms” (Dante, 1998, p. 42).

Dante’s (1998) publication, however, changes the place of application in the movement of constituting meanings of the role of the Problem in teaching practices. We see that the idea of *applying* it stops referring to the application

of the concept in some everyday situations and moves closer to the perspective of applying problem-solving techniques to find answers to any mathematical question without necessarily committing to verisimilitude between the statement and objective reality. In this context, at the end of the work, Dante (1998) presents a collection with one hundred and fourteen suggested Problems and their respective solutions and comments that tend to reinforce the ideas that involve the proposal to apply the techniques to achieve the resolution. The mathematical concept involved in such solutions is secondary, and it is reasonable to assume that the proposal presented by the author admits that such concepts are previously known and, therefore, that Problem is the application of some already known concept and that the solution to the Problem is achieved by applying the resolution techniques, also presented previously.

In summary, so far, we can see that, among the publications of researchers that have circulated in the Brazilian school context related to mathematics teacher education dealing in some way with the role of Problems in mathematics classes, the movements of *meaning-making* of the role of Problems in teaching practices point to *meanings* according to which Problems are opportunities to exercise mathematical lessons already taught, opportunities to apply lessons taught in pragmatic situations or opportunities to develop a supposed ability to solve any Problem.

Thus, in this context, the idea of a good problem for practices related to mathematics teaching involves the cultural and social constitution of the objectives of education as a whole. Different groups' vindication regarding such goals leads to different perceptions of what *ideal* Problems should be like to reach them. It appears, however, that some *meanings* of Problems and their role in teaching practices *mix* and remain impregnated with propositions about mathematics teaching practices, according to what can be seen from texts published by researchers who focused on studies on the role of the Problem in teaching.

As we can see, there are distinct social meanings of the role of Problems in teaching practices. Another of those meanings arises from teaching movements that have as their context the assumptions of the cultural-historical theory. Thus, we now discuss Problems understood as triggers of teaching activity and, therefore, as prominent elements in learning how to organise pedagogical activity.

## The Learning-Triggering Problem (LTP)

Some research deals with problem solving in the context of the applicability of learning theories, such as the theory of meaningful learning (Ausubel, 1963) or the theory of didactical situations (Brousseau, 2008) in which the Problem assumes a role as the purpose of the process educational, with an end in itself, so that “for every mathematical concept there corresponds a situation, a mathematical problem, which somehow will force the student to use it [the concept]” (Moretti & Radford, 2015, p. 9). Agreeing with Moretti and Radford (2015), we believe it is necessary to overcome this perspective in which the Problem has the capacity to bring out a mathematical concept imbued in it.

We understand that, during the formative process, training must offer students suitable tools to learn how to organise teaching and, in this context, according to the assumptions of CHT, Problems must play a leading role as triggers of the teaching and learning processes.

As we have indicated, in mathematics classrooms, *meanings* of the role of Problems often presuppose that the mathematical concepts under study are finished and well-defined objects that, because they exist outside the school (in mathematical science), must be appropriate, *ipsis literis*, inside the school. Thus, in the school context, Problems tend to serve as a didactic strategy that aims to provide opportunities for the exercise of calculations and algorithms that involve these concepts or as opportunities to apply such mathematical concepts in *extracurricular* contexts.

From these *meanings*, we recognise a bourgeois structure of the Problem, in which the assumption is that school would be at *the service* of preparing individuals for experiences that would only begin *after* school life, especially in such a *job market*. Thus, the Problems to be presented to students, and whose resolution processes they should master, would be those that could emulate situations that would allow us to prepare them to enter and remain in the *job market*.

Thus, it is not uncommon to recognise teaching practices that refer to a teaching organisation scheme that starts from the presentation of the definition, detached from its historical production process, goes through the properties and theorems involved, and arrives at the presentation of *exercises and solved Problems* which will serve as models for the next stage: the resolution of *exercises and posed Problems*. In this context, the difference between *exercise* and *Problem* resides in whether any context relates the concept under study to



some pragmatic -supposedly real, useful or interesting- context. Table 1 illustrates a comparative example of what would be, from this perspective, a Problem and an exercise:

**Table 1**

*Distinction between Problem and exercise<sup>3</sup>.*

Problem	Exercise
<i>Joãozinho has 50 reais and wants to buy sweets that cost R\$2.50. How many candies can he buy?</i>	Solve (set up and perform): a) $50 \div 2.5 =$

We realise that, in both cases, it is about the possibility of exercising the algorithm of dividing an integer by a decimal, with the distinction that the Problem presents a supposedly real context in which the concept under study (the division) could be applied.

From a cultural-historical perspective, the understanding of Problem expands, since the subject becomes human through the appropriation of humanity’s cultural production, mediated by the cultural environment, in the historical context, i.e., in the process of humanisation. While the movement is social, it is also marked by the subject’s activity in their interaction with the environment, culture, and historically produced knowledge (Moretti, 2014). In this sense, in the context of mathematics teaching practices, the Problem implies both the subject’s activity and the need that triggers it.

Moretti (2007), for example, in the context of teacher education for the organisation of teaching the concept of function, proposes a Problem contextualised from a game: the Tower of Hanoi<sup>4</sup>. The Problem referred to the request to predict the victory conditions for each game stage. To meet this challenge, teachers needed to represent the movement of variation between the magnitudes involved in the problem (Moretti, 2007, p. 106). Considering that Moretti (2007) adopts methodological aspects of the teaching guiding activity (TGA), understood as an activity “structured to allow subjects to interact,

<sup>3</sup> This distinction is our inference based on the authors/research analysed in this text.

<sup>4</sup> For more about the game we recommend:

[https://www.ibilce.unesp.br/Home/Departamentos/Matematica/labmat/torre\\_de\\_hanoi.pdf](https://www.ibilce.unesp.br/Home/Departamentos/Matematica/labmat/torre_de_hanoi.pdf)

mediated by content, negotiating meanings, to collectively solving a problem situation” (Moura, 2001, p. 155 as cited in Moretti, 2007, p. 96), the proposal is not presented after discussing the definition, concept, or ideas related to the functions.

In TGA, based on the precepts of activity theory (Leontiev, 1978) and the cultural-historical theory (CHT) above, the teacher develops several actions they need to achieve their objective. For example, a possible teaching guiding activity occurs when the teacher studies the history of mathematics (HM) to understand the fundamental aspects of the concept they want to teach. This is because, in this theoretical and methodological perspective, the premise is that to appropriate objects or phenomena that are the product of historical development (that is, they are, historically, *meanings*), it is necessary to develop an *activity* in which the main features of the *activity* incarnated, accumulated in the object (Leontiev, 1978, p. 268).

To this end, movements in the study of the history of mathematics gain importance. With this, however, teachers are not necessarily expected to reproduce the history of mathematics in their classrooms. But they recognise the concept they need to teach as a product of a historical process and not as *something ready*, which would be presented a priori, in the form of a definition or property. While studying the HM, in the organisation of the TGA, the teacher recognises the human needs involved in the historical process of knowledge production and proposes situations from which the same necessities arise for students.

These are learning-triggering situations (LTS) (Moura, 2002; Moura et al., 2010; Moura & Lanner Moura, 1998). The core of an LTS is a Problem that triggers the movements of teaching and learning activities. This core is the learning-triggering problems (LTP) (Virgens, 2019).

When we study the history of knowledge production, we realise that knowledge is produced in human beings’ relationship with each other and nature to enhance human actions of intervention, modification, and control of surrounding phenomena. In this process, the concepts created are objectified as symbolic instruments, i.e., the human work process is fixed in them (Moura et al., 2011, p. 41).

In summary, in this way of understanding, the role of the Problem in mathematics teaching practices is not limited to serving as a source to exercise the application of some previously taught lesson or the simulation of some resolution technique supposedly useful for life. In the example presented in

Moretti's (2007) work, teachers, in their teaching practices, should not start from the definition of function, moving on to the presentation of several *types* and properties of functions that would be applied in solving exercises and Problems supposedly related to everyday life situations.

On the contrary, the *meaning* of Problems, based on Moretti (2007) and the CHT, presupposes identifying the human need involved in the historical process of human production of knowledge that culminated in the concept that we currently recognise as *function*. Thus, *teaching guiding activity* provides that the teacher in activity must recognise the human needs that led to the development of the concept and consider those needs when elaborating and proposing Problems in their teaching practices. In the specific case of the concept of function, we can find records in the HM, according to which:

Nicolas de Oresme (1323–1382), a French mathematician, presents his theory of amplitude of forms (Youschkevitch, 1976, p. 46)<sup>5</sup>. The theory of amplitude of forms consisted of the representation of all quantities and relationships between them through geometric shapes; however, the relationship between variables in a function had not yet emerged. To understand how, at the time, they related domain to image without considering the current way of representing these sets, the literature points us to Oresme. He geometrically represented velocity varying with time by studying motion with constant acceleration. (Barros et al., 2021, p. 57)

In other words, HM points out that, in its origins, the concept of function arises from the need for people to control quantities of magnitudes that vary so that the variation of one quantity influences the variation of the other.

Armed with this knowledge, the teacher can then suggest a situation in which, at some point, this necessity to control variations in quantities also arises. This is the case with Moretti (2007) when he proposed that teachers play the Tower of Hanoi. The game is not proposed as an end but as a context, a *situation* in which, at some point, teachers are faced with the command to *predict* the victory conditions in the following stages.

This command changes the characteristics of the act of playing. This is because prospective teachers, in the context of Moretti's research (2007), now

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<sup>5</sup> Youschkevitch, A. P. (1976) The Concept of Function up to the Middle of the 19th Century. *Archive for History of Exact Sciences*, 16(1), 37-85.

must *control quantities* of disks, which influences *the control of the number* of movements necessary until victory. In other words, the same requirement to control quantities of magnitudes that vary and are related to each other, present in the historical genesis of the concept of function, also arises in the act of playing. It is this command that determines, in the context of the situation, that attention is directed to the control of quantities that constitutes the learning-triggering problem (LTP). And it is the LTP that makes the game –the context– a learning-triggering situation (LTS).

According to Moura and Lanner Moura (1998), an LTS can materialise as a virtual story, thus understanding the fictional situation that aims to simulate the factual historical movement as a game or as situations that emerge from everyday life.

[...] The game with pedagogical purposes can be an important ally in teaching, as it preserves the character of a problem. [...] We must consider the possibility of the game placing the child in a problem situation similar to that experienced by an individual when dealing with mathematical concepts. [...] The problematisation of situations that emerge from daily activities provides educational practice with the opportunity to confront the child with the need to experience the solution of problems they find meaningful. [...] The virtual history of the concept places the child before a problem situation similar to that experienced by an individual (in the generic sense). (Moura & Lanner Moura, 1998, p. 12-14)

We should note that the idea is not to present opportunities to exercise or apply concepts. The role of the Problem in the context of teaching practices, from a cultural-historical perspective, is to raise needs related to the historical needs that led to the production of the concept.

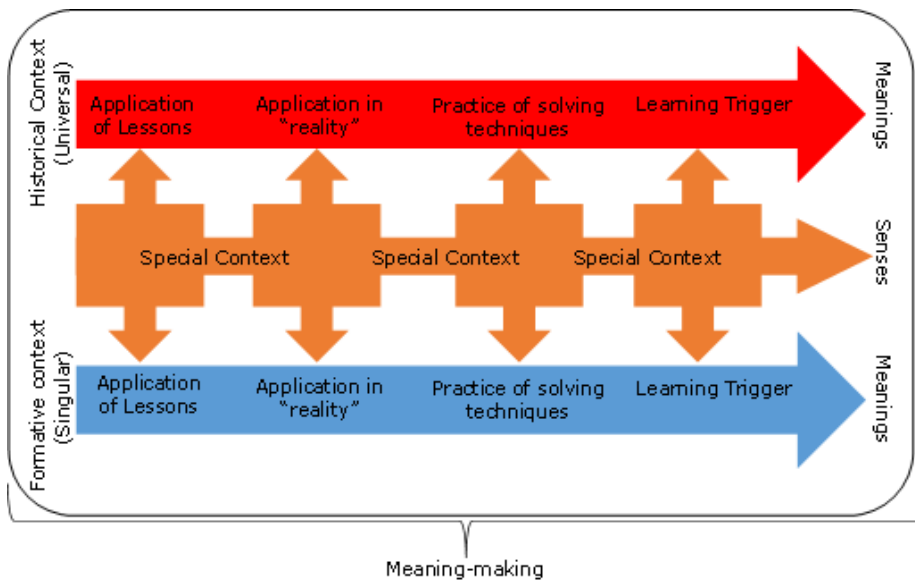
As a result of this discussion, Figure 1, presented below, summarises this movement of meaning of the role of Problems in teaching practices.

In Figure 1, we can then verify the singular-particular-universal dialectical relationship (Oliveira, 2001) and how the *meanings* of the role of the Problem in teaching practices are constituted historically while relating to the personal senses of prospective teachers. Please note that it is not our objective, in this text, to address this relationship between *senses* and *meanings* – we are not dealing with the students' senses – but the image is illustrative of how historically produced meanings relate to the subjects' formative processes. In

short, if the meanings of the role of the Problem in teaching practices are those that we see in the figure – and that we discuss in this text – the *senses* constituted by the prospective teachers will relate to such *meanings*, confirming that knowing the process of *meaning-making* is fundamental for teacher education. Their *senses* will always move in the particular context between the *meanings* present in universal (historical and cultural) and singular (which develops collectively in formative contexts) *meaning-making*.

**Figure 1**

*Summary of the meanings of Problems.* (Adapted from Virgens, 2019)

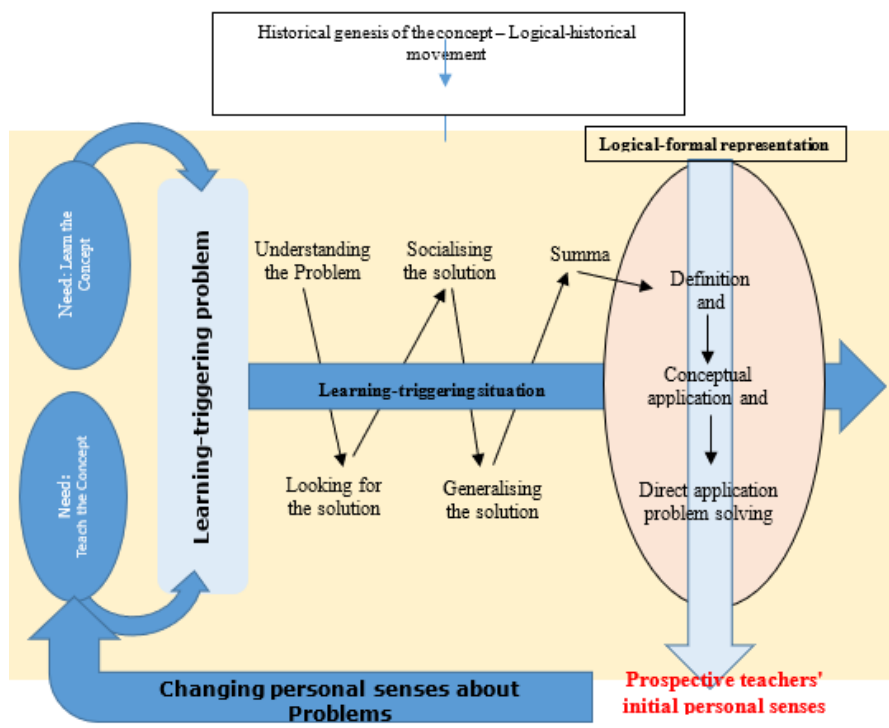


Therefore, if we recognise that the Problem must be the starting point of mathematical activity, as explicitly determined by Brasil (1998), then some movement must emerge from the Problem the subject must carry out that can culminate in learning (and the consolidation of teaching). Problems with this characteristic, therefore, must form unique formative contexts so that they are also established in universal contexts. In this way, particular movements can bring the *senses* of the subjects of *meaning*, according to which the Problem can be recognised as a learning trigger. Figure 2 has the potential to synthesise

the movement of meaning-making from which the LTP becomes the essence of the learning-triggering situation.

**Figure 2**

*Synthesis of the TGA movement. (Adapted from Virgens, 2019)*



In short, the LTP is the essential element that will raise in the LTS the necessity that will unleash the movement of teaching – on the part of the teacher – and learning – on the part of the student – which will characterise that both subjects (teacher and student) are in activity (Leontiev, 1978). Being this essential element, the LTP can appear in the context of the game, in daily situations or virtual stories, but it will always be the “inducement” that, in the context of the conscious organisation of the activity, will subsidise the constitution of the LTS as such, or in other words, it allows, as we see in Moura (2000, p. 35) “an advancement of this subject’s knowledge through the process

of analysis and synthesis and allows them to develop the ability to deal with other knowledge”.

As a pedagogical intention, we expect that this need embedded in the LTP will mobilise subjects in the search for a solution; that is, this need triggers collective activity in the classroom to solve the Problem. And it is in this collective search –in the survey and testing of hypotheses, in the mediations that are established in this context, in the development of actions and, finally, in the conceptual appropriations resulting from this process– that learning takes place. In this way, we verify the presence of a new *meaning* of Problems, which involves triggering the learning movement based on human, historical, and cultural needs. In this context, the Problem assumes a core role, a central, essential element of the learning-triggering situation (LTS), thus becoming a learning-triggering problem (LTP). Without the LTP, the game is just a game. With the LTP the game becomes an LTS.

## CONCLUSIONS

In this text, we discussed the historically produced *meanings* of the role of Problems in mathematics teaching practices in a Brazilian educational context based on Problem-Solving texts.

We could highlight the historical constitution of *meanings* that refer to Problems understood as opportunities for mathematics students to exercise lessons they had seen previously, as opportunities to apply concepts studied in pragmatic everyday situations, classified as situations likely to happen in students’ lives *outside school*, useful to their daily experiences or that have the potential to spark their interest in studying the concept.

We also highlight a *particular meaning* of the concept of Problem, understood as mathematics teaching practices that have CHT as a theoretical and methodological assumption, in which the role of the Problem in mathematics teaching practices is to raise needs that trigger learning movements. That is why we call the Problem that assumes this role as a learning-triggering problem (LTP).

In summary, by recognising the need to teach using Problems as a starting point and not the definition, as indicated by official curriculum documents - for example, the National Mathematics Curriculum Parameters (Brasil, 1998), it becomes essential to understand what it *means* to teach *based*

on the Problem and not *to solve* Problems. While in one context, the Problem resolution process is the starting point, in another, it is the arrival point.

Based on the analysis we carried out in the research (Virgens, 2019) and which we highlight in this section, we can conclude that recognising the process of *meaning-making* of the concept of Problem in pedagogical practices and the *meanings* that are, historically and culturally, impregnated in the term, is fundamental for the mathematics teachers' education. This is because such *meanings* of the role of Problems in teaching practices influence the teachers' teaching practices, contributing to materialising proposals in which Problems emerge either as a mediating element (as a means) or as the purpose of the process. Sometimes as an application resource, sometimes as a triggering element.

We understand that the processes of *meaning-making* that produce ideas according to which Problems should provide opportunities for students to exercise techniques, algorithms, and calculations and ideas that Problems would serve as a possibility to emulate the application of concepts in supposedly real situations tend to bring practices closer to teaching methods in which Problems are the purpose of the educational process; that is, what is taught is taught so that students can solve Problems presented to them.

We also understand that the *meaning* involved in the determination to teach with the Problem as a starting point can place the LTP as the protagonist of teaching practices and the theoretical constructs that relate to it – such as the LTS, the TGA and the CHT that supports it. This is due to the core nature of the LTS and its potential to structure a general way of organising teaching. By developing formative movements that are intentionally organised so that prospective teachers' personal *senses* about the role of the Problem move and get closer to the *meaning* of Problems understood as a trigger for learning, practices that put into effect the idea that the Problem is the starting point of mathematics teaching and learning practices are materialised, supporting that teachers, even in their initial education, can be producers of a new practice where the Problem is understood as a learning trigger.

## **AUTHORSHIP CONTRIBUTION STATEMENT**

WPV is responsible for the original research from the results of which this excerpt was carried out. He was also responsible for writing the first version of the text and making later adjustments, rewriting, and finalisation. VDM was the supervisor of the original research from the results of which this excerpt



was carried out. He contributed to reading the first version, correcting and adjusting the writing and structure of the text and writing the final version.

## DATA AVAILABILITY STATEMENT

Data sharing does not apply to this article as it is a publicly available bibliography search.

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