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How Do Prospective Teachers Find Number Patterns Based on the Laws of Proximity, Closure, and Similarity in a Problem-Solving Activity?

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ABSTRACT

Background: Number pattern is a relevant topic in mathematics. Generally, the problems related to the theme can be solved by applying three principles: proximity, closure, and similarity. Besides, solving number pattern problems is closely related to students' ability to present the problem mathematically**. Objectives**: To examine prospective teachers' mathematical problem-solving processes based on the laws of proximity, closure, and similarity**. Design**: This qualitative research uses a case-study approach to achieve research objectives. **Setting and participants**: The researcher gave ten math questions to 67 prospective teachers (university students), choosing three to participate. The three focal participants were selected based on the categorisation results using indicators of proximity, similarity, and closure approaches from Gestalt theory. **Data collection and analysis**: Besides the test questions, the researcher conducted cognitive interviews with the three focal participants to confirm and explore the thought processes. Researchers used focus group discussion (FGD), cross-section data, and reviews with relevant references to validate the research outcomes. **Results**: The data show that the students could use the law of proximity, closure, and similarity in solving the number pattern problem, which can be seen from their ability to divide the pattern into two parts, complete it into specific geometrical shapes, and to divide it into similar shapes. In this study, prospective teachers' thinking process schemes were also found when solving patterned number problems using the law of proximity, the law of closure, and the law of similarity. **Conclusions:** The students who apply proximity could divide each pattern into two parts: fixed and growth. The pattern of growth difference will become the key in generating the general form of the pattern. The students who applied closure could complete the pattern into a particular shape by

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adding the pattern element. Furthermore, the students who applied similarity divided the pattern into similar shapes. Every student showed a good process in making representation.

Keywords: cognitive structure; Gestalt principles; mathematical representation; number pattern.

Como o futuro professor encontra padrões numéricos com base na lei da proximidade, do fechamento e da similaridade na atividade de solução de problemas?

RESUMO

Contexto: O padrão numérico é um dos tópicos importantes da matemática. Geralmente, os problemas relacionados ao padrão numérico podem ser resolvidos aplicando três princípios: proximidade, fechamento e similaridade. Além disso, resolver problemas de padrão numérico está intimamente relacionado à habilidade dos alunos em apresentar o problema matematicamente. **Objetivos**: Examinar os processos de resolução de problemas matemáticos dos futuros professores com base nas leis de proximidade, fechamento e semelhança. **Design**: Esta pesquisa qualitativa usa uma abordagem de estudo de caso para atingir os objetivos da pesquisa. **Ambiente e participantes**: O pesquisador deu dez questões de matemática para 67 futuros professores (alunos universitários) e escolheu três para serem os participantes focais. Os três participantes dos grupos focais foram selecionados com base nos resultados da categorização usando indicadores de proximidade, similaridade e abordagens de fechamento da Teoria da Gestalt. **Coleta e análise de dados**: Além das perguntas do teste, o pesquisador também conduziu entrevistas cognitivas com três participantes vocais para confirmar e explorar os processos de pensamento que ocorreram. Os pesquisadores usaram dados do grupo focal (GF) e de corte transversal para validar os resultados da pesquisa obtidos. **Resultados**: Os dados mostram que os alunos conseguiram usar a lei da proximidade, fechamento e similaridade na resolução do problema de padrão numérico, o que se observou em sua capacidade de dividir o padrão em duas partes, completá-lo em certa forma geométrica e dividi-lo em formas semelhantes. Neste estudo, os esquemas do processo de pensamento dos futuros professores também foram encontrados ao resolver problemas de números padronizados usando a lei da proximidade, a lei do fechamento e a lei da similaridade. **Conclusões**: Os alunos que aplicaram a proximidade conseguiram dividir cada padrão em duas partes, a parte fixa e a parte de crescimento. O padrão de diferença de crescimento se tornará a chave para gerar a forma geral do padrão. Os alunos que se aproximaram foram capazes de completar o padrão em certa forma adicionando o elemento do padrão. Além disso, os alunos que aplicaram similaridade dividiram o padrão em formas semelhantes. Cada aluno estava mostrando um bom processo de representação.

Palavras-chave: estrutura cognitiva; princípios da Gestalt; representação matemática; padrão numérico.

INTRODUCTION

Mathematics is a fundamental field of knowledge and must be mastered by every human being (Keller et al., 2001). It is a compulsory subject that starts to be learned at the elementary school level or even preschool to build students' engagement and familiarity (Poole & Evertson, 2019). Experts also argue that mathematics is a study of patterns. Based on research by Galante (2014), number pattern is one of the core topics in mathematics. Number pattern problems are usually employed to elaborate the 21st-century skills students should master, including critical thinking, creative thinking, communication, and collaboration (Nugraheni et al., 2022; Santia et al., 2019). The research by Purohit (2016) showed that the number pattern test can dig into the students' creative skills. Based on research by Toom et al. (2015), number pattern problems force the students to determine the general formula. Hence, students' inductive thinking skills will be trained to enable them to generalise and find the unknown number.

Number pattern problems are usually represented visually so students can see the pattern growth (Nurmawanti et al., 2016; Rivera & Becker, 2008; Wagemans et al., 2012). Therefore, to solve the number pattern problem, students need to build good mathematical representation skills. Research by Yang et al. (2016) shows that students' mathematical representation can be used as an indicator of success in solving number pattern problems because, for that, students must analyse the picture to transform the information into a mathematical expression and create the general formula (Garderen & Montague, 2003).

Several approaches, including Gestalt theory, are taken to make patterns(Chang et al., 2014). This approach is better known in the psychological world but has been widely adopted in other fields, such as visual communication design or statistics. Based on research by Nurmawanti et al. (2016), the law of proximity is a pattern formed from the assumption of the location/position of an object. Distant objects will cause random/scattered pattern assumptions. Therefore, moving the object closer is necessary so that it can be understood and become a whole pattern. The law of closure emphasises its assumptions on the completeness of an object. Objects that are incomplete or have negative space (empty) will cause the object to look abstract/not patterned. Then, the closure must be made in the negative space to make it a complete and patterned object. Finally, there is the law of similarity; in this law, the focus of the assumption is on the shape of the object. Abstract objects will be grouped based on the similarity/characteristics of the object. This grouping aims to make the pattern clear and not confusing to see.

Based on these laws, mathematical representational abilities, especially visual ones, really depend on each individual. Assumptions formed from the object being seen will be processed by the individual in accordance with the law they believe (Niehorster & Li, 2017). Moreover, every human being has a different process of representation, so the form of interpretation carried out in the end is not the same, being divided into several laws that have been described. Thus, the red thread can be drawn between a person's mathematical representation (visual) ability and the form of patterns made based on the law of proximity, closure, and similarity.

Several research results show a close relationship between a person's mathematical representation ability and the number pattern formed (King, 2014). The research results by Al-saleem et al. (2020) and Maoto et al. (2014) show that the better the students' mathematical representation ability, the better their ability to find unknown patterns. Qualitatively, the exploration conducted by Chang et al. (2014) and Wagemans et al. (2012) in junior high and high school students found that there were several types of number patterns that someone might do, namely, grouping based on types of numbers (odd-even), grouping based on shapes (arithmetic, geometric), and grouping based on formulas (Pascal and Fibonacci). Unlike the previous findings, the researcher will explore the results of the number patterns carried out by prospective elementary school teachers based on the formation process. Researchers will use Gestalt theory to explore number patterns generated by prospective elementary school teachers.

In Indonesia, the number pattern is introduced as early as the third grade of elementary school (Kaur, 2014). Therefore, the teachers in elementary schools should have an effective method of solving number pattern problems. Reflecting on problems, the aim of this study is to determine students' mathematical representation skills based on the three Gestalt principles of proximity, closure and similarity involved in number pattern-related problems.

THEORETICAL BACKGROUND

Patterns in Math

A pattern is a relationship, namely the shared nature or characteristics of several circumstances, facts, case data, or events. Based on the patterns obtained from several of these circumstances, a person has the basic material for predicting, surmising, or guessing that rules that apply in general, as general statements (generalisations) from several data or facts (Fantz & III, 1975). Yong et al. (2004) explain “pattern as a rule between the elements of a series of constructed mathematical objects”. According to Hausmann et al. (2013), a mathematical pattern can be defined as any predictable regularity, usually involving numerical or geometric shapes.

Yanhui Liu et al. (2010) patterns are usually defined as repeated sequences of objects, actions, sounds, or symbols. Starting with simple patterns, students recognise, represent, extend, and create patterns with different arrangements of the elements. Students can also use geometric shapes to explore patterns. Students learn about patterns with simple activities, but the aim/goal of pattern practice is to find patterns in many situations. Finding and using patterns is an important strategy in life, thinking mathematically and solving problems. Giving challenging problems about patterns can also increase students’ creative thinking and can develop students’ mathematical abilities (Ramirez Rivera et al., 2013).

Yong et al. (2004) describe patterns as the heart of mathematics, which you can find by looking for shapes, numbers, and many other things. In pattern material, students will find and explore patterns and describe them with numbers and formulas. Patterns are also a major element in algebraic thinking. As students mature to develop algebraic thinking, they learn and analyse challenging patterns. They also see patterns in more complex sets of patterns than numbers. Learning to find patterns from various representations and how to explain, translate, and expand patterns is part of doing mathematics and algebraic thinking (Walle et al., 2009, p. 259). Early algebraic thinking involved pattern recognition and general mathematical relationships between numbers, objects, and geometric shapes (Ramirez Rivera et al., 2013).

Meanwhile, according to NCTM (2000), algebraic thinking is indicated by the following components: (1) understand patterns, relations, and functions, (2) representing situational analysis and mathematical structure using algebraic symbols, (3) using mathematical models to represent and understand quantitative relationships, and (4) analysing changes in various contexts.

Number Patterns Identification

Pattern identification is a very important initial activity in finding the type of symbolisation in generalisation (de Walle et al., 2016). In the first three steps (1,2,3), students identify patterns by looking at them and determining temporary rules for each step. Furthermore, students relate the similarities and differences between the observed steps. Based on the relationship between steps 1,2, and 3, students make rules and check whether the rules apply to all three steps.

When identifying patterns, the initial activity that can be done is to identify the structure of the pattern. If it is in the form of an image, then identification can be done by observing the structure or how the constituent forms are from each given image. Students see the structure of the picture by observing their similarities and differences at each step, which can be observed based on the quantity or shape of the constituents.

Toom et al. (2015) explained in detail the activities carried out by students in identifying patterns, including:

a. Observing patterns in positions 1 and 2 to see what is constant and what changes from position 1 to position 2.

b. Students build patterns based on the observations they have made before.

c. Students use the patterns or rules they construct to determine whether they apply to the pattern in the following position.

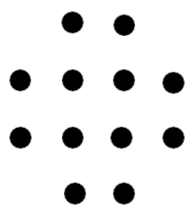
The pattern identification activity is an important activity to be carried out successfully in working with patterns because it is part of the generalisation process. This statement is based on de Walle et al. (2016), who state that there are four steps in the generalisation process: direct modelling, identification of patterns, proof tests of patterns, and finding rules for general cases. Furthermore, de Walle explained that pattern identification is essential in finding the type of symbolisation in generalisation. The importance of pattern identification is also explained by Mason (in Fahle, 2005), who says that the generalisation process consists of four stages: perception of generality, expression of generality, symbolic expression of generality, and manipulation of generality. In the perception of generality, students can recognise patterns and be able to identify patterns. This ability is used to carry out activities at a later stage. Thus, identification is part of the pattern generalisation activity, which is important in determining pattern generalisation.

Based on the explanation above, the strategy used in pattern generalisation can be used in pattern identification. Rivera and Becker (2008) explain that students use different strategies in generalising patterns. The strategies are then grouped into three types: (1) numerical, which only uses cues established from each pattern listed as a sequence of numbers or tabulated in a table to derive rules; (2) figural, which applies only in generalising tasks that describe patterns using images, and relying entirely on visual cues established directly from image structures to derive rules; and (3) pragmatics, a combination of numerical and figural strategies.

Based on the Gestalt laws, the student's perceptual framework identifies the following patterns in Table 1:

Table 1

Characteristics of student perceptions in identifying patterns in mathematics

Gestalt Law	Description	Operational
Law of proximity	Objects in patterns that are close to each other are seen as a group.	For example, in the following image.  Students perceive that there are points that are close to each other, that is, there are 2 points that are close to each other so that there are 6 adjacent points.

Law of similarity	Objects in patterns that have the same characteristics are seen as the same group.	• Students perceive the pattern of the dots into two groups, namely the dots in the middle forming a square and the dots surrounding the square. • Students perceive objects in patterns that have something that is fixed and the same and have something that is different.
Law of closure	Objects in patterns with gaps tend to close to form a complete object.	• Students perceive the dotted image as a square shape by closing the corners by adding four dots. • In order not to change the number of dots in the pattern, you must count the number of dots.

METHODOLOGY

This study aims to examine students' mathematical representation in solving number patterns. Therefore, this is a qualitative-type research with a case study approach. The participants were three focal students out of 67 students of the elementary school education study program (prospective teachers) in Universitas Mataram, Indonesia. The data were gathered from a written test of ten number pattern problems and from student interviews. The instrument is a mathematics problem about number patterns and cognitive semi-interview guidelines. The cognitive interviews were conducted after the students completed the problems given. This interview aimed to capture students' thought processes while solving problems. Interviews were conducted in private, involving only one subject and the researcher.

The data were grouped into three categories of the Gestalt principles: proximity, closure, and similarity. One correct answer will be selected from each category to be analysed further. The focal student was assessed using Gestalt law indicators as provided in Table 2.

Table 2

Gestalt Law Indicators. (Adapted from Nurmawanti et al., 2016)

Process	Indicators (Code Indicators)
Law of proximity	1. The subject groups circle images constantly. 2. The subject represents adjacent images as a group.
Law of closure	1. The subject makes the completion of the circle object to be a recognisable pattern. 2. The subject represents a random picture as a known pictorial pattern
Law of similarity	1. Subjects look for similarities between circle images and then group them. 2. The subject represents the image in the same or similar parts

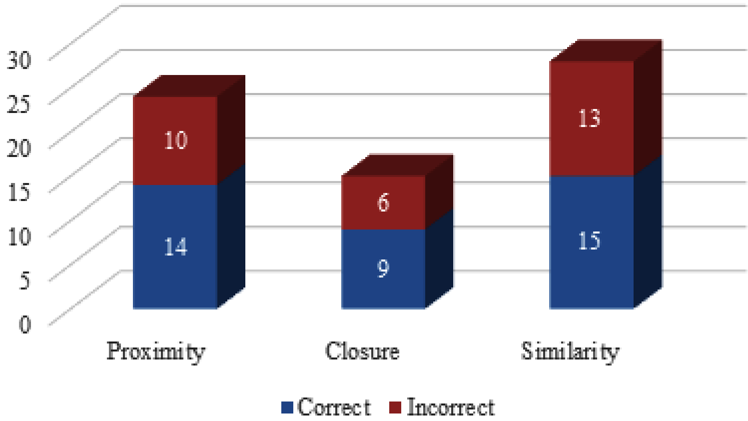
The data were analysed based on three aspects simultaneously, namely similarity, closure, and proximity law, and how the students find the general formula for the arithmetic pattern. Using the focal student's initials, the researchers explored the patterns' results more deeply. The researchers analyse the results of patterns made by students using the formulated Gestalt law indicators. Then, the interview data were analysed through a schematic of the students' thinking processes to explain the flow of students' thinking when solving problems. The exploration results were also used to describe the pattern generation stages carried out by students who fall into the proximity, closure, and similarity categories. To maintain the validity of research data, researchers used several methods, namely, (1) group discussion forum (FGD), (2) cross-checking, and (3) review with relevant references. The FGD activities were carried out with leaders at the university level to confirm the research results obtained. Meanwhile, cross-checking was carried out with each respondent to maintain the accuracy of the cognitive interview transcript conducted by the researcher. After that, the researcher also conducted reference studies related to the research findings results.

RESULTS AND ANALYSIS

From the 67 participants, researchers obtained that the students are applying the principles of proximity, closure, and similarity in solving number pattern problems. The general result of students' works can be observed in Figure 1.

Figure 1

General results



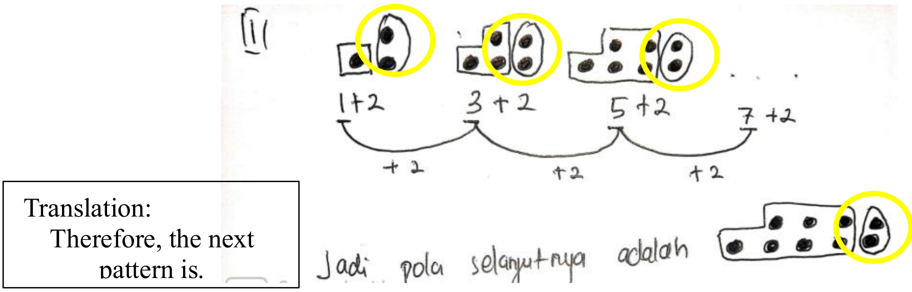
Of the 67 results of the participant work, 38 participants answered correctly, while 29 others gave wrong answers. All correct answers were divided into three categories, and each respondent chose one subject to be discussed in depth. The three respondents whose work results were discussed in depth were given a code to facilitate the analysis process: Subjects using the law of proximity (SP), Subjects using the law of closure (SC), and Subjects using the law of similarity (SS).

Analysis of SP's work

Seven of the 13 subjects who answered the problem correctly applied proximity to determine the following pattern. Research by Rivera and Becker (2008) shows that proximity is done by dividing the pattern into two pieces or parts. The first part is changeable, and the second part is fixed. Observe the example of students' answers in Figure 2.

Figure 2

Proximity principle



The yellow circle in Figure 2 indicates that the SP considered two dots as a fixed value. It is confirmed in the subsequent patterns when the SP keeps the two dots. This fixed value becomes the reference to determine the next one, while the changeable part consists of the additional dots in every pattern. Usually, students tend to group the parts based on the position, in which, if the object is close to another, they are likely to be grouped as one (Hochberg, 2010). Figure 2 also shows the counting result from the dots in fixed and changeable parts. In the first, second and third patterns, the dots can be represented mathematically as "1+2", "3+2", and "5+2", respectively. Based on research by Hidayati et al. (2020), the mathematical expression is usually helpful to enable students to reflect on the counting pattern.

In general, SP's steps can be explained as follows. After distinguishing the pattern as fixed and changeable part, SP looks at the additional dots in every next pattern. SP found that there are two more dots in every next figure. By using that, SP concluded that the next pattern would have two more dots. To enrich the discussion, this study attempted to create a scheme to explain how SP generalised a pattern using the proximity principle, as seen in Figure 3.

Figure 3

Scheme in mathematics representation process by using the proximity principle

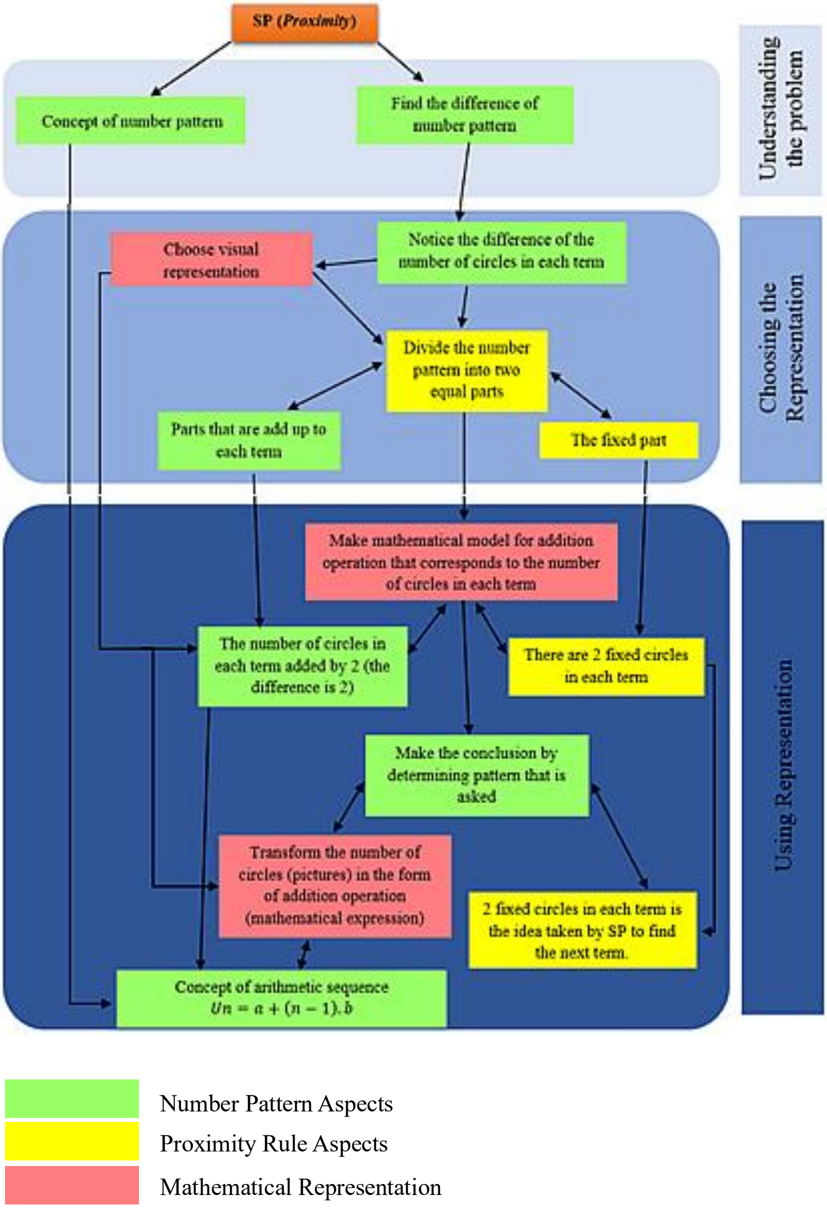


Figure 3 shows the mathematical representation process carried out by SP. It starts by understanding the concept of the number pattern owned by SP to solve the problem. The SP subject chooses a visual representation where the SP observe a given pictorial pattern and then makes a completion strategy. The SP subject divides the image in each pattern into two parts. The first part contains a circle that increases with each pattern, while the second part contains a fixed number of circles. In this second part, there is sufficient proximity used by SP, where SP approaches the two circles found in patterns 1, 2, and 3 that are already known. By looking at the two circles in the second section as key, the SP subject determines the difference in each pattern and finds that the difference between each pattern is two circles. Then, SP changes the image's shape (visual) into the form of addition operations (mathematical expressions). SP does this to facilitate the discovery of number patterns, i.e., two links are added to each pattern (difference = 2). Finally, SP subjects found the 4th pattern correctly using the law of proximity.

Based on the ability of representation, SP sees the pattern of numbers as groups that are close together. From the standpoint of mathematical representation, the information processed by SP is that the pattern of numbers is divided into two parts. The first part is a group that contains a fixed number of circles, and the second group contains increasing circles. This type of representation is identical to the law of proximity approach because SP groups circle images into groups that are close together. The form of the questions was originally shaped like the picture in Figure 4.

Figure 4

Visual mathematical representation with the proximity approach

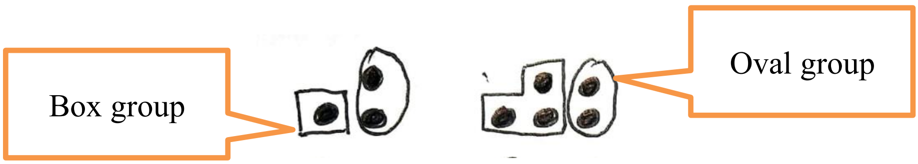


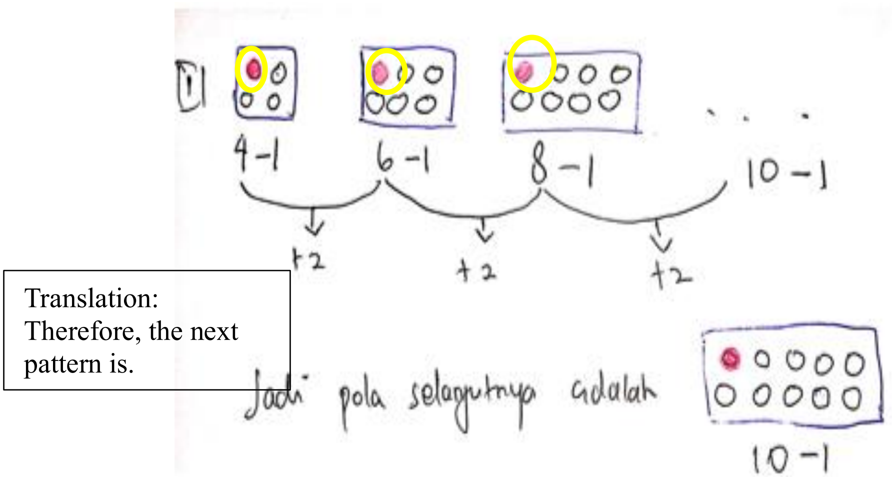
Figure 4 shows that SP views the city and oval groups as two different groups. The grouping process is carried out by SP because SP sees that the circles of the squares are close together. Likewise, the oval groups are also grouped because the object circles are close together. Interestingly, SP does not consider the number of circles in each group in this process. SP only sees circular objects that are close together. This is evident from the number of circles in groups of squares that are not fixed, meaning that it always increases. But as this stage of representation continues to find the subsequent number patterns, SP begins to consider the number of circles in each group. In finding “different” values between numbers, subjects see the difference between the number of circles in a group of squares. This shows that the SP has successfully carried out the mathematical representation process.

Analysis of SC’s work

From 38 subjects who correctly responded to the given problem, nine applied closure to determine the following pattern. Based on research by Rivera and Becker (2008), closure is done by completing the given pattern into a familiar shape. An example of the use of the closure principle can be seen in Figure 5.

Figure 5

Closure principle



As shown in Figure 5, SC saw that the pattern resembles a rectangle if one dot is added. The red dot is an additional one created by the student. After that, SC checks how many dots are in each pattern and then makes a mathematical notation, such as “4-1”, “6-1”, and “8-1”. Based on research by Mhlolo et al. (2012), the mathematical expression plays an important role in enabling students to count. From Figure 5, the students still consider an additional dot in the pattern that they added and write it as “-1”.

In general, the steps in SC can be explained as follows. Students see the pattern and notice it is like a familiar figure. In this problem, the pattern resembles a rectangle. Therefore, one dot is added in each pattern to complete the missing part of the rectangle. Aligned with that, Toom et al. (2015) state that the main idea is fulfilling the space to be a whole figure. Consider the scheme of how the closure principles work in Figure 6.

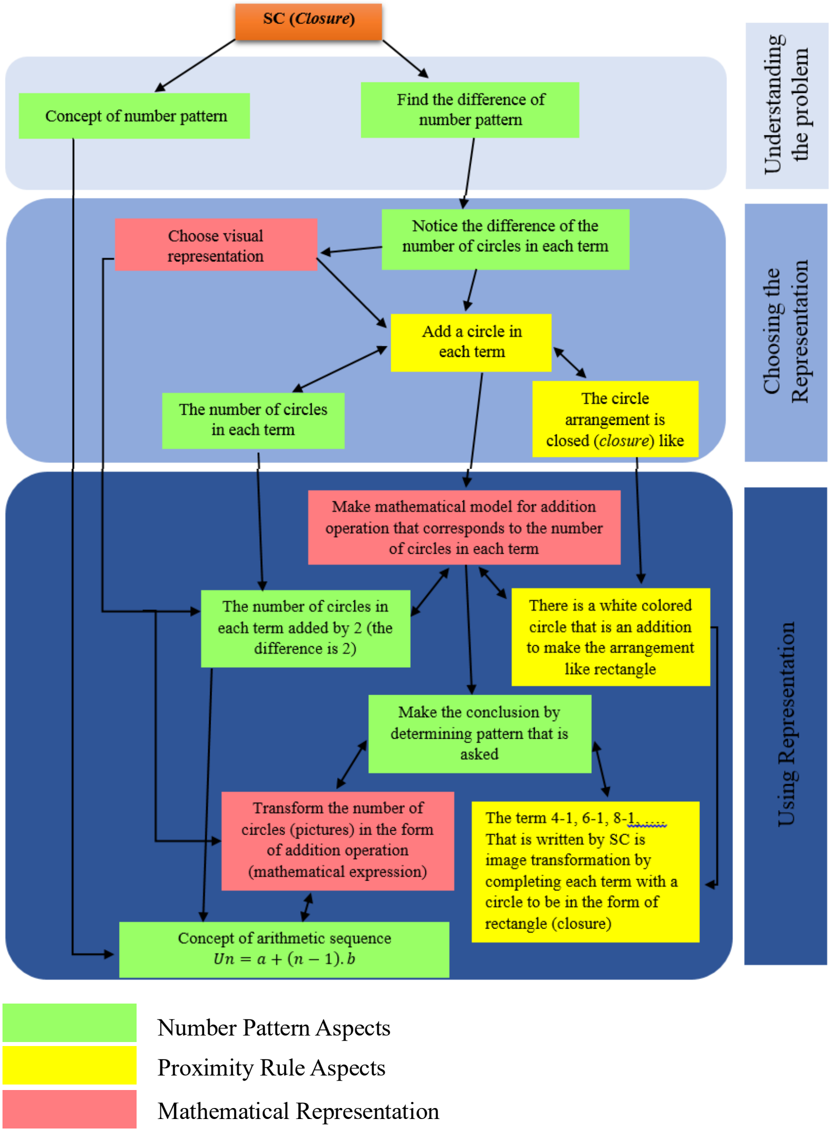
Figure 6 shows the SC process in finding a given pattern. Three aspects are discussed: number patterns, mathematical representations, and the law of closure. Figure 6 shows that SC adds a white circle to complete each term in a number pattern. So, it forms the pattern 4-1,6-1,8-1. SC performs the steps in mathematical representation by making a mathematical model of the pattern given, transforming the image into a mathematical expression.

Based on the mathematical representation capabilities performed, SC views number patterns as random objects. SC sees the image in the process of mathematical representation while completing the drawing of a random pattern to become a recognised object. In drawing a given pattern into a recognisable object, the SC creates all patterns corresponding to a square. The SC subject adds a circle to each pattern so that previously unfinished patterns become associated with the square shape recognised by SC.

Figure 6 presents SC adding one circle to each pattern. Based on the process of mathematical representation, SC pays attention to random image patterns so that the SC begins completing them, as described. In this case, the same as SP, SC subjects do not use the number of circles as a material for consideration of pattern formation. Only then, in the compilation of the process of determining the value of “different”, SC transforms the image pattern information entering the number of circles. Therefore, the pattern of numbers that make up SC is 4-1,6-1,8-1.... If you look at Figure 6, values 4,6,8 are sequentially, while -1 in each pattern is one circle that was previously added. After that, SC sees changes in values 4, 6, 8, to find the “difference” pattern of numbers, i.e., 2. Thus, the mathematical representation made by SC was good.

Figure 6

Scheme in mathematics representation process by using the closure principle

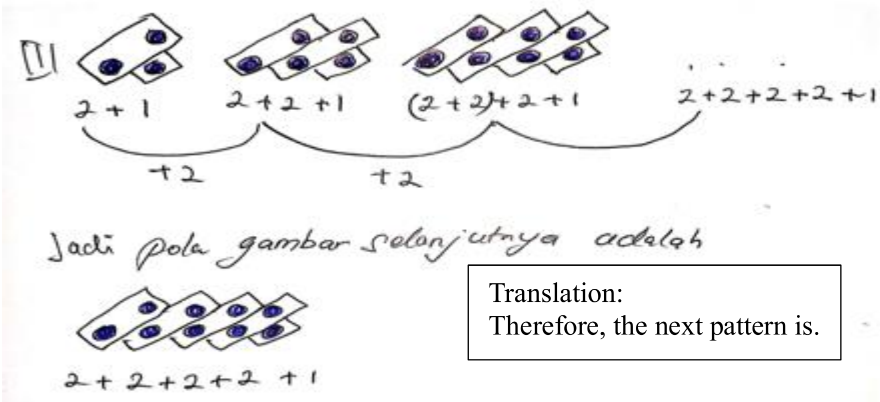


Analysis of SS's work

From the 38 subjects who provided correct answers, 15 applied the law of similarity to determine the next pattern. Research by Chang et al. (2014) shows proximity is done by grouping the pattern into similar shapes. Consider Figure 7.

Figure 7

Similarity principle



As shown in Figure 7, SS grouped the pattern into simpler shapes. From the student's responses, two had been selected as the "main number", in which the student tried to group the pattern into two dots, leaving one. SS found that the difference between the two patterns is "+2"; therefore, in every following pattern, they only need to insert two dots before the last dot. Based on research by Maoto et al. (2014) to restructure the addition into a simpler shape as the student's answer in Figure 6 needs adequate algebraic thinking. Usually, this step is hard, because the student needs to find the proper combination and operation that will be similar to the restructured number or pattern (Wagemans et al., 2012).

Mathematical representation is an important factor in solving number patterns (Purohit, 2016), especially if the pattern is visualised in specific shapes in geometry. When solving a number pattern which is visualised geometrically, the students tend to interpret the figure first before predicting the pattern (Panasuk, 2010). Research by Bikić et al. (2016) found that the geometrical representation can also be used as a mathematical model that supports the students in generalising the pattern.

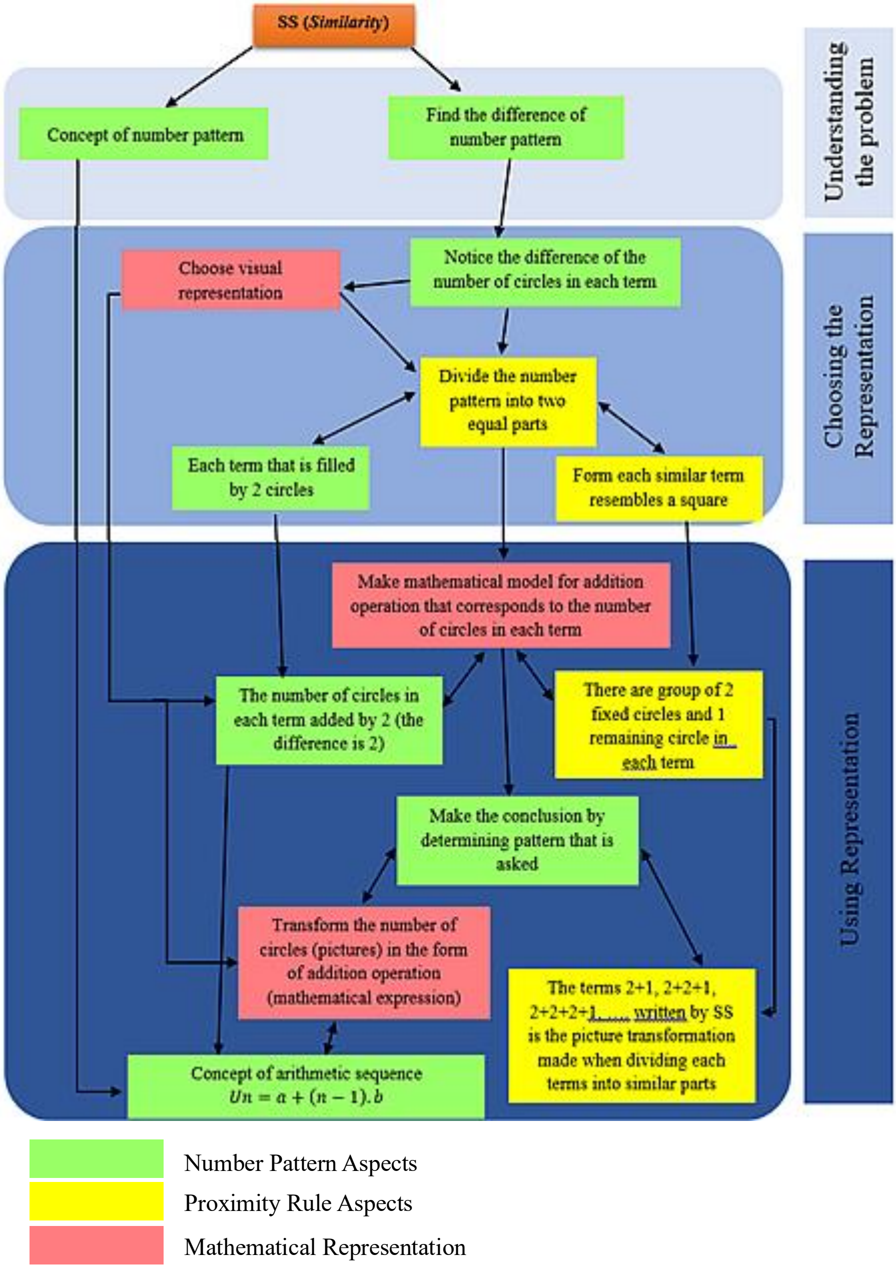
SS's steps can be described as follows. Students interpret the figure to find the pattern. After that, the pictures were converted into numbers. From that, the students looked at the growth of the numbers in relation to the change in the visual representation of the pattern. The complete thinking scheme of SS can be observed in Figure 8.

Figure 8 shows SS divides the pattern image into several equal (similar) parts. These sections contain the same number of circles, except circle students who have not yet been grouped, while we see that SS uses of the law of similarity dim. By looking at two circles in these sections, the SP subject determines the difference in each pattern and finds that the difference of each pattern is two circles. Then, SP changes the image's shape (visual) into the form of addition operations (mathematical expressions). SP does this to facilitate the discovery of number patterns, i.e., two links are added to each pattern (difference = 2). Finally, SP subjects found the 4th pattern correctly using the proximity law.

Based on the mathematical representation process carried out by SS. It starts from understanding the concept of number patterns that SS owns to solve the problem. The SS subject chooses a visual representation where the SS observes a given pictorial pattern and then makes a completion strategy. The representation process carried out by SS is dividing pattern images into similar groups. Unlike SP and SC, in constructing this pattern image, the SS subjects pay attention to the number of circles in each group. This is done because the SS divides this group based on the shape of its resemblance, so the number of circles in it must also be the same. As can be seen from Figure 7, the SS divides the pattern images into groups, each containing two circles and one remaining circle. Only after this process did SS convert it to the mathematical expression "2 + 1,2 + 2 + 1, ...". From this stage, the SS can determine the value of "difference", which is 2.

Figure 8

Scheme in mathematics representation process by using the similarity principle

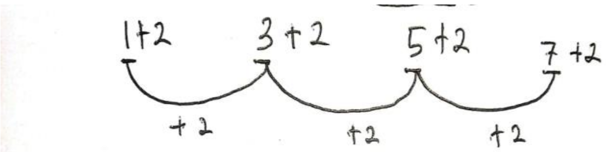


Pattern Generalisation

Pattern generalisation is constructing a general form (mathematical formula) that must apply to the whole word in the agreed universe (Nurmawanti et al., 2016). Drawing conclusions requires analytical skills and number sense from individuals who make pattern generalisations (Chang et al., 2014; Kamid et al., 2020). Based on the results of the SP, SC, and SS work, the three subjects have different styles in determining number patterns. The SP subject generates the following number pattern in Figure 9.

Figure 9

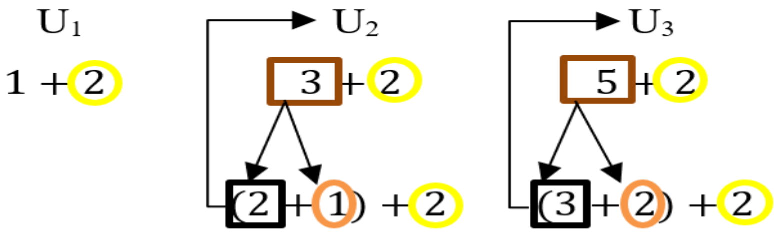
Arithmetic pattern created by SP.



From Figure 9 researchers can see that there is a number which SP always adds to each pattern, which is "+2". Whereas the initial number always increases by 2 for each pattern. This number ultimately becomes the "different" pattern created by SP. To make it easier to understand the generalisation process, the following pattern construction can be done according to the proximity approach in Figure 10.

Figure 10

Pattern construction by SP.



The numbers marked in black and red always appear to have a difference of 1. The next pattern also applies so researchers can generalise n + (n-1). This formula applies to U1, where 1+ (1-1) = 1, whereas the number 2, highlighted in yellow, is always constant in each pattern, so the general form researchers can make is $Un = n + (n-1) + 2$.

$$Un = n + (n-1) + 2$$

$$U_4 = 4 + (4-1) + 2$$

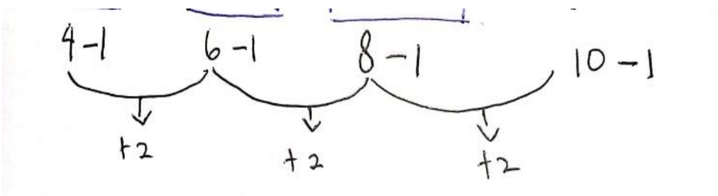
$$U_4 = 9$$

So, the general form of a given number pattern based on the law of proximity is $Un = n + (n-1) + 2$.

Otherwise, The SC subject generates the following number pattern in Figure 11.

Figure 11

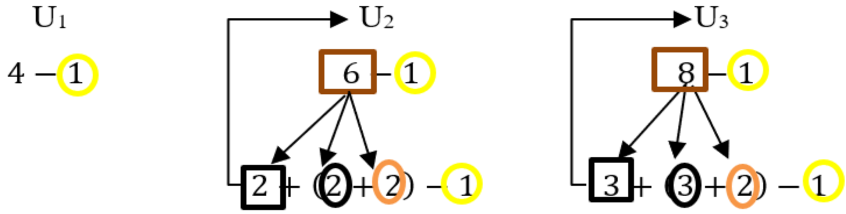
Arithmetic pattern created by SC.



In Figure 10, SC always reduces the value of each pattern by "-1", while the initial number always increases by "2". This increase causes a "difference" found by SC, namely 2. To facilitate an understanding of SC's construction, we can see the following calculations in Figure 12:

Figure 12

Pattern construction by SC.



It appears that SC divides the initial number into three numbers. The first and second numbers marked in black match the line pattern. As if the number planted in red is always “+2”. Finally, on the yellow circle, SC reduces it by “-1”. So, the general form constructed by SC is $Un = [n + (n + 2)] - 1$. Here are the results of the calculation of the SC to find the 4th pattern:

$$Un = [n + (n + 2)] - 1$$

$$U_4 = [4 + (4 + 2)] - 1$$

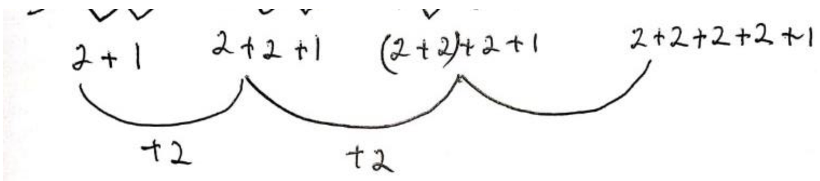
$$U_4 = 9$$

So, the general form of a given number pattern based on the law of closure is $Un = [n + (n + 2)] - 1$

Finally, the SS subject uses the law of similarity to find the 4th-row pattern. Here are the results of the ball pattern performed by SS in Figure 13.

Figure 13

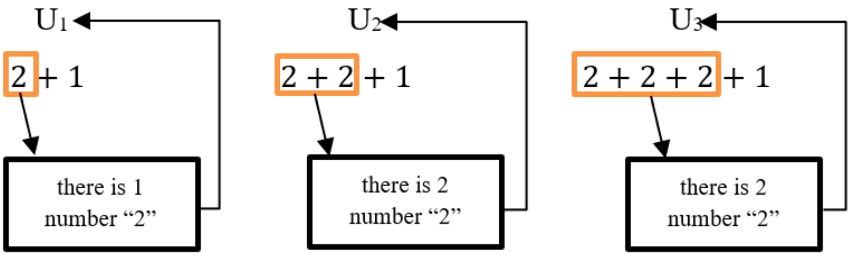
Arithmetic pattern created by SS.



In Figure 13, it appears that the SS divides each part of the pattern into several parts whose values are the same i.e., 2. After that, each number has a constant 1. The SS subject then finds that the difference in each part of the pattern is +2, so the SS finds that the pattern has a “different” value of 2. To understand the pattern construction carried out by the SS, consider the following calculation in Figure 14.

Figure 14

Pattern construction by SS.



Construction carried out by SS tends to be different from SP and SC. This subject pays attention to the number of numbers 2 adjusted to the pattern, which is in the row. The remaining 1 is not grouped and then becomes constant in each pattern. Thus, the form of generalisation produced by SS is $Un = 2n + 1$. The appearance of n here is a representation of how many numbers “2” appear in each pattern. Following are the SS calculations:

$$Un = 2n + 1$$

$$U_4 = 2.4 + 1$$

$$U_4 = 9$$

So, the general form of a given number pattern based on the law of similarity is $Un = 2n + 1$

CONCLUSIONS

Based on the result and discussion, we concluded that the students are implementing all Gestalt principles of proximity, closure, and similarity to solve the number pattern problems. The students who apply proximity could divide each pattern into two parts, the fixed and the growing parts. The pattern of growth difference will become the key in generating the general form of the pattern. The students who apply the closure could complete the pattern into a particular shape by adding the pattern element. Furthermore, the students who used the similarity divided the pattern into similar forms. Every student showed a good process in making representation. They also presented a good number pattern mastery in solving the problem. Moreover, the students could choose the type of representation, modifying it to become more helpful support in creating a mathematical model.

In addition, the results also revealed that the generalisation patterns in each subject were also different. Each approach provides a different assumption so that students who use the law of proximity, of closure, and of similarity make a general shape according to the chosen approach. The general form construction of students who use the laws of proximity and closure is almost similar, where they play in the area of number translation to find the most common form. While students who use the law of similarity are slightly different, these students focus on existing numbers. This is certainly greatly influenced by the similarity mindset that groups students' assumptions into the same numbers.

AUTHORS' CONTRIBUTIONS STATEMENTS

MAM conceived the presented idea. WWD and SGM developed the theory. MAM and YWP adapted the methodology to this context, created the models, performed the activities, and collected the data. MAM and SGM analysed the data. All authors actively participated in the discussion of the results, reviewed, and approved the final version of the work.

DATA AVAILABILITY STATEMENT

Data on student work results and cognitive interview transcripts are owned by MAM. This data can be accessed if needed in the future.

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