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Mathematical Discourse in Research Lessons from a Lesson Study with Two Prospective Teachers

Raquel Vieira [a,b]
João Pedro da Ponte [a]

[a] Universidade de Lisboa, Instituto de Educação, Lisboa, Portugal
[b] Instituto Politécnico de Leiria, Escola Superior de Educação e Ciências Sociais, Leiria, Portugal

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ABSTRACT

Background: Mathematical classroom discourse is fundamental to control mathematical communication. Initial teacher education should support the prospective teacher to prepare and lead discussions that favour mathematical discourse. Lesson study is a professional development process with potential benefits related to developing the prospective teacher's knowledge of mathematics teaching. **Objectives**: We intend to characterise the mathematical discourse in two research lessons led by prospective teachers and understand which aspects of their participation in the process they highlight. **Design**: We follow a qualitative and interpretative participant-observation approach. **Setting and participants**: The study took place in a Portuguese higher education institution. Two prospective teachers, their supervisor and cooperating teacher, and the researcher participated voluntarily. **Data collection and analysis:** Data collection included participant observation, field notes, audio and video recordings, document collection, and semi-structured interviews. Data were transcribed, coded, and analysed according to predefined categories. **Results**: The mathematical discourse in the first research lesson was more favourable to communication than in the second. The prospective teachers' perceptions about their participation in lesson study highlighted aspects of the *teacher's role, questioning, explaining mathematical thinking*, and *mathematical representations*. **Conclusions**: We show that detailed prior preparation of the research lesson can strongly benefit the performance of the prospective teacher. However, this does not guarantee that the lesson is not strongly centred on the teacher.

Keywords: Initial teacher education; Lesson study; Knowledge of mathematics teaching; Mathematical classroom discourse; Prospective teachers' perceptions.

Corresponding author: Raquel Vieira. Email: raquelsvieira@campus.ul.pt

O discurso matemático nas aulas de investigação de um estudo de aula com duas futuras professoras

RESUMO

Contexto: O discurso matemático na sala de aula é fundamental à regulação da comunicação matemática. A formação inicial deve apoiar o futuro professor a preparar e conduzir discussões que favoreçam o discurso matemático. O estudo de aula é um processo de desenvolvimento profissional com potenciais benefícios relativos ao desenvolvimento do conhecimento da prática letiva do futuro professor. **Objetivos**: Pretendemos caracterizar o discurso matemático em duas aulas de investigação, conduzidas por futuras professoras, e compreender que aspetos estas realçam da sua participação no processo. **Design**: Seguimos uma abordagem qualitativa e interpretativa, de observação participante. **Ambiente e participantes**: O estudo ocorreu numa instituição portuguesa de ensino superior e participaram, voluntariamente, duas futuras professoras e os respetivos professor supervisor e professora cooperante, e a investigadora. **Coleta e análise de dados**: A recolha de dados incluiu observação participante, com recurso a notas de campo, gravações em áudio e vídeo, recolha documental e entrevistas semiestruturadas. Os dados foram transcritos, codificados e analisados segundo categorias pré-definidas. **Resultados**: O discurso matemático da primeira aula de investigação foi mais favorável à comunicação do que na segunda aula. As perceções das futuras professoras sobre a participação no estudo de aula evidenciaram aspetos relacionados com o *papel do professor*, *questionamento*, *explicação do pensamento matemático* e *representações matemáticas*. **Conclusões**: Mostramos que a preparação prévia detalhada da aula de investigação pode beneficiar fortemente o desempenho do futuro professor. Contudo, esta não é uma garantia de que a aula não decorra de uma prática de ensino fortemente centrada no professor.

Palavras-Chave: Formação inicial de professores; Estudo de aula; Conhecimento da prática educativa; Discurso matemático na sala de aula; Perceções dos futuros professores.

INTRODUCTION

The mathematical discourse, which "includes the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication" (NCTM, 2014, p. 29), is a crucial element of the development of students mathematical learning (Hufferd-Ackles et al., 2014; Murata et al., 2017). Through mathematical discourse, students can share their thinking, question and be questioned, answering to each other, ensuring their participation in a *math-talk learning community*, situated in the classroom, which supports the mathematical learning of all who participate in it (Hufferd-Ackles et al., 2014). However, for this to

happen, students must learn to base their arguments on evidence that can be explicitly presented to their peers and be able to negotiate and expand them when confronted with the ideas of others (Hufferd-Ackles et al., 2014; Moschkovich, 2015). Thus, the teacher must be able to learn to lead mathematical discussions in the classroom that promote a dialogical discourse (Ghousseini, 2015).

Several investigations have documented that preparing and leading whole-class mathematical discussions in the classroom challenges the teacher (Gomes et al., 2023; Hufferd-Ackles et al., 2014). These discussions tend to demand more from prospective teachers and from those with little professional experience (Ghousseini, 2015; Martins et al., 2023; Smith et al., 2019). Leading classroom discussions requires the teacher to respond immediately to all students' mathematical ideas expressed unexpectedly and be able to relate them to each other, keeping the focus on the goals established for the lesson (Stein et al., 2008). Leading a discussion benefits from careful planning, which includes defining lesson goals and choosing mathematical tasks to address them is a particular aspect of the preparation stage (Smith et al., 2019). A serious challenge that prospective teachers face (Martins et al., 2023; Stein et al., 2008; Vieira et al., 2022), is anticipating student responses, highlighted by several authors as a strategy that supports the teacher in leading discussions (Dunning, 2023; Gomes et al., 2023; Stein et al., 2008). Therefore, studying ways to promote the development of knowledge of prospective teachers is important when preparing and leading whole-class discussions (Ghousseini, 2015; Saylor & Walton, 2018).

In this article, we analyse the participation of two prospective teachers in preparing and leading the *research lesson*, focusing on mathematical discourse. We aim to answer the following questions: (1) What are the characteristics of the mathematical *discourse* in *research lessons* led by prospective teachers? (2) What aspects of their participation in the lesson study related to mathematical discourse do the two prospective teachers highlight?

CLASSROOM DISCOURSE AND KNOWLEDGE OF MATHEMATICS TEACHING

Integrating a wide variety of ideas and modes of representation supports mathematical understanding in the classroom and encourages students to learn (Fujii, 2018; Murata et al., 2017). Students' participation in

classroom discourse allows them to share and consolidate ideas while delving into different types of representation and language to communicate their mathematical arguments (Moschkovich, 2015). Hufferd-Ackles et al. (2014) believe that ensuring that students play an essential role in the construction of their own knowledge and the knowledge of others involves taking into account five aspects of classroom discourse: *teacher role* – the way the teacher participates and supports student involvement; *questioning* - who asks the questions and what type of questions are asked; *explaining mathematical thinking* – who gives the explanations in the classroom and what their nature is; *mathematical representations* – how language and visual and concrete resources are used; and *building student responsibility in the community* – to what extent students share responsibility for their learning and that of their peers.

The teacher must consider regulating communication to maintain a classroom culture favourable to learning (Ponte, 2012). In doing so, the teacher mobilises his/her *knowledge of mathematics teaching* to get students to work productively on the proposed task, organising their productions to present to their colleagues (Gomes et al., 2023). It is the teacher's responsibility to regulate the discursive practices generated and the nature of the discussions held in the classroom, as these strongly determine the communication dynamics (Moschkovich, 2015; Stein et al., 2008). Stein et al. (2008), from the perspective of "making discussion facilitation something manageable for novices, those teachers who are new to this form of teaching" (p. 321), propose a model consisting of five practices: *anticipating likely student responses*, considering the different approaches and challenges that the student may face in solving them and anticipating teacher questioning; *monitoring students' responses*, closely monitoring their work and asking questions that allow the teacher to understand their mathematical thinking; *selecting particular students* that illustrate key strategies and ideas from *the lesson; sequencing the student responses* to benefit the goals of the lesson and the *connections between different students' responses,* making explicit the links between students' mathematical thinking and the goals established for the lesson. Later, Smith et al. (2019) added a practice zero – *setting goals and selecting tasks* – as the basis for the remaining five practices.

We, therefore, highlight two essential ideas: first, mathematical discussions are important in regulating the classroom discourse, and second, managing them requires the teacher to invest heavily in their preparation and leading. When preparing, the teacher must consider the time available, the choice of tasks, and the characteristics of students' thinking, foreseeing

participation structures. When leading, the teacher must select strategies that effectively contribute to productive discussions. Without this selection, some mathematical ideas may not be highlighted in the discussion or even be overlooked, compared to others that are less useful for developing the mathematical classroom discourse (Dunning, 2023).

CHALLENGES FOR THE PROSPECTIVE TEACHER AND THE LESSON STUDY POTENTIAL

Leading a mathematical classroom discussion requires a deep understanding of student thinking and strategies for directing discourse toward immediate learning objectives (Saylor & Walton, 2018). However, prospective teachers' *knowledge of mathematics teaching* is still under development as they have not yet acquired a significant repertoire of classroom experiences. Therefore, *selecting, sequencing, and connecting* student strategies may prove too demanding and critical for prospective teachers.

Lesson study, as a professional development process, allows prospective teachers to be supported in learning how to prepare and lead teaching practice. This is because the lesson study is based on collaborative and practice-oriented work, focusing on student learning. This strategy leads participants to plan and implement a lesson (*research lesson)* aimed at solving a previously identified learning problem (Fujii, 2018). Research has produced evidence that preparing lessons in a collaborative environment, provided by the participation of prospective teachers in a lesson study, mobilises them to construct more detailed and well-founded lesson plans (Leavy & Hourigan, 2016; Martins et al., 2023), which allows them to better respond to the complexity of classrooms (Chen & Zhang, 2019; Norton et al., 2019).

During the planning phase, participants must design the task and anticipate students' responses (Fujii, 2018), ensuring that it meets the learning goals of that lesson. One of the benefits of bringing together a group of experienced in-service teachers and prospective teachers when preparing a lesson is that this environment provides a discussion in which every participant can share and compare their ideas (Chen & Zhang, 2019). On the one hand, prospective teachers benefit from this experience because, by listening to the teachers' experiences, they expand their repertoires of classroom situations and learning styles (Norton et al., 2019). On the other hand, for prospective teachers' enhancement, they must feel free to make

decisions, take risks, and see their ideas valued in the discussion (Ponte, 2017).

One of the focuses of the lesson study is the *research lesson*, which highlights the extent to which lesson planning "is a critical factor in determining the quality of mathematics teaching" (Fujii, 2018, p. 6). During the lesson, the teacher must know all the students' solutions and how the students usually react to the proposed task. This assumption is fundamental to the teacher to ensure the quality of the whole-class discussion, selecting and ordering student presentations in the most convenient order for the lesson goals (Fujii, 2018). Some research suggests that when the context allows prospective teachers themselves to lead it, prior preparation supports the questioning they ask their students, accessing their difficulties more easily (Leavy & Hourigan, 2016; Ni Shuilleabhain & Bjuland, 2019) and monitoring their activity without necessarily reducing the degree of challenge of the task (Martins et al., 2023).

METHODOLOGY

We follow a qualitative (Bogdan & Biklen, 2007) and interpretative approach to participant observation based on the understanding and interpretation of the reality investigated (Erickson, 1986). We counted on the voluntary participation of two prospective teachers, Beatriz and Diana (fictitious names), at a Portuguese higher education institution. The lesson study also involved the teacher educator from the higher education institution, the cooperating teacher from the school where the prospective teachers were doing their practicum, and the researcher (first author). We chose these prospective teachers because they were, at the time, the only ones the teacher educator and collaborator in this study accompanied that semester.

The lesson study took nine sessions, prepared and led by the researcher and the teacher educator. These were integrated into the activities of the Supervised Teaching Practice subject (practicum in a 2nd-grade class, 7/8 years old students). The supervisor selected the mathematical topic, *Sequences and Regularities*. The structure of the sessions included four stages, adapted from the model proposed by Fujii (2018), to which the dissemination of results was added: (i) Study of mathematical and didactic questions [S1-2]; (ii) Preparation of research classes [S3-6]; (iii) Leading research classes and post-class reflection [S5-7]; (iv) Reflection [S8]; (v) Dissemination [S9]. Beatriz led the first research lesson, and the second was

led by Diana. The cooperating teacher, the supervisor professor, and the researcher participated in all sessions, supporting prospective teachers in the preparation, observation, and reflection on the research lessons.

We collected the data from participant observation, field notes, audio and video recordings, documents, and semi-structured interviews. We recorded the sessions on audio and video to complement the written documents and have a detailed transcription of the events. We collected the documents related to the plans for the two research lessons, including the proposed tasks and the students' written productions. We also collected the reflections on the research lessons written by the two prospective teachers [RLx Reflection] and the supervised teaching practice reports they produced at the end of their initial teacher education course [STP Report]. The researcher conducted a final semi-structured interview with each prospective teacher to understand their viewpoints about participating in the lesson study.

The data were transcribed and initially coded by the first author. Five categories were used, which were proposed by the classroom discourse analysis model by Hufferd-Ackles et al. (2014): *Teachers' role* [TR]; *Questioning* [Q]; *Explaining mathematical thinking* [EMT]; *Mathematical representations* [MR]; It is *Building student responsibility within the community* [BSRC]. When presenting the results, the sessions were identified as [Sx] and the *research lessons* as [RL1] and [RL2]. The quality of data analysis was ensured by several conversations between the two authors, who systematically debated the results and data coding. This study guaranteed participants' and institutions' anonymity and confidentiality, and participation was voluntary and informed.

RESULTS

Lesson preparation

Discussing ideas regarding communication in the classroom took a significant part of the sessions. However, S3, which was aimed at building tasks for RL1, was especially productive:

> Teacher educator: *Don't I have to think about anything else when planning and preparing the task?*
>
> Beatriz: *In the way I present the task… Whether I distribute it and tell them to read it; whether they read together, whether I explain and they do it? Whether I walk around, whether I*

> *don't walk around. Whether I know which children have the most difficulty interpreting the task, whether I do more individualised work or not, the type of questioning. What do I need to give them, without giving them everything* [TR] [Q]?
>
> Diana: *Whether we can ask them questions to get answers or we can't say anything. And let them interpret and do it* [TR][Q]. [S3]

Starting from statements proposed by the prospective teachers and keeping in mind the different contributions of the remaining participants, we constructed the first draft of two tasks. The participants solved and shared their solution strategies, giving rise to a moment of comparing answers. The refinement of the statement took place simultaneously with the anticipation of answers and likely teacher-to-student questioning [Q], considering the adequacy of mathematical representations for each context [MR]:

> Researcher: *What types of answers would you expect from students here?*
>
> Beatriz: *"What does this mean?" and we would have to say "the number of the line where it appears"* [Q].
>
> Teacher educator: *At their production level, it would be "they are in the positions I have, if we count by 3, starting from 3". But they would most likely not put "from 3"*. [S3]

In preparing Task 1 of RL1, the adequacy of manipulative material resources was discussed to provide a limited number of shapes. In this way, the materials supported students in the construction but did not allow them to represent all the 15 terms requested in the question [MR]:

> Diana: *First, large group work with manipulative materials would be better* [TR].
>
> Teacher educator: *Being careful not to have 100 [shapes]. Maybe not even 15*.
>
> Diana: *Yes, having 10.*
>
> Beatriz: *5 triangles and 5 squares*. [S3]

The prospective teachers showed that they knew classroom communication theories, as we see in Diana's intervention. However, the need for both to see them illustrated with practical examples became evident

throughout the session. The teacher educator used this idea to extend it to the preparation of the whole-class discussion, and Diana added to the idea, highlighting the importance of monitoring students' autonomous work:

> Diana*: In the last class, in Didactics, we said that... if we want someone to communicate the result, for example, we ask a child who does not have a solution identical to the one I initially asked for, even because we can contrast and debate of ideas ...* [EMT] *This part of communication is very important* [BSRC].
>
> Teacher educator: *When Beatriz asked those questions, how do I present the task? What is my role in monitoring student work? Which productions will I select to discuss? Will I involve all students, or will I only involve some students? Who do I choose first, who one do I choose later? These are all aspects to think about when I am preparing the discussion.*
>
> Diana: *This also depends on the teacher's attitude [TR]. If you don't go through the room and understand the dynamics of each child's solution* [EMT]*, you won't be able to play this role of mediator, of questioning and knowing what to question* [TR]. [S3]

Later, Diana sought to discuss good practices regarding monitoring students' autonomous work and the most productive way to organise this moment of the lesson:

> Diana: *I imagine putting it into practice; if a child told us they didn't understand the question, how can we help [TR]? What questions can we ask so that they can get there?... I'm referring to this one here at* [term of order] 100 [Q].
>
> Teacher educator: *Here, I can ask questions: "In what position are the triangles, in what order?", "And the squares?"; "Does this help us discover some that are on orders further down the line?"* [S3]

After S3, the prospective teachers were responsible for mobilising the ideas discussed in the sessions to construct the RL1 plan, privileging the structure of the exploratory lesson: task presentation, students' autonomous work, whole-class discussion, and summing up, and systematising the ideas mobilised in the first three sessions: anticipating student responses (strategies

and mathematical representations and possible difficulties); strategic teacher actions (including questioning); actions related to leading the discussion and summing up; dynamics of participation and time management.

Mathematical discourse in RL1

Beatriz presented the task (Figure 1) by giving students a set of squares and triangles in cardboard and reproducing the sequence on the board [MR]. She dominated the conversation at the front of the room [TR], leaving no room for students to ask questions [Q].

Figure 1

Task 1(translated from original). [RL1]

a) What will be the next shape?
b) Which group of elements is repeated?
c) What will the 12th element be?
d) Is the 15th element a triangle or a square? Why?
e) What will the 100th element be? Why?

During the autonomous work, Beatriz asked an exploratory question "Miguel, how would we keep repeating if we had a very large sheet?" [Q]. His partner answered and tried to exchange ideas with nearby classmates, sparking students' conversation. The prospective teacher guided this exchange of ideas only on the periphery [TR]. Diogo's motivation to explain his conjecture to his colleagues and his colleagues' motivation to discuss it with him highlights the students' high responsibility in the discursive community [BSRC].

During the solution of question d), as anticipated, a group of students found it difficult to find term 15, as they needed the necessary pictures to construct the first 15 terms. Beatriz encouraged the group to resort to other representation strategies [MR]:

Gaspar: *We all got confused. We have one square less.*

Leonor: *It didn't work out well for us. We don't have 15 pieces.*

Beatriz: *You only have 13 pictures here, and they ask you for the 15th. You have to come up with a strategy.*

Gaspar: *We have a square and a triangle, then a square and a triangle.*

[Gaspar concluded, counting with his fingers.]

Beatriz [addressing Leonor, who seemed not to understand]: *You don't have to use the pieces; you can draw.* [RL1]

The students constructed several mathematical representations and used them to justify their generalisation strategies; however, they showed a low commitment to writing their conclusions. As foreseen in the plan, the prospective teacher took notes regarding students' solutions to organise the whole-class discussion.

Beatriz began the discussion by requesting the collaboration of some students, taking as a starting point the solutions that she selected and ordered [EMT]. To discuss Task 2 (Figure 2), she asked Gustavo to collaborate.

Figure 2

Task 2 (translated from original). [RL1]

a) Complete the missing terms in the sequence.
b) Which group of elements is repeated?
c) What will the 13th element be?
d) In what positions do the circles appear?
e) In 19 elements, how many squares are there?
f) What will the 30th element be?

Beatriz: *Some doubts arose, and I prefer to discuss them with you... so that we can all understand through each other's strategies. Gustavo, what did you do?*

Gustavo*: I drew it.*

[Gustavo addresses the painting and completes the sequence previously constructed by Beatriz.]

Beatriz: *So, what did you understand from the pictures to draw what you did?*

Gustavo: *I saw that it is square, square, circle.*

Beatriz: *Did you understand what Gustavo said? He realized that the sequence is…*

Several students: *Square, square, circle.* [RL1]

In the following questions, Beatriz requested several students to collaborate, organising the discussion in an increasing sense of formalism in the strategies used. She started by calling a student who used active representation to support the counting. She asked several exploratory questions, favouring communication between students [Q] and discussing with them the disadvantages of the mathematical representation used [MR]:

Beatriz: *Ismael, what will be the 13th element?*

Ismael: *It's a square.*

Beatriz: *Why? How did you do it?*

Ismael: *I counted.*

Beatriz: *Come here and explain.*

[Ismael uses his fingers to count, matching each finger to a picture until he reaches the 13th term.]

Beatriz: *This strategy works, but you can be mistaken. We only have ten fingers [smiles]. Ismael and Gonçalo used their fingers. They counted squares... and discovered that the 13th.*

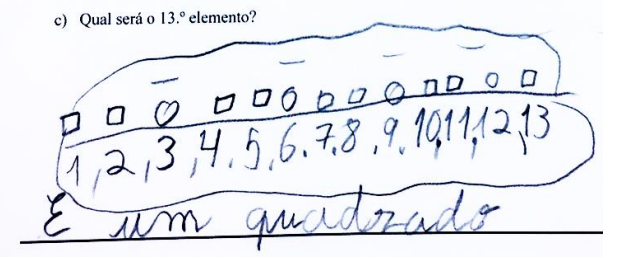
Ismael: *It's a square.*

[Beatriz imitates the gestures used by Ismael, highlighting his strategy.] [RL1]

During the students' autonomous work, Beatriz became aware of Diogo's representation strategies. He created a drawing to describe his mathematical thinking [MR] (Figure 3).

Figure 3

Diogo's solution. [RL1]



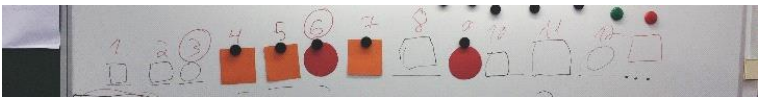
This appropriation allowed him to request student participation in the whole-class discussion. He completed the representation on the board, allowing his classmates to follow his thinking. In this way, the student favoured classroom discourse regarding mathematical representations [MR]. Realising that some students were not following the discussion, Beatriz reinforced Diogo's reasoning [EMT], calling on the rest of the students to actively participate [BSRC]. The discussion continued with the sharing of strategies used by other students, prompted by the prospective teacher's question [Q]:

> Beatriz: *Question d), this also raised some doubts, didn't it, Gaspar and Leonor? After building the sequence, how did you discover the positions in which the circles appeared?*
>
> Gaspar: *In 3.*
>
> Beatriz: *In the third. And the second circle?*
>
> Gaspar and Leonor: *Sixth.*
>
> Beatriz: *And the third circle, in which position does it appear, Maria?*
>
> Maria: *Ninth.*
>
> Beatriz: *And the fourth circle?*
>
> Several students together: *Twelfth.*
>
> Beatriz: *Very good. What does this mean?*
>
> Leonor: *You can do it by threes. Every three.*

Beatriz: *Yes, Leonor discovered that the circles appeared every three positions. So, the first appears in position three and the second in position six.* [Beatriz points out, on the board (Figure 4), orders three and six.] [RL1]

Figure 4:

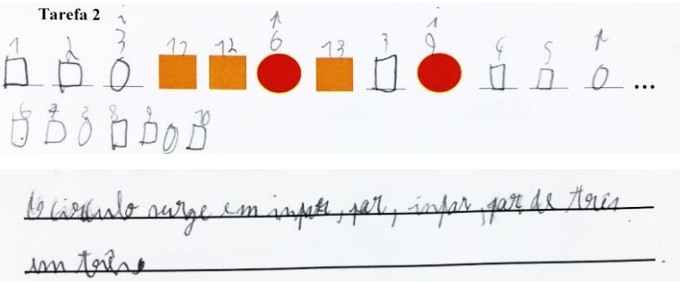
The board during whole-class discussion. [RL1]



Lucas actively listened to the answers and sought to contribute with his strategy, which was similar to Task 1, using the concept of parity [BSRC]. Beatriz let the student share his solution with the other students (Figure 5) and discussed it, proposing some corrections.

Figure 5

Lucas's solution. [RL1]



Beatriz: *But before you say that the circles appear in odd or even positions, you have to say where it starts.*

Diogo: *From 3.*

Lucas: *Ah, every 3, odd, even, odd, even, starting from 3.*

Beatriz: *Your reasoning complements each other.*

[Leonor interrupts Beatriz]

Leonor: *I wrote it another way.*

Beatriz*: In what way?*

Leonor: *"The circles appear every 3".*

Beatriz: *But that's not entirely correct, as in Diogo's solution. They appear every 3 but starting where?*

Leonor: *From 3.*

Beatriz: *Did you understand, Leonor? Did you realise that it is important to say every 3, starting from 3?... Did everyone understand?* [RL1]

Beatriz highlighted the students' thoughts, and they offered to show them [EMT]. By actively listening to other students, they contributed to giving meaning to the discussed solution, creating responsibility for the classroom culture [BSRC]. Beatriz's insistence that students identify the order of the first circle in the sequence was one of the aspects considered in the lesson plan (Figure 6).

Figure 6

Excerpt from plan (translated from original). [RL1]

If it is identified that the circles appear every 3, the teacher can ask the following questions:
- It is by 3s, starting from where?
- In what positions? In what position the 1st circle is? And the 3rd? And the 6th?
This question is important to help students solve question f).

In the discussion of question e), several students expressed interest in responding. However, Beatriz asked the student Gonçalo since, during independent work, he used the strategy of representation and counting. When reproducing the sequence on the board, Gonçalo, by mistake, drew the wrong number of pictures, a situation that he would later correct. To support the counting, Beatriz asked exploratory questions to lead to a mathematical representation (Figure 7) containing the caption of the drawing [MR]:

Beatriz: *Did you draw the pictures until you reached the 19th one?*

Gonçalo: *Yes.*

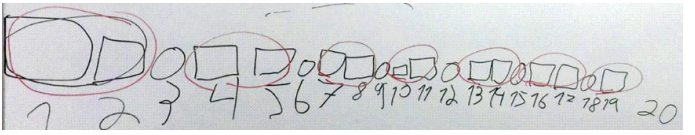
Beatriz: *What can you do there to help you?*

Gonçalo: *The numbers.*

Beatriz: *Circle the squares so you don't get lost.* [RL1]

Figure 7

Gonçalo's participation in the whole-class discussion. [RL1]



To answer question f), Beatriz used the answer to the previous question to justify that, for distant orders, representation and counting might not be efficient [MR]. Thus, she questioned other solution methods, increasingly more formal, that led to the idea of generalisation, using multiples of 3, requesting Francisco's participation:

Beatriz: *Did anyone do it differently?*

Francisco: *I can do it without anything.*

Beatriz: *Come here, Francisco.*

[Francisco gets up and states the sequence of multiples from 3 to 30 while Beatriz writes the sequence on the board.] [RL1]

The lesson ended with Beatriz validating Francisco [TR]'s solution:

Beatriz: [Francisco] *counted by threes mentally and discovered that the 30th picture was a circle because he knew, from the other question, that circles appeared every three positions. Very well. Good job. I am very happy. Now let's tidy up.* [RL1]

Beatriz's perceptions about leading the lesson

In her written reflection on RL1, Beatriz reflected on the *mathematical representations*, focusing on the learning opportunity provided to students through the appropriate use of resources: "The amount [of resources I distributed]… was intentional… did not allow them to discover another next term, which required students to resort to different strategies such as pictorial representation" [Beatriz, Reflection RL1].

Beatriz mentioned two essential purposes regarding her attention to students' autonomous work: (1) *monitor* students' work: "it was up to me to help them with their questions without giving them the answer" and (2) *select and sequence students' solutions*: "I had the opportunity to move around the room, observe, and absorb everything that the pairs were doing and select the productions that revealed different solution strategies to enrich the discussion and synthesis of the task" [Beatriz, Reflection RL1].

In her STP report, Beatriz used an episode from her participation in the lesson study to highlight how vital *questioning* is in student learning:

> *The teacher's questioning... when reflected and considered, can result in moments of pedagogical dialogue between student(s) and teacher or even between student(s) themselves, where ideas, conceptions and reasoning are explored, allowing them to take ownership of interventions from each other. Most students identified that the circles appeared every three. However, students needed to understand that, in fact, the circles appeared every three positions, but from where to start? In the third position.* [Beatriz, STP Report]

Regarding the whole-class discussion, Beatriz highlighted the benefits of sharing ideas and how it encouraged *students to explain their mathematical thinking*, also exposing her perspective on the *teacher's role*:

> Beatriz: *The moment of discussion was so rich, [the students] understood and appropriated each other's reasoning and they transformed the discussion into something that I really like: a dialogue. A pedagogical back-and-forth between me and them and between student and student. I actually like it when I'm in a classroom and they try to explain it to each other because they take ownership of each other's reasoning. I just guided them.* [S8]

Regarding the opportunity that discussion generates with *mathematical representations*, Beatriz stated: "In some cases, it was only during the discussion that it was possible to access representations and oral statements, which illustrated and documented the reasoning carried out [Beatriz, STP Report]. This realisation made her reflect on *the teacher's role* in student's motivation to justify their students' thinking: "Some students could not justify their reasoning on the answer sheets… which leads me to reflect on my action… in encouraging students to implement, on paper, their ideas, their reasoning, and demonstrate what they did. [Beatriz, Reflection RL1]

Regarding the preparation sessions, Beatriz mentioned how valuable it is to *anticipate diversified strategies* when preparing to lead the lesson:

> Beatriz: *Anticipating strategies, foreseeing different forms of solution and, given these forms of solution and strategies, predicting my performance not only gave me confidence but also gave me another appropriation of the content... a student gives an answer that I am not expecting, and as an inexperienced teacher, I tend not to block; in my head, I start to reason faster because it was something I anticipated.* [S8]

Constructing the lesson plan is also another aspect that Beatriz focuses on: "Planning helped a lot… it is a good basis for us to focus on what we want or don't want and the learning we want students to develop" [S8]. However, Beatriz refers to the difficulty in creating such detailed lesson plans: "The level of description we did is not something we often do, due to time constraints... shortly, it is not feasible, however, if you do it, it is good". [S8]

Mathematical Discourse in RL2

Diana built the sequence on the board (Figure 8), using cardboard pictures and asking exploratory questions to encourage students to communicate [Q]. He organised the students so that they were involved in solving the task, encouraging them to share their mathematical ideas [TR].

> Diana: *What do you notice in these three pictures?*
>
> Gonçalo: *Squares of various colours.*
>
> Lucas: *In one, there is an orange square and two blue squares, and [in another], there are four blue squares and*

one orange. It's... [Diana interrupts so that the student does not guess.]

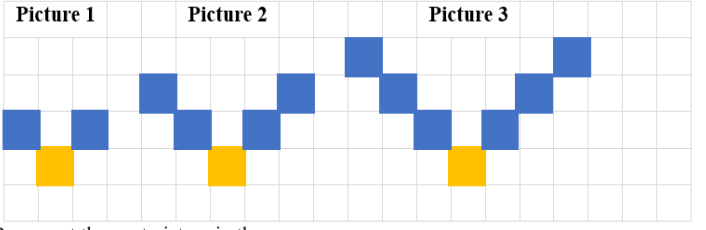
Diogo: *Oh, I know...*

Lucas: *I do, too.*

Gaspar: *It's ascending.* [RL2]

Figure 8

Task (translated from original). [RL2]



a) Represent the next picture in the sequence.
b) How many blue squares will Picture 6 have? And how many orange squares? How many squares will Picture 6 have in total?
c) Can you find out how many blue and orange squares Picture 15 will have? Explain your reasoning.
e) Explain how we can know the number of blue and orange squares needed to construct any picture in the sequence.

Several groups requested Diana's support a few minutes after starting autonomous work, which proved challenging. She walked around the different groups, guiding students through questions or suggesting the use of other representations that would support them. In her approach, she tended to resort to a question that proposed an alternative to the students' strategy, not ensuring that they went on with their mathematical thinking. For example, Gaspar and Leonor chose to represent terms of order 15 (Figures 9 and 10), and Diana advised them not to represent terms of such distant orders. The students did not accept this suggestion and did not explore other strategies [EMT]. When Diana walked away, Gaspar insisted on his strategy, reaching a solution he shared with Leonor, using his colleague's representation to support his explanation:

Diana: *I think that's a really big picture. Let's think of another way, how could we do it?*

Gaspar: *10 + 5.*

Diana: *10 + 5? Think about it.*

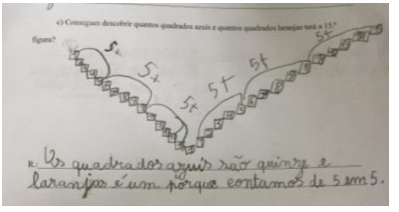
Gaspar: *By twos.*

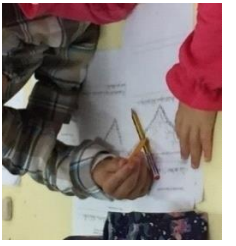
Diana*: So, think by twos...*

Gaspar: *By fives.* [Gaspar uses his fingers to count.] *5, 10, 15.* [RL2]

Figures 9 (left) and 10 (right)

Gaspar and Leonor during autonomous work. [RL2]



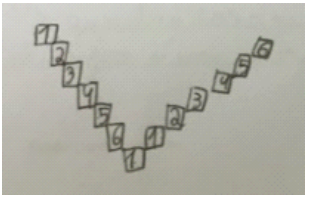


During autonomous work, the students constructed diversified mathematical representations and used them to justify their generalisation strategies. However, there was a noticeable lag in the students' performance in solving the task. Diana struggled to support all the requests, and students' independent work continued with the lesson plan. As a result, Diana had difficulty *selecting* and *sequencing* students' answers.

Diana requested the students' participation to lead the whole-class discussion, starting with Leonor's participation. Despite accompanying the student's group, she seemed unfamiliar with her mathematical thinking, or the mathematical representations used. Leonor used the sequence initially constructed on the board. However, she showed to struggle to explain her thinking and did not have the opportunity to share with her colleagues the mathematical representation (Figure 11) that she had created independently [MR].

Figure 11

Leonor's written production. [RL2]



The following dialogue shows that Leonor is only considered an "arm". The remaining students actively followed the discussion, disagreeing with some of their classmate's answers [BSRC] and actively participating, completing each other's reasoning [EMT]:

> Diana: *You drew and discovered how many blue squares there were?*
>
> [Leonor uses the sequence in the picture to answer the question]
>
> Leonor: *Four.*
>
> [Several students disagree, and Leonor tries to correct it.]
>
> Leonor: *There were six.*
>
> Several students: *Six?*
>
> Diana: *In total, how many squares do you have?*
>
> Leonor: *In total, 13.*
>
> Diana: And the blue ones?
>
> Leonor: *Of the blue ones, six.*
>
> Diana: *Count them.*
>
> [Leonor counts, pointing to each of the squares, answering six.]
>
> Diana: *And on the other side?*
>
> [Leonor counts again]

Leonor: *12.*

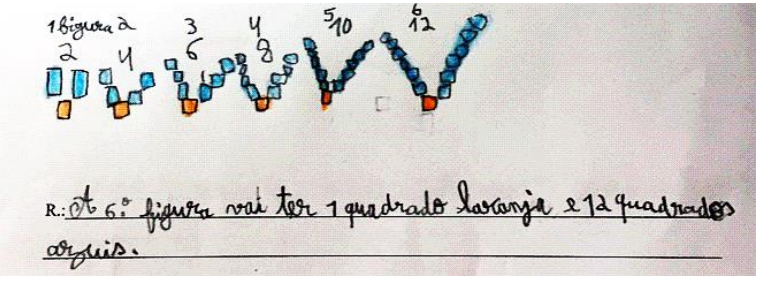
Afonso: *And with the orange one, there are 13.*

Diana: *There are 13 squares, aren't they?* [RL2]

Francisco sought to participate, presenting a solution process that used the double (Figure 12), which, from his perspective, facilitated the solution [BSRC]. Even though Francisco's intervention had been spontaneous, Diana agreed:

Figure 12

Francisco's written production. [RL2]



Francisco: *There is an easier way.*

Diana*: Is there an easier way? Come here and show us.*

Francisco: *So, if we add 6 and 6, we get 12.*

Diana: *How did you do it?*

Francisco: *I added 6 and 6.*

Diana: *And then, did you add one more? Was it 13? OK, thank you. Has anyone else done it differently? Lucas? Come here and explain how you did it.* [RL2]

Francisco's intervention was very brief, and Diana completed the student's reasoning without asking for his collaboration or asking him to share his representation [MR]. Neither did she use his mathematical thinking to expand the discussion to other students in the class [EMT], opting to ask

another student to collaborate. The recursive generalisation strategy Lucas presented, although correct, proved to be less effective than Francisco's:

> Lucas: *We did it from the 4th picture, we counted, and it came to 9. From 1 to 2, two changed; from 2 to 3, two squares changed… It was 13: 9 plus 2, 11 and 11 plus 2, 13.*
>
> Diana: *Ok.* [RL2]

Once again, Diana did not explore the mathematical thinking of the student [EMT], asking for Gaspar's collaboration, who also presented a recursive strategy with an integer object for terms whose order was a multiple of 5. Gaspar relied on the representation previously constructed by Diana in the painting to justify his reasoning (Figure 13):

> Gaspar: *Plus 5, plus 5, 3 times 5, in a straight line like this…* [Gestures to suggest extension.] [RL2]

Figure 13

Gaspar, during whole-class discussion. [RL2]

Diana tried to question Gaspar, but Diogo intervened to present his solution, in which he used a global strategy. Gaspar's reasoning ended up not being analysed, and the whole-class discussion ended with interventions from Diogo and Francisco:

> Diana: *But there is still another way of thinking; say it, Diogo.*
>
> Diogo: *I thought, 15 + 15 = 30.*
>
> Diana: *Then, tell us what you thought.*
>
> [Diogo hesitates, saying he resolved it together with Francisco.]

Diana: *Francisco, help your mate.*

Francisco: *We did 15 + 15.*

Diana: *But why is it 15 and not 20?*

Francisco: *Because it's the picture, each side must be the same. 15 + 15 = 30*

Diana: *And what is this regarding the squares?*

Francisco: *Blue. Then, we added another one from orange, and it made 31.* [RL2]

Diana sought to systematise the solution strategy that considered twice as many blue squares in relation to the order of the picture, using addition. To do this, she organised the data by completing a table she had previously constructed on cardboard. This moment of synthesis happened 2 minutes before the lesson ended. Diana asked for oral collaboration from two students who had already presented their solutions.

Although the students constructed diverse mathematical representations during independent work, the prospective teacher did not highlight these during the whole-class discussion [MR]. On the board, from the beginning to the end of the discussion, there was only the representation initially constructed by Diana, which supported the students to justify their generalisation strategies.

Diana's perceptions about leading the lesson

Diana's perceptions highlight her challenges regarding her *teacher's role* when leading the class. This highlights the *monitoring* of students' work, which would condition the management of the whole-class discussion:

> Diana: *In my class, I felt a bit unmotivated... some children understood very well, but some didn't. As much as I gave feedback, I couldn't give everyone the time they needed... I like that the children come with me, walking towards the same goal, and following me. I need to feel like the whole-class understands, or we have to step back at some point... a group of students couldn't move forward. I don't know if it was a problem of interpretation or if I didn't ask the right questions... indeed, there were others, and this emerged in the*

> *whole-class discussion... many of them couldn't understand, so I questioned my role.* [S8]

Diana added that the *questioning* was critical, highlighting her lack of professional experience leading it: "[I had] difficulty knowing what questions I could ask to move them forward... I think this happens because we have little experience. A more experienced teacher would know, and it wouldn't need to be in the lesson plan" [S8]. Another aspect highlighted was the management of available lesson time, which valued the resource she had prepared to systematise the solution strategy she intended to present at that time *[mathematical representation*]: "I think I should have given more time to the discussion when I used the table. The table was fundamental, but I should have had more time to discuss it" [S8]. Regarding what would change in the lesson preparation phase, the prospective teacher points out the growth sequence used in the task:

> Diana: *The plan was well made. I felt it was a very comfortable basis for putting it into practice. It was very clear... I think it would completely change our sequence because I immediately realised they faced problems. I felt very lost… but I was very confident when I went to class. I thought the group would enjoy the task.* [S8]

Diana highlighted, as positive aspects of her participation in the lesson study, the work of preparing lessons in a collaborative environment: "I highlight the way we planned, as well as the dialogue and participation of all teachers in the discussions and carrying out the tasks" [Diana, STP Report]. Regarding *anticipating students' answers*: "The meetings we had helped me a lot to gain an understanding… of children's possible difficulties, of solutions, of the children themselves" [Diana, Final Interview]. Despite the advantages, she points out the time limitation to continue the construction of detailed lesson plans: "The level of detail of the tasks planned for the study is not viable in a teacher's day-to-day life, as this is an exercise that they do on their own and it takes much time" [Diana, Final Interview].

DISCUSSION

Preparation and leading of research lessons

The *two research lessons* analysed here culminate a preparation process where various aspects related to regulating communication in the

classroom were considered (Ponte, 2012). Beatriz and Diana had similar opportunities and identical involvement and participation. Both showed theoretical knowledge and predisposition to address communication issues. The preparation sessions resulted in two plans with detailed instructions for leading research lessons, the product of a discussion that combined theoretical knowledge with practical examples aimed at the students in the class, and where the teacher educator and the cooperating teacher actively intervened. The lesson plans privileged the viewpoint of a teacher who encourages students to engage in conversation, leading them to *challenge* each other. The plans *anticipated students' responses* aimed at various perspectives of mathematical thinking to support prospective teachers in *monitoring*, *selecting* and *sequencing* the answers. The discussion about *mathematical representations* included the use of manipulative materials. However, the analysis of the two lessons highlights two discursive practices in the classroom with notable differences.

During RL1, Beatriz called on students to present their thinking and encouraged them to share their mathematical ideas. She used exploratory *questioning* that favoured communication between them. She encouraged students to use manipulative materials and discussed possibilities for using more effective *mathematical representations*, encouraging mathematical drawing. The way she *selected* and *sequenced* students' participation allowed her *to connect students' mathematical thinking* and guide it towards the planned focus of the lesson. The students shared the *mathematical representations* they constructed to support the explanation *of their mathematical thinking*. In several situations, individual *responsibility* in constructing classroom discourse was put in evidence, as students remained participatory, actively listening to each other to make meaningful contributions. In short, RL1 tended to develop in an environment in which the prospective teacher gave students space to share their mathematical thinking, and they assumed an important role in the *discourse community*, proving to be a way they could use to share and consolidate ideas and to meet different mathematical representations (Moschkovich, 2015).

In RL2, during autonomous work, the students made several requests to which Diana struggled to respond. Despite closely following all the groups, the prospective teacher used *questioning* directed toward correction, using her solution perspective, and ignoring students' mathematical thinking. This action may stem from her difficulty following some *explanations of students' mathematical thinking*, even if she accompanied them during their independent work. The sequence of answers presented in the whole-class

discussion did not follow a progressive order of sophistication, not favouring a *relationship* between students' mathematical ideas and lesson goals. Although the students had constructed drawings, during their independent work, all participants used Diana's mathematical representation previously constructed on the board to justify their solutions. Therefore, the students did not have the opportunity to share their own representations with their classmates. The lesson summing-up was also carried out with the table created by the prospective teacher, who asked for students' contributions in filling it out. In other words, in RL2, the mathematical discourse was centred on the teacher to the extent that, even though the students shared their ideas, they had few opportunities to argue about them or base themselves on *mathematical representations* they had built.

In RL1, Beatriz easily completed the five steps suggested by Stein et al. (2008), while in RL2, Diana found it challenging to *select, sequence, and connect* students' ideas. This limitation affected Diana's performance in leading the whole-class discussion and her ability to highlight students' mathematical ideas in her classroom discourse. These results support the argument that the nature of discussions facilitated in the classroom strongly determines communication dynamics (Dunning, 2023; Moschkovich, 2015; Stein et al., 2008).

Aspects the prospective teachers highlighted about classroom discourse

Beatriz underscored the *mathematical representations* as a relevant aspect of student learning to be considered in their teaching practice. She emphasised oral communication when she mentioned that only in the whole-class discussion she could access students' reasoning, which aligns with the standpoint that the whole-class discussion is a privileged moment for the teacher to access students' mathematical thinking (Fujii, 2018). She also highlighted the relevance of mutual *questioning*, which is fundamental to students' sharing ideas and appropriating reasoning. The prospective teacher also highlighted the importance of the *teacher's role* in *explaining students' mathematical thinking*: "On their own initiative, they transformed the discussion into something that I really like, a dialogue… I just guided them" [S8].

According to Beatriz, a decisive factor in leading the whole-class discussion was *monitoring, selecting, and sequencing students' solutions*,

which she put into practice during her independent work, which she believes allowed her to enrich the whole-class discussion. This note illustrates the argument of Stein et al. (2008) that *anticipating, monitoring, selecting, and sequencing* are practices that support the leading of whole-class discussion.

The perspective highlighted by Diana presents the *teacher's role* centred on oneself: "I like the children to come with me, that we all walk towards the same goal and that they follow me" [S8]. Her perception of the *mathematical representations* is consistent with this way of understanding classroom discourse. She cited the table she used in the final recap as an important resource, regretting not having had time to explore it in greater depth with the students. However, she never mentioned the importance of exploring the representations the students created. Regarding students' *explanation of mathematical thinking*, she highlighted monitoring as an opportunity to learn about their strategies despite admitting difficulty in monitoring students' independent work. She also added how difficult it was for her to lead *questioning* to support them in finding a solution. From her perspective, this fact demotivated her and made her role in leading the whole-class discussion difficult. In other words, as in other studies (Ghousseini, 2015; Martins et al., 2023), leading the whole-class discussion was very challenging for Diana. The prospective teacher pointed out two reasons to justify the constraints she encountered when leading RL2: the growth sequence, which served as the basis for the task, and the lack of experience in managing the *questioning*. She added, "A more experienced teacher would know, and it didn't need to be in the lesson plan" [S8]. Note that this statement differs from others in which Diana highlights the importance the lesson plan created for RL2 had for her as support for leading the class.

The prospective teachers highlighted aspects in line with other research as the main benefits of their participation in the lesson study: the anticipation of students' answers, which allowed them to delve deeper into content and the prior identification of their difficulties, enabling them a more immediate answer (Martins et al., 2023; Vieira et al., 2022); the supported construction of a lesson plan with instructions that direct the teacher's action towards the lesson goals and the students' difficulties (Leavy & Hourigan, 2016; Ni Shuilleabhain & Bjuland, 2019) and the collaborative nature of the work carried out in the lesson study, where they could compare their ideas with the ideas of more experienced teachers (Chen & Zhang, 2019; Norton et al., 2019; Ponte, 2017). The only limitation mentioned was the low expectation regarding this level of detail in future lesson plans.

CONCLUSION

The results lead us to conclude that the mathematical discourse generated in the first *research lesson* benefited student-teacher and student-student communication when compared to the second lesson. Beatriz had the opportunity to select and sequence the students' work, while Diana could not complete this step foreseen in the lesson plan. Regarding prospective teachers' perceptions, it was possible to verify two distinct positions regarding classroom discourse. Beatriz perceived herself as a teacher who highlights the students' mathematical thinking, thus seeking students to learn from sharing and questioning each other. Diana's perspective aligns with the self-centred teacher's, despite being receptive to the importance of questioning students to support them. The prospective teachers' perceptions of mathematical discourse revealed aspects of the *teacher's role, questioning, explaining mathematical thinking*, and *mathematical representations*. The prospective teachers also indicated that the main benefits of their participation in the lesson study were anticipating students' responses and having support to construct a lesson plan.

These results make us question the argument presented by several authors: that classroom discourse benefits from meticulous planning of mathematical discussions. In fact, in the first *research lesson*, like the studies by Leavy and Hourigan (2016) and Ni Shuilleabhain and Bjuland (2019), prior preparation clearly supported Beatriz in leading the lesson. However, in Diana's lesson, this was not evident, even more so if we consider that the investment made in preparing the second *research lesson* was identical to the first lesson. Thus, although detailed planning can benefit prospective teachers' leading of whole-class discussions, the results suggest that we must consider other aspects related to their profile and prior preparation.

This research shows the benefits of integrating lesson studies into initial teacher education (Ponte, 2017) and adds new data regarding its usefulness in studying mathematical classroom discourse. This process allowed prospective teachers to be supported in preparing their teaching practice, of which we highlight the anticipation of students' mathematical thinking and the construction of lesson plans. Regarding lesson leading, we highlight how lesson study practices are close to the practices that support mathematical classroom discourse, making this formative process an appropriate means to study the challenges prospective teachers face in leading communication in class.

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AUTHORSHIP CONTRIBUTION STATEMENT

RV outlined the first draft of the theoretical framework and methodology and collected the data. RV and JPP analysed the data and actively discussed the results, and reviewed and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The corresponding author, RV, will make the data supporting the results available upon adequately justified request.

REFERENCES

Bogdan, R. & Biklen, S. (2007). *Qualitative research for education: An introduction to theory and methods*. Pearson.

Chen, S. & Zhang, B. (2019). *Improving prospective teachers' lesson planning knowledge and skills through lesson study* (pp. 549–575). https://doi.org/10.1007/978-3-030-04031-4_27

Dunning, A. (2023). A framework for selecting strategies for whole-class discussions. *Journal of Mathematics Teacher Education*, *26*(4), 433–454. https://doi.org/10.1007/s10857-022-09536-5

Erickson, F. (1986). Qualitative Methods in Research on Teaching. In M. C. Wittrock (Ed.), *Handbook of Research on Teaching* (pp. 119–161). MacMillan.

Fujii, T. (2018). Lesson study and teaching mathematics through problem solving: The two wheels of a cart. In M. Quaresma, C. Winslow, S. Clivaz, J.P. Ponte, & A. Ni. Shuilleabháin (Eds.), *Mathematics lesson*

study around the world (pp. 1–21). https://doi.org/10.1007/978-3-319-75696-7_1

Ghousseini, H. (2015). Core practices and problems of practice in learning to lead classroom discussions. *The Elementary School Journal*, *115*(3), 334–357. https://doi.org/10.1086/680053

Gomes, P., Quaresma, M., & Ponte, J.P. (2023). Leading whole-class discussions: From participating in a lesson study to teaching practice. *International Journal for Lesson & Learning Studies*, *12*(2), 139–151. https://doi.org/10.1108/IJLLS-02-2022-0022

Hufferd-Ackles, K., Fuson, K.C., & Sherin, M. G. (2014). Describing levels and components of a math-talk learning community. In E.A. Silver & P.A. Kenney (Eds.), *More lessons learned from research, Volume 1: Useful and usable research related to core mathematical practices* (Vol. 1, pp. 125–134). NCTM.

Leavy, A.M. & Hourigan, M. (2016). Using lesson study to support knowledge development in initial teacher education: Insights from early number classrooms. *Teaching and Teacher Education*, *57*, 161–175. https://doi.org/10.1016/j.tate.2016.04.002

Martins, M., Ponte, J.P., & Mata-Pereira, J. (2023). Preparing, leading, and reflecting on whole-class discussions: How prospective mathematics teachers develop their knowledge during lesson study. *European Journal of Science and Mathematics Education*, *11*(1), 33–48. https://doi.org/10.30935/scimath/12432

Moschkovich, J.N. (2015). Scaffolding student participation in mathematical practices. *ZDM*, *47*(7), 1067–1078. https://doi.org/10.1007/s11858-015-0730-3

Murata, A., Siker, J., Kang, B., Baldinger, E.M., Kim, H.-J., Scott, M., & Lanouette, K. (2017). Math talk and student strategy trajectories: The case of two first grade classrooms. *Cognition and Instruction*, *35*(4), 290–316. https://doi.org/10.1080/07370008.2017.1362408

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to actions: Ensuring mathematical success for all*. NCTM.

Ni Shuilleabhain, A. & Bjuland, R. (2019). Incorporating lesson study in ITE: Organisational structures to support student teacher learning. *Journal*

of Education for Teaching, *45*(4), 434–445. https://doi.org/10.1080/02607476.2019.1639262

Norton, J., Helgevold, N., & Bjuland, R. (2019). The role of collaborative planning: How to use joint planning as a learning process in lesson study. In P. Wood, D.L.S. Larssen, N. Helgevold, & W. Cajkler (Eds.), *Lesson Study in initial teacher education: Principles and practices* (pp. 61–73). Emerald. https://doi.org/10.1108/978-1-78756-797-920191005

Ponte, J.P. (2012). Estudiando el conocimiento y el desarrollo profesional del profesorado de matemáticas. In N. Planas (Ed.), *Teoría, crítica y prática de la educación matemática* (pp. 83–98). Graó.

Ponte, J.P. (2017). Lesson studies in initial mathematics teacher education. *International Journal for Lesson and Learning Studies*, *6*(2), 169–181. https://doi.org/10.1108/IJLLS-08-2016-0021

Saylor, L.L. & Walton, J.B. (2018). Creating a math-talk learning community with preservice teachers. *School Science and Mathematics*, *118*(8), 348–357. https://doi.org/10.1111/ssm.12302

Smith, M., Bill, V., & Sherin, M.G. (2019). *The five practices in practice [elementary]: successfully orchestrating mathematics discussions in your elementary classroom*. Sage. https://books.google.pt/books?id=Pf-pDwAAQBAJ

Stein, M.K., Engle, R.A., Smith, M.S., & Hughes, E.K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*(4), 313–340. https://doi.org/10.1080/10986060802229675

Vieira, R., Ponte, J.P., & Mata-Pereira, J. (2022). Conhecimento matemático de futuros professores: Aprendizados realizados num estudo de aula. *Bolema: Boletim de Educação Matemática*, *36*(73), 822–843. https://doi.org/10.1590/1980-4415v36n73a10