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Reflections on Continued Training in Fractions: Influence on Teachers' Professional Development

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ABSTRACT

Background: The teaching of fractions is important due to its everyday applicability and relevance in different fields. However, its teaching is still challenging, and students often encounter difficulties. Given this reality, it is necessary to promote continuous training on this topic to enhance teachers' pedagogical practices. **Objectives**: The objective of the research was to analyze the activities developed in continued training for teachers in the area of Mathematics in the initial years of Elementary School, aiming to examine their contributions to the development of Fraction content. **Design**: The methodology adopted was qualitative, with an interpretative analysis of the collected materials. **Environment and participants**: The continued training for teachers was carried out remotely, with synchronous moments via Google Meetings and the Moodle platform. The research participants were 24 teachers from the initial years of Elementary School in the city of Taquara, in Rio Grande do Sul. **Data collection and analysis**: Data collection and analysis were carried out based on materials obtained during continued training for teachers, being analyzed in a descriptive and interpretative way. **Results**: The training offered to teachers allowed them to expand their knowledge on the proposed topic, enabling them to develop plans that enhance the teaching and learning process of Fractions content. **Conclusions**: The training meetings were considered important to enable teachers to improve their mathematical and methodological knowledge, allowing them to approach the concepts judiciously, promoting a comprehensive understanding. It is understood that continued training, such as the proposal, is fundamental to improving teaching and strengthening teaching practice.

Keywords: Mathematics Education; continuing teacher training; teaching Fractions; initial years of Elementary School.

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Reflexões sobre a formação continuada em Frações: influência no desenvolvimento profissional dos professores

RESUMO

Contexto: O ensino de Frações é importante devido à sua aplicabilidade cotidiana e em diversos campos. No entanto, seu ensino ainda é desafiador, e os alunos frequentemente apresentam dificuldades. Diante dessa realidade, torna-se necessário promover formações continuadas com esse tema para aprimorar as práticas pedagógicas dos professores. **Objetivos:** O objetivo da pesquisa foi analisar as atividades desenvolvidas em uma formação continuada na área de Matemática por parte de docentes dos anos iniciais do Ensino Fundamental, visando examinar suas contribuições para o desenvolvimento do conteúdo de Frações. **Design:** A metodologia adotada foi qualitativa, com análise interpretativa dos materiais coletados. **Ambiente e participantes:** A formação continuada foi realizada de forma remota, com momentos síncronos pelo *Google Meeting* e a plataforma *Moodle*. Os participantes da pesquisa foram 24 professoras dos anos iniciais do Ensino Fundamental do município de Taquara, no Rio Grande do Sul. **Coleta e análise de dados:** A coleta e análise de dados foram realizadas a partir dos materiais obtidos durante a formação continuada, sendo analisados de forma descritiva e interpretativa. **Resultados:** A formação oferecida aos docentes permitiu a ampliação de conhecimentos sobre a temática proposta, habilitando-os a elaborar planejamentos que potencializam o processo de ensino e aprendizagem do conteúdo de Frações. **Conclusões:** Os encontros formativos foram considerados importantes para possibilitar aos professores o aprimoramento dos conhecimentos matemáticos e metodológicos, permitindo-lhes abordar os conceitos de maneira criteriosa, promovendo uma compreensão abrangente. Entende-se que as formações continuadas, como a proposta, são fundamentais para aperfeiçoar o ensino e fortalecer a prática docente.

Palavras-chave: Educação Matemática; formação continuada de professores; ensino de Frações; anos iniciais do Ensino Fundamental.

INTRODUCTION

The teaching and learning of positive rational numbers in the early years of Elementary School present difficulties that encompass different representations, such as decimals, percentages, and fractions, as well as their meanings. Fractional numbers are applicable in everyday situations, such as making purchases and payments, in addition to being fundamental in various areas of knowledge, such as economics and mathematics, where they are related to concepts of ratio, proportion, and probability. Given the importance of this knowledge and the difficulties faced, it is necessary to carry out research on the teaching of fractions and promote continued training for teachers in the initial years, aiming to overcome these challenges and improve teaching practice in the area of Mathematics.

In this article, we seek to present the results of a master's degree investigation that aimed to understand: what are the contributions to teaching planning when teachers in the initial years of Elementary School participate in continued training in the area of Mathematics, specifically focused on the topic of Fractions?

The objective of the research was to analyze the activities developed in continuing education in the area of Mathematics by teachers in the initial years of Elementary School, aiming to examine their contributions to the development of Fractions content. This research is carried out through investigation and analysis of the impact of this training on the teaching planning of a group of teachers who work in the municipality of Taquara, in Rio Grande do Sul.

In the next sections, reflections are presented on the importance of continuing education and its role in developing teachers' professional knowledge, as well as the challenges faced and teachers' knowledge in the context of teaching Fractions.

CONTINUING TRAINING AND PROFESSIONAL KNOWLEDGE OF THE TEACHER

Teacher training has been the subject of analysis and discussion in recent studies, highlighting its complexity and scope, which go beyond the mere acquisition of specific knowledge. This theme involves several aspects that are fundamental to the constitution of the teacher's professional knowledge, transcending the scope of teaching (Imbernón, 2010; Ponte, 1994).

According to Nóvoa (1992), being involved in a training process implies the construction of a personal identity, which demands a significant investment, as such construction involves free and creative work on one's trajectory, seeking connections with professional dimensions. Therefore, training is a journey that requires critical reflection and collaboration, aiming to expand knowledge and practices. In this sense, it is recognized that continuing education plays a fundamental role in the teacher's professional development. The author also highlights the importance of innovating in training practices, providing teachers with interactions with pedagogical and scientific knowledge, to stimulate the improvement of teacher training.

According to Imbernón (2010), in the process of continuing teacher training, they must develop a teaching identity, recognizing themselves as active subjects of their training process, instead of being treated as mere passive recipients or manipulable objects in the hands of others. Given this, it is imperative to promote continuing education programs that offer opportunities for interaction between teachers, in addition to presenting concrete proposals that can be applied in the context of teaching practice.

Ponte (1992) adds that teacher training cannot be restricted to existing practices, but must provide moments in which teachers can contemplate new concepts, requiring a perspective open to new horizons, enabling the discussion of their ideas and experiences in teaching. The author highlights some fundamental elements to be considered in the training process: a) the comprehensive theoretical basis, with specific reference to the didactics of the discipline; b) the dynamics of the training process, involving group work and a healthy relationship between all participants, including those with training responsibilities; and c) activities, which promote interaction with teachers' practices and provide adequate opportunities for reflection.

However, training alone is not capable of promoting change in conceptions and practices, since its effectiveness strongly depends on the context in which it occurs. In this sense, in a training process, it is essential to take into account collective needs, different methodological strategies, and specific knowledge of the content to be taught. However, when conceiving training in this way, it is essential to incorporate moments of interaction and exchange of experiences, reflection, and the search for expanding knowledge relevant to the development of teaching practice. We agree with Nóvoa (1992) that it is insufficient to simply seek professional transformation; It is necessary to promote change in the environment in which we operate. It is understood that one cannot separate training from the production of knowledge, nor dissociate the professional context from the context that surrounds it, that is, schools cannot be transformed without the commitment of teachers, and they cannot change without a transformation of the institution.

Given the above, it is clear that the teacher's professional development must be closely linked to the school and its projects, as training occurs amid changes, driving innovation and improving educational practices. Therefore, it is through the transformation of professionals and the environment that a new meaning is attributed to teacher training, as well as the search for new knowledge in the school context.

Therefore, when preparing a training proposal, it is essential to consider the real demands and check whether they are aligned with the needs of the group involved. Therefore, it becomes important to develop a didactic proposal, in this case for teaching Fractions in the initial years of Elementary School, to address the concepts and procedures of the content using different methodologies (Simões, 2022).

According to Ponte and Oliveira (2002), theoretical contact with knowledge is not sufficient to guarantee effective professional training, since professional knowledge has a personal dimension intrinsically related to action and reflection. Therefore, the development of teachers' knowledge requires creative and diversified approaches, preferably close to situations encountered in practice.

For Ponte (1994, p. 10), "teachers' professional knowledge manifests itself mainly in action, especially in pedagogical practice, but also in other school and even extracurricular activities in which they are involved". Therefore, teachers must have their conceptions about education and their professional role, adopting an active stance to avoid settling into routines and practices that increasingly distance themselves from curricular guidelines and students' needs.

Teacher professional development occurs in two interconnected spheres. One of them concerns improving the knowledge and professional skills necessary to perform routine activities, such as solving problems in different areas. The other is related to the construction and consolidation of professional identity, which plays a significant role in the teacher's social identity (Ponte & Oliveira, 2002).

In this context, professional development is influenced by several conditions, including individual characteristics of the subject, educational context, and availability of resources, whether human or material (Ponte, 1994). In this sense, it is essential to acquire different types of knowledge, each corresponding to a specific social practice, such as academic knowledge, professional knowledge, and common sense knowledge (Ponte & Oliveira, 2002; Ponte, 1992).

Still according to the same authors, academic knowledge, for example, is related to the creation and validation of scientific, humanistic, and philosophical knowledge, seeking to understand and explain rigorously and systematically, often with a mathematical basis. On the other hand, common sense knowledge meets the basic needs of survival and satisfaction, not being so concerned with logical coherence, being formed by the internalization of dominant social representations and individual experience, allowing a certain margin of maneuver. In turn, professional knowledge differs from the previous ones, as it is related to socially recognized activities, carried out in specific domains of practice. This knowledge is essential for the performance of professional activity and covers both routine processes and the resolution of concrete problems in specific social contexts, presenting challenges that are different from those found in the academic or everyday environment.

Ponte (1994) highlights that, in addition to mastering the content that is taught, pedagogical training is also extremely important. However, it is important to highlight that different types of knowledge cannot be taught directly in institutions, as they are developed through individual practices and the professional experience of teachers.

TEACHING FRACTIONS AND TEACHERS' KNOWLEDGE

In everyday life, numbers are used for various purposes, such as quantification, location, ordering, measurement, and identification. However, in addition to counting, there is a need to represent parts, giving rise to fractions, an idea that dates back to ancient people. With the development of science in response to social and economic demands, the concepts of fractions have expanded and enriched over time (Parcianello, 2014).

Yet, according to the author, it is crucial to make teachers aware of the importance of using appropriate and meaningful methodologies in teaching Fractions in the first years of Elementary School. This approach is based on the understanding that it is necessary to understand the teaching of Mathematics to achieve the desired objectives. Therefore, the teacher must play his role in a critical, planned, and intentional way, to enable students to develop critical thinking and social and cultural participation consciously and actively.

It must be taken into account that the difficulties presented by students when developing a certain concept may not be the same in another. Because certain situations presented to the student may not make sense. It is necessary for them to be presented, or revisited through different situations and problems so that the concept can acquire meaning for the student. It is essential to recognize that the difficulties students encounter when understanding a given concept may vary in different contexts. Some situations presented to students may not make sense, initially seeming disconnected, making it necessary to approach them again through different situations and problems, so that the concept acquires meaning for them (Vergnaud, 1986; 1993).

To promote the teaching of Fractions effectively, the teacher must reflect and recognize the importance of this content, as well as its applicability in different contexts. To achieve this objective, the teacher must propose learning situations that encompass the concept of fractions, their reading, the different meanings associated with them, as well as the notions of equivalent fractions and operations involving fractions.

In this context, fractions have a specific way of reading, which differentiates them from other known numbers due to their representation. When reading fractions, it is essential to initially understand that the denominator assigns a name or type to the parts, while the numerator indicates the quantity of these parts. To read correctly, one must read "first the numerator and then the denominator," that is, first the number of parts of a given type, followed by the name or type of the parts (Parcianello, 2014, p. 6).

Magina and Campos (2008) emphasized that the process of learning the concept of fractions can be enhanced when this concept is approached in its five essential meanings: number, whole part, measure, quotient, and multiplicative operator. Furthermore, it is crucial to have clearly defined operational invariants when addressing each of these meanings. Parcianello (2014) presents seven meanings, namely: percentage, part-whole, measure, number, quotient, multiplicative operator, and ratio.

As pointed out by Magina and Campos (2008), teachers who teach in the early years of Elementary School generally adopt the predominant approach of part-whole situations when teaching Fractions. This approach involves the technique of double counting the parts. However, these practices lead students to develop conceptions about Fractions based on sensory perceptions, instead of exploring the logical-mathematical relationships that can be established.

Thus, from the division of flat geometric figures into equivalent parts, the teaching process begins, introducing a sequence of procedures. Furthermore, the concept of fractions must acquire meaning for the student, which can be achieved through a comprehensive approach, exploring situations that allow the student to move between different ways of constructing meanings. This will provide students with the acquisition of logic as a basis for building this concept, in addition to understanding that there are different operations related to the same mathematical symbol (Simões, 2022).

Silva (2005) highlights that fractions, as well as percentages and decimal numbers, are representations of rational numbers. The author also describes that the concept of a rational number is constructed from different interpretations, which include the notions of part, whole, measure, quotient, ratio, and operator.

This perspective emphasizes that Fractions are more than simple mathematical symbols; they have a conceptual meaning that also encompasses various interpretations. For example, a fraction can be seen as a part of a whole, as a measurement relative to a reference unit, as a quotient of two quantities, or as a ratio between two quantities. Furthermore, Fractions can also be used as operators in mathematical calculations.

Therefore, understanding fractions involves recognizing the variety of possible interpretations and understanding how these different perspectives relate to each other, contributing to the construction of the broader concept of rational numbers.

That way, it is essential to highlight the importance of understanding the different meanings of Fractions, as they serve as a guide for teaching practice in the classroom. However, in the literature, the same problem can be approached in several different ways. This occurs because, when dealing with problem-solving, each individual mobilizes their knowledge and strategies. These approaches can reveal different meanings and lead to different paths to achieve success in solving the proposed problem (Mocrosky *et al*., 2019).

In Table 1, a brief description of the different meanings is presented, according to Silva (2005), Magina and Campos (2008) and Parcianello (2014).

Simões (2022) highlights that, during training, the different meanings attributed to Fractions were addressed, highlighting the need to provide students with diverse situations. Thus, teachers were able to develop lesson plans including activities that allowed exploring these different aspects of fractions, stimulating a comprehensive conceptual understanding.

Table 1

Different meanings that Fractions can have. (Simões, 2022)

Meanings of Fractions	
Part-whole	This interpretation of fractions involves the idea of dividing a whole into equal parts, where each part represents a fraction of the whole. It is an approach that allows us to understand the proportional relationships between the parts and the whole.
Measure	In measurement situations, a fraction is used as a unit of measurement to quantify another quantity. This raises the question of how many times the unit of measurement is contained in the quantity being evaluated.
Quotient	The concept of quotient is related to the use of fractions as a representation of a division and the result obtained from that division. Fractions have two variables, the numerator and the denominator, which indicate the quantity to be divided and the divisor, respectively.
Ratio	From this perspective, fractions are considered as multipliers of the indicated quantity. They enlarge or reduce a quantity, depending on the numerical values of the numerator and denominator of the fraction.
Multiplicative operator	The ratio is an interpretation of fractions that expresses the relationship between two variables. Fractions are used to represent the proportion or relationship between these variables, providing a comparative measure between them.
Number	In addition to the previously mentioned interpretations, fractions are also considered numbers in their own right, regardless of a specific relationship or context. They have a numerical value and can be represented on a number line, contributing to the representation

and understanding of different quantities and values.

Part-Whole: We often hear the terminology "Fraction" used to describe subdivisions of something or parts of a whole. As a result, it is common for students to associate fractions only with their parts. However, it is important to highlight that there are fractions that are greater than 1 (fractions in which the numerator is greater than the denominator). There is a significant tendency to introduce the concept of Fractions through exploration of the meaning of whole parts, in which students are led to count the total number of parts and the parts being considered, often without fully understanding the true meaning of this new type of number being presented (Parcianello, 2014).

According to Magina and Campos (2008), even though teachers predominantly use part-whole situations as the main context in teaching Fractions, it is likely that, through their own experiences with fractions, they have developed an understanding in other contexts as well. The aforementioned authors also report that their research highlights difficulties related to the concept of fractions, both in teaching and learning. Therefore, there is often a strong tendency to approach the concept only through the meaning of part-whole, using the representation $\frac{a}{b}$, where a and b are natural numbers and $b \neq 0$, encouraging students to employ the double counting procedure (which involves counting the total number of parts and the highlighted parts). However, students often do not fully understand the meaning of this new type of number.

Measure: When carrying out length measurement activities, it is possible to highlight the limitation of natural numbers and the need to introduce "new numbers" for an adequate quantification of length. These tasks are intrinsically related to the conception of measurement, which, in turn, is directly linked to the nature of the quantity in question, measuring continuous quantities and counting discrete quantities. In this context, a certain unit is used as a reference to measure another (Silva, 2005; Parcianello, 2014).

Quotient: The fraction representation indicates a division and the result of that division, involving two variables, the numerator and the denominator (Parcianello, 2014). According to Magina and Campos (2008), problems involving the meaning of quotient can be used to help children understand the order of fractions through logical reasoning.

An example of this type of problem is dividing a cake, where the more divisions are made, the smaller the portion of each piece becomes. This inverse relationship between the divisor and the quotient can help students understand that the larger the denominator of the fraction, the smaller the part represented. Furthermore, when using the concept of quotient, it is possible to use reason to help students understand the invariance of equivalence of fractions. For example, if the same ratio between the number of children and the number of cakes is maintained, the corresponding fractions will be equivalent (Magina & Campos, 2008).

Ratio: As mentioned by Parcianello (2014), the Fraction represents the relationship between two variables. In this sense, the ratio can also be applied in situations where fractions are used to describe intensive quantities, as is the case with paint mixtures. For example, if two paint mixtures are made with the same ratio of blue paint to white paint, the resulting color will be the same and the fractions will be equivalent, even though the total amount of paint in each mixture is different (Magina & Campos, 2008).

Multiplicative operator: According to Magina and Campos (2008), the fraction has the meaning of a multiplicative operator. In this context, the fraction represents a scalar value applied to a quantity. For example, about a whole number, one can say that one got twelve bullets, while in the case of a fraction, one can say that one got $\frac{3}{4}$ of a set of bullets. The underlying idea is that the number works as a multiplier of the quantity, allowing us to assert that tquartos of $\frac{3}{4}$ of bullets from a pack of 16 units.

Number: According to Parcianello (2014, p. 9), the fraction is a number in itself, not requiring a specific relationship or context to be understood in a given situation.

METHODOLOGY

Research in Mathematics Education, especially about teacher training, has experienced notable growth in recent times, driven by the growing demand for pedagogical improvement and the search for knowledge. Given this context, the study consists of contributing to the advancement of the field of study, presenting an analysis of part of the activities developed in continuing education aimed at teachers in the initial years of Elementary School, carried out in the municipality of Taquara, RS, throughout the year 2021 (Simões, 2022).

The methodological option used in this research adopted a qualitative approach, based on the naturalistic perspective proposed by Gray (2012), which aims to understand the phenomena in their context of occurrence, through an interpretative analysis of the data, considering the theoretical contribution. The design of this study was based on the investigation of continued training aimed at teachers in the initial years of Elementary School.

The activities analyzed were developed during the training process, which took place in five meetings, in which the objects of knowledge and skills established in the National Common Curricular Base (BNCC) and in the Guiding Document of the Municipality of Taquara, RS (DOM), related teaching Fractions [1]. During this process, Epistemological Obstacles and difficulties in teaching Fractions, concepts and meanings of Fractions, and methodological resources for their teaching were also discussed. Furthermore, didactic sequences were developed that sought to address the aspects above.

In this article, specifically, the analyses and results of activities developed during training are presented, which are related to the different meanings of Fractions. The results of the proposed continuing education are presented in the following section.

RESULTS AND ANALYSIS OF CONTINUING TRAINING

The data presented in this section comes from the planning of a group of teachers who participated in Continuing Training involving Fractions. Table 2 presents the objects of knowledge, resources, and methodology proposed for the development of Fractions content in the 5th year of Elementary School.

[1] The research was submitted to the ethics committee for research involving human beings and received approval from the Ethics committee at Plataforma Brasil, with Certificate of Presentation for Ethical Assessment (CAAE) number 39805620.8.0000.5349 and opinion number 4.428.346.

Table 2

Example of Planning developed in training. (Simões, 2022, p. 111)

Planning description	
Thematic unit BNCC	Fractional representation of rational numbers
Year of Teaching activity hours	5th year 4 hours
BNCC knowledge object (content)	Fractional representation of rational numbers: recognition, meanings, reading, and representation on the number line.
BNCC skill (objective)	(EF05MA03) Identify and represent fractions (larger and smaller than unity), associating them with the result of a division or the idea of part of a whole, using the number line as a resource.
DOM Skills	(EF05MA03RS-1) Identify, represent, and translate orally or in writing, a fraction associated with the idea of a whole, with an understanding of the meaning of the numerator and denominator, in different contextualized situations.
Resources used	Poster with illustrations; Activity in a notebook.
Methodology	The planning was prepared thinking about the needs of the class, thinking about the BNCC's object of knowledge, the planning can be carried out through technological means, and it can also be applied in the classroom. The planning is organized into 6 moments: History of fractions, cooking with fractions, activity relating the fraction and its graphic representation, identifying fractions greater than unity, activity representing fractions greater than unity, and representation of fractions on the number line.
Assessment	The assessment can be carried out by observing students during the class and carrying out the proposed activities, checking whether the initial objectives of the class were achieved.
Didactic contract (previously established combinations)	The concept of division must be well established for the student to understand the concept of fractions. Students often do not understand the format presented, so the teacher must look for different teaching strategies to understand the

student better.

During the planning process, the group highlighted its concern with clearly presenting the object of knowledge and the skill to be addressed, as well as the methodology, assessment, and didactic contract, following the model previously provided in the previous training meeting. The objective was to ensure that the planning was structured in a way that was understandable and applicable to other professionals. In this sense, it is important to consider that teacher training is not just restricted to existing practices, but rather allows moments in which teachers can visualize new conceptions, covering a solid theoretical basis regarding the didactics that involve the discipline (Ponte, 1992).

Next, an introduction to the teaching of Fractions was presented through a methodological approach to History, as shown in Table 3.

Table 3

Example of activity developed in training. (Simões, 2022, p. 112)

Activities for 5th grade
1st Moment: Objective of the activity: Remember the history of fractions and their definition. Duration: 30 minutes. **History of fractions** The word fraction comes from the Latin *fractus*, which means party, and began to be used in ancient Egypt. Egyptian mathematicians delimited the lands, but during the rainy season, the river flooded many lands and consequently ended the demarcations. As a result, mathematicians decided to demarcate the lands with ropes to solve the flooding problem. However, they noticed that many terrains were not just made up of whole numbers, some of them measured part of a whole.

Vergnaud (1986; 1993) highlights that the difficulties faced by students when understanding a specific concept may vary depending on the context, and certain situations presented to students may have no meaning. Therefore, it is crucial to approach the concept through different situations, opening up a range of possibilities and going beyond a restricted set of circumstances to give it meaning. Given that, the historical approach can prove to be an effective method to achieve this goal. After introducing the topic, as illustrated in Figure 1, the suggestion for the second moment involves activities based on procedures, based on the use of a recipe.

Figure 1

Everyday situations. (Simões, 2022, p. 112-113)

2nd Moment:
Objective of the activity: Explore together the ideas of fractions in everyday situations.
Duration: 2 hours
Fractions are quotients between two whole numbers and are used to represent rational numbers.
The main idea of using fractions is to indicate quantities smaller than unity, although fractions can indicate numbers greater than 1, as is the case with whole numbers.
Cooking with fractions:
Marta wanted to make an Orange cake and looked for the recipe on the internet.

ORANGE CAKE
1 ½ cup (tea) of orange juice.
2 ¾ cup (tea) of wheat flour.
1 cup (tea) of butter.
1 ½ spoon (tea) of baking powder.
4 eggs
2 cups (tea) of sugar

-Fractions are used in many everyday situations, such as in recipes. In which everyday situations can we find fractions? Discuss with the class some examples: Hours, pizza, gas tank, music, money, tools (labor), and calculator.
- What it mean $1\frac{1}{2}$ teaspoon?
1 and a half spoons, or 1 spoon plus half a spoon.

- How do you find out what $\frac{3}{4}$ of a cup of flour is?

1 cup divided into 4 parts, only 3 parts will be used.

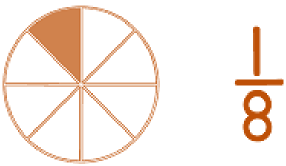
To continue, the proposal involves the contextualization of the topic, aiming to facilitate the understanding of the applicability of the concept in everyday situations and highlight the intrinsic importance of acquiring substantial knowledge in this field. Then, in Figure 2, the suggestion of dividing a cake into 8 equal parts is presented to determine the fractions of each slice. This approach exemplifies the concept of quotient since problems related to this concept can help children understand the ordering of fractions through logical reasoning. (Magina & Campos, 2008).

Figure 2

Everyday situations. (Simões, 2022, p. 113)

2nd Moment:

-After the cake is ready, it will be divided into 8 equal pieces. What fraction can we write that indicates each slice?



-200 grams represents $\frac{1}{3}$, $\frac{1}{4}$ or $\frac{1}{5}$?

To find the fraction that represents 200g. We must carry out a division.

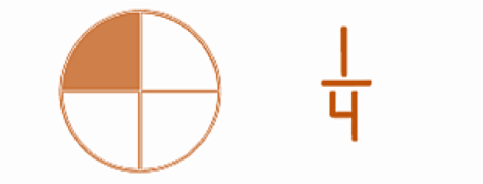
Where 1kg=1000g = which represents 1 integer.

So our division will be $\frac{1000}{200}$ = 5.

I am arriving at the conclusion that 200g = 5th part.
Thus representing the fraction $\frac{1}{5}$.
1kg=1000g= que representa 1 inteiro.
Então nossa divisão será: $\frac{1000}{200}$ = 5.
Chegando à conclusão que 200g = 5ª parte.
Representando assim a fração $\frac{1}{5}$.

Remembering:
Numerator: How many parts do we take of the whole?
Denominator: Indicates how many parts the whole was divided into.

Graphic representation:



When analyzing the activities presented in Figure 2, it is evident that there was a concern in selecting situations that connected with the student's daily lives. At the end of the activities, the fundamental meanings of the numerator, denominator, and graphic representation of fractions were revisited (Simões, 2022).

However, this type of activity can still be deepened by showing students that the more divisions are made, the smaller the part of each piece becomes. This inverse relationship between the divisor and the quotient can help students understand that the larger the denominator of the fraction, the smaller the part represented.

Based on the activity illustrated in Figure 3, additional activities are proposed that include different representations of fractional numbers, including representation as division of fractions and exploration of the visual part. Such activities are fundamentally relevant for introducing concepts, as well as improving writing and manipulating fractions. This comprehensive and diverse approach contributes to the effective learning of fractional numbers by students.

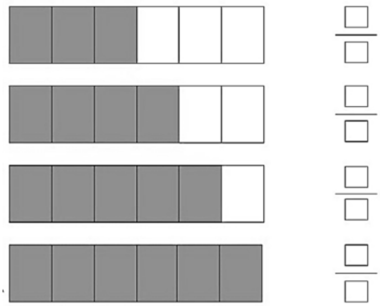
Figure 3

Fractional representations. (Simões, 2022, p. 114)

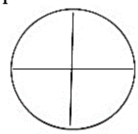
3rd Moment:
Objective of the activity: Put into practice the concepts previously worked on and fix the content. Duration: 20 minutes.
Activity 1:

ACTIVITY

Write the fraction that indicates the colored part in each figure below:



color $\frac{3}{4}$ of the picture below:



Through symbolic representations and writing, it is possible to relate the colored parts with fractional numbers that express specific quantities, as described by Parcianello (2014). In the context of fractions, their peculiar way of reading is distinguished from other known numbers due to their representation.

According to Magina and Campos (2008), the most commonly used approach when teaching Fractions in the early years of Elementary School is the part-whole approach, which employs the technique of double counting the parts. However, this approach can lead students to develop conceptions about Fractions based on sensory perceptions, neglecting the exploration of the underlying logical-mathematical relationships. That said, it is essential to present situations that involve other meanings that fractions can assume. In the following activities, it is observed that situations are presented that address fractions greater than unity, as exemplified in Figure 4.

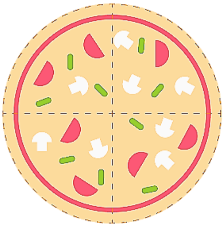
Figure 4

Fractions greater than one unit. (Simões, 2022, p. 114-115)

4th Moment:
Objective of the activity: Use fractions to indicate parts of a unit.
Duration: 1 hour 30 minutes

Fractions greater than unity
Let's look at an example:
João says he is very hungry and can eat $\frac{5}{4}$ of pizza. Do you know how to represent the amount of pizza that João could eat?

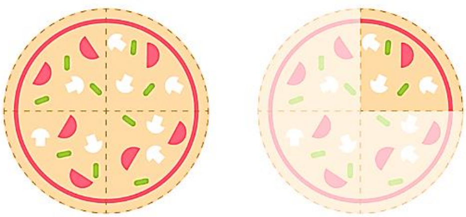
➢ First, we must divide the unit by the number of equal parts indicated in the denominator. In this case, we must divide the pizza into four.



➢ After that, we must take the parts indicated by the numerator, that is, five.
But there is a problem, as we divide the pizza into four we only have four parts, how can we solve this problem?

➢ Simple, we take another pizza and divide it in the same way as the first, this way we will get the piece we are missing. Note that the resulting fraction was greater than unity, this happened precisely because 5 is greater than 4.

➢ Now we can see the number of pieces of pizza that João will eat in the image below $\frac{5}{4}$:



The situations presented in Figure 4, involving fractions greater than unity, where the numerator is greater than the denominator, have the potential to deepen the understanding of the concept, because, as highlighted by Vergnaud (1993), it is these situations that give meaning to the mathematical concepts. However, it is important to highlight that this approach allows for the exploration of different meanings of fractions, including situations involving the quotient indicating a division.

Problems that use this meaning can be used to help children understand concepts of order invariance and equivalence of fractions through logical reasoning. For example, when further discussing, problems involving dividing a cake or pizza, in which the number of pieces increases as more divisions are made, leading to a reduction in the size of each piece, can help students realize the inverse relationship between the divisor and the quotient, highlighting that the larger the denominator of the fraction, the smaller the part represented (Magina & Campos, 2008). For the fifth moment, the activity returns to the written and graphic representation of fractional numbers (Table 4).

Table 4

Reading and graphical representation. (Simões, 2022, p. 116)

Atividades para o 5º ano
5th Moment: **Objective of the activity:** Put previously worked concepts and content fiction into practice. Duration: 20 minutes Activity 2: Write the numbers in full and graph each fraction: $\frac{10}{8}$: $\frac{3}{2}$: $\frac{9}{3}$: $\frac{6}{3}$:

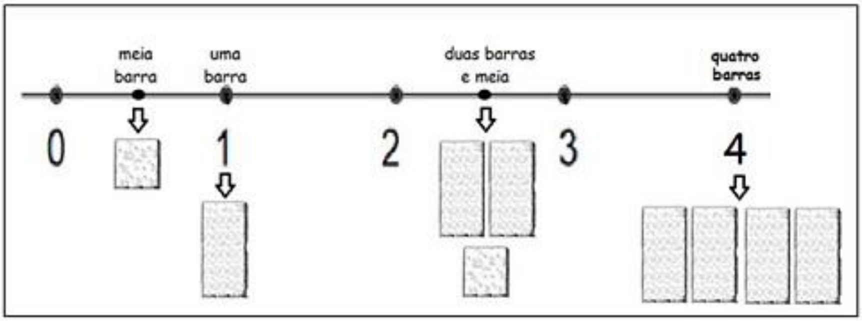
It is recommended to approach different representations of fractions, such as graphic and written ones, and encourage the reading of these representations, as exemplified in the mentioned activity. This gradual approach will allow the student to build the concept more solidly over time. Even if the language is not initially precise, understanding can be improved over time (Simões, 2022). To contemplate the ability to associate the result of a division or the idea of part of a whole, using the number line as a resource, the research group presented a sequence of activities, which are presented in Figure 5.

Figure 5

Represent Fractions on the number line. (Simões, 2022, 116)

6th Moment:
Objective of the activity: Understand and represent fractions on the number line.
Duration: 1h 30 minutes.
Let's work with chocolate bars!

➢ On the number line it is possible to indicate Fractions which, as we have already seen, are numbers that can represent a part of the unit or more than one unit. Look at the image below that illustrates the statements above.



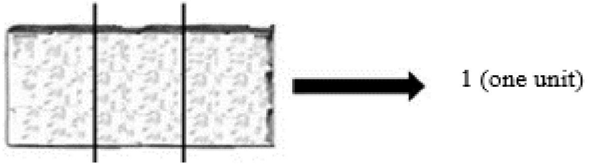
The initially proposed activity indicates the possibility of working with one unit or more than one unit. These situations presented, according to Silva (2005), constitute a suitable environment for addressing fractional numbers greater than one, also enabling the introduction of mixed number notation. In Figure 6, one can observe an activity contemplating the division of a unit into equal parts and the representation on the number line.

Figure 6

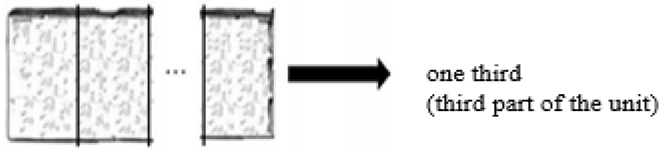
Represent Fractions on the number line. (Simões, 2022, p. 117)

6th Moment:

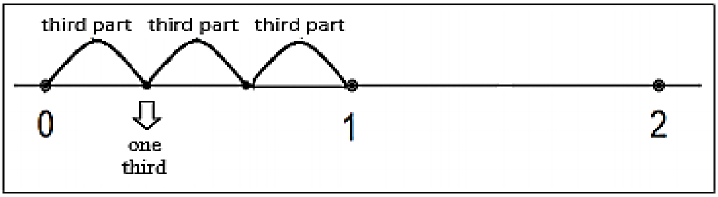
➢ For the following examples, the chocolate bar will be our unit. Therefore, each bar will represent one unit.



➢ If we divide our bar into three equal parts, and consider only one part of the three, that is, the third part of the bar, we will have the fraction one-third of the bar. If the entire bar is represented by 1, one part of the three parts into which it is divided will be one-third.



➢ On the number line we will represent it like this:



➢ Observing the previous line, discuss in pairs, but each one making their records, and answer:

a) How many thirds, that is, how many thirds are there in a whole?

____

b) If there are four children, is it possible to divide a chocolate bar into three parts and give exactly one-third to each child?
( ) Yes ( )No Justify your answer:

____

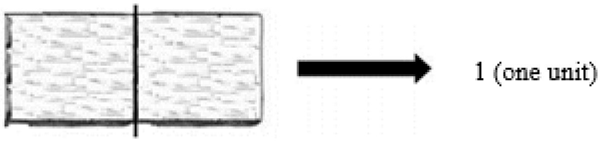
Next, we seek to explore different partitions within an integer. This approach aims to allow students to understand the content broadly and practically, as can be seen in Figure 7.

Figure 7

Represent Fractions on the number line. (Simões, 2022, p. 118)

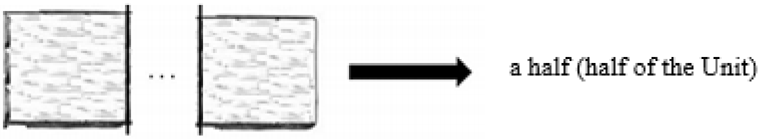
6th Moment:

➢ You will continue to observe, exchange ideas, and work on your material...

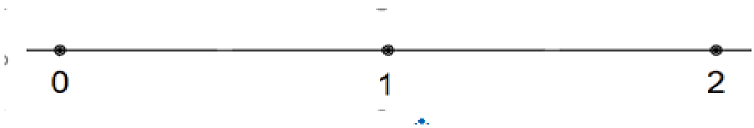


If we consider half of the bar above, we will have a fraction one one-half of the bar.

If the entire bar is represented by 1 (one), half of 1 (one) will be a half.



➢ How would you represent this on the number line?



a) How many halves, that is, how many halves are there in a whole?

____

b) If there are three children, is it possible to divide a chocolate bar in half and give exactly one half to each child?
( ) Yes ( )No

Justify your answer:

____

After carrying out activities with a third and a half, as shown in Figure 8, the next step is to challenge students to mark a quarter on the chocolate bar and draw this representation on the number line.

Figure 8

Represent Fractions on the number line. (Simões, 2022, p. 119)

6th Moment:

➢ What would you do to find the next quarter or quarter of the chocolate bar? Show by drawing on the chocolate bar.

a) Color on the bar how many quarters a whole chocolate bar has.

b) How many quarters does a whole chocolate bar have?

____

c) Using a ruler, draw a number line and mark the point that corresponds to a quarter.

As can be seen, in addition to the activity itself, a sequence was presented that uses the chocolate bar as a unit and the number line as a tool. According to observations by Simões (2022), training participants mentioned difficulties in finding resources to develop the ability to work on the content of fractions using the number line (Table 2). Therefore, this sequence represents an excellent opportunity to address this issue.

Given the above, the need to expand the range of activities is evident, going beyond the approach that is restricted to a single entire unit in teaching fractions. However, it is pertinent to point out that this aspect of teaching still lacks comprehensive exploration, thus demanding deeper investigation at a subsequent time. It is important to note that, during the training, the teachers involved expressed challenges, specifically about the formulation of activities that involve content on the number line for application in the classroom.

It is worth noting that, according to Simões (2022), to carry out the activities intended for the 5th year, a moment of reflection was previously provided on the aspects to be considered in the planning. The analysis of these proposed activities revealed an advance in teaching planning compared to the situations presented in the previous training meeting.

Furthermore, it is important to emphasize the relevance of exploring a diversity of situations when developing the concept of Fractions, avoiding restricting oneself to a single form of representation, as a comprehensive and in-depth understanding of these concepts requires the transition between different ways of producing meanings, promoting more solid learning (Parcianello, 2014).

According to Simões (2022), approaching the content of Fractions in the school environment requires careful analysis on the part of the educator, understanding the relevance of this topic and its applicability in different contexts. To achieve this objective, the teacher must recognize the need to employ appropriate and meaningful methodologies for teaching.

In this sense, providing students with varied situations that involve not only the concept of fractions but also their reading and interpretation, as well as the exploration of different associated meanings, such as the notions of equivalent fractions and operations involving fractions, makes the learning process more comprehensive and enriching. This approach contributes to a solid and in-depth understanding of this important mathematical content.

CONCLUSION

When investigating the contributions of continuing education in the area of Mathematics, with a specific focus on Fractions, to the teaching planning of teachers in the initial years of Elementary School, relevant aspects stand out. Initially, the training promoted moments of discussion and reflection on the activities carried out for the 4th year, aiming to contribute to the construction of the next plan (5th year). This involved reflection on the importance of understanding the skills to be explored, to adapt them to the students' level of development.

However, when analyzing the activities proposed for the 5th year, we see the opportunity to deepen them, exploring the different meanings of fractions (Simões, 2022). During the training, concepts and meanings of fractions were discussed, as well as methodological resources, including the use of virtual games, a relevant approach especially in the context of remote classes during the pandemic.

However, it is important to highlight that the sequence of activities did not incorporate the suggestion of using games, although these were presented during the training. Furthermore, it would be beneficial to delve deeper into activities involving mixed numbers, exploring situations in which the unit is greater than one.

Despite this, the planned activities covered the proposed skill, highlighting the concern with presenting meanings and representations of fractions beyond the part-whole model. This reflects the effort to diversify the approach, enriching students' understanding.

In this context, the relevance of offering training in Mathematics for teachers working in the initial years is highlighted, allowing them to share their experiences, challenges, and difficulties when developing mathematical concepts. Such moments of interaction and reflection are suitable spaces for clarifying doubts and deepening knowledge on important topics. In this way, continued training is fundamental for improving the teaching of Fractions and Mathematics as a whole.

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AUTHORS’ CONTRIBUTIONS STATEMENTS

All authors contributed to the research and writing of the article. DGS carried out the research, data analysis, and writing of the text, and CAO guided the field research and contributed to data analysis writing and reviewing of the article.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author (DGS) upon reasonable request.

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