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Didactic-mathematical knowledge in the education of mathematics teachers: A study from the perspective of the onto-semiotic approach

Paulo Cesar Pereira Napar[a] ORCID iD: 0000-0001-8234-2562
Carmen Teresa Kaiber[b] ORCID iD: 0000-0003-1883-230X

[a] Universidade Federal do Rio Grande do Sul (UFRGS), Instituto de Matemática e Estatística (IME), Porto Alegre, Brasil.
[b] Universidade Luterana do Brasil (ULBRA), Programa de Pós-Graduação em Ensino de Ciências e Matemática (PPGECIM), Canoas, Brasil.

ABSTRACT

Background: The appropriation of knowledge by prospective mathematics teachers is based on studies, experiences, and actions that make them experience practical and theoretical elements necessary to make them reflective and critical professionals about mathematical practices. **Objectives:** To investigate perceptions and understandings of prospective mathematics teachers who responded to hypothetical teaching and learning situations of a professional dimension in the light of analysis within the scope of didactic-mathematical knowledge arising from the onto-semiotic approach regarding teaching and learning situations professional dimension. **Design:** The investigation consists of a qualitative perspective that lists three teaching and learning situations of a professional dimension, elaborated from a mediational, affective, interactional, and ecological perspective that were subjected to analysis by prospective teachers. **Setting and participants**: The investigation was constituted from a critical-reflective analysis perspective that considers the perceptions of three prospective teachers attending a mathematics teaching degree course. **Data collection and analysis:** Data were collected from problem situations of a professional dimension submitted to prospective teachers and analysed against didactic-mathematical knowledge of an interactional, mediational, affective, and ecological nature, within the scope of the onto-semiotic approach. **Conclusions:** In teaching learning based on ecological knowledge, prospective teachers recognised the importance of adapting the curriculum to the needs of students. Regarding the situation based on interactional and affective knowledge, they are sensitive to understanding that affective relationships and direct interaction with students direct the teaching process towards effective learning. Finally, in the mediational dimension, they converge on the need for collaborative, integrated teaching that is attentive to technological changes.

Keywords: Onto-semiotic approach; Professional dimension; Didactic-mathematical knowledge.

Corresponding author: Paulo Cesar Pereira Napar. E-mail: paulo.napar@ufrgs.br

Conhecimentos didático-matemáticos na formação de professores de matemática: Um estudo na perspectiva do enfoque ontossemiótico

RESUMO

Contexto: a apropriação de conhecimentos por parte de futuros professores de Matemática se constitui a partir de estudos, vivências e ações que os fazem experimentar elementos práticos e teóricos necessários a torná-los profissionais reflexivos e críticos sobre as práticas matemáticas. **Objetivos**: investigar percepções e entendimentos de futuros professores de Matemática que responderam a situações de ensino e aprendizagem de dimensão profissional hipotéticas à luz de uma análise no âmbito dos Conhecimentos Didático-Matemáticos oriundos do Enfoque Ontossemiótico, no que se refere a situações de ensino e aprendizagem de dimensão profissional. **Design**: a investigação se constitui em uma perspectiva qualitativa que elenca três situações de ensino e aprendizagem de dimensão profissional, elaboradas sob a ótica mediacional, afetiva, interacional e ecológica que foram submetidas a análise dos futuros professores. **Ambiente e participante**: a investigação se constituiu sob uma ótica de análise crítico-reflexivo que considera as percepções de três futuros professores, acadêmicos de um curso de Licenciatura em Matemática. **Coleta e análise de dados**: os dados foram coletados a partir de situações-problemas de dimensão profissional submetidas a futuros professores e analisadas frente aos Conhecimentos Didático-Matemáticos de cunho interacional, mediacional, afetivo e ecológico, no âmbito do Enfoque Ontossemiótico. **Conclusões**: na situação de ensino de aprendizagem baseada no conhecimento ecológico, os futuros professores reconheceram a importância de adaptar o currículo às necessidades dos estudantes. No que se refere a situação baseada em conhecimento interacional e afetivo, se mostram sensíveis a entenderem que as relações afetivas e interação direta com os estudantes direciona o processo de ensino para uma aprendizagem eficaz. Por fim, na dimensão mediacional, convergem para a necessidade de um ensino colaborativo, integrado e atento as mudanças tecnológicas.

Palavras-chave: enfoque ontossemiotico; Dimensão profissional; Conhecimento didático-matemático.

INTRODUCTION

Mathematics prospective teachers' qualification is a complex process that involves appropriating theoretical and practical knowledge, guided by legal regulations and educational guidelines, and knowledge produced in teacher education. In the Brazilian context, this qualification is undergoing significant transformations, especially in the implementation of the National Common Curriculum Base (BNCC) (Brazil, 2018) and the National Curriculum Guidelines for Initial Teacher Education for Basic School (Brazil, 2019).

In this context, to understand how theoretical-practical relationships and skills development are presented in formative processes and influence prospective teachers' visions, we must identify and investigate their perceptions of situations that may be present in their future work environments. Thus, proposing that prospective mathematics teachers in initial education express their thoughts on problem situations that may emerge in school contexts is a valuable resource for analysing the mentioned elements.

This perspective was addressed in a phase of the doctoral research "A Mobilização de Competências e Conhecimentos Didático-Matemáticos de Licenciandos em Matemática para a Docência no Ensino Médio" [The mobilisation of didactic-mathematical competencies and knowledge of mathematics undergraduates for teaching in secondary education] (Napar, 2022). This qualitative research investigated how mathematics teaching degree students mobilised their didactic-mathematical competencies and knowledge throughout a Supervised Practicum for Secondary Education from the perspective of the onto-semiotic approach to mathematical knowledge and instruction.

Within the research scope, the guiding question to which the data and analyses presented in this article refer is how academics in a mathematics teaching degree course perceive and give their opinion on 'professional teaching and learning situations' as for ecological, interactional, affective, and mediational issues (Godino et al., 2017) that can potentially be developed in school contexts. Thus, this work presents a reflective analysis of the manifestations of prospective mathematics teachers who responded to hypothetical problem situations in the light of an analysis within the scope of didactic-mathematical competencies and knowledge arising from the onto-semiotic approach (Godino et al., 2017) as to the teaching and learning situations of a professional dimension already mentioned.

The following presents elements of the onto-semiotic approach (OSA) as guidelines for the research and analysis produced.

ONTO-SEMIOTIC APPROACH: DIDACTIC-MATHEMATICAL KNOWLEDGE

The onto-semiotic approach to mathematical knowledge and instruction (OSA) introduces an ontological discussion centred on mathematical objects that conceives mathematics as a socially shared problem-solving activity and a logically organised conceptual system endowed with its

own language (Godino, Batanero, & Font, 2008). The constructs presented within the scope of the OSA focus on improving and expanding knowledge related to mathematics teaching and learning processes, which apply to analysing, reflecting, and guiding educational proposals in mathematics education.

Currently, the set of theoretical notions that make up the onto-semiotic approach to mathematical knowledge and instruction (OSA) is organised into five groups: (1) Systems of practices (operational and discursive), (2) Configuration of emerging mathematical objects and processes that influence mathematical practices, (3) Didactic configurations, (4) Normative dimension and (5) Didactic suitability (Godino et al., 2017).

The theoretical groups mentioned intertwine in search of the potential of how mathematics teaching and learning can be interpreted, analysed, and put into practice. In this context, the first four groups provide tools for a didactic-explanatory analysis, allowing the description of the functioning of teaching systems and relationships with objects (Godino, Batanero, & Font, 2008). On the other hand, the fifth group, referring to didactic suitability, is configured as a harmonious relationship between partial suitability, also known as epistemic, affective, ecological, mediating, interactional, and emotional dimensions. This group offers empirical components and indicators, constituting tools that enable the investigation of elements aimed at effective intervention in the classroom (Godino, 2009) and mathematics teaching and learning processes.

The context of the onto-semiotic approach to mathematical knowledge and instruction (OSA), from the perspective of didactic suitability, values an approach to mathematics teaching and learning based on epistemic, cognitive, ecological, mediational, affective, and interactional principles. These principles can be observed, studied, interpreted, understood, evaluated, and shared to improve and qualify the processes above.

However, for the success of this approach, in-service and prospective teachers must have adequate knowledge and develop competencies that cover the didactic-mathematical aspects and the integral didactic analysis of the teaching and learning processes.

Thus, in mathematics teacher education, it becomes relevant to consider an approach that allows education professionals to develop an integrated set of competencies and knowledge aimed at practice, reflection, and self-reflection on teaching action in the school environment. The onto-semiotic approach to mathematical knowledge and instruction (OSA), in this sense,

based on the principles already outlined and the theoretical foundations that support it, proposes a discussion on the modelling of knowledge and essential competencies for mathematics teachers' practice.

Godino et al. (2017) suggest a systematisation of this knowledge and competencies, anchored in aspects derived from the theoretical framework, especially those that emanate from the perspective of didactic suitability. This proposal was called didactic-mathematical knowledge and competencies (DMKC). Considering that this work proposes using didactic-mathematical knowledge, we will not highlight the model for didactic competencies.

The modelling of didactic-mathematical knowledge postulates that the teacher must have appropriate skills in mathematics teaching for the educational level in which he/she works and, additionally, must be able to articulate this knowledge with its equivalents in subsequent levels (Godino et al., 2017). Brazilian basic education and mathematics teachers' education require that teachers who teach the subject know mathematical practices relevant to their professional performance, covering from early childhood education to high school (Napar, 2022).

To achieve this objective, those professionals must have adequate knowledge of the principles of mathematical knowledge, the teaching of mathematics, curriculum (standards, guidelines, and laws), the interaction between subjects, and the ability to mediate effectively in their school environment. However, considering the expansion of this knowledge, it is necessary that they also acquire knowledge that corresponds to subsequent educational levels. This knowledge could include, for example, mathematical practices emerging from advances in research in mathematics education, as well as integrating these objects with knowledge from other contexts and teaching areas.

The highlighted emphasis reinforces the connection between the dimensions of didactic suitability, which consider the different contexts of educational practice, and a perspective focused on mathematics teachers' education. Table 1 presents the assumptions of the teacher knowledge model (Godino, 2009; Pino-Fan; Godino, 2015; Godino et al., 2017), called didactic-mathematical knowledge of the teacher (DMKT). This model covers the facets of the different types of teacher knowledge, also called knowledge dimensions: epistemic, cognitive, affective, mediating interactional, and ecological.

Table 1

Didactic-Mathematical Knowledge

Facet	Description
Epistemic	It comprises teachers' knowledge of mathematical objects shared by them and the students (common content), and of objects at later levels, such as research and advanced knowledge (expanded content).
Cognitive	Teachers' knowledge of how students learn, their prior knowledge necessary to learn new objects, and the specific learning needs of different students.
Affective	Teacher's knowledge of the affective and emotional aspects, attitudes, and beliefs related to mathematical objects in the teaching and learning process.
Interactional	Teachers' knowledge of the epistemological obstacles that students experience and the organisation of tasks according to students' needs.
Mediating	Teachers' knowledge of the mastery of technological, material, and temporal resources that are appropriate to improving students' learning.
Ecological	It allows for understanding knowledge of mathematical content in its interaction with other areas of knowledge and curriculum components. Furthermore, it encompasses socio-professional, political, and economic aspects that influence mathematics teaching and learning processes.

Source: Godino et al. (2017)

This modelling considers the knowledge mobilised by mathematics teachers in their professional contexts. For example, cognitive and affective suitability regard the teacher's knowledge of the teaching and learning processes in which students are involved. When considering a formative context in which the "students" are mathematics teachers, affective suitability encompasses teachers' beliefs about educational processes, while cognitive suitability addresses the metacognitive processes involved in the mathematics teachers' performance (Godino et al., 2017) (referring to mathematical knowledge, didactic situations, and problems related to teaching, among others).

In the context of mathematics teachers' education, the OSA proposes that teachers have skills ranging from mathematical knowledge to the ability to mediate effectively at different educational levels. The emphasis on the connection between the dimensions of didactic suitability and teacher education highlights the relevance of an integrated approach to developing competencies and knowledge essential to teaching practice. Thus, when addressing epistemic, cognitive, affective, interactional, mediating, and ecological aspects, didactic-mathematical knowledge modelling provides a comprehensive guide for teacher education, aiming at a more effective and contextually appropriate educational practice. Based on the OSA principles, this approach stands out as a valuable contribution to mathematics education, allowing for a deeper and more integrated understanding of how mathematics teaching and learning processes develop based on the elements that must be in the domain of teaching knowledge.

METHODOLOGICAL ASPECTS

The work presented in this article is linked to the doctoral research "A Mobilização de Competências e Conhecimentos Didático-Matemáticos de Licenciandos em Matemática para a Docência no Ensino Médio" [The mobilisation of didactic-mathematical competencies and knowledge of mathematics undergraduates for teaching in secondary education] (Napar, 2022), as already highlighted. This qualitative approach research aimed to investigate how students in a mathematics teaching degree course mobilise their didactic-mathematical knowledge and competencies in the organisation, constitution, and implementation of teaching practices for high school. This research was submitted for ethical assessment and was approved by the Certificate of Presentation of Ethical Assessment (CAAE) under number 15443019.0.0000.5349.

This analysis and discussion use data produced in the research above that were not analysed in the thesis production. The data in question were collected using a research instrument made available on Google Forms so that participating academics could respond to problem situations related to teaching contexts that may occur in the day-to-day work of teachers in basic education schools. The instrument was organised according to a Likert scale of agreement (I totally disagree, I partially disagree, I partially agree, and I totally agree) in which academics had to check one of the options. Moreover, participants had an empty textbox to justify their choice freely.

The tool included thirteen teaching and learning situations related to the professional dimension, developed based on the dimensions of the DMK system (Godino et al., 2017). The term "professional teaching and learning situations" refers to issues related to teaching and learning that future or current mathematics teachers need to develop throughout their teacher education. In Brazil (2019), the professional dimension must be developed and deepened because academics can form their cultural perceptions of how to act within school environments through such maturing. However, given the need to establish a cut-off range due to the many discussions that could be raised, we will bring forward only four dimensions of knowledge: ecological, interactional, affective, and mediational. We will analyse data from three questions. The analysis will be limited to three prospective teachers: A, B, and C.

Thus, this work presents a critical-reflective analysis of the prospective teachers' answers, seeking to identify their opinions and perceptions on how they would act in the situations presented. As already highlighted, we use the constructs of the onto-semiotic approach, specifically in mathematics teachers' education, to analyse and reflect on the manifestations of the prospective teachers, always aiming to understand their mobilisation of didactic-mathematical knowledge.

DATA ANALYSIS

We take as a reference the contributions of the onto-semiotic approach and organise the analysis into categories, considering the question presented to the undergraduates as a "teaching and learning situation of a professional dimension" and the ecological, affective, interactional, and mediational facets of the didactic-mathematical knowledge model (Godino et al., 2017). The following presents the analysis constructed.

Ecological Didactic-Mathematical Knowledge

Within the scope of the onto-semiotic approach, which is taken as the basis for the analysis, the ecological dimension refers to the sociocultural issues involving school contexts and didactic relationships in the teaching and learning processes. Therefore, this category comprises the relationships between the curriculum and practices inherent to teachers' professional performance. In this sense, the question posed to prospective teachers so that they could take a stand on aspects of this category was:

- Cris is a teacher at a school in Rio Grande do Sul countryside. The school administration informed him that he must use the curriculum prescribed for the 9th grade of elementary school to plan his classes for the 1st grade of high school. The teacher refuses to do as told because the school has a prescribed curriculum for the 1st grade of high school, and he understands that it must be followed.

This first question aimed to investigate how prospective teachers would deal with that kind of atypical situation, which somehow reflects the contextual demands of schools that face significant challenges in the search to establish a teaching and learning process that addresses gaps in the face of possible complications during curriculum development. This thought highlights the importance of asking how academics would behave before a critical situation, such as the need to adopt a curriculum that, theoretically, should have already been addressed but which, in practice, brought several atypical situations to the school's reality. This situation would require a curriculum setback that allowed students to learn concepts in which they did not yet have the expected proficiency.

The analysis of students' answers about teacher Cris's stance on the imposition of using the 9th-grade curriculum to plan his classes for the 1st grade of high school reveals a range of perspectives of prospective teachers about their perceptions of the world and their experience developed throughout their qualification course. The prospective teachers' complete answers are presented in Table 2:

Table 2
Prospective teachers' answers to the question addressing the ecological dimension

Prospective teacher	Agreement level	Answer
A	I partially agree	If it is a public school, the teacher's inflexible behaviour is not justified since public education in Brazil is of very poor quality. It may have happened that the students did not see the ninth-grade content. What he could do is merge the two contents because, in his inflexibility, he may be leaving students without content.
B	I partially disagree	The teacher needs to check where his students are to move forward

C	I partially agree	I think that both curricula can be studied, and if something from the ninth grade can be reinforced in the first-grade curriculum, I think it's valid.

Prospective teacher A's advocacy of mixing content to avoid possible gaps in public education shows sensitivity toward the socio-professional reality and possible deficiencies in the quality of teaching. It is coherent to say that the multicultural reality encountered even in public schools in a same region is ordinary in the Brazilian scenario, even if it was expected, for the sake of righteousness, that the curriculum of the first grade of high school would be implemented as it should. When the prospective teacher highlights knowing the scenario mentioned, we understand that his knowledge and experiences, typical of a reflection on the practices he has already experienced, corroborate his direct and honest perception of how schools should deal with their educational proposals. Godino et al. (2017) mention that teachers must know the school context and consider how cultures affect their pedagogical practices and professional environments. Even though prospective teacher A's argument that "public education in Brazil is of very poor quality" seems inflexible, it reflects his good sense in being a reflective teacher capable of understanding the complexities of his teaching practice (Schön, 1992) and the need for adaptability that his professional context demands. Therefore, it is indisputable that, given this didactic situation, the prospective teacher demonstrated adequate ecological didactic-mathematical knowledge.

On the other hand, prospective teacher B, when evaluating students' prior knowledge, demonstrates regard for the individual factors that influence learning processes, highlighting the interaction of mathematical knowledge with social and cognitive aspects. Although the prospective teacher does not explicitly say whether or not he would follow the management's guidance in the teaching situation, he mentions considering the need to verify students' prior knowledge. We understand that despite the problem portraying a didactic situation that, in essence, reflected on ecological aspects, the prospective teacher, by caring about the student's learning process, demonstrated cognitive didactic-mathematical knowledge, as cited by Godino et al. (2017). This stance reflects the ability to adapt to challenging contexts and a genuine concern for student development, underscoring the relevance of a reflective approach (Schön, 1992) in teaching practice. Thus, prospective teacher B, by emphasising the assessment of prior knowledge, highlights the importance of an approach centred on the student's needs, as expected from cognitive

didactic-mathematical knowledge (Godino et al., 2017), indicating that he is concerned with a more personalised learning process for his students.

Prospective teacher C proposes an integrated approach, suggesting a comparative study between curricula. We infer that the objective indicated by the prospective teacher is to reorganise a curriculum that considers students' needs in light of what the two prescribed curricula have in common. Thus, the curriculum could be adapted by observing the elements missing from the ninth grade of elementary school and the first grade of high school. Considering that such an argument reflects what the prospective teacher thought, it would be logical to say that he made an ecological assessment of the situation, seeking to resolve the institutional conflict presented by meditating on the objects of knowledge to be taught to the students. Therefore, it would be coherent to say that this person demonstrates adequate ecologically based didactic-mathematical knowledge to organise a curriculum that meets students' needs, as well as due to the expressed understanding of how the institutional situation could be resolved. Based on Godino (2009), we understand that in the didactic-mathematical knowledge of epistemic dimension, the teacher must be flexible and keep in mind the culture and school reality because only then can he/she understand and exercise a pedagogical proposal attentive to students' needs. As Schön (1992) and Napar (2022) point out, reflection on practice must be institutionalised in concrete and objective planning, but following the best and most personalised teaching for the student.

These diverse perspectives, articulated by ecological didactic-mathematical knowledge, highlight the complexity inherent in decisions about prospective teachers' curriculum, connecting mathematical knowledge to a web of factors that permeate the educational scenario, as outlined by Godino et al. (2017). In this sense, within the ecological dimension, the respondents demonstrated sensitivity to the need to adapt the curriculum to the needs of students and the school's culture, seeking ways to resolve a conflict emerging from the confrontation between the institutional and the personal.

Interactional and Affective Didactic-Mathematical Knowledge

Didactic-mathematical knowledge of an interactive nature refers to teachers' understanding and skills to interact effectively with students during mathematics teaching and learning. This knowledge involves establishing meaningful dialogue, creating an atmosphere of collaborative learning, and promoting constructive classroom interactions. Continuous interaction with students is not enough; teachers must also understand students' needs, identify

epistemological obstacles that negatively impact learning (Godino et al., 2017), and provide adequate and personalised planning for different groups. Affective didactic-mathematical knowledge values the affections that teachers and students must have so that the teaching processes are constituted of affective, emotional aspects, attitudes, and beliefs about mathematical objects in the teaching and learning process (Godino et al., 2017).

The question chosen for this category considers the following situation:

- A teacher resolves all conflicts with students through strict behaviour demands, even if these actions cause difficulties in the relationship between them and in the implementation of their educational proposals. Prospective teachers' answers are presented in Table 3.

Table 3

Prospective teachers' answers about the interactional/affective dimension

Prospective teacher	Agreement level	Answer
A	I strongly disagree	The teacher is failing as an educator. Fear is not respect. If students are not carrying out their activities, this is not only disrespectful to the teacher, but also a failure in teaching, because without carrying out the proposals, students will not learn. The teacher must review his conduct and learn other ways of resolving conflicts, as a good teacher must be respected and admired, but never feared.
B	I strongly disagree	The teaching-learning process takes place through dialogue; forcing it will not benefit either party.
C	I partially disagree	Demands are necessary but must be applied with caution and care.

The prospective teacher vehemently criticises the approach adopted, highlighting that generating fear is ineffective for gaining students' respect. He highlights that the failure to complete activities is not disrespectful to the teacher but points to a failure in teaching, emphasising the need to review the educator's conduct. Prospective teacher A aligns his perspective with Godino et al.'s (2017) and Napar's (2022) interactive principles of didactic-mathematical knowledge, which highlight the importance of a reflective and

sensitive approach to the complexities of the educational environment. It is important to note that teachers must seek ways to establish a respectful relationship with students, as we understand that truculent attitudes do not create an adequate basis for building the teacher's authority, as A points out. His position reflects an adequate understanding of the teacher-student relationship, indicating that the construction of a positive learning environment is intrinsically linked to the quality of affective interactions between educators and students. This statement is contemplated in what is understood by affective didactic-mathematical knowledge, which aligns with what Godino et al. (2017) present, showing that prospective teachers have a perception that meets this need for teaching and learning practices.

By completely disagreeing with the teacher's position, prospective teacher B demonstrates a vision centred on dialogue as the basis for the teaching and learning process. His justification highlights that forcing an authoritarian approach does not benefit both parties. This positioning agrees with Schön's (1992) premise on reflection in teaching practice, indicating a more participatory, collaborative, and mainly reflective approach on the part of the prospective teacher. This understanding completely contradicts the interactional prerogative, which involves aspects of the interaction between actors in the educational process, and the affective prerogative, which refers to the feelings these same actors share in the teaching and learning processes.

On the other hand, prospective teacher C entirely agrees with the need for demands but emphasises that these must be applied carefully. This stance reflects an understanding of the importance of balance between demands and sensitivity in pedagogical practice, considering aspects of affective didactic-mathematical knowledge (Godino et al., 2017), which leads to an acknowledgement of the importance of affection in conducting relationships in the classroom. The position suggests balancing imposing limits and considering students' emotional aspects, reflecting an emotionally intelligent approach.

The analysis of prospective teachers A, B, and C's stances reveals similar conceptions about the appropriate approach to dealing with conflicts in the classroom. It indicates how they perceive the importance of affectionate relationships with students and how positive interactions are necessary for this to happen. Critical reflection on these approaches contributes to a deeper and more reflective understanding of the teacher's role in building an effective and enriching educational environment. It is undeniable to mention, in this context, that the prospective teachers showed signs of having adequate didactic-mathematical knowledge regarding the affective and interactional dimensions,

corroborating a critical analysis of how the academics, faced with the proposed scenario, integrate their knowledge that contemplates interpersonal relationships, emotional care and the search for more inclusive and reflective pedagogical practices. This approach strengthens the performance of these prospective teachers, contributing to a harmonious educational environment conducive to more meaningful learning.

Didactic-Mathematical Mediational Knowledge

Didactic-mathematical knowledge of a mediational nature guides the teacher's ability to adequately use the time available in class and the material resources, whether digital or not, that interfere and are essential in educational practices (Godino et al., 2017). Such a dimension is essential for the teacher to have the necessary means and resources to conduct a didactic transposition coherent with the objectives and skills that he/she wants his/her students to develop and mobilise.

With this dimension in mind, the questioning we asked the prospective teachers considers:

- A teacher, K, is constantly checking for new technologies that can be used in his classes. This educator seeks not only to replicate their use but also thinks of ways for students to use them to stimulate creativity and produce new knowledge, among other elements.

The prospective teachers' answers tended, unanimously, to fully agree with the situation presented, as shown in Table 4.

Table 4

Prospective teachers' answers to the question on the ecological dimension

Prospective teacher	Agreement level	Answer
A	I totally agree	This is a teacher who thinks outside the box. He keeps up to date and tries to integrate his subject with new technologies. He is an excellent professional who is not stuck in time.
B	I totally agree	The use of technology allows for dynamic and participatory teaching, making teaching something meaningful and current.
C	I totally agree	Teachers must seek knowledge, the use of technologies is essential.

The participants' answers indicate a positive view of integrating technology in teaching, which aligns with one of the main elements of mediational didactic-mathematical knowledge. Teachers' consideration of technological objects as a participatory and collaborative focus in teaching practices shows how sensitive they are to seeking a more dynamic environment in the face of students' cultural need to learn with digital technologies (Redecker, Punie, 2018, & Napar, 2022).

Participant A highlights the teacher's ability to stay up to date and integrate knowledge objects with new technologies, praising the teacher's updating and flexibility in the situation. This perspective reflects the importance of what can be called 'technological didactic-mathematical knowledge' (Napar, 2022), which involves the ability to incorporate educational innovations, such as new technologies, to make teaching more dynamic and effective. According to Redecker and Punie (2018), teachers must have mastery of how technologies can positively impact teaching, both as a collaborative element, encouraging groups of students to share their data and results arising from the use of technology, as well as from the perspective of a teaching and learning proposal in which the teacher considers guiding students not only to use technology, as cited by Napar (2022) but also leading them to produce evidence of their learning with these same resources.

Participant B highlights that technology provides dynamic and participatory teaching, making learning meaningful and current. This vision aligns with the understanding that didactic-mathematical knowledge must consider how teaching, learning, and technologies integrate and corroborate a continuous progression of study processes, as proposed by Godino et al. (2017). Furthermore, the teacher's answer revisits a crucial concept presented in Redecker and Punie (2018): the teacher's ability to conduct a reflective practice, considering whether they are, in fact, reflecting critically, individually and collectively, on their own pedagogical practice with technologies. Considering a reflective practice, in this case, means conducting multiple teaching that deals with what has already been done so that future practices can be improved. This concept is directed towards Schön's (1992) understanding when he mentions "reflection on reflection in practice."

Participant C completely agrees and highlights the indispensability of using technologies, emphasising that teachers must seek knowledge in this area. This answer reinforces the importance of mediational didactic-mathematical knowledge in a technological field as an essential dimension in teacher education, aligning with the principles of Godino et al. (2017). Redecker and

Punie (2018) highlight that the area of professional development for teachers in the area of technology allows the construction of the capacity to use different technologies for communication, collaboration, and professional development. In this context, as Napar (2022) mentions, continuing education is the main foundation for updating teaching skills with digital technologies so that the teacher "[uses] different sources and resources for continuing digital education" (p. 138).

Given the scenario presented by the prospective teachers, the manifestations put forward the participants' understanding of the relevance of technology in mathematics teaching, pointing to the need for teachers to develop specific skills in this area. This analysis can contribute to discussing and deepening pedagogical practices that integrate technologies into mathematical practices, considering the perspective of mediational didactic-mathematical knowledge when considering the technological context. These prospective teachers' stances reflect that their teaching degree course and all the experiences surrounding their initial education direct them to understand that technologies are necessary in the context of today's schools.

FINAL CONSIDERATIONS

The analysis of data collected from the prospective mathematics teachers' statements reveals a wealth of perceptions and positions that offer valuable indications about mobilising didactic-mathematical knowledge in the context of these teachers' education.

In the ecological dimension of didactic-mathematical knowledge, academics showed sensitivity toward the multicultural reality of teaching and learning situations with a professional dimension, highlighting the importance of adapting the curriculum to students' needs. Therefore, constructing situations presented to prospective teachers through the ecological dimension allowed them to demonstrate how they would develop their practices when faced with problems arising from day-to-day school life, providing indications of their actions as future professionals. In this sense, we highlight the importance of ecological didactic-mathematical knowledge that considers a flexible curriculum attentive to a school community's cultural and learning needs.

In the interactional dimension, we observed significant convergences in the academics' conceptions about the importance of affectionate relationships in constructing a positive educational environment. The emphasis on the need for dialogue and the collaborative approach highlights prospective teachers' understanding of the relevance of interactions and their impact on the

teaching and learning process. Regarding the dimension of affections, the questioning revealed what academics think about the importance of balancing what is required of students and sensitivity in pedagogical practice, highlighting their understanding of the influence of emotional aspects in the conduct of relationships in the classroom. Prospective teachers' search for a caringly intelligent approach highlighted their concern for students' emotional well-being, strengthening the proposal for a harmonious educational environment. Based on this context, we observed that the didactic situation developed from the affectionate and interactional dimensions of didactic-mathematical knowledge has allowed prospective teachers to reflect on how relationships and interactions in the classroom can be relevant for a harmonious teaching and learning process with more significant learning necessary for school success.

The analysis revealed a positive stance toward integrating technology in teaching and learning regarding the mediational dimension. The undergraduates recognised the importance of using technological resources in a collaborative and reflective manner, aligning with the mediational didactic-mathematical knowledge proposed by the OSA. In the mediational context, the positive acceptance of technology integration highlights how crucial mediational didactic-mathematical knowledge is, especially in the ability to select, use, and reflect on technological resources. This understanding aims to recap how important it is to prepare prospective teachers to deal with the demands of an increasingly digital world. In this sense, the didactic situation we presented to these prospective teachers helped them reflect on the arguments highlighted here, enabling a critical immersion in technological didactic-mathematical knowledge. The academics' responses evidenced an approach to integrating technologies in the educational context that emphasises the need for teachers to think strategically about using technological resources. Thus, when faced with a situation involving a teacher who constantly seeks new technologies to enrich their classes, we noted that the participants demonstrated sensitivity beyond the simple use of these tools, considering the importance not only of replicating technologies but of devising innovative ways of involving students, stimulating creativity and the production of knowledge.

The critical and reflective analysis of these dimensions highlights the need and urgency to invest in solid didactic-mathematical knowledge education. This knowledge enables teachers to address practical challenges and promote a more effective approach to building pedagogical practices that meet the complexities of teacher education in mathematics as an object of teaching and construct new possibilities for knowldege production.

AUTHORSHIP CONTRIBUTION STATEMENT

PCPN and **CTK** contributed to constructing and validating this work, reviewing and discussing the analyses and theoretical elaboration.

DATA AVAILABILITY STATEMENT

The data is controlled by PCPN and is available for access by any reader interested in *Acta Scientia*.

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