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Analysis of resources in an integrative curriculum material that fosters professional teaching knowledge in Mathematics

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ABSTRACT

Background: Curriculum materials incorporate resources that help teachers expand what they know and build teaching knowledge. This implies that such materials can also enable Mathematics undergraduate students to build knowledge of Mathematics and its teaching. **Objective**: Identify and discuss resources of integrative curriculum materials that induce professional teaching knowledge in Mathematics. **Design**: Document analysis. **Setting and participants**: Discussion based on the analysis of a Teacher's Manuals of curriculum materials that did not involve participants as research collaborators. **Data collection and analysis**: This section included reading guidelines in the introductory part of the Teacher's Manuals, the teaching projects incorporated therein, and the corresponding guidelines for implementation in class, considering the concept of curriculum integration as a theoretical reference. **Results**: The weakness is evident in terms of explaining the difference between the disciplinary and integrative organisation, as well as the concept of interdisciplinarity and the variation in mathematical representations and degree of complexity of tasks that can be categorised, increasing or decreasing according to students' level of learning. **Conclusions**: The Teacher's Manuals incorporates resources that enhance mathematics teaching degree students' learning, but its weaknesses must be critically evaluated and problematised, which is also a learning process.

Keywords: Mathematics Education; Curriculum Integration; Curriculum Materials; Teacher Learning.

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Análise de recursos em um material curricular integrador que induzem ao conhecimento profissional docente em Matemática

RESUMO

Contexto: Materiais curriculares incorporam recursos que colaboram para professores ampliarem o que sabem e construir conhecimentos em relação à docência. Isso implica considerar que tais materiais também podem possibilitar estudantes de Licenciatura em Matemática a construir conhecimentos sobre a Matemática e seu ensino. **Objetivo**: Identificar e discutir recursos de um material curricular integrador que induzem o conhecimento profissional docente em Matemática. **Design**: Análise documental. **Ambiente e participantes**: Discussão baseada na análise de um Manual do Professor de material curricular que não envolveu participantes como colaboradores de pesquisa. **Coleta e análise de dados**: Leitura de orientações na parte introdutória do Manual do Professor, dos projetos de ensino nele incorporados e das orientações correspondentes para a implementação em aula, considerando o conceito de integração curricular como referencial teórico. **Resultados**: Evidencia-se a fragilidade quanto à explicitação da diferença da organização disciplinar para a integradora, como ao conceito de interdisciplinaridade, e a variação das representações matemáticas e ao grau de complexidade das tarefas que podem ser categorizadas, aumentando ou diminuindo de acordo o nível de aprendizado dos estudantes. **Conclusões**: O Manual do Professor incorpora recursos que potencializam o processo de aprendizagem de Licenciandos em Matemática, mas requer avaliação crítica e problematização de suas fragilidades, sendo a discussão destas, também, um processo de aprendizagem.

Palavras-chave: Educação Matemática; Integração Curricular; Materiais Curriculares; Aprendizagem Docente.

CONTEXTUALISING THE STUDY

In Brazil, the reform of secondary education and the publication of the National Common Curriculum Base (Base Nacional Comum Curricular — BNCC) impacted the National Textbook and Teaching Material Program (Programa Nacional do Livro Didático — PNLD). In 2019, call notice no. 3/2019, referent to the PNLD 2021, was published to provide for the register and evaluation of didactic and literary works and digital resources aimed at high school students, teachers, and managers. The notice alludes to five types of works presented as Objects.

The Integrating Projects (Object 1) are prepared in a single volume by areas of knowledge, composed of six projects, to implement students' learning by articulating different curriculum components and areas of knowledge.

Considering discussions about curriculum materials, Remillard (2005) highlights their importance in Mathematics classes and how teachers are

influenced by the resources presented in their guidance texts for planning and developing classes. As Collopy (2003) highlights, curriculum materials are directly linked to teachers' daily practices and their teaching process; as they interact with the materials, these professionals develop new beliefs and expand their understanding of the content.

Therefore, discussing the implications of integrative materials in developing the Mathematics curriculum and discussing professional teaching knowledge is pertinent. Remillard and Kim (2017) state that the mathematical ideas in the curriculum materials aim to guide and activate fundamental knowledge for teaching.

Working with integrative curriculum materials is a challenge and a novelty. Contact with this work format (book) — favouring the approach of projects with themes of social relevance and the valorisation of active methodologies — gives conditions for professional teaching knowledge to be (re)signified. As discussed in Januario (2022), it is an opportunity to understand and experience alternatives to break with limiting and standardising learning practices.

These discussions show the relevance of research that analyses such materials to help us understand their conception, proposed pedagogical practice, and principle for students' education, as well as investigate aspects that can be perceived as affordances for teachers to expand what they know and to build Mathematics learning and teaching, planning, and class conduction. Thus, this article[1] is guided by the objective of *identifying and discussing resources of integrative curriculum material that induce professional teaching knowledge in Mathematics*. The study that gave origin to the work was developed in the Research Group on Curriculum in Mathematics Education (Grupo de Pesquisa Currículos em Educação Matemática — GPCEEM).

CURRICULUM INTEGRATION AND PROFESSIONAL TEACHING KNOWLEDGE

The integrative curriculum seeks to align educational experiences with everyday situations, expanding one's own knowledge and knowledge of reality linked to curriculum components, constituting a project that aims at students'

[1] This article is part of the first author's master's degree multi-paper-format thesis, supervised by the second author, defended in the Postgraduate Program in Education at the State University of Montes Claros.

integrated education. In the integrative curriculum material — which Machado (2023) defines as a resource developed "to promote the articulation of knowledge from different school subjects and other knowledge relevant to student learning" (p. 17) —, the curriculum organisation is integration, and the methodological axis is the project.

Curriculum integration aims to relate situations of social coexistence to solve real problems that might interest students. The baseline is the centre of interest from which the knowledge necessary for understanding and solving the problems students elaborate is raised. Santomé (1998) and Aires (2011) collaborate to distinguish curriculum integration from interdisciplinarity: interdisciplinarity is related to knowledge and how it is organised in scientific disciplines (or science), while curriculum integration refers to everyday issues integrated with the knowledge discussed in school subjects or, even, to the way of organising and approaching knowledge in the school curriculum.

Beane (2003) says the curriculum organised by integration is permeated with social and personal issues to put teachers and students into action, committing to integrating knowledge. This practice brings knowledge closer to students' realities, reflecting a broad view of their environment and favouring new knowledge, experiences, beliefs, values, and (re)significations.

According to Alonso's (2002) contributions, curriculum integration enables teachers to become critical curriculum builders in partnership with students, teaching with themes, subjects, and problems characteristic of their places. So, integrating knowledge with projects requires a vision of students' context and strengths and weaknesses.

Aligned with curriculum integration is the work with projects, which, as Hernández (1998) argues, originates from a problem situation. In the classroom, working with projects goes beyond teaching through subjects, which enables the study of concepts and strategies based on experiences and social situations.

Projects are central to curriculum integration; through them, we learn to think critically, give meaning to information, plan, analyse, and create strategies to solve problems. Hernández (1998) considers that a project is not situated in a finished process but rather in a proposal that relates, explores, and materialises, connecting the school to the outside world.

Studying mathematics through projects brings the content closer to the students' reality, fostering interest in the topic being addressed and involvement in learning. According to Alonso (2002), this project perspective leads students

and teachers to investigate themes and problems linked to different areas of knowledge.

Understanding curriculum integration for Mathematics teaching means considering the different themes and social contexts that originate and interconnect with others at the same level of importance in solving a given problem.

When interacting with integrative curriculum materials, teachers activate knowledge by reading, interpreting, and planning through projects incorporated into those materials. The discussion about curriculum integration is a way of reflecting on students' experiences as a resource in solving socially relevant problem situations based on content from different subjects; it is important to understand that when studying materials characterised as an integrative project, teachers take advantage of the resources present in them to teach. The relationship with integrative materials allows teachers to build knowledge related to mathematics and its teaching and learn about pedagogical innovations incorporated into the materials.

Teachers' beliefs and knowledge of Mathematics, teaching, and students may influence how they respond to those materials (Collopy, 2003), which can induce professional teaching knowledge. The resources in the materials also allow teachers to understand them as sources of knowledge for their pedagogical practice; according to Beane (2003), as teachers and students take ownership of curriculum integration to solve their particular situations and desires, they also develop their meanings.

Mobilisation of professional teaching knowledge can be activated as teachers become familiar with integrative curriculum materials, perceiving their affordances through analysis, reflections, and problematising, with readings, interpretations of their projects, and also through the reproduction, adaptation, and improvisation of the tasks present in the materials.

We understand that the study of integrative curriculum materials and professional teaching knowledge is relevant to the academic community to investigate which knowledge these professionals can mobilise, understanding the resources present in the materials for the construction of knowledge for teaching. The following section addresses the model Remillard and Kim (2017) proposed regarding the knowledge that teachers mobilise and construct when interacting with materials.

PROFESSIONAL TEACHING KNOWLEDGE BASED ON THE RELATIONSHIP BETWEEN TEACHER AND CURRICULUM MATERIALS

The way teachers use materials to plan their classes, the search for the one that best identifies with their teaching purposes, the interaction through reading and interpreting their guidelines, and the evaluation and selection of tasks that promote the conditions for the construction of students' learning, they all refer to the teacher-curriculum materials relationship. According to Remillard (2005), this relationship can occur when teachers use their own resources and skills when reading, assigning senses and meanings, evaluating the material, and adapting and replacing according to daily classroom demands.

Curriculum materials are considered to support pedagogical practice, with various guidelines, instructions, and tasks that help them direct actions towards curriculum development. Therefore, it is essential to understand how teachers develop their mathematics learning and teaching as they read, interpret, evaluate, select, and plan based on curriculum materials, regarding the "resources that the materials offer and that can enhance the mobilisation of teachers' knowledge and, as a consequence, their learning" (Januario & Lima, 2019, p. 418).

Mathematics teachers regularly use curriculum materials in their profession. Remillard and Kim (2017) argue that the resources in mathematics materials used to guide educational processes establish specific demands for teachers' use of knowledge that can be inserted into how knowledge for teaching is conceptualised, studied, and developed.

When interacting with the materials through reading, interpreting, and evaluating the guidelines and tasks in them, teachers mobilise knowledge and can change their conceptions. Teachers bring their beliefs and experiences to the materials, creating their own meanings in interpreting their authors' purposes (Collopy, 2003). Hence, the materials help construct new beliefs and knowledge about teaching, learning, and content.

When planning their classes, teachers look to the materials for guidance on improving curriculum, selecting, and adapting the content and tasks according to the teaching objectives and the students' demands. The ideas incorporated in the materials activate teachers' mathematical knowledge, which Remillard and Kim (2017) theorise in the *Knowledge of Curriculum Embedded Mathematics* (KCEM) model. Curriculum materials are composed of tasks that embed content aimed at student learning. They also present resources that

provide teachers with knowledge of pedagogical innovations, differentiated approaches, justifications for specific procedures, criteria for organising content, and cognitive demands, among other things, activating new knowledge.

Januario and Lima (2019) state that when using curriculum materials, teachers select the most appropriate tasks, assess complexity, and mobilise knowledge to understand the possible difficulties that may arise, adapting them for their students.

When planning lessons, curriculum materials help achieve learning objectives and induce changes in Mathematics teaching practice. In this aspect, materials can help teachers learn about anticipating answers to students' questions and predicting how students think when working on the proposed tasks.

Collopy (2003) argues that teachers' learning corresponds to changes in their beliefs related to teaching and learning mathematics and their practices encouraged by curriculum materials. Based on discussions about professional teaching knowledge through the relationship between teachers and curriculum materials, we present the methodological procedures of the research.

METHODOLOGICAL PROCEDURES

The research path takes the curriculum material, especially the Teacher's Manuals, as the object of analysis, as it is a facilitator for the development of the school curriculum, accessible to the teachers and that they regularly use in their planning and construction of task repertoires, which impacts research in mathematics education and professional teaching knowledge.

When we return to the proposed objective, the study portrayed here is characterised as documentary research, which Marconi and Lakatos (2003) define as data collected from primary source documents. In other words, the study uses data or information that has not yet been scientifically reported. It does not start from already prepared and analytical materials nor a scientific analysis of its information.

This investigation is classified as documentary research because it involves the analysis of curricular material, specifically the Teacher's Manuals. The book analysed is *Práticas na Escola — Matemáticas e suas Tecnologias* [School Practices — Mathematics and its Technologies]. Collectively authored,

it was produced by Editora Moderna and published in 2020. The decision to choose this material was based on work we had previously carried out with books from the same publisher and the themes covered in the projects.

Composed of six integrating projects, the schedule assigned around three months for each project. The handbook includes a general part, with texts introducing the material and giving didactic and methodological guidelines to help readers understand the material's design lesson planning when working on projects. It also consists of a specific part with guidelines for structuring the integrating projects and the steps related to each of the six proposed projects. Below, we will reproduce the pages of the student's book, present each project, and put them in small text boxes with class development guidelines.

The analysis of this curriculum material considers the proposal for curriculum integration for project work, the potential for integrating mathematics with content from other subjects for student learning and for achieving the skills and objectives proposed for teaching, as well as the dimensions of KCEM, discussing the resources that can induce professional teaching knowledge.

CURRICULUM INTEGRATION PROPOSAL

The Teacher's Manuals opening pages include guidelines for planning and carrying out project-based classes, with integration being the method of curriculum organisation. The material invites teachers to learn about the proposal based on integrative projects that aim to enable them to explore students' knowledge and critical education, as discussed by Hernández (1998), Alonso (2002) and Beane (2003), focusing on themes and problem situations of the contemporary world, mobilising specific skills and abilities in mathematics and other areas of knowledge.

The Teacher's Manuals contextualises the flexibility of the Mathematics curriculum in favour of the solution and communication of results to established interest centres. However, the analysis points out weaknesses in explaining the difference between disciplinary and integrative organisation to teachers. The material refers to integration as:

> *A pedagogical approach that integrates different areas so that students develop various skills present in the Common Curriculum Base (BNCC), working on socio-emotional issues and preparing students for*

> *future challenges.* (Práticas na Escola, 2020, p. IX)
>
> *Assembled by themes or competencies, not by subjects. From the original perspective of integrating subjects, it makes no sense to include a STEAM teacher and leave him/her isolated from the other subjects, from the Math, Science, and Physics teacher...* (Práticas na Escola, 2020, p. IX.
>
> *Designing requires openness to the unknown, to the undetermined, and flexibility to reformulate goals and paths as the designed actions reveal new problems and doubts.* (Práticas na Escola, 2020, p. VII)
>
> *Learning and teaching through projects requires a new approach from the teacher and the students. New roles are played. The teacher ceases to be the centre of attention and the holder of all knowledge and becomes the mediator and guide for students on the path to be followed.* (Práticas na Escola, 2020, p. VI)

Based on the analysis, it is clear that the concept of interdisciplinarity is mentioned in the guidance texts, but with the meaning of curriculum integration. In his study, Aires (2011) addresses the difference between these two concepts, which can be mistakenly interpreted as synonyms, in which interdisciplinarity begins and ends based on scientific disciplines, while curriculum integration begins and ends in the centres of interests focused on school subjects. The approach is not enlightening enough for teachers to understand the differentiation of these concepts; indicating complementary readings could be a strategy for teachers to understand the meaning of each concept, which could impact their reading and interpretation of the curriculum material and the Mathematics proposal incorporated into it.

The Teacher's Manuals guidelines offer an approach to how the different mathematics contents are integrated, as well as the contents of other subjects, intending to grant students autonomy, creativity, and responsibility in the search for knowledge and achievement of the skills and objectives proposed in a process connected with their culture, habits, and aspirations. During the specifications, we notice approaches that contribute to understanding the role of mathematics in this integration proposal. Figure 1 shows the proposal of a theme related to a real-life problem.

The material encourages teachers to reflect on strategies for teaching and learning through project works, which lead students to (re)construct their knowledge with active involvement in the process. The themes of the projects

are close to students' identity in social transformations and knowledge, with critical dialogues on these situations. We infer this conception presents a proposal organised and centred on students and their social demands.

Figure 1

Organisation of an integrative project (Práticas na Escola, 2020, p. VII)

ITEMS	BRIEF DESCRIPTION
Subject	Quantities and measures
Problematisation	Do you know how to choose a cell phone plan based on the amount of data it offers?
Common-sense knowledge	Measurements used in the digital world.
Defining the problem	Measurements that use the decimal system and measurements that do not; new ways to measure audience on social media.
Collecting Information	Research on data packages to be used on smartphones, in Wi-Fi connections, measurement by likes on social networks, and cost of advertising in different media.
Systematising and reflection	Final product 1 Organizing comparative tables of decimal and non-decimal measurements; organising a comparative chart between audience measurements on social networks and television channels about the cost of advertising broadcast on these media.
Scientific knowledge	Final product 2 Solving practical problems on all types of measurements using the tables built by the groups.

The Teacher's Manuals guidelines, in short texts, present resources that allow teachers to expand their knowledge of the curriculum, such as working through projects with themes that are relevant to the environment in which students live and active methodologies that lead these professionals to understand the integration of mathematics and understand specific competencies and abilities of the different areas for curriculum development. The authors approach this work as follows:

In the classroom and outside, students "work" in search of knowledge

> *that allows them to execute a final product that presents answers to a research problem. Along the way, students research, plan actions, develop hypotheses, draw conclusions, redefine their path, defend their ideas, organise a product (not necessarily physical) and present the work process based on written texts or images.* (Práticas na Escola, 2020, p. VI)
>
> *The title of the integrative project is the opening for the generating theme that will be developed during the proposed stages. As a stimulating element, it arouses readers' interest and curiosity, inviting them to learn about and understand the relevance and purposes of the project. It also aims to establish a relationship between the topic discussed and the final product developed. Therefore, the title must always be highlighted in the project's publicity strategies and in the presentation of the final product to the school and/or local community.* (Práticas na Escola, 2020, p. XXXIII)
>
> *Designing requires openness to the unknown, to the undetermined, and flexibility to reformulate goals and paths as the designed actions reveal new problems and doubts.* (Práticas na Escola, 2020, p. VII)

The integrative projects of the analysed curriculum material include themes that impact society, such as the creation of innovative, economical, and sustainable packaging, the creation of a community cultural space, water scarcity, the promotion of a culture of peace in conflict situations, financial planning, and depression in adolescence. Although such topics and their guidelines may have the potential for teachers to expand their knowledge, the guidelines alone are not enough to promote learning and changes in practices; among other aspects, it is necessary to invest in teachers' working conditions and in the development of materials that are addressed to teachers and students who face situations that are adverse to the scenarios idealised in the teaching projects incorporated in them.

When projects are mentioned, they can be considered as a means with the potential to rethink, reorganise, and redefine school knowledge (Hernández, 1998); in the presentation of the projects, the guidelines explain aspects that allow teachers to design the mathematics class beyond the disciplinary organisation of the curriculum. Figure 2 shows how Project 1 is organised, and the related skills. It is called *Qual é a melhor embalagem?* [What is the best packaging?].

We can consider that working with projects promotes teaching for understanding a problem based on a relational approach, which links the main idea with the skills and abilities of the areas of knowledge (Hernández, 1998). Each project in the curriculum material includes initial mobilising questions to encourage students to reflect on and motivate them regarding the subject to be developed; teachers are encouraged to facilitate and promote debates. This means that teachers are fundamental to help students exchange ideas, arguments, and opinions on the topics studied.

Figure 2

Skills for various mathematics contents (Práticas na Escola, 2020, p. XXXVI)

(EM13MAT302) Building models employing 1st or 2nd degree polynomial functions to solve problems in different contexts, with or without the support of digital technologies.
(EM13MAT307) Employing numerous methods to obtain the measurement of the area of a surface (reconfigurations, approximation by cuts, etc.) and deducing calculation expressions to apply them in real situations (such as the relocation and distribution of plantations, among others), with or without the support of digital technologies.
(EM13MAT309) Solving and developing problems involving calculating total areas and volumes of prisms, pyramids, and round bodies in real situations (such as calculating the cost of material for coating or painting objects whose shapes are compositions of the solids studied) with or without the support of digital technologies.
(EM13MAT314) Solving and developing problems involving quantities determined by the ratio or product of others (speed, population density, electrical energy, etc.).
(EM13MAT406): Building and interpreting frequency tables and graphs based on data obtained in statistical sample research, including or not using software that interrelates statistics, geometry, and algebra.

The projects integrate competencies and abilities, enabling students to relate the problem situation to their lives and understand its multiple objectives for learning. Figure 3 from Project 5, called *Planejamento financeiro* [Financial planning], represents the format of the tasks suggested in the curriculum material.

Figure 3

Knowing your individual practices (Práticas na Escola, 2020, p. 116)

Knowing your expenses

You have already discussed some issues with the class, which must have triggered new questions about your financial life.

Making a general estimate of the types of expenses

(1) Now, you should reflect on your monthly expenses by yourself. First, could you tell me the total amount of your monthly expenses? If you don't know, make a general estimate to find a value and write it in your notebook in an analysis chart like the model below. Based on this value, estimate the percentage of essential expenses, the percentage of significant but plannable expenses, and the percentage of superfluous expenses, recording them as well.

Initial estimate of individual financial situation	
Monthly expense (value): ———————	
Essential/necessary expenses (%)	———
Major (but plannable) expenses (%)	———
Superfluous (%)	———
Total	———

(2) In pairs, share your monthly expenses with your partner and explain what types of expenses you considered in each line. Can you both think about which categories you could include each expense in, such as food and transportation?

Therefore, the Teacher's Manuals offers an opportunity to design student education projects for learning with significant and enlightened potential for citizenship (Alonso, 2002). The projects correspond to the conception discussed in texts in the introductory part of the material, in addition to presenting resources that allow teachers to expand their knowledge through reading and interpreting subsidies presented in the project stages, the organisation and theoretical approach, through mediation of mobilising questions, suggested tasks, evaluation and self-evaluation of tasks, and presentation of the final work proposed in each project. However, we infer that the Teacher's Manuals limits teachers' role in coordinating classes because there is no indication for these professionals to develop projects with their students or no guidance for creating other development strategies for the proposed projects.

THE DIMENSIONS OF KCEM AND ITS RESOURCES FOR PROFESSIONAL TEACHING KNOWLEDGE

The point of analysis considered in this section is based on understanding the bases of professional teaching knowledge in integrative activities, considering the Teacher's Manuals of the material *Práticas na Escola* and the dimensions of the KCEM proposed by Remillard and Kim (2017). We sought to understand the presentation of mathematical ideas in this material, which induces teachers' knowledge when reading and interpreting the project resources for later evaluation, selection, and creation based on what they present as teaching proposals.

According to these authors, teachers, with their knowledge, must recognise the mathematical ideas present in the procedures, evaluate them and adapt them so that students can understand the mathematical justification in the construction of learning. From the perspective of the first dimension of the KCEM, *fundamental ideas of mathematics*, which corresponds to how mathematics can be presented and justified in a given procedure, the texts in the general and introductory part of the material present guidelines regarding mathematical procedures that justify different ways of solving the tasks incorporated into the projects, allowing the understanding of other strategies in the exploration of mathematical relationships, aimed at the exploration of investigation procedures and scenarios, integrating mathematics with themes from reality and other subjects.

Thus, when it comes to mathematical procedures, the texts guide the development of each project in the specific part, proposing mobilising questions that give teachers information for the projects' forwarding and knowledge about the theme that will be worked on, with the potential to lead students to reflect on how to resolve the proposed tasks. However, they do not suggest the organisation of this work based on the workload of the subjects, limiting themselves to the duration period, which may influence the development of their proposal. In this context, teachers may feel unguided about reconciling conventional mathematics classes and working with projects in an organisational proposal based on curriculum integration.

The curriculum material explicitly suggests a single way of solving the problem, which, given the discussions held and prior knowledge regarding Mathematics and the topic addressed, discourages students from presenting different mathematical procedures and justifications and deciding on the most favourable way to solve the problem. Presenting different forms of resolution would expand teachers' repertoire when monitoring students and analysing their strategies.

Explicit language, in the justification of mathematical ideas, leads students to the ability to solve tasks with effective procedures. When analysing the projects in the Teacher's Manuals, even if a single way of resolution is privileged, it is clear that some mathematical procedures justify different ways to solve a problem, so that part of the tasks in the material focus on indicating personal responses composed of real-life data, to generate reflections, critical thinking, and creativity in students regarding mathematical ideas, exploring their points of view, and promoting exchanges and questions. In this sense, the procedures presented in the possible resolution of the task and the guidelines

for the development of the project allow teachers to learn about what mathematically justifies the different forms of resolution supported, at times, with what we can call *rules*. However, such justifications are implicit in the guidelines, which require teachers to read them carefully and may go unnoticed and limit the learning experiences expected for students.

Figure 4 from Project 3, called *A escassez da água o que eu posso fazer?* [Water shortage, what can I do?] represents the format of the tasks suggested in the material, which proposes a mathematical procedure and guides teachers on how to solve them.

To the *representations and their connections*, the second dimension of the KCEM, refers to how mathematics is represented visually and symbolically to provide students with coherent learning. In this sense, the general and specific parts provide evidence regarding guidelines related to the options for representing content and its connections, considering mathematics and integration with other subjects. The authors of the analysed material consider that mathematical concepts can be represented by different resources such as tables, charts, mathematical language, diagrams, and other figurative elements.

Figure 4

Daily individual consumption (Práticas na Escola, 2020, p. 74)

There is no correct answer to this question. The objective is to generate reflection, initially identifying in absolute numbers (percentages) where consumption is highest and then thinking about whether a certain instrument demands high water consumption (washing machine, dishwasher, etc.) or whether it is used incorrectly (long showers, running taps to wash dishes or brush teeth, etc.).

3. They are expected to calculate the proportion: $\frac{110}{x}$ (if consumption is greater than 110).

Example: If actual consumption is 240 ℓ, calculate $\frac{110}{240} = 0.46$.

Calculate the consumption amount (in litres) and the consumption unit (minutes of use or number of flushes) by applying the rule of three for each instrument.

Example: If the shower consumes 60 ℓ, then the new consumption should be 60 ℓ × 0.46 = 27.6 ℓ.

If the shower consumption unit is 5 minutes, the new unit should be 2.3 minutes.

Is it possible to identify consumer villains in your routine? What are they? Is the high consumption a result of the instrument itself or its misuse? How could consumption be adjusted?

(3) Maintaining the consumption proportion between the instruments, estimate how much your consumption would be (in litres and consumption units) if you followed the recommended limit of 110 litres per day.

Your daily individual water consumption, adjusted to the recommended amount (110 ℓ)			
Water usage instruments	Litres	Percentage	Consumption units (minutes or number of flushing)
Washing machine	— — —	— — —	— — — — —
Tank usage	— — —	— — —	— — — — —
Toilet flushing	— — —	— — —	— — — — —
Bathroom sink	— — —	— — —	— — — — —
Shower	— — —	— — —	— — — — —
Kitchen sink	— — —	— — —	— — — — —
Washing machine	— — —	— — —	— — — — —
Result	— — —	— — —	— — — — —

The representations used in approaching the project contents are centred on tables for organising data and graphs of different classifications for analysis and discussion, both used in the six projects; in some stages, algebraic expressions and figural mathematical elements focused on geometry are presented. Through the analysis, we considered that the projects could present variations in mathematical representations, which would broaden teachers' vision for teaching and exploring new strategies so that students could be instigated by the later stages and in understanding the breadth of mathematics teaching.

Therefore, some connections between different contents in the same project allow teachers to understand the differentiation of representation and carry out the articulation to form learning. Some historical, social, and cultural figurative representations linked to the proposed theme are presented for illustrative purposes. Figure 5 and Figure 4 exemplify the recurring representation format in projects with tables that integrate mathematics with a theme from students' realities.

Figure 5

Communicating through statistics (Práticas na Escola, 2020, p. 45)

Pop: 48 students
K-Pop: 24 students
Country: 36 students
Pagode: 20 students

Funk: 42 students
Rock: 30 students
Recording this data in a frequency table, we have:

Preferred musical style	Absolute frequency (AF)	Relative frequency (RF)
Pop	48	$\frac{48}{200} = 0.24$
K-Pop	24	$\frac{24}{200} = 0.12$
Country	36	$\frac{36}{200} = 0.18$
Pagode	20	$\frac{20}{200} = 0.10$
Funk	42	$\frac{42}{200} = 0.21$
Rock	30	$\frac{2}{8} = 0.15$
TOTAL	200	1.00

We can transform the relative frequency data into a percentage, allowing us to conclude that Pop is the preferred musical style of 24% of those interviewed, just as K-Pop is preferred by 12%, Country by 18%, *Pagode* by 10%, Funk by 21% and Rock by 15% of them.

Attention! When we research a quantitative variable whose data is unlikely to be repeated, such as a person's height, we can create the table by intervals.

In the direction of this dimension — *representations and their connections* — which considers learning beyond the subject matter for teaching, and through ways of representing ideas that make the subject understandable for students, the projects propose tasks with tabular representations using data from students' lives, allowing teachers to reflect on representations and their influence on working with integrative projects, establishing connections between them and enabling students to learn.

The third dimension of the KCEM, *complexity relative to the problem*, is concerned with the levels of cognitive demands of the tasks. The introductory section of the teachers' handbook proposes aspects related to the degree of complexity and the requirement for different reasoning to resolve the tasks that make up the projects. It discusses the general and specific competencies and the various tasks that comprise each project.

Within the scope of the cognitive demands of the tasks that make up the integrative projects of the specific part of the Teacher's Manuals, teachers are guided to instigate, if they consider it pertinent, in-depth questions regarding the mathematical approach and its potential for significant student involvement. In this regard, more efficient dialogue is necessary to understand what can be advanced or returned to in order to understand and resolve the proposed tasks. The relationships for constructing mathematical ideas are determined by the sequence or level of complexity of the tasks.

The analysis of the material allows us to understand that there is a variety in terms of low or high levels of cognitive demand; the tasks that suggest personal answers have the main objective of making teachers understand the importance of encouraging students to use different procedures to arrive at an estimate and make possible decisions, being encouraged to formulate other questions related to the topic. Teachers are guided throughout the projects to encourage students to be the leading players in the construction of their own knowledge, as exemplified by the following excerpts:

> *Use mathematical strategies, concepts, definitions, and procedures to interpret, build models, and solve problems in different contexts, analysing the plausibility of the results and the adequacy of the proposed solutions to build consistent arguments.* (Práticas na Escola, 2020, p. XXXVI)
>
> *Let them bring their prior knowledge about the topic and identify what they know and what they still need to know.* (Práticas na Escola, 2020, p. XLIX)
>
> *Once they have recalled the probability calculations for the situations seen, guide them [students] to apply them [calculus] to several numbers of participants and situations so that they can see which scenario is ideal for the game.* (Práticas na Escola, 2020, p. LXXXIX)
>
> *During the presentation of answers and ideas, other questions may be formulated and reformulated, enriching the discussion. Therefore, encourage the act of questioning based on your own reflections and those of your colleagues.* (Práticas na Escola, 2020, p. XLII)

As the fourth dimension of the KCEM, the *mathematical learning paths* refer to the paths of mathematical learning for ideas and skills in several grades of schooling. The introductory guidelines of the Teacher's Manuals

suggest an organisation of the stages that can be considered in projects throughout high school, as well as the role of mathematics content and other subjects. We can understand that the proposed sequence in approaches to student education and the stages do not follow a rigid chronology; teachers can think of it as a cycle that can generate other cycles of discussion and studies of problem situations.

The specific part of the Teacher's Manuals's guidelines provides evidence of these paths for learning. It is clear that, according to the theme of the project to be developed, the authors highlight the organisation of the competencies and skills prescribed in the BNCC, the role of the contents of the different areas of knowledge for students' education, and the stages of each project throughout its development to achieve the proposed objective. These steps are subdivided into classes and serve as guidance for teachers, with instructions for their completion. In developing an integrative curriculum, teachers must give meaning to the established sequence, how mathematical ideas are constructed and their proper connections. In this aspect, the material is weak because it does not explain the importance of a flexible organisation of teaching proposals nor provides guidance on teachers' autonomy to plan classes and approach projects according to what they experience and identify as their students' demands. Figure 6 from Project 1, called *Qual a melhor embalagem?* [What is the best packaging?], represents the organisation of projects into stages and division of classes.

Figure 6

The projects and their referrals (Práticas na Escola, 2020, p. XLVI)

STAGE 4

As these classes will require computers with mathematical applications for graphs, such as Geogebra, schedule the computer room in advance so the classes can take place there.

Classes 1 and 2

Mobilising questions (Page 26)

At the beginning of the class, ask the groups to come together to discuss the mobilising questions. Set aside ten minutes for this first discussion. Explain that each group will have to develop a joint answer for each question, as they will then share their positions with the class.

Before starting the activity, watch the video indicated in *Assista+* [Watch+] with the class to encourage them to realise that research, calculations, tests, and adjustments are part of the engineering process behind the design of each package.

In the group activity, remind them that we must listen to be heard and always respect opinions that are different and/or contrary to their own.

Even with different approaches to contemporary themes, some mathematical content is repeated throughout the projects, such as percentages and measures of central tendency and dispersion; the approaches used in previous projects contribute to the understanding and resolution of the tasks of the current project; the organisation and approaches of these contents help teachers connect students' learning in mathematics to other areas covered in the project.

One challenge that teachers may encounter due to the workload is the preparation of lesson plans that consider the work with mathematics content in conventional classes, the approach to integrative projects, and the articulation with teachers of other subjects to outline the tasks to be developed. In this process, the support of school management and the pedagogical team is essential in organising moments of dialogue, ongoing education, and collective planning.

All material is organised with questions of supposed interest to students. Teachers are allowed to learn about their presence and role in teaching strategies. This approach highlights the relevance of teachers' actions in connecting mathematical topics with content studied previously or later for the construction of knowledge.

FINAL CONSIDERATIONS

Mathematics teaching is usually seen from a disciplinary perspective, with conventional curriculum material being a resource that reproduces and maintains this perspective. According to the National Textbook and Teaching Material Program (PNLD), the materials characterised as an integrative project propose another teaching logic. They propose a format that allows teachers to coordinate tasks in which students take a position as active subjects. However, they are limiting since they impose themes of supposed interest to students when they could present possibilities for teachers and their classes to develop projects with themes from the reality of the school community.

Curriculum integration involves teaching and learning built on understanding oneself and the world, enabling mathematics teachers to use new methodologies and practices to educate mathematically. Teaching based on integration provides learning for teachers so that knowledge is integrated, establishing a curriculum different from that commonly practised.

The analysis makes us consider that teachers can build knowledge by

interacting with integrative curriculum material. However, the relationship alone does not guarantee these professionals' learning process. Although the Teacher's Manuals presents resources that help teachers expand their knowledge, it is necessary to invest in continuing education actions; in particular, it is necessary to create conditions for the school to become a formative space in which problems are discussed, and solutions are sought for the issues that involve teaching and the learning process.

The material *Práticas na Escola* [Practices at School] presents a significant approach to curriculum integration but does not clarify interdisciplinarity differences. Although we consider that mentioning divergent terms will not imply antagonistic teaching practices, explaining what they are, or indicating material that addresses them, certainly helps teachers learn about such concepts and understand how both can be present in the curriculum and teaching proposals.

The Teacher's Manuals suggests guidelines for working with projects, allowing teachers to understand their role as coordinators and students as authors throughout the development process and collaborative work in the sense that mathematics integrates themes of personal, social, and cultural relevance. Regarding the above, we infer that the projects have the attribute of providing the understanding that the themes are focused on students' realities and that mathematics learning has the potential to be constructed through its integration with other curricular areas.

The article led us to highlight that the Teacher's Manuals of the *Práticas na Escola* can be a source of knowledge about an integrative approach and think about and implement teaching proposals that can be developed based on topics of real interest to students and the school community. Teachers have the opportunity to learn about a non-linear curriculum organisation so that mathematics can be understood as a subject focused on centres of interest.

The analysis of the Teacher's Manuals impacts the professional knowledge of teachers who teach mathematics because it helps them understand the fundamental ideas of mathematics incorporated in each project, realising that these ideas come from effective student participation.

As for representations and their connections, the curriculum material *Práticas na Escola* could expand how mathematical ideas and relationships are presented, leading teachers to construct mathematical learning with projects in a more relevant way. As the material aims to integrate mathematics with other areas of knowledge, representation is fundamental because it leads teachers to

explore connections between mathematical ideas to investigate a problem situation.

Regarding the cognitive demands of project tasks, teachers can have contact with tasks ranging from the lowest to the most advanced level. When coordinating the final work of projects using the material analysed, teachers need to pay attention to the skills developed, requiring well-defined planning to achieve the objectives. In this aspect, when interacting with such material, they may understand that working through projects involves tasks requiring greater or lesser cognitive effort, implying the mobilisation of different mathematical reasoning.

The curriculum material presents a learning path in which teachers can identify possible ways to learn mathematics, even with content repeated throughout the projects. In the analysis, we identified the lack of explanation on how to approach and resume content and how to organise the teaching proposal in a given school period.

To enhance the resources of *Práticas na Escola*, we must offer conditions for teachers to study, exchange ideas with their peers, and be autonomous to adapt and develop other projects with themes that interest students and the school community. To attribute solely to the curriculum material the source of professional teaching knowledge, whatever it may be, is to exempt the education department management from creating formative actions for teachers' development and improving teachers' working conditions.

DECLARATION OF AUTHORSHIP CONTRIBUTIONS

RCOS and GJ conceived the idea presented. GJ conceived the organisation of the article. RCOS developed the theoretical proposal, analysis, and discussion. RCOS actively participated in the writing. RCOS and GJ actively participated in the critical reading and discussion of the ideas and reflections set out in the article.

DATA AVAILABILITY STATEMENT

The data discussed in the article will be made available upon reasonable request and will be provided by the authors.

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