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Digital Didactical Design Alternatif Based on Learning Obstacles in Topic of Proportion: A Study of Pre-Service Mathematics Teachers

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ABSTRACT

Background: The objectives of this study are to (1) identify learning obstacles for pre-service mathematics teachers (PMTs) in understanding the concept of proportion and (2) design alternative digital didactic designs based on learning obstacle findings. **Methods:** In this study, an interpretive paradigm was used to conduct Didactic Design Research (DDR). This study involved 25 pre-service eighth-semester mathematics teachers at a private university in Indonesia. Data from test results, interviews, and document studies were analyzed using the triangulation method. **Results:** The results of this study show that PMT experiences an ontogeny type of learning obstacle. Mentally, students are not prepared to receive basic knowledge because the didactic design does not account for the relationship of prerequisite material to proportions, resulting in didactic obstacles. Barriers to epistemological learning were also identified as a result of a lack of understanding and mastery of the concept of ratio, which is only related to the context of cross-product strategy. Then, an alternative digital didactic design based on stages of the theory of didactic situations (TDS) was used, with e-learning-based Moodle as a hypothetical didactic design to minimize learning obstacles on the concept of proportion for PMT. **Conclusion:** PMT identified learning obstacles and then developed a hypothetical digital didactic design to minimize learning obstacles related to the concept of proportion for PMT.

Keywords: Digital didactic design, learning obstacle, pre-service teacher, proportion, ratio, mathematics.

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DESIGN DIDÁTICO DIGITAL ALTERNATIVO BASEADO EM OBSTÁCULOS DE APRENDIZAGEM NO TÓPICO DE PROPORÇÃO: UM ESTUDO COM PROFESSORES DE MATEMÁTICA EM FORMAÇÃO INICIAL

RESUMO

Contexto: Os objetivos deste estudo são (1) identificar obstáculos de aprendizagem para professores de matemática em formação inicial (PMTs) na compreensão do conceito de proporção e (2) projetar designs didáticos digitais alternativos com base em descobertas de obstáculos de aprendizagem. Métodos: Neste estudo, um paradigma interpretativo foi usado para conduzir a Pesquisa de Design Didático (DDR). Este estudo envolveu 25 professores de matemática em formação inicial do oitavo semestre em uma universidade privada na Indonésia. Dados de resultados de testes, entrevistas e estudos de documentos foram analisados usando o método de triangulação. Resultados: Os resultados deste estudo mostram que o PMT experimenta um tipo de obstáculo de aprendizagem ontogenético. Mentalmente, os alunos não estão preparados para receber conhecimento básico porque o design didático não leva em conta a relação do material pré-requisito com as proporções, resultando em obstáculos didáticos. Barreiras à aprendizagem epistemológica também foram identificadas como resultado da falta de compreensão e domínio do conceito de razão, que está relacionado apenas ao contexto da estratégia de produto cruzado. Em seguida, foi utilizado um design didático digital alternativo baseado em estágios da teoria de situações didáticas (TDS), com o Moodle baseado em e-learning como um design didático hipotético para minimizar obstáculos de aprendizagem sobre o conceito de proporção para PMT. Conclusão: PMT identificou obstáculos de aprendizagem e então desenvolveu um design didático digital hipotético para minimizar obstáculos de aprendizagem relacionados ao conceito de proporção para PMT.

Palavras-chave: Design didático digital, obstáculo de aprendizagem, futuro professor, proporção, razão, matemática.

INTRODUCTION

Proportion refers to the equivalence of two ratios (Musser, Gary L.; Peterson, Blake E.; Burger, 2014). Proportion serves as the foundation for higher mathematics materials, including algebra, calculus, geometry, and probability (Lamon, 2020). In addition, proportion material in schools is important because it is one of the prerequisites for students to learn further material (Bintara & Suhendra, 2021). The thought process in learning the concept of comparison can also influence students' success in developing other mathematical knowledge and abilities (Arican & Kiymaz, 2022).

Despite the importance of the material, several learning obstacles arise in learning proportion. Based on several studies, there are various kinds of obstacles that arise in learning proportions. Students experience learning obstacles in epistemological obstacles when understanding contextual applications (Andini & Jupri, 2017), as well as the concept of proportion ontogeny, didactic, and epistemological (Bintara & Suhendra, 2021; Wahyuni et al., 2019). In addition, students also experience epistemological obstacles in understanding concepts, using problem-solving procedures, and operational techniques for solving proportion problems (Biori et al., 2022). They also have difficulties determining strategies in proportion relationships (Özen Yılmaz, 2019). Additionally, according to Karli & Yildiz, (2022), students are also unable to distinguish indirect proportion and inverse proportion, as well as proportions involving additive and multiplication relationships (Bintara & Suhendra, 2021; Karli & Yilidiz, 2022).

Learning obstacles occur when students use cross-multiplication methods and are unable to explain the meaning of the solution procedures they use (Arican, 2018). These students solely rely on procedures and formulas without understanding, such as the scale formula on a map or the formula for the relationship between distance and time. In fact, students can only understand problems that the teacher models in class (Arican, 2015). One of the main problems when teaching proportion relationships is the emphasis on memorizing rules to solve problems (Izsák & Jacobson, 2013). Standard phrases, such as, "When one quantity increases, the other also increases, and if one decreases, the other also decreases," is often used when a teacher presents different proportions, or in the case of inverse value comparisons, "When one increases, the other decreases" ” (Cabero-Fayos et al., 2020).

This failure demonstrates that students and teachers do not understand the term “equivalent ratio in proportion.” Universities educating pre-service mathematics teachers must identify their learning obstacles in understanding proportion material. The learning process will reveal the extent to which students understand proportion concepts. Students’ prior experience with school mathematics undoubtedly influences their learning of proportion. In general, there have been many studies related to learning obstacles in proportion to pre-service teacher students. Among them are the findings of Arican et al. (2018) and Osana & Royea (2011), who found that pre-service teachers experienced obstacles in providing representations of ratios and providing inappropriate arguments in explaining their solutions. In addition, students have difficulty distinguishing proportional relationships from nonproportional relationships, even after being given instructions about these relationships (Arican, 2019; Valverde & Castro, 2012). One of the causes of the lack of proportional reasoning ability of prospective teacher students is that they are accustomed to focusing and memorizing the steps to attain the results of solving a problem (Valverde & Castro, 2012).

The concept image created by the teacher differs from the concept definition of proportion material as an equivalent ratio. Since pre-service mathematics teachers will be the driving force behind the success of the educational process in schools, research on learning barriers in these individuals is imperative. Understanding the learning obstacles of pre-service mathematics teachers will assist lecturers in developing hypothetical didactic designs.

Considering the significant importance of the proportion material, this study focuses on the learning obstacles of pre-service mathematics teachers especially on proportion. This study aims to (1) identify various types of learning obstacles experienced by pre-service mathematics teachers on the topic of proportion and (2) design alternative didactic designs based on the learning obstacle findings. The results of previous studies explain that pre-service mathematics teachers still experience learning obstacles in the material of proportion. However, studies on what contributes to the design of alternative didactical designs for pre-service mathematics teachers on the topic of proportion based on learning obstacles are still limited. Therefore, this study attempts to describe and identify the learning obstacles of pre-service mathematics teachers in solving proportion problems.

THEORETICAL BACKGROUND

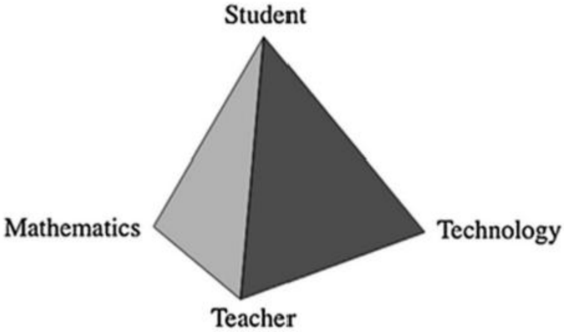
Digital Didactical Design

Digital didactical design (DDD) is the development of teaching materials made by using digital technology in learning practices. Using the Didactic Tetrahedron framework, digital didactical design takes into account the elements of teachers, students, mathematics, and technology. The integration of technology in the didactic situation is the pinnacle of design development (H. S. Lee & Hollebrands, 2006; Nopriana et al., 2023). The incorporation of technology into Didactic situations is proven to have a significant impact on the activity system underlying educational studies (Rezat et al., 2022).

Lee & Hollebrands (2006) developed the idea of the didactic tetrahedron by viewing technology as the fourth component of the didactic triangle. The introduction of technology into a didactical situation can have a transformational effect on the improved didactical situation. Mathematics is abstract, so representations make it more accessible. Technology offers new and different representations for students and teachers to use in the learning process. Figure 1 depicts a tetrahedron with the base triangle Teacher-Mathematics-Technology and the apex being the student component. (Dasari et al., 2023).

Figure 1

Didactic Tetrahedron



The digital didactic situation (DDD) design takes into account four aspects: learning obstacles, learning trajectory, theory of didactic situations, and prospective teachers' abstraction process when studying school mathematics. The developed didactic design is then implemented and developed to build a concept, followed by overcoming and minimizing learning obstacles that pre-service mathematics teachers experience during learning.

DDD provides a new perspective on didactics and design activities (Jahnke et al., 2014). In this digital era, technology affects many layers of education, from classroom reactions, learning content, out-of-school activities, and agendas to local and national decision-making (Jahnke et al., 2017). DDD uses the term “digital” because, in an Internet-driven world, teaching practices are usually technology-based. However, the extent to which they support different forms of learning varies depending on the quantity and quality of technology integration (Jahnke et al., 2014, 2017). The integration of technology in teaching affects the existing didactic design. The utilization of technology is extended cognition because it provides advantages for teachers to deliver knowledge and for students to receive knowledge. The main components of DDD are Teaching Objectives, Process-Based Assessment, and

Learning Activities delivered through digital media. According to Jahnke et al. (2014, 2017), there are five design elements — learning objectives, learning activities, process-based assessment, and social relationships —and their ideal characteristics for deep and meaningful learning.

Previous research shows the integration of technology in digital didactical design using iPad (Jahnke et al., 2017), Virtual Reality (Vallance, 2021), and (Nopriana et al., 2023) emphasize the importance of considering student learning obstacles and user prerequisites in the design of digital learning modules. This research incorporates technology that can be accessed easily by lecturers and students from anywhere and anytime using only their laptops, tablets, or cell phones. The learning mobile system (LMS) Moodle, which is based on learning obstacles and includes DDD components, is used to present the technological integration in this study. First, the design of learning objectives and expected outcomes is clear and visible to students. Learning objectives in this study are based on essential competencies and indicators of competency achievement. Second, the design of learning activities helps students to achieve learning objectives. The learning activities of this study are based on the theory of didactical situations (Brousseau, 2006)Third, the design of process-based assessment allows students to receive guided reflection for performance or skill development. This research presents a process-based assessment by raising problems that can lead to group discussion activities and hands-on worksheets. Fourth, social relationships. For example, lecturers are experts, process mentors, and learning buddies.

Learning Obstacle

Some students may struggle to understand the material for a variety of reasons. Externally caused problems are referred to as learning obstacles. Students' knowledge, the manner in which material is taught or presented in textbooks, and their limited understanding all contribute to obstacles to student learning (Brousseau, 2002). These barriers are classified as ontogeny barriers, didactic barriers, and epistemological barriers. Meanwhile, other researchers have distinguished four types of barriers: cognitive barriers, psychological barriers, didactic barriers, and epistemological barriers. This study focuses on ontogeny and epistemological learning obstacles.

The difficulties and problems encountered in learning materials, particularly comparison, can be categorized into three categories: ontogeny obstacles, didactical obstacles, and epistemological obstacles. Ontogeny obstacle is a learning obstacle that occurs due to the limitations of students in self-development or related to the mental readiness of students to learn. (Suryadi, 2019) suggested that ontogenically, obstacles can be divided into three types, which are psychological, instrumental, and conceptual obstacles. Psychological ontogeny obstacles include student unpreparedness related to learning motivation and interest in the material being studied. Instrumental ontogeny obstacles are student unpreparedness related to the technical nature of the learning process. This can be revealed through student responses and completion errors in the learning process. Ontogenically conceptual obstacles are student unpreparedness related to previous learning experiences, for example, a lack of understanding of concepts in prerequisite material.

The second type of obstacle, didactical obstacles, are obstacles that arise from the methods or approaches used by a teacher. Finally, epistemological obstacles are obstacles that occur due to the limited knowledge that students have in a particular context. During the learning process, students often experience difficulties in learning. These difficulties are commonly referred to as learning obstacles. Numerous factors cause learning obstacles to arise. The three main factors in students include ontogeny obstacles, didactical obstacles, and epistemological learning obstacles. (Brousseau, 2002).

Proportion

Proportion is the statement that two given ratios are equivalent. A ratio is the comparison of two quantities, a and b, which are both rational numbers. A ratio is denoted as a/b or a:b and is read as "a to b." A pure ratio occurs when the comparison involves two similar quantities. In comparison, a mixed ratio occurs when the comparison involves two different quantities, such as phi (π) or rate (speed, discharge, etc). In this context, the proportion theorem states that if a, b, c, and d are rational numbers with $b \neq 0$, then $a/b=c/d$ if and only if $ad=bc$.

There are two types of proportion, including direct proportion and inverse proportion (Nicholson, 2014). Direct proportion, which states that for every rational number a/b and c/d, with $a\neq0$ and $c\neq0$, $a/b=c/d$ if and only if $b/a=d/c$. Whereas, inverse proportion states that for every rational number a/b and c/d, with $c\neq0$, $a/b=c/d$ if and only if $a/c=b/d$. If the variables x and y are related by the equation $y=kx$ ($k=y/x$), then y is proportional to x, and k is the proportionality constant of y and x.

There are several methods for dealing with proportions, including the cross-product strategy, unite rate strategy, fraction strategy, equation strategy, and building-up strategy (Arican et al., 2018; Tunç, 2020). Each strategy has a specific approach to solving different proportion problems.

METHODOLOGY

Design

This study employed Design Didactical Research (DDR) (Suryadi, 2019) within an interpretive paradigm (Creswell, 2017). DDR is a research design that seeks to understand problems related to the teacher's thought process in three phases which are before learning, during learning, and the phase after learning (Suryadi, 2019). An interpretive paradigm is a research approach used to deeply understand and explore the essence of a reality experienced by subjects (Creswell, 2017; Suryadi, 2019). In this study, the interpretive paradigm aims to comprehend and explore every phenomenon related to the learning obstacles faced by pre-service mathematics teachers and to develop alternative didactic designs based on these obstacles. Therefore, this study used a phenomenological approach. Phenomenological studies do not measure received reality, but rather seek to understand out how subjects perceive their surroundings (Eddles-hirsch, 2015; Williams, 2021).

The mathematics material in this study was limited to proportion. The main reason is that the material is required for students to learn further material in school mathematics (Bintara & Suhendra, 2021), as well as the foundation for learning higher mathematics, such as algebra, calculus, geometry, and probability (Lamon, 2020). Based on the results of the study, students experienced various obstacles when learning the proportion material, including ontogeny obstacles, didactical obstacles, and epistemological obstacles (Andini & Jupri, 2017; Biori et al., 2022; Riehl & Steinthorsdottir, 2019).

Participants

A total of 24 eighth-semester students of a Mathematics Education study program at one of the universities in Cianjur Regency, Indonesia, were participants in this study. Participants were selected using the purposive sampling method (Campbell et al., 1995; Guarte & Barrios, 2006). Participant selection was based on the fact that pre-service mathematics teachers have studied proportion material, and are the main source of learning obstacle data acquisition. The proportion material is part of school mathematics and one of the early algebra. Problems in this material have solutions to further mathematical concepts. This study was carried out at a private university in Indonesia. The research location was chosen with the researcher's convenience in mind. All participants first read about the research protocol. In addition, they must sign a letter of willingness to become a participant. The Helsinki guidelines conducted this research, and the protocol was approved by the Ethics

Committee of Universitas Pendidikan Indonesia with number 6003/UN40.F4. D1/KM/2023.

Instrument

The researcher acted as the main instrument in this study and was directly involved in all data collection processes. Supporting instruments include proportion diagnostic tests, student interview guidelines, and documentation (audio recordings).

Diagnostic Test

Data collection began with administering a diagnostic test to 25 pre-service mathematics teachers. Diagnostic tests are an integral part of decision-making (Ketterlin-Geller & Yovanoff, 2019; S. Lee & Robinson, 2005). In this study, the diagnostic test consisted of four questions, which aimed as a starting point to capture students' proportional reasoning in solving proportion problems. The questions given to students were selected using three criteria based on the criteria of didactical situation theory (Brousseau, 2002): (a) presentation of the type of representation of the proportion relationship, (b) the type of proportion used in each problem, namely direct proportion and inverse proportion, and (c) the type of context used in each problem that has the possibility of various student answer strategies.

Three experts in mathematics education theoretically validated this diagnostic test to ensure its suitability for prospective mathematics teachers and the characteristics of proportion relationships. In addition, the test lasted for about sixty minutes. In this test, students were not allowed to use calculators or engage in other forms of cheating. Aspects of diagnostic test validation consist of content aspects, question construction, and language aspects. The validation results show that the instrument is suitable for use with revision. Validators provided various inputs, among others, on the aspects of clarity of information on the questions, aspects of language clarity, and aspects of the answer key for diagnostic questions. The diagnostic test used to collect data for this study was developed with input from the validator team.

Interviews

Once the test answers of the students were obtained, the interview instrument was used. This study used semi-structured interviews (Magaldi & Berler, 2020; Schmidt, 2004), meaning that researchers did not use systematically prepared interview guidelines; instead, the guidelines used were in the form of an outline of the problems to be asked. The researcher explored various information from participants in an effort to clarify the answers given on the diagnostic test. This was done to find the root causes of the difficulties experienced and to strengthen the allegations of the types of obstacles experienced by pre-service mathematics teachers.

Eight students were selected for additional individual interviews based on pre-selection by observing all students' answers on the written test. This was done to collect more detailed information about the learning obstacles on the proportion material produced by students in each problem. Then, the researcher conducted interviews and recorded the interview process for approximately 15-25 minutes per student. During the semi-structured interviews, each student's written answers were presented, while being encouraged to provide explanations for their answers.

Documentation

Documentation instruments are a method used to collect important information in qualitative research. Documentary studies gather qualitative text-based data. According to Creswell (2016), the collected documents can include both public and private documents related to the location or participants in a study. In this study, several documents have been collected as data sources for analyzing student learning obstacles, including interview transcripts, selected materials for middle school mathematics, and student notebooks.

Data Analysis

The qualitative data analysis software ATLAS.ti was utilized throughout the entire data analysis process in this research. This software has been employed in qualitative educational research, including phenomenological studies (Lukman et al., 2021). Specific student responses were selected based on learning obstacles in Table 1 (Brousseau, 2002), which were expounded upon in-depth in the discussion section of this article.

Table 1

Learning Obstacle Characteristics

Learning Obstacle Factors	Criteria
Ontogeny Obstacles	- Students lack interest in learning the proportion material (Ontogeny psychological obstacles). - Students make mistakes in the solution process for arithmetic calculations (Ontogeny instrumental obstacles).

Learning Obstacle Factors	Criteria
	- Students have not mastered the prerequisite material (Ontogeny conceptual obstacles).
Didactic Obstacle	- Students have difficulty learning the concepts taught due to the limitations of teaching lecturers or the limitations of the teaching materials used - Students have difficulty understanding concepts due to jumping material or prerequisite material that has not been learned.
Epistemological Obstacles	- Students struggle to develop their existing concepts due to insufficient practice with non-routine problems. - Students face difficulties in developing their concepts due to a lack of understanding of prerequisite material concepts.

After participants were selected, interviews were conducted to confirm their responses. The interviews were recorded and transcribed for 45 to 60 minutes. The next step was data triangulation, by comparing the written test results with the interview results. The data analysis process was completed by concluding the research results based on the triangulation process and didactic design on the topic of proportion based on the learning obstacles experienced by students.

Ethics Statement

This study has received consent from the research subjects, including both lecturers and students, who signed an informed consent form (ICF). The research also obtained approval from the Dean of the institution where the research subjects are affiliated through an official letter from the university's Vice-Rector of Academics. Therefore, the research team did not seek a prior ethical assessment from the relevant review board for this project. Through this statement, we release Acta Scientiae from any resulting consequences, including full assistance and potential compensation for any harm experienced by research participants, in accordance with Resolution No. 510, dated April 7, 2016, from the Brazilian National Health Council.

RESULTS AND ANALISES

Pre-service math teachers must comprehend a variety of ratio rules to be able to identify proportion relationships. The concept of proportion is usually addressed in reference books following the ratio. In addition, the notion of proportion comes before the ratio. For these reasons, the author created a diagnostic exam to pinpoint pupils' challenges. The test comprised four equivalent ratio, direct proportion, and inverse proportion problems. The test findings were used to identify the different learning challenges that pre-service math teachers faced. Two different kinds of learning obstacles were identified in this study.

Learning Obstacles for Students in the Ontogeny Obstacle Type

Pre-service mathematics teachers face challenges because of their mental preparedness for learning. They have encountered this difficulty while working on the problems in Table 2.

Table 2

Table, Graph, and Equation Representation Problems

<table>
<tr><th>Number</th><th>Questions</th></tr>
<tr><td>1</td><td>Look at the table that depicts the relationship between the weight of rice and the price of rice!

<table>
<tr><td>Weight of rice (Kg)</td><td>1kg</td><td>2kg</td><td>3kg</td><td>...</td><td>10kg</td></tr>
<tr><td>Price (IDR)</td><td>12.000</td><td>24.000</td><td>...</td><td>...</td><td>...</td></tr>
</table>

a. Complete the table that depicts the relationship between the quantity of rice and the price of rice above!
b. On cartesian coordinates, draw a graph representing the relationship between the weight of rice (x) and the price of rice (y)!
c. Write the equation that expresses the relationship between the weight of rice (x) and rice's price (y)!</td></tr>
<tr><td>3</td><td>If a cake tin filled with equal-sized cakes is distributed to 32 children, each receiving an equal share, each child will get four cakes.
<table>
<tr><td>Number of Children</td><td>Number of cakes each child receives</td></tr>
<tr><td>32</td><td>4</td></tr>
<tr><td>16</td><td>….</td></tr>
<tr><td>8</td><td>….</td></tr>
</table></td></tr>
</table>

Number	Questions	
	4	….
	1	….

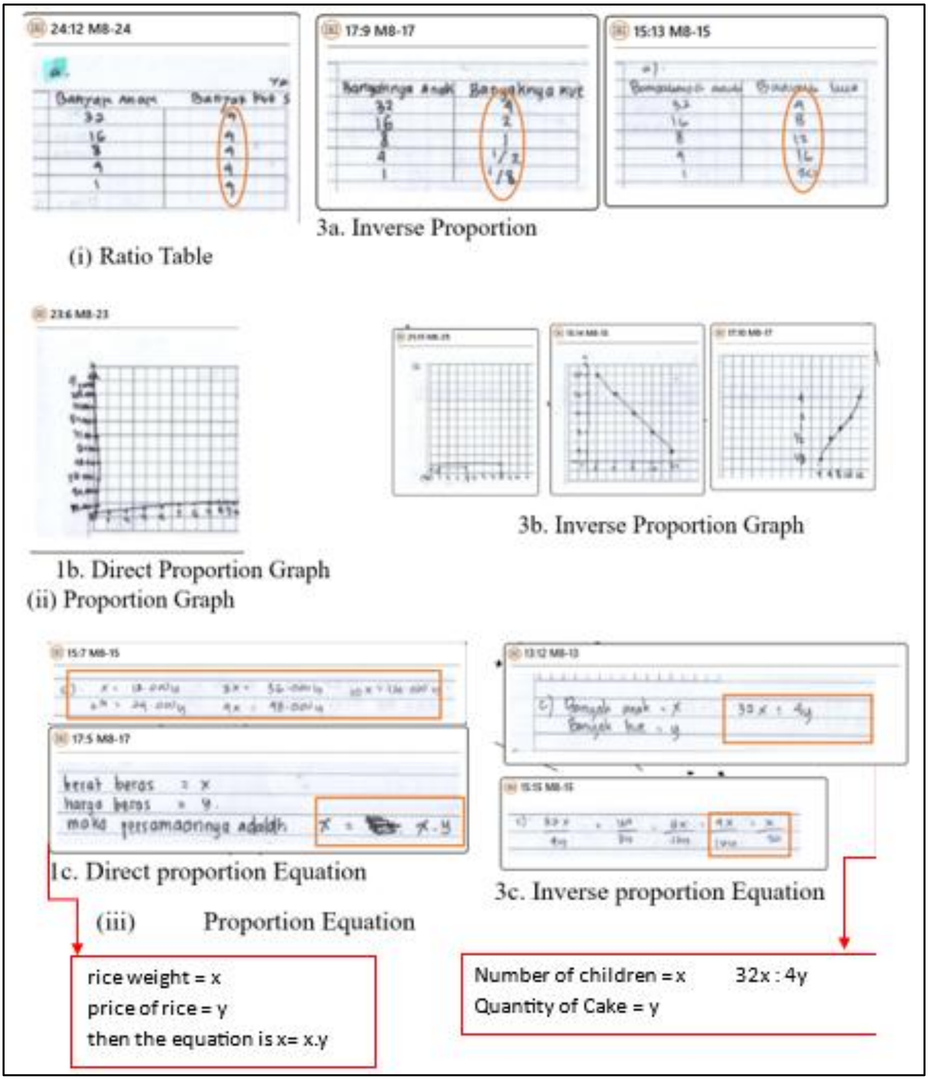
a. Complete the table that depicts the relationship between the number of children and the number of cookies each child receives.
b. On cartesian coordinates, draw a graph representing the relationship between the number of children (x) and the number of cookies each receives (y).
c. Write an equation that expresses the relationship between the number of children (x) and the number of cookies each child receives (y).

Various instrumental ontogeny was found in the problem of interpreting tables, graphs, and equations for proportion material. Students cannot solve the problem properly. These include (Figure 2a) students who did not represent technical matters related to the ratio rule presented in the table for the inverse proportion type. It was seen that students performed poorly when it came to representing the direct proportion graph (M8-23). Weaknesses in representing inverse proportion graphs were also found in various forms, ranging from vertical, linear, and meaningless graphs (M8-25, M8-15, M8-17), as a result of pre-service mathematics teachers not understanding proportion relationships, preventing them from adequately representing graphs. Students also demonstrated how to express algebra equations. This indicates that students identified ontogeny concepts (for example, M8-15). Students showed a weakness with the symbol "equal to" without understanding the proportion relationship $x = 12000y, 2x = 24000y, 3x = 36000y, 4x = 48000y, 10x = 120000y$, and concluded an equation formula from the relationship in general.

Figure 2 demonstrates that students struggle to comprehend the concept of ratios when using tables to solve inverse proportion problems, so they cannot draw graphs from the tables provided. This indicates that students face ontogeny obstacles. Students struggle to understand the ratio relationship and proportional rules when representing equations. Students can find the proportion relationship when solving direct proportion with a cross-multiplication strategy.

Figure 2

Representation of Table, Graph, and Equation on the Concept of Proportion



Students demonstrated a lack of prerequisite knowledge in their responses to daily life problems presented in Table 3.

Table 3

Proportion Problem-Solving Questions

Number	Questions
2	The price for a kg of rice is IDR 6,000. If Hamzah paid IDR 36,000 to buy the rice, how many kg of rice did Hamzah get? Solve using 2 different strategies!
4	A car departs from city A to city B in 2 hours, averaging 50 km/hour. If the average speed of the car is 60 km/hour, how long does it take to cover the distance from city A to city B? Solve using 2 different strategies!

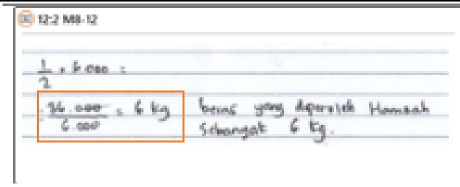
Based on their responses to question number two, students were identified as making errors in determining the final answer to the direct proportion problem. Students answered 6-kilogram as a solution without knowing what was known in the problem where the price of half the rice was known (for example, M8-12 Figure 3 part i). Meanwhile, in the inverse proportion problem, students were identified as incorrectly calculating, revealing the ontogeny concept.

In addition to the results of the pre-service mathematics teacher test answers, interviews were also conducted with the following transcript:

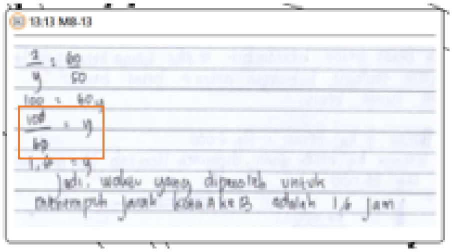
How do you complete an inverse proportion table?	M8-15: 32 gets 4, 16 gets 8, and 8 gets 12 because 16 is from 32: 2 and 16: 2, so Miss. If 12 is from 4 plus everything, 4 becomes 8, then add 4, which equals 12.
How do you draw a proportion graph?	M8-23: I do not know, Miss, I forgot. M8-17: Just draw it, Miss; according to the table, it is curved because of the fraction.
How do you write the inverse proportion equation?	M8-12: $32x = 4y$, that is it, Miss. $32x, 16x\ 8x\ 4x, x$.
How do you solve Hamza's problem (direct proportion)?	M9-12: So $\frac{1}{2} \times 6000$, then straight away, $\frac{36000}{6000} = 6\ kg$, so the rice that Hamzah got was $6\ kg$, Miss.
How do you solve the problem of car speed (inverse proportion)?	M8-8: The way to do it is to use the known (information) and make it like this: Miss, $\frac{2}{y} = \frac{60}{50}$, then by crossing it, you get $100 = 60y$, so $y = \frac{100}{60} = 0.6$.

Figure 3

Proportion Problem



(i) Problem Number 2. Direct proportion
Translation
1/6 x 6000 =
36,000/6,000 = 6 kg
Hamzah got 6 kg of rice.



(iv) Problem Number 4. Inverse proportion`
Translation
2/y = 60/50
100 = 60y
100/60 = y
So, the time obtained to cover the distance from city A to B is 1.6 hours

The interview results from the students provide the following information: (a) students had difficulty representing proportions that involve tables, graphs, and equations; (b) some students did not state ratios; (c) students did not understand the meaning of ratios; (d) students made errors in finding final answers, such as miscalculations, and (d) some students were aware of the cross-multiplication strategy, but were weak in prerequisite material, specifically ratios.

The students' difficulty is an ontogeny learning obstacle; the student has not understood the related technical matters, as shown in Figure 2. This is classified as an instrumental kind of ontogeny learning. The didactic circumstance offered in the student's notes document does not alter the ratio from what is typically encountered, causing pupils to be unable to solve the problem effectively, as shown in Figure 2. This is classified as a conceptual form of ontogeny obstacles. Weak prerequisite knowledge is the cause of student abstraction, which is classed as an ontogeny barrier, as illustrated in Figures 1 and 2. Learning hurdles can be produced by students' mental preparation, cognitive maturity in obtaining knowledge, and the complexity of didactic situations that impede their ability to follow the learning process.

Learning Obstacles Experienced by Students are Epistemological Obstacles

The obstacle arises due to the limited understanding of students regarding the concept, issues, or other elements related to the concept of proportion. Their understanding is narrowly linked to their experiences within a context. From Figure 4, in the direct proportion situation, students failed to comprehend the question, and the variables in the question were not interpreted correctly (e.g., M8-23 Figure 4i), stating the quotient of the relationship between the price of rice $\frac{36000}{6000}$ without connecting it to the known equivalent ratio in the problem, resulting in an epistemological obstacle. The misunderstanding of the concept of proportion occurred in students who solved indirect proportion problems by multiplying 6000 by 36000 without understanding the meaning of the multiplication (e.g., M8-11 Figure 4ii). The weakness in the concept of proportion arises because students cannot model it correctly.

In the inverse proportion situation, most students were identified as experiencing epistemological obstacles. This is due to the concept of proportion students attain using the cross-product strategy, $\frac{2}{50} = x = 60$, without understanding the equivalent ratio in inverse proportion (M8-25 in Figure 4). The wrong concept of inverse proportion in prospective mathematics teacher students is solved by using the difference between mileage, $120 - 100 = 20$, $100 - 20 = 80$ kilometres. This indicates that students do not understand the concept of proportion well.

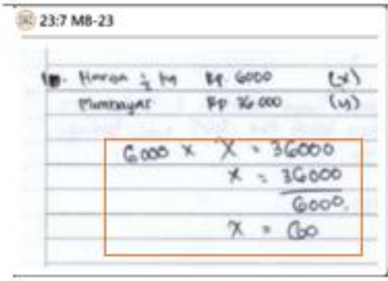
The results of the following interview strengthen this indication:

How do you solve Hamzah's problem (direct proposition)?	M8-11: The way is, ½ kilo is 6000, and Hamzah pays 36000, so you just have to multiply it, Miss, $\mathbf{6000 \times 36000}$, and the result is 216000.
How do you solve the car speed equation (inverse proportion)?	M8-21: I forgot how to do it, the best thing to do is just look at the *difference*, 120-100 becomes 20, then 100-20 becomes 80 km, that is all.

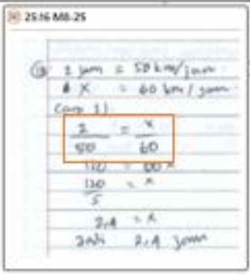
The interview results show that students are familiar with the cross-product strategy with the function notation $a/b = b/c$ which is then solved by the cross-product method. Students cannot model proportion well due to the limited concept of proportion. Most of the students were unfamiliar with the equation $y = m/k$ or $y = m$ because it was not presented in the students' notes document during the lecture, students mentioned that they only guessed the relationship in proportion in solving proportion problems.

Figure 4

Identification of Answers to Proportion Epistemology Problems



(i)

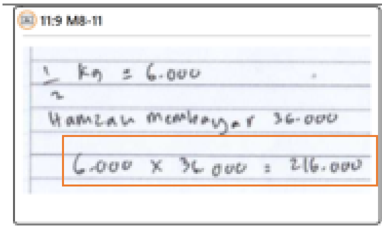


(iii)

Translation
½ kg price IDR 6000 (x)
Paying IDR 36000 (y)

6000 x = 36000
x = 36000/6000
x = 60

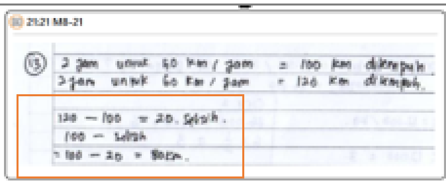
Translation
2 hours = 50 km/h
6x = 6 km/h
Method 1).
2/50 = x/60
120 = 50x
120/5 = x
2,4 = x
So 2.4 hours.



(ii)

Translation
½ kg = 6,000
Hamzah paid 36,000

6.000 x 36.000 = 216.000



(iv)

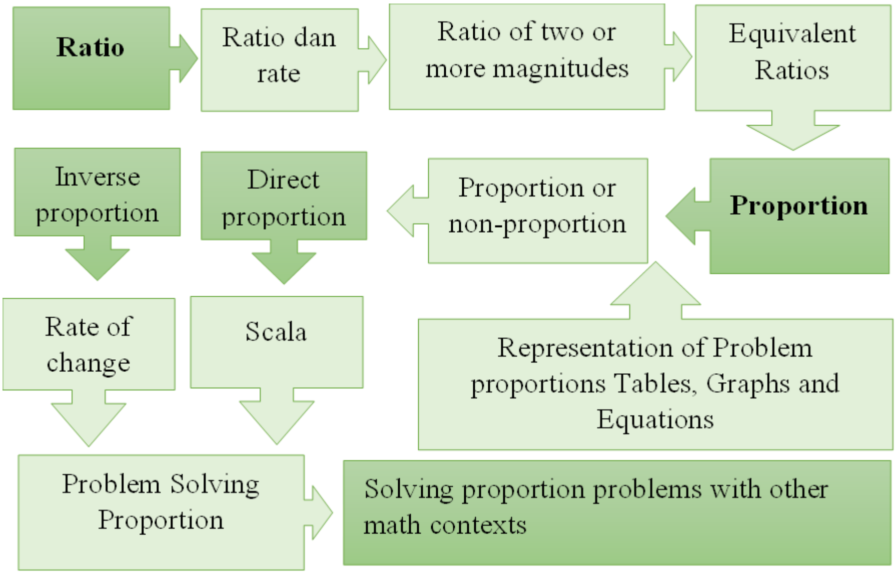
Translation
2 hours for 50 km/h = 100 km traveled
2 hours for 60 km/h = 120 km traveled
120 - 100 = 20. Difference.
100 - difference
= 100 - 20 = 80 km

Hypothetical Learning Trajectory of Proportion Concept

The learning trajectory of pre-service mathematics teachers is depicted in Figure 5. The material of proportion concerning the organization of praxeology must be strengthened with the learning trajectory of ratio, rate, equivalent ratio, proportion, non-proportion, the concept of direct proportion and scale, and solving direct proportion problems. Furthermore, the concept of inverse proportion and rate of change and its solution, tabular representations, graphs, and equations on proportion and proportion solutions are associated with other mathematical contexts such as geometry.

Figure 5

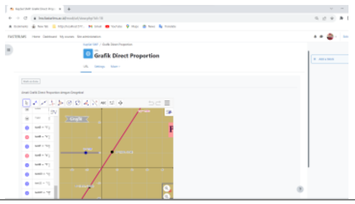
Learning Trajectory of The Proportion Concept

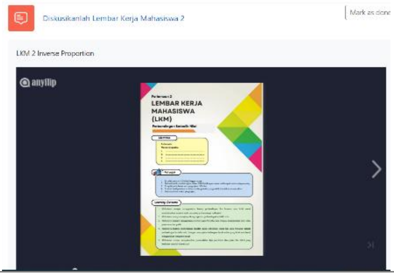
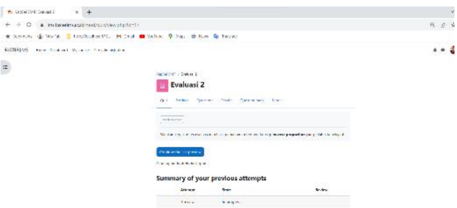


The stages of the theory of didactic situations (TDS), specifically action, formulation, validation, and institutionalization of situations adapted to digital didactic design are designed for digital didactic design based on the concept of proportion. Teaching based on digital didactic design is one of the most effective ways to reduce learning difficulties (Nopriana et al., 2023). The designed digital didactic design includes: (1) subjects and subtopics; (2) forecasting student replies; (3) didactic and pedagogical anticipation; and (4) objects and mathematical skill development. The didactic and pedagogical anticipatory component includes TDS-based learning phases as well as the use of the learning mobile system, Moodle. TDS is used in stages: (a) action situation, presenting problems related to composite functions to stimulate students to think and realize the importance of chain rules in e-learning; (b) formulation situation, directing students to understand the concept of chain rules; (c) validation situation, the process of repairing or strengthening certain concepts if some students have different formulations or appear to be incorrect constructions; and (d) formulation situation, stu The Socratic Questioning technique helps students understand topics at each stage of the alternative didactic design. The didactic design format incorporates didactic and pedagogical anticipation, artifacts, and mathematical skills (refer to Table 2).

Table 2

Design of Digital Didactic Design on the Proportion Concept

Digital Activity	Goals	Mathematics Objects
Pre-class Activity: Action Situation ✓ Analyzing the understanding of proportion in school books and teaching modules  	✓ Students can analyze the understanding of ratio and proportion through the observation of teaching e-modules and school books on e-learning.	✓ Ratio Definition The ratio of two quantities a and b, where a and b are rational numbers, denoted a/b or a:b, and read a to b. ✓ A pure ratio is a comparison of two similar quantities.
✓ Representing proportion with data tables, equations, and graphs using GeoGebra 	✓ Students can represent proportions with data tables, equations, and graphs using GeoGebra. ✓ Students can solve proportion problems in e-learning task design.	✓ Mixed Ratio is the ratio of two different quantities e.g. phi (π), Rate (Speed, discharge, etc.). ✓ Proportion Definition A statement that two given ratios are equivalent. ✓ Proportion Theorem If a,b,c and d are rational numbers and $b \neq 0$ then $a/b = c/d$ if and only if $ad = bc$. ✓ Theorem of Direct Propotition

Digital Activity	Goals	Mathematics Objects
✓ Solving the proportion problem in e-learning task design  		For any rational number a/b and c/d, with a ≠ 0 and c ≠ 0, a/b = c/d if and only if b/a = d/c. ✓ Teorema inverse proportion For any rational numbers a/b and c/d, with a ≠ 0 and c ≠ 0, then a/b = c/d if and only if b/a = d/c.
In-class Activity: ***Formulation Situation*** ✓ Explaining mathematical objects: definitions, theorems, proofs of theorems, proportion problems and solutions ✓ Representing proportion with data tables, equations and graphs using GeoGebra. ***Validation Situation*** ✓ Solve proportion problems in task design with various strategies	✓ Students are able to explain mathematical objects: definitions, theorems, proofs of theorems, problems and solutions in proportion through zemi presentations in class. ✓ Students are able to represent proportions on graphs using the GeoGebra. ✓ Students are able to solve proportion problems in task design with various strategies	✓ Definition of constant of proportionality ✓ If the variables x and y are related by the equation y= kx (k= y/x) then y is proportional to x and k is the proportionality constant of y and x. ✓ Proportion Strategy - *Cross Product Strategy* - *Unite Rate Strategy* - *Fraction Strategy* - *Equation Strategy* - *Building Up Strategy*
Out of class Activity: ***Institutionalization situation*** ✓ Reflecting on understanding of ratio and proportion mathematics objects  	✓ Students are able to solve the concept of proportion on the test.	

Hypothetical Didactical Design

Following the completion of the learning obstacle analysis, a Hypothetical Didactic Design was created, which is a didactic situation design based on the newly identified learning hurdles. According to DDD theory, this didactic situation is one of the learning activities offered utilizing technology integration. Table 5 shows an example of the outcomes of learning barrier analysis, digital didactic design, and didactic anticipation.

Table 3

Hypothetical Digital Didactic Design

Digital didactic situation	Response prediction	Pedagogical didactical anticipation
Action Situation		
Presenting learning objectives as a reference for learning the material.	Students observe the learning objectives as a reference for learning the material.	The lecturer gives directions through WhatsApp group for learning instructions.
Presenting material presented in books taught at school and digital teaching modules on e-learning media through mind mapping to be explored in class.	Students map the material presented in books taught at school and digital teaching modules on e-learning media through digital or manual mind mapping.	Lecturers give feedback to students who have done mind mapping in WhatsApp groups.
Representing proportion with data tables, equations, and graphs using GeoGebra.	Students represent proportion with data tables, equations, and graphs using GeoGebra.	The lecturer provides a situation that requires students to make observations about proportions with data tables, equations, and graphs using GeoGebra media on e-learning media.
Discuss completing some design tasks on the e-worksheet provided by e-learning.	Students discuss completing several design tasks on the e-worksheet provided by e-learning by asking lecturers and friends via chat/telephone / Zoom/gmeet/e-learnings.	The lecturer reminds students via WhatsApp group to discuss on e-learning media by starting to ask questions about the meaning of several design concepts on the e-worksheet with friends via chat/telephone/zoom/gmeet/e-learning.
offline Present mathematical objects: definitions, theorems, proofs of theorems, problems, and solutions on the topic of proportion through Zemi presentations in class. Before presenting them, discuss all the design tasks presented in the e-	offline One of the students presented mathematical objects: definitions, theorems, proofs of theorems, problems, and solutions on the topic of proportion through Zemi presentations in class.	Lecturers provide a situation about proportion with data tables, equations, and graphs using GeoGebra on e-learning media.

Digital didactic situation	Response prediction	Pedagogical didactical anticipation
worksheet again with a group of friends in class.	Students discuss all the design tasks presented on the e-worksheet with a group of friends in class before presenting them by asking friends and the lecturer.	The lecturer gives interesting questions about mathematical objects: definitions, theorems, theorem proofs, problems and solutions. What is the definition of proportion? What rules are used to solve proportion problems? What strategy is used to solve the proportion? How to differentiate direct proportion from inverse proportion? Etc.
Formulation Situation Offline Present each design proportion task on the e-worksheet for each group ratio. 1. proportion 2. proportion and non-proportion 3. direct proportion 4. inverse proportion 5. representation proportion 6. problem solving	Present each completion of the proportion design task on the e-worksheet for each group. Support the solutions of the presenting group with the solutions of the others.	The lecturer invites students to present problem-solving in the design task. The lecturer invites other groups to respond to the presentation of the design text to each group.
Validation Situation Provides conclusions about the definition of ratio, proportion theorem, procedures for solving proportions, problems and proportion solutions based on discussions in completing design tasks.	Several students reached conclusions about the definition of ratio, the proportion theorem, and procedures for solving proportions, problems and proportion solutions based on discussions while completing the design task.	The lecturer invites students to conclude a design task based on the findings of mathematical objects. The lecturer directs the conclusions of several students to scholarly knowledge.
Institutionalization situation Online Reflecting an understanding of the concept of proportion through evaluating questions number 1-10 on e-learning media.	Students work on evaluation questions via e-learning media.	The lecturer invites students to fill in evaluation questions number 1-10 on e-learning media. Lecturers provide feedback to students who have completed the evaluation.

DISCUSSION

Based on the results of this study, lecturers can make a suitable design for their students, demonstrate the praxeology organization of proportion material, and minimize learning obstacles for students. Praxeology organizations can present mathematical materials in different teaching methods, such as visualization and mathematical modeling (Pansell, 2023; Rothbard, 1973). Technology plays a role in understanding mathematical concepts and can be used as a medium for the theoretical construction of different students' understanding of mathematical concepts (Pansell, 2023; Semenikhina et al., 2018).

The digital didactical design (DDD) in this study is proposed to minimize learning obstacles in learning proportion for pre-service mathematics teachers in Indonesia, which can be effective in helping them understand the concept of comparison. The development of appropriate digital didactical designs can facilitate students in making "conceptual changes" and allow teachers to be able to evaluate the actual designs commonly used in classrooms (Amelia et al., 2024; Jahnke et al., 2014), allowing for deeper learning. One example of the success of a design is that it can make students think critically, self-reflect, and have multi-perspectives (Jahnke et al., 2014); moreover, technology acts as a medium so that students can see the meaning of mathematics (Barzel et al., 2005). This implies that good design can facilitate the possibilities of learning.

The practical implications of this research can be an input for lecturers and other researchers to reapply it in school mathematics learning. This research proposes a trajectory of mathematics learning, specifically, how to present the comparison material topics and how to arrange and interrelate topics to achieve a continuous learning sequence. A study from (Clark-Wilson et al., 2014) shows that teachers who focus on pedagogical content knowledge in the learning trajectory enable them to understand knowledge in the subject matter and it is mediated by the teacher's previous knowledge.

CONCLUSIONS

Nam suscipit convallis elit a convallis. Ut cursus odio a justo pulvinar, eget maximus ante imperdiet. Vivamus ac tempus velit. Donec sit amet neque non leo dignissim blandit. Aenean at orci nisl. Nunc faucibus mattis viverra. Duis non vestibulum mi. Vestibulum bibendum leo vel neque ullamcorper, quis tincidunt mi venenatis. Proin cursus aliquet efficitur. Cras augue sapien, tincidunt eu congue id, varius in nunc. Nunc consequat erat non purus elementum sollicitudin. Vivamus lacus turpis, sodales id fermentum in, iaculis ut felis. Fusce lorem metus, dictum eget ullamcorper id, posuere vitae ipsum.

AUTHORS’ CONTRIBUTIONS STATEMENTS

RS carried out conceptualization, developing theory, writing the original draft, editing, and visualization; TH was responsible for reviewing, editing, and developing theory; DD was responsible for the review, formal analysis, and methodology; SP was responsible for result and discussion. SM worked on editing and proofreading the article.

DATA AVAILABILITY STATEMENT

Anyone making a reasonable request to the first author of the article, RS, will be provided with the supporting data for the research findings.

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