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Preservice primary teachers' definitions of the area of 2D figures

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ABSTRACT

Background: Defining mathematical concepts is critical to preservice teacher education, as it reflects their foundational mathematical knowledge and instructional capacity. However, research shows that preservice primary teachers often encounter challenges when formulating robust definitions of geometric concepts. **Objectives:** This study aims to characterise how preservice primary teachers define the concept of area, focusing on identifying the objects and processes that underpin their definitions. Tools from the onto-semiotic approach of mathematical knowledge and instruction are employed for this purpose. **Design:** A qualitative approach was adopted, utilising content analysis to explore the definitions provided by preservice teachers. **Setting and participants:** Data were gathered from 70 preservice primary teachers enrolled in a teacher education program at a Spanish university during the 2020–2021 academic year. All participants were in their third year of study and had completed coursework on geometry and measurement in prior semesters. **Data collection and analysis:** Definitions were collected via a semi-structured questionnaire that tasked participants with defining the concept of area for fifth-grade students. Content analysis was conducted to identify the primary objects, processes, and partial meanings mobilised by participants and to evaluate the alignment between their personal and institutional meanings of an area. **Results:** The findings indicate that most definitions are based on a single partial meaning (e.g., area as space bounded by a closed line). Few participants integrated multiple partial meanings (e.g., additive measurement or multiplicative structures). Definitions that combined all three partial meanings of area tended to align more closely with institutional meanings but often lacked abstraction or generalizability. **Conclusions:** This study underscores the need to strengthen preservice teachers' ability to formulate robust definitions through targeted formative activities

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that enhance their global personal meaning of area. Future research should explore how these definitions develop in real classroom settings to support the creation of effective pedagogical practices better.

Keywords: Onto-semiotic approach; area; definitions; preservice primary teachers.

Definições de professores em formação inicial para o ensino fundamental sobre a área de figuras 2D

RESUMO

Contexto: Definir conceitos matemáticos é um componente crítico na formação de professores em formação inicial, pois reflete seus conhecimentos matemáticos fundamentais e sua capacidade de instrução. No entanto, pesquisas mostram que professores em formação inicial frequentemente enfrentam desafios ao formular definições robustas de conceitos geométricos. **Objetivos:** Este estudo tem como objetivo caracterizar como professores em formação inicial definem o conceito de área, com foco na identificação dos objetos e processos que sustentam suas definições. Para isso, são utilizadas ferramentas do Enfoque Ontossêmico do Conhecimento e Instrução Matemáticos. **Método:** Foi adotada uma abordagem qualitativa, utilizando análise de conteúdo para explorar as definições fornecidas pelos professores em formação inicial. **Contexto e participantes:** Os dados foram coletados de 70 professores em formação inicial matriculados em um programa de formação docente em uma universidade espanhola durante o ano acadêmico de 2020–2021. Todos os participantes estavam no terceiro ano do curso e já haviam cursado disciplinas sobre geometria e medição em semestres anteriores. **Coleta e análise de dados:** As definições foram coletadas por meio de um questionário semiestruturado, no qual os participantes foram solicitados a definir o conceito de área para alunos do quinto ano do ensino fundamental. Foi realizada uma análise de conteúdo para identificar os objetos primários, processos e significados parciais mobilizados pelos participantes, bem como para avaliar o alinhamento entre seus significados pessoais e institucionais de área. **Resultados:** Os resultados indicam que a maioria das definições é baseada em um único significado parcial (por exemplo, área como espaço delimitado por uma linha fechada). Poucos participantes integraram múltiplos significados parciais (por exemplo, medição aditiva ou estruturas multiplicativas). As definições que combinaram os três significados parciais da área tenderam a alinhar-se mais estreitamente com os significados institucionais, mas frequentemente careciam de abstração ou generalização. **Conclusões:** Este estudo destaca a necessidade de fortalecer a capacidade dos professores em formação inicial de formular definições robustas por meio de atividades formativas direcionadas que ampliem seu significado pessoal global de área. Pesquisas futuras devem explorar como essas definições se desenvolvem em contextos reais de sala de aula para apoiar melhor a criação de práticas pedagógicas eficazes.

Palavras-chave: Enfoque ontossêmico; área; definições; professores em formação.

INTRODUCTION

Defining is a distinctive process in mathematical practice, as definitions provide a negotiated foundation for mathematical work and are closely tied to developing and revising new concepts (Kobiela & Lehrer, 2015). The National Council of Teachers of Mathematics (NCTM, 2010) recognises the importance of definitions in mathematics education, emphasising that mathematically competent students understand and use definitions to construct arguments, engage in reasoning, and communicate mathematical ideas. Similarly, some studies highlight that learning relevant definitions should be essential to the mathematics learning process (Avcu, 2022; Miller, 2018; Zaslavsky & Shir, 2005). For example, Zaslavsky and Shir (2005) argue that the importance of definitions in mathematical practice lies in their ability to introduce ideas, describe objects and concepts, identify fundamental properties of mathematical objects, support problem-solving, and aid in proof construction. In this context, the set of definitions used to specify basic mathematical concepts influences the structure of mathematics curricula, the sequencing of mathematical concepts, and, consequently, the way teachers address these concepts in the classroom (Brown, 1999). Despite this, definitions often have a limited presence in school mathematics (de Villiers, 1994; Kobiela & Lehrer, 2015).

Mathematical concept definitions are a fundamental component of the mathematical and pedagogical knowledge of mathematics teachers (Ball et al., 2008; Zazkis & Leikin, 2008), as they reflect a range of logical relationships, such as those between propositions, didactic learning sequences, mathematical connections, or mathematical communication (Leikin & Winicki-Landman, 2001). The diversity of perspectives on the structure of a mathematical definition, along with the challenges associated with it, has led researchers to propose that definitions should be explicitly addressed as part of both preservice and in-service teacher education (Leikin & Winicki-Landman, 2001; Zazkis & Leikin, 2008). This is crucial because teachers play a decisive role in the definitions used in the classroom and the role those definitions play in mathematical activities. However, research on teachers' knowledge of mathematical definitions is limited, with most studies highlighting the challenges associated with this process (Leikin & Winicki-Landman, 2001). Specifically, some studies document the difficulties experienced by both practising and preservice teachers in defining geometric concepts, such as quadrilaterals or polygons (e.g., Avcu, 2022; Fujita & Jones, 2007; Leikin & Zazkis, 2010; Miller, 2018).

Concerning the concept of area, research is scarce and suggests that preservice teachers tend to define an area as “length × width” (Livy et al., 2012), mirroring the definitions given by students. Within this context, this paper aims to characterise the definitions of the concept of area formulated by a group of preservice primary teachers (PPTs). For this purpose, tools from the onto-semiotic approach of mathematical knowledge and instruction (OSA) are employed, allowing an analysis of how PPTs articulate the definition and how they select and present examples to support their definitions.

THEORETICAL BACKGROUND

Definitions in the teaching of geometry

According to Sinclair et al. (2012), definitions are one of the four key “big ideas” in school geometry, alongside diagrams, proofs, and theorems. Definitions are a fundamental tool for geometric exploration, as they influence the understanding of a geometric concept’s properties and act as a bridge between visual representations (images) and verbal descriptions. However, some studies highlight that PPTs often face challenges in this area. For instance, they frequently associate definitions with the physical representations of a concept, without considering that definitions and representations are complementary ways of communicating geometric concepts (Kuzniak & Rauscher, 2005).

Research has underscored the need to study how PPTs construct their own definitions, focusing on identifying the elements that constitute them (e.g., Fujita & Jones, 2007; Miller, 2018), as well as the examples, counterexamples, and non-examples that help distinguish the critical features of the concept being defined (Watson & Mason, 2005). In this sense, mathematical definitions in instructional settings explicitly include examples—whether stated or represented—that facilitate understanding the defined concept. Given this complexity, this paper delves into the definitions formulated by PPTs regarding the concept of area, utilising tools from OSA, which adopts an anthropological and pragmatist perspective on mathematical concepts (Godino et al., 2019).

Onto-semiotic Approach of Mathematical Knowledge and Instruction

Godino et al. (2007) conceptualise the meaning of a mathematical object (e.g., area, perimeter, volume) in terms of the systems of practices undertaken to solve problem situations. A practice is defined as any action or manifestation, whether linguistic or otherwise, performed by an individual to solve problems and communicate the solution to others (Godino & Batanero, 1998; Godino, 2019). These systems of practices encompass operative facets (e.g., actions and procedures) and discursive facets (e.g., propositions, definitions, and arguments), which interact and complement each other. Mathematical definitions emerge as discursive components derived from operative practices as mathematical activity develops (Wilhemi et al., 2007). Language is fundamental in this process, articulating operative and discursive facets. Through language, individuals communicate procedures and results and create and assign meanings to new mathematical objects (Wilhemi et al., 2007). In this way, definitions interact in a complex and recursive manner with problem situations, employed procedures, and established rules, generating new questions and fostering the development of increasingly complex systems of practices (Wilhemi et al., 2007). Definitions evolve and formalise through engagement in mathematical activity, becoming critical tools in concept articulation and understanding.

Godino et al. (2007) distinguish between personal and institutional meanings, attributing operative and discursive practices to individuals or broader institutions. The alignment of personal and institutional meanings significantly indicates student progress in understanding mathematical concepts (Godino et al., 2019). Personal meaning can be further categorised into global personal meaning, encompassing all practices a student can perform concerning a concept, whether explicitly demonstrated or not; declared personal meaning, reflecting the practices explicitly shown by the student when solving problems, regardless of accuracy; and achieved personal meaning, which corresponds to practices aligned with institutional meaning, indicating an understanding consistent with established mathematical standards. Conflicts may arise between declared personal meaning and institutional meaning, highlighting discrepancies in understanding. For example, a student might formally state that a parallelogram is a quadrilateral with two pairs of parallel sides but fail to recognise rectangles, squares, or rhombi as parallelograms due to limitations in their global personal meaning. These conflicts illustrate gaps between formal articulation and deeper operational understanding. In such cases, mathematical definitions play a crucial role in resolving these inconsistencies, enabling precise classification of geometric figures and reinforcing institutional meaning. The use of accurate definitions helps students consolidate their achieved personal meaning, linking their existing knowledge with institutional expectations (Vinner, 1991).

The interplay between personal and institutional meanings is influenced by the practices from which mathematical objects emerge. These objects are embedded within specific contexts and are shaped by personal or social systems of practices. Mathematical activity is inherently complex, as it requires the mobilisation of primary objects, including linguistic elements, problem situations, concepts, definitions, propositions, procedures, and arguments (Caviedes et al., 2024; Caviedes & Pallauta, 2024; Godino et al., 2019). These objects interact dynamically, forming networks that underpin either epistemic configurations (institutional systems of objects) or cognitive configurations (personal systems of objects). The relationships among these objects are further developed through communication, problem formulation, definition, statement, algorithmisation, and argumentation. These processes are analysed through dual facets, including materialisation and idealisation, particularisation and generalisation, decomposition and reification, and representation and signification. Such dualities are fundamental to understanding how students construct mathematical meanings and articulate concepts in practice.

This study considers that definitions involve the cognitive configurations of PPTs, reflecting their personal meanings. As primary mathematical objects, concepts and definitions contribute to articulating area as a referential object. The definition of area aligns with the institutional meaning, which consists of a network of interconnected primary objects (Font et al., 2013). Definitions, in this context, function as expressions of either personal or institutional meaning within the duality of institutionalisation and personalisation (Godino et al., 2017). This framework underscores how PPTs adapt institutional meanings to their own understanding and the specific educational contexts in which they operate.

Definitions associated with the concept of area

The concept of area is deeply rooted in cultural practices and everyday life while also playing a critical role in science and technology (Kordaki & Potari, 1998). This centrality explains why area is mandatory in most primary education curricula worldwide (Tatto et al., 2012). However, as Freudenthal (1983) noted, one of the most striking features of the concept of area is the richness of contexts in which it applies, contrasted with the relative poverty often seen in its teaching. Freudenthal conceptualised area as a magnitude used to measure various surfaces, requiring both length and width for its assignment. He also emphasised that as a geometric object, the area is constructed from mental representations that organise geometric figures, with definitions derived from observable properties. For example, in his work *The Elements*, Euclid defined a line as “a length without breadth” and a surface as “that which has length and breadth only.” These definitions, focusing on absence (“without” or “only”), separate the concept from the mental representation of physical objects (Freudenthal, 1983). Thus, the definition of area is conditioned by its two-dimensional nature, dependent on both length and width. Area can also be defined in terms of the procedures used to measure it or as a function over sets (Freudenthal, 1983), such as the set of two-dimensional surfaces bounded by a closed line and the set of positive real numbers. This perspective is particularly relevant in geometry and mathematical analysis, where methods like integral calculus allow for the determination of areas of complex or irregular surfaces, transcending the limitations of purely geometric procedures.

Sarama and Clements (2009) complement this view by defining area as the amount of two-dimensional surface enclosed within a boundary. To fully grasp the two-dimensional nature of area, they highlight the need to understand length and its measurement, as calculating area involves establishing a multiplicative relationship between two linear dimensions (Barrett et al., 2017; Sarama & Clements, 2009). This multiplicative relationship compels the understanding of concepts, properties, and procedures. Conceptually, area involves assigning a numerical value to a given surface, achieved by spatial structuring, which entails covering a surface with squares aligned in rows and columns. Units of measurement must be reproducible and divisible without gaps. The properties of area include accumulation and additivity, which allow for the composition and decomposition of figures into surfaces with equivalent areas; conservation, enabling the cutting and rearranging of a surface without altering its total area; and transitivity, facilitating the comparison of two surfaces’ areas by referencing a third. Procedures for understanding area include equitable partitioning, which involves dividing a surface into equal parts either physically or visually, and unit iteration, which covers a surface with a two-dimensional unit without overlaps or gaps.

Building on previous studies, Caviedes et al. (2021) identified three partial meanings (Pm) of area: Pm1, which defines area as the space enclosed by a closed line; Pm2, which defines area as the two-dimensional units covering a surface; and Pm3, which defines area as the product of two linear dimensions. These partial meanings are interrelated and are constructed progressively and systematically. According to Caviedes et al. (2021), the institutional meaning of area in primary education is determined by articulating these three partial meanings and the complexity inherent in their integration. Consequently, the definition of area as a referential object in the educational context is shaped by its two-dimensional nature, whether covering a surface with measurement units or calculating it using formulas.

METHODOLOGY

This study is grounded in an interpretative paradigm and follows a qualitative approach (Cohen et al., 2007), employing content analysis with a deductive coding strategy. The analysis is based on an epistemic configuration of the concept of area, adapted from Caviedes et al. (2021), to identify how the cognitive configuration emerging in the definitions proposed by PPTs relates to the institutional meaning of area (Godino et al., 2019). Institutional meaning is the definition constructed through the coordination of the partial meanings of area, defined as: (Pm1) area as space delimited by a close line; (Pm2) area as the number of two-dimensional units that cover a surface; and (Pm3) area as a two-dimensional linear product. It is important to note that institutional meanings are dynamic and can vary depending on the reference institution or the group formulating the definition. For example, a formal definition of area in a school context may differ from one used in an academic setting, reflecting differences in mathematical practices across institutions. This variability implies that what is considered a formal definition in one context might not align in another, creating multiple interpretations and approaches to teaching the same concept.

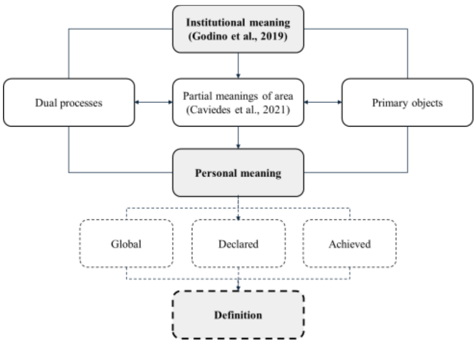
Deductive coding facilitates the identification of objects emerging from the cognitive configurations of PPTs, the personal meanings they associate with area as a referential object, and the processes involved. Specifically, the analysis considers the process of representation/signification, which focuses on how PPTs attribute content or meaning to a representation of area. This process explores the relationship between a geometric figure or representation and the meaning assigned to it. Additionally, the analysis examines the process of idealisation/materialisation, which centres on how PPTs transform abstract concepts into concrete representations, such as translating mathematical formulas into graphical representations.

Within this framework, the partial meanings (Pm1, Pm2, and Pm3) are identified based on the primary objects mobilised by PPTs to construct their definitions of area. These partial meanings manifest as expressions of the personal meanings of PPTs and vary depending on the objects they use to construct their personal definitions. In line with Godino et al. (2017), personal meaning is expressed at three levels: global meaning, encompassing the set of practices and knowledge that PPTs possess about area, even if not explicitly expressed in their definitions; declared meaning, reflecting the practices explicitly demonstrated in PPTs' definitions; and achieved meaning, representing practices and knowledge aligned with institutional meaning and present in PPTs' definitions. Thus, the definitions provided by PPTs reveal these personal meanings.

The mobilisation of partial meanings triggers the emergence of dual processes, which, in turn, influence how these meanings are selected and utilised, potentially enriching or limiting the definitions offered by PPTs. This interplay suggests that partial meanings not only guide the emerging processes but that dual processes also affect how PPTs mobilise and articulate these meanings, ultimately shaping the complexity and precision of their definitions. In particular, the dual processes of representation/signification and idealisation/materialisation integrate with the mobilised partial and personal meanings. For instance, the idealisation/materialisation process becomes evident when PPTs transform abstract concepts, such as the formula for area, into concrete representations, like the iteration of square units or the measurement of surfaces. These processes enable PPTs' definitions to explicitly convey how they conceptualise and operationalise area based on their global personal meaning. The interaction between partial meanings and dual processes contributes to the explicit articulation of PPTs' definitions. This dynamic highlights the interconnectedness of conceptual understanding and procedural practices, emphasising the importance of examining the cognitive and epistemic configurations shaping PPTs' understanding of area. Figure 1 illustrates the analysis of definitions formulated by PPTs, grounded in the theoretical constructs of the OSA approach.

Figure 1

Analysis of definitions based on the constructs of the onto-semiotic approach of mathematical knowledge and instruction [Compiled by authors]



Discrepancies between declared personal meaning and institutional meaning reflect the depth or superficiality with which PPTs approach partial meanings and how they mobilise them in their definitions (jointly or independently). These discrepancies may also indicate potential limitations in the content or pedagogical knowledge of PPTs, which impacts their ability to formulate more robust definitions. Given this, we understand that PPTs adjust their declared personal meaning to make it understandable in a primary classroom, simplifying or modifying objects from their global personal meaning to facilitate student understanding. This adjustment reflects the PPTs' ability to select elements from their global personal meaning and mobilise them in their definitions and their pedagogical skill in adapting this knowledge to an effective teaching context. Consequently, the definitions provided by PPTs also illustrate their capacity to apply this knowledge in the classroom, making the educational functionality of their definitions explicit.

Instrument and procedure

A semi-structured open-ended questionnaire was designed (Bailey, 1994) and validated by experts in mathematics education and in-service and preservice teachers. The questionnaire was administered individually and in written form to a group of 70 PPTs enrolled in the third year of a primary education degree program during the 2020–2021 academic year at a university in Spain. In previous courses, these students had received instruction on various geometric concepts, as well as on magnitudes and their measurement (see Table 1). The questionnaire, part of a graded activity, consisted of eight tasks. The first five tasks required the application of different types of procedures. Task 6 asked participants to classify a set of statements, while Task 7 required them to define the concept of area for fifth-grade students. Task 8 involved analysing students' responses to a task related to area. The course instructor administered the questionnaire online due to the COVID-19 pandemic, and the PPTs were given one week to complete it. For this study, the written responses corresponding to Task 7 were analysed. This task presented the following problem: "If you had to introduce the geometric concept of area in fifth grade, how would you define area?"

Table 1

Courses completed by PPTs during their first three years of training

Academic year	Courses	Content related to the partial meanings of area

1	Geometry for Understanding Space	Basic geometric constructions: planar representation of space. Measurement for understanding the environment: concept of magnitude.
2	Curriculum Organisation: Space and Shape	Knowledge of plane shapes: lines, polygons, and puzzles. Classification of basic geometric elements. Relationship between 2D and 3D: orientation in planes and space. Mazes, paths, and coordinates. Use of various materials for teaching geometry.
3	Mathematical and Didactical Analysis of Primary School Mathematics Curriculum	Geometry: geometric transformations, symmetry, similarity. Measurement: magnitudes and units, measurement procedures.

ANALYSIS

The analysis was conducted using the software MAXQDAplus, which facilitated the organisation and categorisation of data. A system of a priori categories was established based on the primary objects and processes of OSA and the epistemic configuration proposed by Caviedes et al. (2021). The definitions the PPTs provided allowed the inference of a global personal meaning derived from the mobilisation and emergence of objects associated with the three partial meanings of area described by Caviedes et al. (2021) (see Table 2). The emergence of partial meanings enabled the identification of how PPTs' definitions promoted two key processes: representation/signification and idealisation/materialisation. The first process, related to the duality of expression-content, implies that a representation not only reflects an object but also communicates its internal structure (Font & Rubio, 2017). The second process, associated with the duality of ostensive-non-ostensive, refers to how mathematical objects, though typically imperceptible, are utilised in mathematical practices through their ostensive representations, such as notations or graphs (Font & Rubio, 2017).

During the analysis, three types of definitions formulated by PPTs were identified: (1) definitions involving two dual processes; (2) definitions involving a single dual process; (3) definitions that do not involve dual processes. These definitions allow for the differentiation of cases in which PPTs mobilised objects from their global personal meaning that aligned with the institutional meaning of area, providing a deeper understanding of why some PPTs' definitions were robust while others appeared superficial.

Table 2

Emerging primary objects in PPTs' definitions. (Adapted from Caviedes et al. 2021)

Primary object	Descriptor
Procedures	Pm2 - Iterating two-dimensional units of measurement to compare and measure areas
	Pm1 - Using isometries (e.g., rotation, translation).
	Pm2 - Measuring areas as an additive process by counting units and/or subunits covering the surface
	Pm1 - Decomposing two or more surfaces graphically or visually in a convenient way
	Pm2 - Using the multiplicative structure of rectangular area models.
Properties	Pm1 - Conservation.
	Pm2 - Covering property of measurement units (reproducible and divisible without gaps when covering the surface with units or their fractions).
Definitions / Concepts	Pm2 - Spatial structuring as the alignment of a surface with squares in rows and columns.
	Pm3 - The magnitude of area assigns a quantitative meaning to a surface, expressed as the product of two lengths ($a \times b$).
	Pm2 - Square units express the bidimensional nature of area as the product of two lengths.
	Pm2 - Non-standard measurement units correspond to a plane region bounded by a square with side length U.
	Pm2 - The two-dimensional measurement unit relates to the distance unit by convention. For any distance unit (U), the area is measured in the corresponding squared unit (U^2).
	Pm3 - Perimeter is a 1D magnitude, corresponding to the distance measured around the boundary of an object.
	Pm3 - The measurement unit is a fixed quantity of a physical magnitude that enables the standardised expression of area.
	Pm3 - A surface is the property of objects that allows them to be measured, possessing both length and width. Area can be measured if it makes sense to describe the objects as wide or narrow.

	Pm1 - The quantity of surface corresponds to the extent occupied by a closed space (planar figure-polygon).
Linguistic Elements	Pm1 - Geometric representations using convenient decompositions to compare, estimate, and/or calculate surface quantities.
	Pm2 - Graphical representations using grids and partitions into congruent figures (squares and/or triangles) for units of measurement and surfaces.
	Pm3 - Symbolic (numerical/algebraic) representations: the set of positive real numbers ($\mathbb{R}^+$) for counting square units or summing areas; for indirect area calculation.

Definitions involving two dual processes

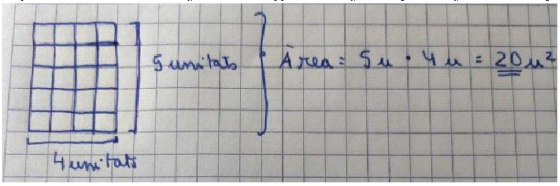
Figure 2 illustrates the mobilisation of a representation/signification process by PPT 55, who employs a 4×5 grid to assign content (area) to a rectangular surface. This approach assigns a multiplicative meaning to the row-and-column structure of the area model. The use of the grid highlights the emergence of Pm2, as PPT 55 defines area as the number of two-dimensional units (squares) that cover a surface. The grid allows inferring the objects of the PPT's global personal meaning—using the grid as a method for calculating area—and translates this into their definition. We infer that the declared personal meaning does not fully align with the institutional meaning of area, suggesting that PPT 55's global personal meaning is limited. The rigid use of the grid (without delving into the relationship between the measurement units and the general area formula) shows a superficial approach to measurement through squares, which limits the conceptualisation of area as the product of two linear dimensions (Pm3). While the PPT uses the grid as a visual tool to support their definition, their exclusive focus on covering the surface with squares, without fully integrating the 4×5 formula, results in an incomplete definition based on numerical examples.

A process of idealisation/materialisation is identified, as PPT 55 evokes the properties of measurement units, using the grid to materialise the concept of area as a measurable space with square units. This materialisation gives tangible meaning to the area formula but remains confined to visual and numerical examples, revealing a difficulty in generalising the formula to more complex shapes. The analysis suggests the emergence of the concept of surface as a two-dimensional characteristic enabling the measurement of objects through length and width, along with the use of square units as a standard of measurement in the plane. These objects support the identification of the three partial meanings. Although PPT 55 provides a functional definition in an educational context, it does not fully address the components necessary for the definition to be robust and aligned with institutional meaning. The conflict arises because, while the approach is useful for specific situations (such as measuring areas with regular grids), it does not adequately represent the more abstract and general nature of the concept of area, which involves formulas and the understanding of area as a multiplicative magnitude. This conflict between the PPT's declared personal meaning and the institutional meaning of area explains the observed limitations in their definition.

Figure 2

PTT 55's definition [Compiled by authors]

PPT 55: *First, I would define it as the amount of space or surface covered by a flat (two-dimensional) figure. Area is measured in square units of fixed size, such as square centimeters, square inches, square miles, etc. Thus, if we draw a figure on a plane, to find its area, we count how many squares of a given size will cover the region inside the polygon. For example, consider the following case of a square formed by these unit (5x4):*



In this case, you can count the squares and obtain 20; therefore, the area is 20 square units. However, this method can be inefficient if the rectangle has larger dimensions or if the units are smaller... in such cases, multiplication can be used, 5 x 4, as there are 4 rows of 5 squares.

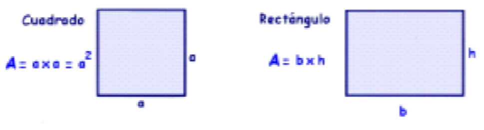
The definition provided by PPT 62, shown in Figure 3, shows a representation/signification process as the PPT uses a square and a rectangle, relying on the formulas a^2 y $b \times h$, respectively, to assign content (area) to these figures. However, the PPT does not consider the square to be a particular case of a rectangle. Using these formulas indicates that PPT 62's global personal meaning is connected to area as a magnitude calculable through algebraic expressions—in this case, the product of two linear dimensions. Nevertheless, there is an observable difficulty in fully aligning their declared personal meaning with the institutional meaning of area, which restricts the generalisation of their definition beyond these specific examples. The conflict between the PPT's declared personal meaning and the institutional meaning is evident in their definition, as the PPT employs formulas without evidencing a deeper understanding of their elements. The definition focuses on specific cases, whereas the institutional meaning encompasses a generalisable magnitude applicable to various geometric shapes and contexts.

Simultaneously, we can infer the presence of an idealisation/materialisation process because the PPT uses surface measurement units (two-dimensional) to materialise the concept of square units. The representation of the square and rectangle suggests an achieved personal meaning connected to area as a bounded and enclosed space, as the formulas used refer to the entirety of each figure's internal space. Pm1 becomes apparent through the geometric examples, where area is defined by the boundaries of the figures. The application of the formulas a^2 y $b \times h$ reflects a global personal meaning linked to area as the product of two linear dimensions. PPT 62 does not explore the relationship between dimensions and formulas in depth, limiting him/herself to applying specific formulas to particular shapes without attempting to generalise this concept to other geometric forms. Consequently, PPT 62 mobilises Pm1 and Pm3, but their definition remains confined to simple figures. This highlights a conflict between their declared personal meaning and the institutional meaning, which impacts their ability to generate a more comprehensive and institutionally aligned definition of area.

Figure 3

PTT 62's definition [Compiled by authors]

PPT 62: *... we would define area as the space or surface occupied by a bounded figure, that is, the interior part of the figure. Additionally, we would clarify that area is expressed in measurement units called surface units (for example, square meters, square decameters, square kilometers, square centimeters, etc.). Measurement units are always raised to the power of 2 because they represent two-dimensional figures. I would use examples of how to calculate the area of various geometric figures and also employ counterexamples to help students understand that area is not the same as perimeter.*



The analysis of PPT 7's definition (Figure 4) reveals an idealisation/materialisation process, as the PPT validates the product formula using the rectangular model of multiplication. This process materialises the formula for the area of a rectangle by structuring the rows and columns that compose the figure. Additionally, the emergence of the concept of magnitude is evident, understood as a property of planar figures that enables their measurement, alongside the concept of a unit of measurement, represented by a region of the plane bounded by a square covering the rectangle and interpreted as a fixed quantity of a physical magnitude. From this definition, we can infer a global personal meaning associated with Pm3.

The structuring of rows and columns in the rectangular multiplication model also reflects a representation/signification process in which the PPT assigns the content of area to the rectangular surface and gives a multiplicative meaning to the arrangement of figures in space, suggesting a global personal meaning associated with Pm2 and, to a lesser extent, with area as the internal space of a figure linked to Pm1. This aspect of the definition is less explicit because the PPT's definition focuses on the multiplicative model without delving into the geometric notion of area as a magnitude applicable to various shapes. We infer that the PPT's global personal meaning limits their ability to generalise their declared personal meaning beyond this specific approach and, consequently, align it more closely with the institutional meaning of area. Although the PPT's definition is grounded in a global personal meaning of area as the product of two linear dimensions, their reliance on the rectangular model prevents them from exploring how area applies to more complex or diverse figures. This limited perspective affects their ability to formulate a more robust and adaptable definition suitable for other contexts.

Figure 4

PTT 7's definition [Compiled by authors]

PPT 7: *magnitude of a bounded surface, the interior size of a figure. I would teach examples... one can start with the rectangular model of multiplication. By fifth grade, students will likely have memorized multiplication tables and can see those tables represented as rectangular figures, where to find the number of parcels (or squares of chocolate, or pieces, etc.), they only need to multiply one side by the other. This action consists of understanding the interior of the figure. Gradually, the shape is varied to work on the area of any figure.*

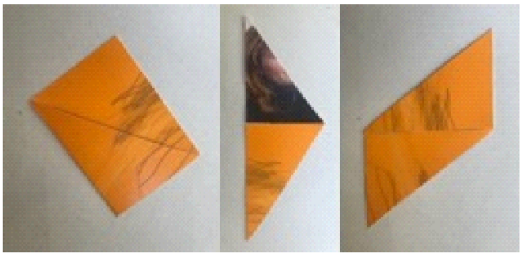
Definitions involving a single dual process

The definition provided by PPT 11 (Figure 5) reveals the emergence of a representation/signification process. PPT 11 assigns content (area) to a polygonal figure, interpreting it as a bounded space that can be decomposed into simpler figures. Additionally, the PPT attributes an operational content to the figure, implying that the area can be partitioned into congruent units. The definition further suggests introducing the concepts of isometry and fraction, with the latter associated with the "part-whole" relationship, enabling the PPT to relate area to the subdivision of a figure into congruent parts. Manipulative representations and reference to the conservation of area imply a global personal meaning linked to the concept of surface and Pm1. However, the focus on perimeter and the absence of references to mathematical formulas indicate that PPT 11's global personal meaning is limited to concrete examples and does not extend to more complex or diverse figures. This limitation prevents the declared personal meaning from fully aligning with the institutional meaning of area due to the constraints of the objects mobilised in the definition. PPT 11 chooses to emphasise the decomposition of geometric figures and the conservation of area, concepts that are effective for explaining area through concrete examples in an educational context. For instance, by mentioning the possibility of decomposing a rectangle into a triangle and a parallelogram while maintaining the same area, the PPT introduces a manipulative representation that supports teaching and defining the concept. However, the definition relies exclusively on physical manipulations and omits the mathematical formulas integral to the institutional meaning of area, which requires a more robust understanding of the concept.

Figure 5

PTT 11's definition [Compiled by authors]

PPT 11: *is the space occupied by a figure on the plane. It is important to understand that the shape of the area can change. To maintain the same area, the shape does not need to remain the same; we can decompose it, altering its shape while keeping the measurement of the area constant. It will occupy the same space but in a different form. For example, we can modify the perimeter by decomposing a rectangle along one of its diagonals, creating a triangle and a parallelogram with the same area. This is not the only way to decompose a figure. We divide/partition the figure, and its area → area as a fraction. To divide or partition → isometries.*



In the case of PPT 34 (Figure 6), the definition of area as "the space that a figure occupies on a plane," illustrated through the example of tiling a classroom, reflects an idealisation/materialisation process. Here, area is conceptualised through the iteration of measurement units (tiles) that cover a surface. This approach materialises the idea of area as a measurable space in square units, suggesting the emergence of procedures related to decomposing surfaces into congruent units. The example of using tiles to cover the classroom's surface indicates a global personal meaning linked to Pm2.

In their definition, PPT 34 explicitly states that to determine the number of tiles required, the surface area of the classroom and each tile must first be known, implying an achieved personal meaning connected to the concept of square units as the two-dimensional measure of area. The tiling process establishes an equivalence between the classroom's surface and the area covered by the tiles, providing evidence that the PPT recognises area as a bounded space. Although the limits of the surface are not explicitly mentioned in the definition, the notion that the area is delimited by the classroom's boundaries suggests a global personal meaning associated with Pm1. This meaning aligns with the measurement of area through the iteration of measurement units (tiles). However, the declared personal meaning still lacks particular essential objects (e.g., geometric properties, formulas derived from the general area formula) required to align fully with the institutional meaning of area. The institutional meaning entails a more abstract understanding of area as a two-dimensional magnitude, extending beyond concrete applications like tiling. The absence of these objects in the definition limits the PPT's ability to conceptualise area as a generalisable and abstract magnitude.

Figure 6

PTT 34's definition [Compiled by authors]

PPT 34*: to explain what area is, I would start with an example. Imagine that we want to tile the classroom floor with new tiles. To determine how many tiles we need to buy, we first need to know the surface area of the classroom and the area of each tile. Based on this, we can calculate the number of tiles required. Thus, area is the space that a figure occupies on a plane, and its unit of measurement is square meters (m^2)*

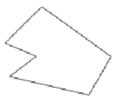
The definition provided by PPT 65 (Figure 7) shows the emergence of a representation/signification process. In this definition, the PPT assigns content (area) to a polygonal figure and attributes meaning to area as a closed space that can be measured. A notable aspect of this definition is the use of a counterexample: the PPT draws a concave and irregular surface, suggesting that their global personal meaning extends beyond simple polygonal shapes to include more complex surfaces, which indicates that PPT 65's declared personal meaning of area is not confined to conventional geometric representations. This counterexample reflects a global personal meaning associated with Pm1 and the concept of surface as a characteristic of objects that allows for measuring length and width, implying a definition of area as a two-dimensional magnitude. The explicit reference to the impossibility of calculating the area of non-closed figures, illustrated by an irregular open polygon drawing, reinforces this understanding, which makes us infer that the declared personal meaning mobilises a broader range of objects from the global personal meaning, such as complex shapes and the recognition of the geometric boundaries required for area calculation.

Despite these strengths, the definition has limitations. While it encompasses a variety of geometric shapes, it does not address more abstract aspects of the area concept, such as the application of formulas for calculating the area of different figures. For instance, the PPT does not mention specific formulas applicable to particular geometric figures or explain how these formulas could be used to measure areas. This omission suggests that, although the global personal meaning includes a variety of figures, the declared personal meaning lacks the necessary objects (e.g., geometric properties) to fully align with the institutional meaning.

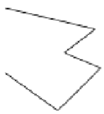
Figure 7

PTT 65's definition [Compiled by authors]

PPT 65*: is the surface found within boundaries; therefore, we could say it is the surface of a closed space. Additionally, this surface is flat (two-dimensional). We can calculate the area of polygons because they are closed spaces. For example:*



However, we CANNOT calculate the area of surfaces that are not bounded. For example:



Definitions that do not involve dual processes

The definition provided by PPT 56 (Figure 8) does not indicate the presence of dual processes. The statement: “area is the surface occupied by something” suggests a global personal meaning associated with Pm1 (area as a space bounded by a closed line), allowing the inference of an achieved personal meaning related to area as the space occupied by a two-dimensional object. Although the PPT proposes a definition linked to the concept of surface, there is no explicit reference to the idea of a closed or internal space within a figure. The definition “surface that something occupies” does not clarify the need for specific geometric boundaries to calculate the area of a figure, which leads us to infer that the global personal meaning of the PPT lacks the objects necessary to construct a more robust definition of area. The PPT’s definition lacks sufficient precision because it omits explicit references to two-dimensionality and the delimitation of a figure. This aspect suggests that the declared personal meaning does not align with the institutional meaning of area. This lack of precision prevents the definition from being translated into a robust formulation of the concept of area, indicating that the PPT has not developed a sufficiently comprehensive declared personal meaning of area as a two-dimensional magnitude applicable to various geometric contexts.

Figure 8

PTT 56’s definition [Compiled by authors]

> **PPT 56:** to *introduce the concept of area in a fifth-grade classroom, I would define it as follows: area is the surface that something occupies.*

RESULTS

The definitions provided by PPTs reveal how they select and mobilise objects from their global personal meaning associated with the three partial meanings of area (Caviedes et al., 2021). The declared personal meaning reflects the mobilised objects from the global personal meaning, adjusting them in a specific educational context and adapting the definition for classroom teaching purposes. Table 3 indicates that the most common meaning in the PPTs’ definitions is Pm1, where area is defined as a space bounded by a closed line. PPTs who select objects from this partial meaning tend to introduce geometric examples and manipulations, explicitly emphasising the concept of surface as the extent occupied by a flat figure. For PPTs whose definitions align with Pm2, they select objects from their global personal meaning related to measuring area through grids and counting square units. These definitions highlight the structure of area as the result of summing two-dimensional units, suggesting a global personal meaning centred on the iteration of these units over a surface. When formulating their definitions, PPTs adjust this notion to make it more accessible to students, often using visual or manipulative examples to simplify the process. PPTs frequently employ grids to illustrate how square units cover a surface, reinforcing the concept of area as a measure obtained by accumulating equivalent units. This adaptation to an educational context reflects how PPTs adapt their declared personal meaning by mobilising objects that facilitate the definition. Finally, PPTs mobilising objects from their global personal meaning linked to Pm3 often use formulas to express area as the product of two linear dimensions. These definitions reflect the direct application of formulas, while the declared personal meaning adjusts these objects to explain the multiplication process for calculating area. This adjustment involves precise notations and simplifications tailored to the educational level.

The analysis highlights how the PPTs' global personal meaning influences the selection and mobilisation of objects that shape their declared personal meaning and how these meanings are reflected in their definitions. When PPTs mobilise more than one partial meaning (Pm1, Pm2, or Pm3), their definitions tend to be more robust and precise, aligning more closely with the institutional meaning of area. However, when the selection and mobilisation of objects are limited to a single partial meaning, the definition may not fully align with the institutional meaning, suggesting a possible influence of a restricted global personal meaning of area.

Table 3

Emerging Meanings in PPTs' Definitions (N=69)

Partial meaning of area	**Frequency**
Pm1 - Area as a space bounded by a closed line	69
Pm2 - Area as the number of two-dimensional units covering a surface	9
Pm3 - Area as the product of two linear dimensions	24

Pm1 and Pm3	16
Pm1 and Pm2	7
Pm2 and Pm3	3
Pm2 and Pm1	7
Pm1, Pm2, and Pm3	3

Table 4 highlights that most PPTs use the representation/signification process, which involves attributing content (area) to a polygonal surface, giving it meaning as a closed space that can be measured. Based on Pm1, this process reflects that PPTs select objects from their global personal meaning associated with area as a bounded two-dimensional surface. However, their definitions often remain limited to a simple conception of area, focusing on closed surfaces without progressing toward a more complex representation of the concept.

A minority of PPTs engage additional representation/signification or idealisation/materialisation processes, allowing them to select objects from more than one partial meaning of area, thereby producing more robust definitions. When PPTs mobilise more than one partial meaning, they demonstrate a stronger ability to align their achieved personal meaning with the institutional meaning of area. This enables them to formulate definitions that incorporate formulas, multiplicative structures, and additive measurement. PPTs who select and mobilise objects from their global personal meaning connected to all three partial meanings of area tend to align their achieved personal meaning more closely with the institutional meaning. These PPTs successfully connect the iteration of measurement units to a more abstract definition of area, incorporating geometric formulas that extend beyond conventional examples. The analysis also shows that idealisation/materialisation processes only emerge when PPTs select objects from two or more partial meanings. In such cases, PPTs materialise the concept of area by iterating measurement units, connecting area to the use of geometric formulas, and enhancing their ability to offer well-rounded definitions. Conversely, PPTs who focus solely on objects from Pm1 rely on more elementary representation/signification processes, such as decomposing polygonal surfaces, which limits their ability to present broader and more detailed definitions of area.

Table 4

Emerging Meanings and Processes in PPTs' Definitions (N=69)

Emergent Processes	Descriptor	Partial meaning	Frequency
Idealisation-Materialisation	(1) The emergence of the unit iteration procedure (standard and non-standard) and its properties materialises the idea of area as a measurable space in square units.	Pm1 and Pm2	1
		Pm1, Pm2, and Pm3	2
	(2) The emergence of the product formula procedure, through the rectangular multiplication model, materialises the rectangle formula via row-and-column structuring.	Pm 2 and Pm 3	1
Representation-Signification	(1) Content is attributed to a square polygonal surface, giving meaning to its property as a bounded/closed space that can be decomposed into other shapes.	Pm1	1
	(2) Content is attributed to a polygonal figure, giving meaning to its property as a bounded/closed space with a measurable surface extension.	Pm1	26
		Pm3	2
		Pm1 and Pm2	6
		Pm1 and Pm3	17
		Pm1, Pm2, and Pm3	1
	(3) Content is attributed to a rectangular surface, giving multiplicative meaning to the row-and-column structure of the area model.	Pm1, Pm2, and Pm3	3

DISCUSSION AND CONCLUSIONS

This study aimed to characterise the definitions of the concept of area formulated by a group of PPTs. The results indicate that the most robust definitions emerge when PPTs select and mobilise objects from their global personal meaning associated with the three partial meanings of area: (Pm1) area as space delimited by a close line; (Pm2) area as the number of two-dimensional units that cover a surface; and (Pm3) area as a two-dimensional linear product. However, the selection of these objects is not consistent across all definitions. In many cases, the achieved personal meaning reflected in the PPTs' definitions aligns with one or more partial meanings but rarely integrates all three coherently. Robust definitions, where objects from all three partial meanings are mobilised together, approximate the institutional meaning of area but do not encompass all its aspects, suggesting a potential mismatch between the PPTs' declared personal meaning and the institutional meaning, resulting in definitions that, while operationally functional, remain incomplete.

The analysis also highlighted that limitations in PPTs' ability to mobilise a higher number of primary objects may stem from insufficient mathematical content knowledge or difficulties in articulating these elements within an instructional context, which affects the quality of the definitions PPTs provide. Consequently, the global personal meaning they possess not only indicates what they know but also what they should know to mobilise a broader range of objects and processes in their definitions. The ability to select, articulate, and adapt these objects effectively in a definition is directly linked to the extent and depth of their global personal meaning. Therefore, strengthening this aspect is crucial for enabling preservice teachers to develop more accurate definitions that reflect theoretical knowledge and classroom application.

We also observed a bidirectional relationship between the mobilisation of partial meanings and the emergence of the dual representation/signification and idealisation/materialisation processes. As PPTs mobilise multiple partial meanings, more complex processes emerge, enriching and strengthening their definitions. Dual processes such as materialising abstract concepts into concrete representations (e.g., using a grid to explain area) act as a "bridge" that translates abstract elements of global personal meaning into more explicit and accessible representations. However, potential discrepancies between the declared personal and institutional meanings reflect the depth or superficiality with which PPTs engage with these partial meanings and how they mobilise them jointly or independently in their definitions.

Using the configuration of objects and processes as a theoretical-analytical framework made it possible to articulate the inherent complexity of defining area in teaching-learning contexts (de Villiers et al., 2009). In agreement with previous research, the process of defining not only involves describing objects and concepts but also identifying mathematical properties (Zaslavsky & Shir, 2005), communicating ideas clearly and effectively (NCTM, 2010), and coordinating various representations (Sinclair et al., 2012). This complexity is crucial in mathematics learning, as definitions require the coherent articulation of multiple interrelated objects and processes that address this complexity. Thus, defining an integral part of initial teacher education is an essential process (Zaslavsky et al., 2003), as it enables preservice teachers to develop a richer, more integrated global personal meaning that can lead to more comprehensive definitions.

This study provides an initial exploration of the process of defining area within the context of teacher education. However, using a semi-structured questionnaire to collect PPTs' definitions represents a limitation, as it does not allow observing how PPTs adjust their definitions during direct interactions with students. The lack of real-time feedback limits PPTs' opportunities to refine their definitions according to specific classroom needs. Future research could address this limitation by investigating the process of defining in real teaching settings, offering a more comprehensive view of how PPTs adapt their definitions to contextual and didactic demands. Additionally, it would be relevant to explore in greater depth how PPTs develop their global personal meaning of area and how it evolves into an achieved personal meaning that manifests in more precise and complete definitions.

AUTHORS' CONTRIBUTIONS STATEMENTS

S.C., G.D.G., E.B., and L.P.F. contributed to the conceptualisation of the idea, the development of the theory, and data analysis and actively participated in the discussion of the results. S.C., G.D.G., E.B., and L.P.F. reviewed and approved the final version of the manuscript.

DATA AVAILABILITY STATEMENT

The data associated with this study will be made available upon request.

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