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Trigonometric knowledge mobilised by students in an evaluation after a pedagogical intervention

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ABSTRACT

Background: Poor students' performance on large-scale assessments prompted an intervention in a high school. This study investigates the trigonometric knowledge mobilised by students who participated in a trigonometry-focused hypothetical learning trajectory (HLT) when taking state-wide end-of-high-school exams. **Objectives:** To identify the trigonometric knowledge mobilised by students after engaging in an HLT and to assess its impact on students' performance in large-scale assessments. **Design:** A mixed-methods study, employing both quantitative and qualitative data. Data was collected from 2019 to 2023 using digital platforms and a doctoral research protocols. **Setting and Participants:** A public high school in Linhares, Espírito Santo, Brazil. Data was collected from 14 high school students. **Data Collection and Analysis:** Data were analysed in three phases: pre-intervention, during the HLT, and post-intervention. **Results:** The HLT significantly enhanced students' ability to mobilise trigonometric knowledge in large-scale assessments and improved their overall performance on trigonometry-related items. **Conclusions:** This longitudinal study underscores the importance of sustained research and its long-term impacts. It provides a model for researchers to compare students' performance before, during, and after interventions.

Keywords: Trigonometry; Trigonometric Knowledge; Hypothetical Learning Trajectory; Large Scale Assessments; Design Experiment.

Conhecimentos trigonométricos mobilizados por estudantes em uma avaliação após intervenção pedagógica

RESUMO

Contexto: Resultados insatisfatórios dos estudantes em avaliação de larga escala justificaram o processo de intervenção em uma escola. Após a pesquisa surgiram questionamentos, tais como: que conhecimentos trigonométricos, explorados na trajetória hipotética de aprendizagem (THA) em trigonometria os alunos participantes da pesquisa mobilizaram na avaliação de larga escala das escolas estaduais capixabas

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ao término do ensino médio? **Objetivos**: Identificar conhecimentos trigonométricos mobilizados por um grupo de estudantes em avaliações de larga escala do Programa de Avaliação da Educação Básica do Espírito Santo (PAEBES) após terem participado de uma THA em trigonometria. **Design**: Este estudo é misto, qualitativo e quantitativo. A produção de dados ocorreu de 2019 a 2023 e utilizando dados das plataformas digitais de divulgação dos resultados dessas avaliações e os protocolos da pesquisa de doutoramento. **Ambiente e participantes**: Uma escola estadual, localizada na cidade de Linhares, no estado brasileiro do Espírito Santo. Os dados são de 14 alunos do Ensino Médio, sendo as fontes analisadas os dados das pesquisas e os portais oficiais de divulgação dos resultados. **Coleta e análise de dados**: Foi realizada em três etapas: resultados antes da intervenção; intervenção com THA em trigonometria; e resultados da escola após a intervenção. **Resultados**: A THA em trigonometria proporcionou ao grupo de alunos a mobilização de conhecimentos trigonométricos quando participaram em avaliações de larga escala, bem como promoveu expressiva melhoria de desempenho nos indicadores acerca dos conceitos de trigonometria. **Conclusões**: Este estudo demonstrou a importância da continuidade da pesquisa e seus impactos a longo prazo, demonstrando caminhos para os pesquisadores realizar comparações antes, durante e após suas intervenções.

Palavras-chave: Trigonometria; Conhecimentos Trigonométricos; Trajetória Hipotética de Aprendizagem; Avaliações de Larga Escala; *Design Experiment*.

INTRODUCTION

Large-scale assessments are official instruments for evaluating students' performance in several Brazilian states, enabling the measurement of various indicators, including the performance of public education. Hojas and Manfio (2015) emphasise the importance of this type of assessment, as it provides an individualised mapping of the student, the class, the school, and even the entire educational network.

Specifically, in the state of Espírito Santo, state schools administer external assessments through programs such as the Learning Monitoring Assessment (Avaliação de Monitoramento da Aprendizagem - AMA) and the Espírito Santo Assessment Program (Programa de Avaliação do Espírito Santo - PAEBES), which assign ratings of fully satisfactory, satisfactory, and unsatisfactory.

It is essential to emphasise that analysing the results of these programs enables us to more broadly visualise possible gaps in learning in certain subjects and, thus, plan strategies to overcome them. Thus, the analysis of 2019 PAEBES mathematics data from a state school located in the city of Linhares (ES), Brazil, categorised the students' knowledge of trigonometry as being at an unsatisfactory level. Given this fact, it was necessary, according to Martins (2023), to undertake an interventionist process via hypothetical learning trajectory (HLT) to facilitate learning of this object of knowledge by high school students, with this HLT being based on the results of this large-scale assessment applied in state schools in Espírito Santo.

For this purpose, we applied an HLT in the last academic quarter of 2021 to a group of 14 students in their 2nd grade of high school. This intervention was part of the data collection for a doctoral research project conducted by the first author of this text, under the supervision of the second author. At the end of 2022, the group of students participating in this research, who were then in their final year of basic education, carried out the external evaluation of PAEBES at the state level.

To carry out the analysis, data produced in the application of the HLT from 2022 to 2023 were transcribed, organised, tabulated, and analysed. While the data were being analysed, the good results of the HLT application were evident. However, the researcher raised some questions, including: What trigonometric knowledge explored in the HLT in trigonometry did the students participating in the research mobilise in the large-scale assessment of Espírito Santo state schools at the end of high school? What level of proficiency did the students participating in the research achieve in the PAEBES assessment?

However, given the limitations imposed by the methodological timeframe of doctoral research, as well as considering the publication of the 2022 PAEBES results in June 2023, after the completion of the doctorate, a proposal was made for future research, which would involve analysing the results in external assessments.

Given these facts, this study aimed to identify the trigonometric knowledge mobilised by a group of students in large-scale assessments of the Espírito Santo Basic Education Assessment Program (PAEBES), after having participated in an HLT in trigonometry.

To answer these questions, this study was developed with a mixed quantitative and qualitative approach, in three stages: 1) to analyse the results in external assessments of a group of students from a state school in Linhares (ES) before the application of the HLT; 2) to explore the results of the application of the HLT; and 3) to analyse the results of the students participating in the research in the assessment of high school graduates.

In the next section, we present the theoretical framework guiding the research, followed by the methodology and results of the three stages mentioned above.

THEORETICAL FRAMEWORK

The National Education Plan (PNE) (Brasil, 2014) emphasises the importance of large-scale assessments at all levels of Brazilian education, prioritising the improvement of teaching, school management, and data dissemination.

Especially throughout secondary education, these types of assessments are essential. When applied throughout the education network, they aim to monitor students' performance in various curriculum components, and the statistical methodology used in tabulating data should enable the promotion of equality in basic education (Werle, 2011; Tavares, 2012).

For Martins and Lobo da Costa emphasise that,

> In Brazil, the application of external assessments in schools has been increasing at different federative levels. Examples include: National High School Exam (Exame Nacional do Ensino Médio - ENEM); Brazil Assessments for the First Years and for the Last Years of Elementary School (Provinha Brasil and Prova Brasil); São Paulo State School Performance Assessment System (Sistema de Avaliação do Rendimento Escolar do Estado de São Paulo - SARESP); Ceará Permanent Basic Education Assessment System (Sistema Permanente de Avaliação da Educação Básica do Ceará - SPAECE); Curitiba Municipal School Performance Assessment System (Sistema Municipal de Avaliação de Rendimento Escolar de Curitiba - SIMARE), among other educational assessment systems (Martins & Lobo da Costa, 2021a, p. 25).

Specifically, in the Brazilian state of Espírito Santo, the Espírito Santo Basic Education Assessment Program (PAEBES) was established in 2000, to individually diagnose students' performance throughout elementary and secondary education (Pereira, 2015).

PAEBES is an assessment applied at the end of the fifth and ninth grades of elementary school, as well as the third grade of high school. Until 2019, students enrolled in high school took the assessment in three school quarters. Given the pandemic period, the quarterly assessment was no longer applied in 2020, leaving the PAEBES to students graduating from each cycle.

However, in 2023, the large-scale assessment was renamed Learning Monitoring Assessment (Avaliação de Monitoramento da Aprendizagem - AMA), being applied to 5th and 9th grade students in elementary school and 3rd grade students in high school (Espírito Santo, 2023). In 2024, AMA continued to be applied to students in the 5th grade of elementary school and to all students in the state education system from the 8th grade of elementary school to the 3rd grade of high school.

The Espírito Santo State Department of Education provides teachers with two tools for accessing and disseminating results: 1) Power BI, available virtually in the State Schools Management System, providing statistical reports on performance at the class, school, region, and state levels; 2) The Espírito Santo Education Assessment and Monitoring Platform, also available virtually, under the control of the Center for Public Policies and Education Assessment (Centro de Políticas Públicas e Avaliação da Educação - CAEd) of the Federal University of Juiz de Fora.

Thus, to disseminate results, in addition to the traditional line, pie, and column graphs, the results monitoring system provides a map that relates the pedagogical complexity of a given descriptor with the students' degree of mastery. Figure 1 below shows a model of this map.

Figure 1

Model map of the relationship between pedagogical complexity and degree of mastery. (Martins & Lobo da Costa, 2021a)

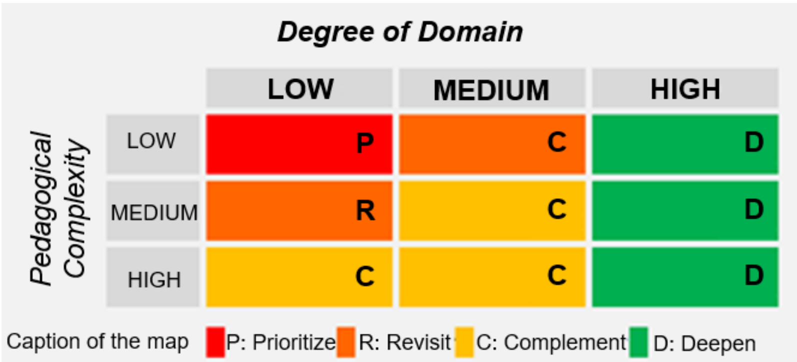


Figure 1 illustrates the relationship between pedagogical complexity and degree of mastery, which can be adapted and applied to any other series or grade of basic education. The pedagogical complexity is defined by the institution promoting the exam, considering the construction of the item evaluated and the knowledge to be mobilised to solve it. In turn, the degree of mastery includes characteristics of performance patterns in accordance with the item response theory (IRT) (Espírito Santo, 2021; CAEd Digital, 2022). In this way, the monitoring system itself presents the results for teachers to analyse.

This map, which relates pedagogical complexity and the student's degree of mastery of the descriptor, is accompanied by suggestions for teachers' actions, according to the student's performance, such as prioritisation [of content], resumption, complementation, or deepening.

After understanding the process of quantitatively measuring the results of the analysed external evaluation, it is also essential to understand the hypothetical learning trajectory (HLT) conceived by Simon (1995), which was developed for the pedagogical intervention applied to the research participants.

The hypothetical learning trajectory is part of the mathematics teaching cycle (MTC). Simon (1995) states that the MTC refers to a conceptual framework describing the relationships between teacher knowledge, student objectives, anticipation of student learning, planning, and interaction with students (Simon, 1995). Simon (1997) proposes two objectives for the teacher: respond to students' mathematics while presenting tasks and managing discourse, but focusing on particular issues.

HLT is the learning path designed by the teacher with the aim of teaching something to their students. The term "trajectory" refers to the path along which learning can occur; it is "hypothetical" because it precedes the moment of student learning. That is, while teachers make their plans, they generate hypotheses that can occur in the classroom during teaching and learning (Simon, 1995).

Three parts constitute the HLT: 1) the teacher's objectives for learning, which direct the HLT on the learning path; 2) the teacher's plan for learning activities, with a description of the tasks to be performed by the students; 3) the teacher's hypotheses regarding the learning process, i.e., possible solutions developed by the students when performing the tasks, what doubts may arise, different ways of solving them, as well as possible understandings of them throughout the tasks. Thus, to build the HLT, it is crucial to maintain focus on learning (Martins & Lobo da Costa, 2021b).

Simon (1995) also emphasises the importance of the teacher's role in determining the content and the tasks to be used in the classroom, as well as paying attention to the particularities of each situation, which must be recorded individually.

According to Martins (2023), Simon's interests were based on three factors: 1) believing that it would increase the understanding of the concepts covered; 2) providing the opportunity for learning based on the juxtaposition of strategies; and 3) deepening teachers' own understanding of the mathematical connections involved as they observe students' mathematical thinking.

When reflecting on the development and discussions of the proposed tasks, Simon (1995) emphasises that: teaching becomes guided by the conceptual objectives to be constructed by the students and these objectives must be constantly modified; mathematical understanding and the teacher's hypotheses regarding the students' knowledge are two factors that are related and underpin the objective and design of the lesson; the teacher must infer the nature of the students' understanding by interpreting the students' behaviours (Martins, 2023).

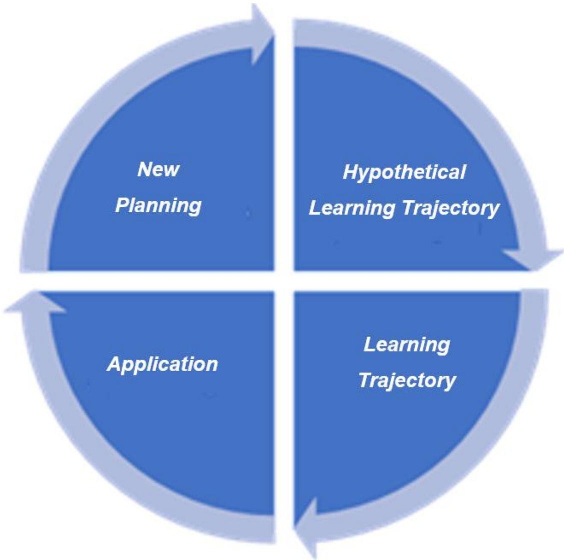
In this sense, Rodrigues, Bisognin, and Bisognin (2019) emphasise the hypothetical nature of teaching planning, highlighting that unforeseen events may occur throughout the process, as well as reinforcing the teacher's performance, especially regarding the systematic review of HLT aspects.

It is worth noting that the construction of hypotheses put forward by teachers may differ depending on their classroom experiences and, especially, their experiences with the proposed tasks. In this way, a hypothetical learning trajectory after its application becomes a learning trajectory, so even if another teacher uses it, it will again be hypothetical to the new group of students, with new hypotheses, with adjustments in the number of tasks, and after being applied, it will again be a learning trajectory.

From this perspective, Martins (2023) proposes the cycle of relations between the hypothetical learning trajectory and the learning trajectory, which is illustrated in Figure 2.

Figure 2

Model map of the relationship between pedagogical complexity and degree of mastery. (Martins, 2023)



As shown in Figure 2, each reapplication of a task will again be an HLT, and at the end of its application and/or reapplication, it will once again become a learning trajectory. This cycle does not end because, before a new application, the teacher can modify the hypothetical solutions of their students, according to the teacher's previous assessments of their knowledge.

The following section outlines the methodological procedures employed in the study.

METHODOLOGY

The mixed research (qualitative and quantitative) in this study is exploratory and characterised as documentary (Gil, 2008), which enables the collection of reliable data through the reproduction of existing information. The objective was to identify the trigonometric knowledge mobilised by a group of students in the large-scale evaluation of the Basic Education Assessment Program (PAEBES) of Espírito Santo after having participated in an HLT.

This study was developed by analysing results obtained by students at a state public school located in Linhares, a Brazilian city in the state of Espírito Santo. For this purpose , data collection and production involved three stages:

The first stage presents the descriptors and results related to trigonometry in PAEBES prior to the application of the intervention. For this, we analysed Martins and Lobo da Costa's (2021a) research, which presented the results of the group of students participating in the external assessments carried out up to 2019.

The second stage had as its object of study Martins' (2023) doctoral research developed in November 2021, in which a hypothetical learning trajectory (HLT) was applied to a group of students at the school.

This stage was carried out in a natural environment, involving students from a second-grade class of a regular state high school in Linhares (ES), where the researcher (first author) taught mathematics. The group of students ranged in age from 16 to 18 years old. We considered the data from 14 students who voluntarily participated in the application of an HLT during which trigonometric concepts were discussed. To ensure the confidentiality of the participants, the students' names were replaced with random codes, ranging from A01 to A14.

The third stage took place in November 2023 with the analysis of the results published by 2022 PAEBES. The third-grade high school students at this school who participated in the 2022 PAEBES were the subjects of the research applied in 2021. These data were collected through the assessment results platform made available by CAEd/UFJF.

To develop the third stage, we used the filters available on the platform. Among the available selections were: descriptors, all state schools, by regions (superintendencies), municipalities, schools, classes, and even by students.

We opted for: 1) a school filter, presenting the school data as a whole and allowing comparison with the initial data. 2) a filter by student, with the possibility of displaying the data of the research participants in the assessment of the third grade of high school in the state school network. In both cases, we used descriptors related to the mathematical object under study.

The research was authorised by the Ethics Committee under CAAE: 246711221.5.0000.5493 and Opinion No. 4.808.527, dated June 26, 2021. The research subjects participated voluntarily, with authorisation and consent via the Free and Informed Consent Form.

The results and discussion of this study will be presented in the following section.

RESULTS AND ANALYSIS

The first stage of this study aims to present the descriptors corresponding to trigonometry in PAEBES, as well as the results obtained in this field of knowledge up to the year 2021. In consultation with the PAEBES reference matrices, Martins and Lobo da Costa (2021a) identified the descriptors related to trigonometry, which are summarised in Chart 1.

Chart 1

Trigonometry Descriptors in the PAEBES Reference Matrix. (Martins & Lobo da Costa, 2021a)

Code	Descriptor
D39	Using trigonometric ratios in a right triangle to solve problems
D40	Using the law of sines or the law of cosines to solve problems
D28	Matching a trigonometric function to its graph
D29	Determining the solution set of a trigonometric equation

As shown above, four descriptors related to trigonometric concepts in the large-scale assessment of state schools in Espírito Santo involve trigonometric ratios applied to the right triangle, the law of sines and cosines (D39 and D40), the trigonometric function, and its graph and trigonometric equations (D28 and D29).

Researchers Martins and Lobo da Costa (2021a), when observing the history of this state school in Linhares (ES) in relation to the percentages of correct answers in descriptors related to trigonometry, could measure the simple average of the results in Chart 2.

Chart 2

PAEBES hits until the year. (Martins & Lobo da Costa, 2021a)

Code	Percentage of correct answers before HLT in trigonometry
D39	29.20%
D40	27.40%
D28	22.88%
D29	10.38%

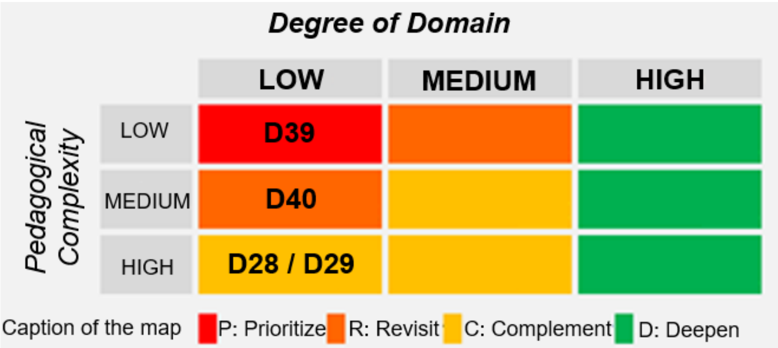
Thus, when analysing the data in Chart 2 above, we determined the simple average of correctness, with 22.47% hits in questions related to trigonometry concepts in the PAEBES assessments.

The data presented in Chart 2 also reveal a lower percentage of correct answers in problems with low pedagogical complexity levels (D39 and D40), achieving almost three times the success rate in solving them, when compared to those with high complexity levels (D28 and D29). Even with these results, the percentage obtained by students is insufficient to be considered at a higher level of mastery.

For a better visualisation of this issue, we created a map of the relationship between pedagogical complexity and degree of mastery of the descriptors, as shown in Figure 3 below.

Figure 3

Result of trigonometry descriptors in the map of the relationship between pedagogical complexity and the degree of mastery. (Martins & Lobo da Costa, 2021a)



As illustrated in Figure 3, the map of the relationship between pedagogical complexity and the degree of mastery for the trigonometry descriptors shows that, regardless of the pedagogical complexity of the descriptor, the school analysed received a low degree of mastery classification in all the descriptors evaluated.

Given these results, Martins and Lobo da Costa (2021a) examined the indicators and found, *in situ*, the difficulty that school students face in mobilising trigonometric concepts in subsequent grades. Furthermore, historical records of the school's external assessments demonstrate a decline in student performance in these four descriptors over previous years.

The data presented on the Espírito Santo Education Assessment and Monitoring Platform, illustrated in Figure 3, indicated the need to prioritise trigonometric ratios, revisit the concepts of the law of sines and the law of cosines, as well as complement trigonometric functions and their graphs.

However, the percentage of correctness, as presented in Chart 2, demonstrated the need for intervention in the teaching and learning processes of that school. These indices, aligned with what was observed in the research environment, reinforced the notion that teaching trigonometry was insufficient for student learning, indicating the need to prioritise prior knowledge, formalise essential concepts for learning, and mobilise trigonometric knowledge.

Based on these results, we initiated the second stage of this study, which motivated Martins (2023) to develop a trigonometry-based HLT. Through a diagnostic task, it was possible to:

> [...] identify commitments regarding the concepts of ratio, similarity of triangles and figures, Pythagorean theorem, and naming of the sides of the right triangle. Furthermore, difficulties in trigonometry, particularly with right triangles and notable angles, were also encountered. (Martins, 2023, p. 215)

Chart 3 outlines this HLT proposal in trigonometry, detailing the meetings and themes that have been developed.

Chart 3

Hypothetical learning trajectory developed. (Martins, 2023)

Meeting	Subject
01	Diagnostic task
02	Trigonometry, historical context
03	Similarity
04	*GeoGebra* and trigonometric ratios
05	Accessibility and trigonometry
06	From problems to models I
07	From problems to models II
08	Any triangle: Law of cosines
09	Any triangle: Law of sines
10	Trigonometric cycle
11	Fundamental relation of trigonometry
12	Ferris wheel: Introduction to trigonometric function

13	Sine function and cosine function
14	Final task

According to Chart 3, the HLT consisted of fourteen meetings on content involving the four descriptors assessed by PAEBES. It is worth noting that, in the research, trigonometric equations were addressed with functions; however, in the last PAEBES, some trigonometry descriptors, such as D29 and D40, were not evaluated.

The diagnostic task consisted of nine questions focusing on prior knowledge about: ascending and descending slope angle; nomenclature referring to the sides of a right triangle; ratio, proportion, and similarity of triangles; recognition that in a right triangle, in which one of the internal angles is 30°, the measure of the leg opposite this 30° angle is half the measure of the hypotenuse; identification of supplementary and complementary angles; development of strategies to solve problems of calculating the height of objects and the Pythagorean theorem.

When analysing the data from the diagnostic task for the group of students who were subjects of the research, we observed that 36% of the questions were left unanswered, 43% presented incorrect solutions, and 7% were incomplete. The results of the diagnostic task highlighted the relationship between the participants' knowledge and the school's external assessments from previous years, achieving an average percentage of correct answers of 14% for this area of knowledge.

To develop this HLT in trigonometry, Martins (2023) employed the following tools and methods: historical texts on trigonometry, GeoGebra, a trigonometric circle, models, manipulative materials, instruments for measuring angles and lengths, individual research protocols, dialogues, and voice and video recordings. At the last HLT meeting, a "Final Task" consisting of nine tasks was applied.

The results of the HLT "Final Task" regarding hits, errors, and incomplete tasks are shown in Chart 4 below.

Chart 4

Quantitative summary of the Final Task. (Martins, 2023)

Task	Mathematical knowledge	Hit (%)	Error (%)	No Response (%)
1	Trigonometric Ratios	100	0	0
2	Trigonometry in Right Triangle (Sine)	93	7	0
3	Trigonometry in Right Triangle (Tangent)	86	7	7
4	Trigonometry in Right Triangle (Cosine)	86	14	0
5	Trigonometry in Any Triangle (Law of Sines)	93	7	0
6	Trigonometry in Any Triangle (Law of Cosines)	72	21	7
7	Relate the trigonometric function to the graph (Cosine function)	72	28	0
8	Relate the trigonometric function to the graph (Sine function)	100	0	0
9	Mind Map of the studied Trigonometry	100	0	0

Chart 4 highlights the dimension of correctness achieved by research participants in the final task of the interventionist process with the HLT in trigonometry. It is possible to observe the reduction in the percentage of unsolved tasks, from 36% of the diagnostic task to 7% in the final task; a reduction from 7% to 0% of incomplete tasks; a reduction in the percentage of errors; and, consequently, a significant increase in the percentage of assertiveness in the proposed items.

In short, throughout the meetings, it was evident that students consistently applied the knowledge they had acquired in previous meetings to attempt to solve the proposed problems. In other words, concepts were mobilised, demonstrating a change in the levels of thinking in mathematical tasks.

Martins (2023) highlights that:

> In addition to the concepts addressed as the focus of this study, algebraic knowledge was mobilised in the solutions of equations, proportionality, the reduction or enlargement of polygons, the reading and extraction of data from tables, the importance of technical standards, especially accessibility,

> percentage, and the use of models to represent reality. (Martins, 2023, p. 215)

Thus, the development of the HLT reveals the mobilisation of knowledge beyond trigonometric knowledge. The results presented lead us to infer that there were significant advances in the knowledge of trigonometry, as demonstrated by the successful application of the research carried out. For example, task number 8 of the “Final Task” – Relating the trigonometric function to the graph (Sine function) – showed 100% correct answers.

It is also important to highlight that the execution of the HLT raised some questions, such as: considering that these students are in the second grade of high school, what will their results be like in the PAEBES one year after the conclusion of this investigative process?

To answer this question, the third stage of the research that supports this article presents the results obtained by students participating in the HLT in trigonometry during the 2022 PAEBES assessment, conducted at the end of high school.

Regarding the 2022 PAEBES, of the 14 subjects who voluntarily participated in the research and the HLT developed in the second stage described above, two did not participate in the PAEBES assessment: one (A03) was transferred from the school, and one (A13) was absent on the day of the PAEBES application.

It is worth clarifying that one of the characteristics of this large-scale assessment is the arrangement of the questions, in which the assessment booklets have: 1) different questions; 2) different descriptors; 3) different numbers of questions per descriptor; and 4) the names of the students in the booklets follow a random order. These characteristics align with the item response theory (IRT) used in PAEBES.

Chart 5 below displays the items answered and quantified by each subject in this study, as well as the participants in the HLT intervention.

It is worth highlighting that in this edition of PAEBES, only two of the descriptors related to trigonometry knowledge were addressed, namely: D28 (Match a trigonometric function to its graph) and D39 (Use trigonometric ratios in a right triangle to solve problems. Therefore, our observations refer to this mobilisation of knowledge.

Chart 5

Distribution of questions per student and corresponding hits. (Authors, 2024)

Participating Student	D39		D28	
	Number of items	Hits	Number of items	Hits
A01	-	-	1	1
A02	1	1	-	-
A03	*Transferred to another school*			
A04	-	-	2	2
A05	1	0	-	-
A06	-	-	1	1
A07	-	-	1	0
A08	2	2	-	-
A09	-	-	1	1
A10	-	-	2	2
A11	1	1		-
A12	2	1		
A13	*Transferred to another school*			
A14			2	1

As observed in Chart 5, a randomisation process prevents students from answering the descriptor on trigonometric ratios (D39) and graphs of the trigonometric function (D28) in the same assessment. Furthermore, there is no standard number of items for each student, so students either respond to one, two, or none.

Thus, the data presented in Chart 5 reveal that five students answered seven problems involving descriptor D39, while another seven students answered ten problems involving descriptor D28. Additionally, none of the students answered two descriptors in the PAEBES assessment, totalling 18 questions (items) related to trigonometry concepts in the evaluation.

There is a lack of data for students A03 and A13. Both participants were involved in the intervention process using the hypothetical learning trajectory; however, their PAEBES results, obtained at the end of high school, were not made available because they were transferred to another school unit. In this situation, the students' data are disclosed to the school of origin and are restricted to the school of destination of the transfer, as long as it is part of the Espírito Santo state schools network. Furthermore, the data presented in Chart

5 enabled us to measure the statistics of the group of students who voluntarily participated in the study.

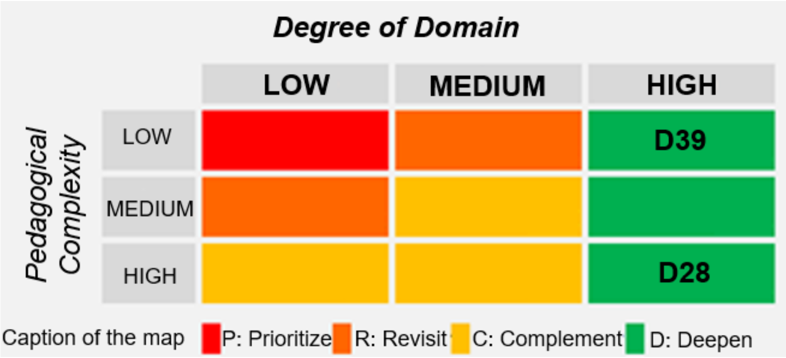
In conclusion, after applying the methodology used in the general statistics of PAEBES, we can state:

- Regarding descriptor D39, five students out of the 12 who took the assessment (A02, A05, A08, A11, A12) answered the items related to this descriptor; of these five students, three answered all the proposed items correctly, which represents 60% of the responding students; and of the seven total items evaluated by this descriptor, five were answered correctly, with 71.43% being the total correctness in the item.
- Regarding descriptor D28, seven students (A01, A04, A06, A07, A09, A10, A14) answered the items related to this descriptor; of these, five answered all the items proposed in their evaluation correctly, representing 71.43% of the responding students; and of the ten total items evaluated by this descriptor, eight were answered correctly, with 80% being the total correctness in the item.

These data were directly reflected in the map of the relationship between pedagogical complexity and the degree of mastery of these students who participated in the intervention process, as represented in Figure 4.

Figure 4

Map of the relationship between complexity and students' level of mastery after participating in the HLT. (Authors, 2024)



The data presented in Figure 4 generally demonstrate the mastery of the trigonometry descriptors D28 and D3 assessed in the PAEBES, concatenating them with the results obtained in the “Final Task” of the HLT, as well as enabling us to verify the mobilisation of students’ knowledge at the end of the basic education cycle, even with one year of application of the interventionist process.

In summary, this group of students, who participated in the intervention process with the HLT in trigonometry, remained at a high mastery level, and, consequently, the recommendation for them is to deepen their knowledge in using trigonometric ratios in a right triangle to solve problems and to match a trigonometric function to its graph.

Furthermore, the results of this group of students had a positive impact on the school’s performance in relation to the object of knowledge under study. On the data collection platform, when filtering by school with the trigonometry descriptors evaluated in 2022 PAEBES, it was possible to create a map showing progress by relating the degree of pedagogical complexity to the degree of mastery, as shown in Figure 5.

Figure 5

Map of the relationship between pedagogical complexity and degree of mastery in PAEBES after application of the intervention (school). (Authors, 2024)

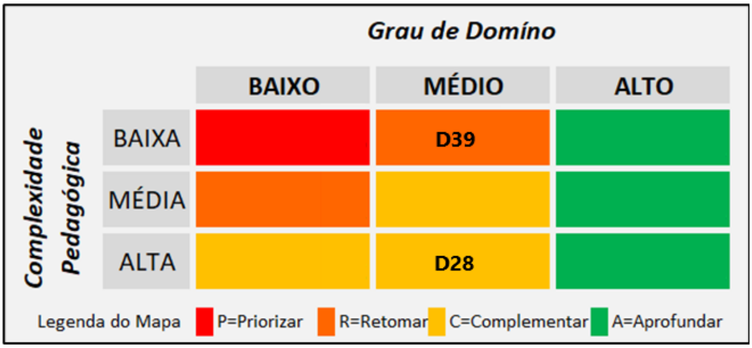


Figure 5 presents the school’s results regarding trigonometry after the intervention process. When comparing it with the previous result shown in Figure 3, which was one of the drivers for planning and implementing an interventionist process through an HLT, an improvement in students’ performance regarding this object of knowledge can be observed. In this comparison, descriptors D28 and D39 demonstrated a medium degree of mastery with the application of HLT in trigonometry, whereas they had previously shown a low degree of mastery. The two other descriptors related to trigonometric knowledge were not evaluated in the 2022 PAEBES edition.

CONCLUSION

Large-scale assessments have several functions in educational management and public policies. They are used by educational institutions, government agencies, national and international non-governmental institutions, as well as by teachers, researchers, and other professionals to provide indicators of the quality and effectiveness of teaching.

Public access to the results of these assessments has been a valuable documentary source for researchers, particularly in the field of mathematics education, serving to justify their research and provide support for proposing interventions and comparing results.

The first author, under the guidance of the second author, developed a doctoral study using open-access data, with trigonometry as its mathematical and learning object, duly justified, and compared the results achieved.

The results of this doctoral research highlighted an important advance in student performance in the external assessment of PAEBES in 2022. When comparing the students' results in the 2021 PAEBES assessment with the research results, we observed percentage indicators of mastery in trigonometry ranging from 10.38% to 29.20%. However, after applying the HLT, they achieved 72% to 100%.

The success scenario, characterised by improved performance in indicators through the intervention, was the research assumption. Regarding the confirmation of the hypothesis, even when referring to research involving people, some factors provided an acceptable level of controllability on the part of the researchers involved, namely: the set of tasks, geometric manipulation software, models, questions about the research protocols, and the researcher's actions in accordance with the theory. Such controllability in research procedures and instruments is necessary and must strictly adhere to the precepts related to the theoretical and methodological contribution, since, within the scope of the chosen theories, there will be varying degrees of intervention by the researcher.

In this context, Martins and Lobo da Costa (2024) emphasised the importance of qualitative research in the area of mathematics education, as well as exposed issues of reliability and validity of the methodological process undertaken in doctoral research, in which they underscored that, regarding qualitative research, it is necessary to highlight the contingencies and procedures through which they were mitigated.

Thus, the present study, methodologically of a mixed nature, explored the open data from the PAEBES assessment applied at the end of 2022, identifying for each student participating in the pedagogical intervention (HLT) the descriptors of their tests; the number of questions per descriptor and their correctness.

In the PAEBES large-scale assessment, two descriptors related to trigonometric knowledge were identified: D39) Use trigonometric ratios in a right triangle to solve problems; and D28) Match a trigonometric function to its graph.

Given the absence of descriptors D40 and D29 in the PAEBES assessment in 2023, it became unfeasible to design analyses regarding the mobilisation of knowledge of these students in external assessments after the application of an HLT. In turn, considering the results obtained in the doctoral research and comparing them with the results of other descriptors, the students likely made progress in their proficiency regarding the concepts of the law of sines and the law of cosines, as well as the solution set of a trigonometric equation.

According to the results presented, it was evident that each student randomly answered a single descriptor and may have solved one or two problems involving trigonometry out of the total number of questions in the assessment. Given this analysis, when comparing the results of all PAEBES tests before the intervention, the results of the final intervention task and the PAEBES after the intervention, we observed:

- Regarding the mobilisation of students' knowledge on how to "use trigonometric ratios in a right triangle to solve problems" (D39), there were 71.43% correct answers.
- There was a 16.9 percentage point reduction compared to the thesis results in D39. However, when comparing the PAEBES results released in 2021 and 2023, we observed a growth from 29.2% to 71.43%, an increase of 42.23 percentage points.

- Regarding the descriptor "Match a trigonometric function to its graph" (D28), we observed an 86% hit rate in solving the problems proposed in the doctoral research.
- When comparing the results of Descriptor D28 of the research with the PAEBES results released in 2023, a reduction of six percentage points was observed. In turn, when compared with the 2021 edition, growth increased from 22.88% to 80% in these two editions under analysis, demonstrating a 57.12 percentage point increase.

We can therefore infer that even after a year of applying the HLT, students were able to mobilise trigonometric knowledge in the state assessment of third-grade high school students.

The hypothetical learning trajectory in trigonometry facilitated the knowledge construction path throughout the intervention process carried out at the school, contributing to the achievement of positive results by both the participating students in the research and the school as a whole.

As a result, students participating in the intervention process via HLT in trigonometry achieved a high level of mastery in trigonometry descriptors, and, in general, these students improved the school's indicators regarding this area of knowledge. Thus, they demonstrated that HLT is an efficient instrument, as it produced positive results throughout its application in the classroom and in external assessments, even after a long period.

Regarding the school's results and applying the item response theory (IRT), which related other mathematical skills of students throughout the development of the assessment itself, it can be stated that these results managed to leverage the school's results in the level of mastery in both descriptors (D39 and D28), in which students went from the low level of mastery to the medium level.

Thus, concerning the first knowledge, it was evident that the students could mobilise the trigonometric knowledge in the state of Espírito Santo curriculum, including trigonometry in the right triangle and trigonometric functions.

The level of mastery achieved by the students participating in the research changed from low to advanced, but the school, in general, remained in the average category, despite the progress.

Thus, as explained above, this study achieved its objective, which is to identify the trigonometric knowledge mobilised by a group of students in large-scale assessments of the Espírito Santo Basic Education Assessment Program (PAEBES) after having participated in an HLT in trigonometry.

However, beyond this achieved objective, we emphasise, as researchers in mathematics education, the importance of continuing research that aims to truly understand the long-term impacts of pedagogical interventions carried out in the classroom.

Furthermore, due to the absence of questions involving two trigonometry descriptors in the 2022 PABES assessment after the application of the pedagogical intervention (HLT), it was unattainable to measure the results regarding the knowledge mobilised by students on the law of sines and cosines (Descriptor D40), as well as the set of solutions of a trigonometric equation (Descriptor D29). However, considering the results in the PAEBES before the intervention, the intervention data, and the results of the PAEBES after the intervention, it is likely that the students obtained a result similar to that of the other descriptors analysed in the evaluation.

In this study, we also emphasise, especially for researchers in the field of mathematics education who develop qualitative and mixed research, the importance of providing a detailed description of qualitative methods, especially in the descriptions of contingencies and how they were adapted to the research. Such details must be relevant and highlight the validation and reliability of the method.

Finally, we describe ways to continue research and the studies themselves, which, when completed, are often posted in repositories and, like a sheet of paper in a drawer, remain there forgotten. However, there is hope of returning to the same justifying instrument used in the main study, and in some way, it has presented results to compare with the analysed results again.

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AUTHORS' CONTRIBUTIONS

LM and NMLC conceived the presented research and planned the methodological approach. LM drafted the document, and NMLC performed revisions. LM organised and processed the data. LM and NMLC analysed and discussed the data, as well as approved the final version.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author upon reasonable request.

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