




Preservice teachers' mobilisation of reflective writing in online *Vaivém*

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ABSTRACT

Background: The context that motivated the research discussed in this article involves using a written communication instrument between educator and mathematics preservice teacher called *Vaivém*, in which a variation of some of its characteristics was proposed, involving the configuration called online *Vaivém*. **Objectives:** to analyse evidence of mobilisation of reflective writing by preservice mathematics teachers in the online *Vaivém*. **Design:** Qualitative research of an interpretative nature. **Data collection and analysis:** To produce the data, 12 *vaivéns* produced in the context of this discipline were considered. Of these, three were selected for specific analysis, in which we sought to identify and analyse the evidence of mobilisation of reflective writing. **Results:** The online *Vaivém* enhanced the development of reflections by future teachers, enabling them to move from descriptions of experiences to more in-depth analyses, characterised by questioning conceptions about teaching mathematics, the challenges of teaching practice and decision-making. The interactions guided by the trainer influenced these reflections, encouraging preservice teachers to deepen their analyses and broaden their perspectives. **Conclusions:** The online *Vaivém*, as a new configuration of the traditional *vaivém*, presented potential characteristics for the learning process of preservice teachers. The fact that it was online allowed the undergraduates to get even closer to the instrument. From the educator's point of view, this new configuration made the instrument more practical, and the interventions happened more quickly and naturally.

Keywords: Mathematics Education; Mathematics Teacher Education; Reflective Writing; Written Production; *Vaivém*.

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A mobilização da escrita reflexiva de futuros professores de matemática no Vaivém on-line

RESUMO

Contexto: O contexto que motivou a pesquisa discutida neste artigo envolve a utilização de um instrumento de comunicação escrita entre formador e futuro professor de matemática denominado vaivém, no qual foi proposta uma variação de algumas de suas características, envolvendo a configuração denominada “vaivém on-line”. **Objetivos:** analisar indícios de mobilização da escrita reflexiva por futuros professores de Matemática no vaivém on-line. **Design:** Pesquisa de natureza qualitativa de cunho interpretativo. **Ambiente e participantes:** Participaram da pesquisa, 12 futuros professores, matriculados em uma disciplina do 2º ano do curso de licenciatura em matemática de uma universidade pública do estado do Paraná. **Coleta e análise de dados:** Para produção dos dados, foram considerados 12 vaivéns, produzidos no contexto dessa disciplina. Dentre eles, três foram selecionados para uma análise específica, em que se buscou identificar e analisar os indícios de mobilização da escrita reflexiva. **Resultados:** o vaivém on-line potencializou o desenvolvimento de reflexões pelos futuros professores, possibilitando que passassem de descrições sobre as experiências para análises mais aprofundadas, caracterizadas pelo questionamento das concepções sobre o ensino de matemática, os desafios da prática docente e as tomadas de decisão. As interações direcionadas pelo formador influenciaram tais reflexões, incentivando os futuros professores a aprofundarem suas análises e ampliarem suas perspectivas. **Conclusões:** O vaivém on-line, como uma nova configuração do vaivém tradicional, apresentou características potenciais para o processo de aprendizagem do futuro professor. O fato de ser on-line permitiu com que os licenciandos se aproximassem ainda mais do instrumento. Do ponto de vista do formador essa nova configuração deixou o instrumento mais prático, e as intervenções aconteceram de maneira mais rápida e natural.

Palavras-chave: Educação Matemática; Formação de professores de Matemática; Escrita Reflexiva; Produção Escrita; Vaivém.

INTRODUCTION

Teacher education, particularly in mathematics, is of utmost importance to society. According to the National Institute of Educational Studies and Research (Instituto Nacional de Estudos e Pesquisas Educacionais - INEP), there is a growing shortage of teachers in all regions of Brazil. The lack of these professionals significantly impacts the quality of education, resulting in the sudden replacement of temporary teachers, resignations, and classes with full student enrollment. Teachers become overworked, and some leave the profession. Besides all that, teachers are socially discredited in a growing context of devaluation of science.

That said, investing in the education of preservice mathematics teachers is crucial for enhancing various educational processes and mitigating the disparity in opportunities. Concerning such investment, reflection has been the focus of numerous investigations that problematise teacher formative processes.

Based on Schön (1983), discussions suggest that professional learning should be anchored in teachers' reflection on classroom practice (Day, 1999; Muir & Beswick, 2007). In this context, learning is not based solely on theories but on reflection. In teacher education, reflection is often regarded as an essential tool for connecting theory and practice, facilitating conceptual development, and enabling preservice teachers to articulate practical experiences in conjunction with new theoretical understandings (Ottesen, 2007). When (preservice) teachers reflect on their practices, it is possible to identify challenges, evaluate their pedagogical methods, seek alternatives to overcome them and find other strategies to meet each student's needs. Reflection serves not only as self-analysis but also as a means of building knowledge. Reflection must be continuous, allowing the teacher to use the analysis to improve their teaching.

In this direction, narrative has been utilised as a potential instrument for the professional learning of (preservice) mathematics teachers, primarily because it enables, among other aspects, critical reflections on specific dimensions of teaching practice (Coura, 2019; Coura & Passos, 2019; Cristovam & Fiorentini, 2021). The *Vaivém*, because it has, among other aspects, a narrative characteristic (Rodrigues & Cyrino, 2020), has also been used as an instrument to investigate preservice teachers' reflections in training (Silva, Innocenti & Zankum, 2002; Rodrigues & Cyrino, 2020; Silva & Gardin, 2023; Silva, 2023).

Vaivém is an assessment tool created by Professor Regina Buriasco, which has been used in undergraduate and graduate classes since 1978. The tool serves as a space for written communication between teachers and students. In general, we can say that, in *Vaivém*, the teacher asks a question to the whole class, and each student answers on a sheet of paper. From each student's answer, the teacher asks follow-up questions and provides individualised feedback to the student. (Silva, 2018, p. 56)

With this, the *Vaivém* becomes a potential instrument for developing these reflections, encouraging the constant exchange of ideas among those

involved in the formative process, in which reflection is encouraged and structured. Through the *Vaivém*, preservice teachers can reevaluate their methods and decisions, adjusting them based on feedback. Silva et al. (2022) state that *Vaivém* enables students to review and refine their reflections, thereby promoting continuous learning. For this reason, we justify its use in our investigation, as it enables the mobilisation of preservice mathematics teachers' reflections in an interactive professional learning context. Furthermore, we propose a variation of this instrument called "online *Vaivém*."¹

Below, we present the theoretical foundations of reflective writing in the education of teachers who teach mathematics. Next, we detail the methodological procedures adopted. Next, we present our analysis and then provide our final considerations.

REFLECTIVE WRITING IN THE EDUCATION OF MATHEMATICS TEACHERS

For Hampton (2010), reflective writing analyses, explores, and explains what happened and why. It involves constructively critiquing oneself, the event, and often others, using evidence to support one's claims. Therefore, having clear records of your ideas and thoughts becomes crucial. According to Liu (2013), critical reflection is a form of reflection that directs teachers to think about and improve student learning. These points highlight how reflective writing, in the context of preservice mathematics teachers' education, goes beyond a simple description of events, promoting critical thinking about pedagogical practices and teaching actions.

In recent years, different teacher education proposals have approached reflection to expand teachers' learning possibilities, particularly in the critical analysis of pedagogical practices (Day, 1999; Muir & Beswick, 2007; Van Manen, 1977; Hampton, 2010; Pontes, 2011). Writing is one way to develop reflections.

We can understand reflective writing as a way for teachers and students to build their "mathematical knowledge while they speak, draw, express their ideas, reflect on their own words, and establish relationships through writing" (Miné, 2011, pp. 6-7). Reflective writing encourages students or preservice teachers to think about and record different approaches to a study or topic.

¹ In the research methodological procedures section, we present more details of this proposed variation.

Reflective writing is a form of personal writing that allows authors to express their feelings, opinions, and understandings within the themes explored. Through various instruments, reflective writing within the scope of initial teacher education has been recognised as an enhancer of the teacher education process (Galiazzi & Lindemann, 2003; Passos, 2008; Fioravante, 2014; Silva & Passos, 2016).

To this end, some authors consider that there are categories of reflection² that vary in complexity (Day, 1999; Muir & Beswick, 2007; Van Manen, 1977). Van Manen's (1977) study considers three categories of reflection in educational contexts: technical, practical, and critical or emancipatory. Each category of reflection has its own degree of complexity and varies gradually, ranging from the least complex to the most complex. The technical category refers to applying knowledge practically to achieve a specific objective, following basic educational methods without questioning or reflecting deeply on them. The practical category involves [...] identifying problematic aspects in the learning situation. And the critical or emancipatory category involves broader questions about the value of knowledge and the mobilisation of incidents associated with ethical, political, and social issues.

Additionally, in the initiative to categorise reflection for analytical purposes, Day (1999) and Muir and Beswick (2007) formalised a framework of possible thematic units for reflection: descriptive, deliberate, and critical reflection.

Chart 1

Categories of reflection, according to Muir and Beswick (2007)

Level 1: Technical description

The participant describes general accounts of classroom practice, often focused on technical aspects, without considering the value of the experiences.

For example: The lesson went well; I didn't ask enough questions; All students could do the task.

Level 2: Deliberate reflection

² When we use the term "category," we are not referring to mutually exclusive groups (one of its meanings). In other words, it is possible that there are elements between the groups that can be associated.

The participant identifies the “critical incidents” and provides a justification or explanation for the action or behaviour.

For example: Joãozinho was not quite himself today; I think the question was too difficult for him; the way he was working the area showed me that he was confusing it with the perimeter; I really wanted them to use concrete materials, as I felt they didn’t have a good conceptual understanding of how the addition algorithm works.

Level 3: Critical reflection

The participant goes beyond identifying “critical incidents” and providing explanations and begins to consider the perspectives of others and offer alternatives.

For example:

I shouldn’t have put Jack in a difficult situation by asking him to explain a square number; It’s obvious that he felt uncomfortable. Perhaps I could incorporate a “think in pairs-share” strategy, where students could talk to each other before sharing more publicly; I’ve always taught division this way, but I could see their eyes glaze over and thought there must be a better way; I need to get them more involved in the process, maybe using concrete materials could help.

Here, the categories of reflection also gradually increase in complexity.

Hampton (2010) presents a possible structure for reflective writing, where it can be divided into three groups: description (brief, explains what happened/is being examined); interpretation (what is most interesting/relevant from a personal point of view, how it can be explained) and; result (what you learned from the experience and what it means for your future).

In Chart 2 below, we summarise these categories.

Chart 2

Summary of reflection categories and their respective characteristics

Categories of reflection	Characteristics	Authors
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Technique	<ul style="list-style-type: none"> • Application of knowledge in a practical way without questioning 	Van Manen (1977)
Practical	<ul style="list-style-type: none"> • Identification of problematic aspects 	
Critical or emancipatory	<ul style="list-style-type: none"> • Mobilisation of broader aspects associated with moral, political, and social issues; 	
Descriptive	<ul style="list-style-type: none"> • Identification of general aspects; • Temporal account of the experience lived; 	Day (1999), Muir & Beswick (2007)
Deliberate	<ul style="list-style-type: none"> • Identification of critical incidents; • Presentation of justifications/explanations for the observed actions; 	
Critical	<ul style="list-style-type: none"> • Mobilisation of explanations, suggestions, articulations, and problematisation; 	
Description	<ul style="list-style-type: none"> • Temporal explanation; 	Hampton (2010).
Interpretation	<ul style="list-style-type: none"> • Focusing on more interesting or relevant aspects from a personal point of view; 	
Results	<ul style="list-style-type: none"> • Learning from experience; • Planning perspectives; 	

The discussion on categories of reflection is fundamental in the education of preservice mathematics teachers, as it enables educators to use reflective writing as a reference, allowing undergraduates to understand their pedagogical practice and teaching approach. These reflection categories have a structure that allows preservice teachers to identify and adapt the most effective approaches within the classroom, promoting and implementing continuous improvements in their practices. All category templates are designed to support

reflective writing, involving structured discussions and critical analyses of pedagogical practices. Essentially, these models are intended to provide an organised basis that encourages (preservice) mathematics teachers to reflect in-depth on their education and professional practices (Mulryan-Kyne, 2020).

Writing reflectively enables meaningful connections with diverse perspectives and facilitates dialogues between educators and preservice teachers, as it promotes the exchange of ideas and experiences. Furthermore, reflective writing between teachers and students allows teachers to carry out a self-evaluation of their own practice, which makes them reflect on whether or not they are achieving their objectives since, according to Pontes (2011, p. 4), when one writes and reflects on one's own experiences, one "exercises his/her authorship, reconstructs his/her relationship with writing and rebuilds his/her identity."

To analyse the reflective writing of preservice mathematics teachers using the "Vaivém" instrument, we chose Muir and Beswick's (2007) approach. This choice was made due to the clarity and organisation with which the levels of reflection are structured, facilitating the separation between different depths of analysis. Furthermore, the method aligned with the author's preferences and understanding, providing a more intuitive and accessible approach to conducting reflective analysis.

METHODOLOGICAL PROCEDURES

This research, of a qualitative and interpretative nature, was conducted within the context of the subject *Mathematics Teaching Methodology*,³ taught in the 2nd year of the mathematics degree course at a public state university in the northwest region of Paraná in 2024. The second author of this article is the professor in charge of this subject. Among the activities developed by preservice teachers in the *Mathematics Teaching Methodology* subject, the *Vaivém* was used from the first day of class, but with a variation: we chose to develop it online with the undergraduates.

On the first encounter, in April 2024, the class professor presented the possibility of using the *Vaivém* with the preservice teachers and suggested two configurations: online or traditional writing. About the online version, he

³ The subject syllabus: Conceptions on teaching and learning in mathematics. The mathematics teacher and his professional development. Pedagogical trends in mathematics education. Didactic and pedagogical resources for teaching mathematics. Planning and assessment in mathematics.

highlighted that it would be his first experience in this format, unlike traditional writing, with which he had already worked on other occasions, as a student and teacher. When proposing the choice to preservice teachers, they decided to develop the *Vaivém* online. Given this, the online *Vaivém* configuration was negotiated as follows:

Chart 3

Online Vaivém configuration

- Several WhatsApp⁴ groups are created, one for each preservice teacher.
- Each WhatsApp group is called “xxxxxxx’s *Vaivém*,” and “xxxxxxx” corresponds to the name of the preservice teacher. For example, “Larissa’s *Vaivém*.”
- Each WhatsApp group has a head professor and a preservice teacher as members.
- The groups are saved on each smartphone so that notifications do not interfere with the personal use of the application by the preservice teacher and the head teacher.
- The preservice teachers are instructed that subjects not associated with the *Vaivém* should not be mentioned in the respective groups.
- In each *Vaivém*, the head professor launches an initial question, which each preservice teacher will answer according to their knowledge and beliefs. Then, the head professor intervenes or asks follow-up questions, generating a new phase to be answered by the preservice teacher.

For this research, the *vaivéns* were considered until November 11, 2024⁵, and the initial/generating question was: “What does it mean to be a good math teacher?” In total, 12 *vaivéns* were produced, and the same number of people enrolled in the subject. The preservice teachers signed an informed consent form⁶ to use the *Vaivém* for research purposes, in which we emphasised

⁴ *WhatsApp* is an instant messaging application from the conglomerate *Meta Platforms*.

⁵ For educational purposes, the *Vaivém* continued to be used. However, for this research, we chose to make this cut to enable the construction of the article.

⁶ The project at stake is not registered with the ethics committee because, initially, it had only a didactic intention for the subject in question. Over time, as the quality of the data improved, we decided to conduct an investigation based on it. However, after this

that their real names would be preserved. Therefore, the names of the participants are fictitious, chosen by them.

Having gathered all the *vaivéns*, we selected and organised evidence of reflections from three preservice teachers according to the following criteria:

- Student involvement in the *Vaivém* process;
- The quality of the topics developed during the reflections;
- Their adequacy to the theoretical perspective chosen for the analysis;
- Students who had more stages of interaction with the *Vaivém*.
- The selected students were Tiago, Claudia, and Antônia. Each presented different levels of involvement and depth in their reflections.

The categorisation of the evidence of reflections will be carried out according to Muir and Beswick's (2007) framework, which identifies three categories of reflection. The written productions of the preservice teachers will be analysed using the following categories as lenses:

Category 1: Technical description – Participants describe their classroom practices, focusing on technical aspects without considering the value of the experiences, whether as students or teachers in training.

Category 2: Deliberate reflection – Participants identify “critical incidents” and justify or explain their actions and behaviours. Critical incidents are specific moments in the trajectory of preservice teachers that generate essential questions or challenges, leading them to reflect on their practices.

Category 3: Critical reflection – Participants reflect beyond critical incidents, considering the perspectives of others and proposing alternatives for future practices.

With this approach, we could identify signs of reflection in each student and observe how reflective writing contributed to the critical and pedagogical development of preservice mathematics teachers.

decision, the informed consent form was signed by the preservice teachers. We exempt Acta Scientiae from any resulting consequences, including full assistance and eventual compensation for damages to any research participant, as per Resolution No. 510, dated April 7, 2016, of the National Health Council of Brazil.

REFLECTIVE WRITING OF PRESERVICE MATHEMATICS TEACHERS IN THE VAIVÉM

In this section, the analysis of the written productions of preservice teachers will be divided into subsections, organised gradually according to each one's reflective writing.

In the following figures, the numbers (1, 2, 3, etc.) indicate different questions asked by the teacher educator during the Vaivém, organised within a context, to facilitate the identification and analysis of the preservice teachers' answers.

Antônia's reflective writing⁷

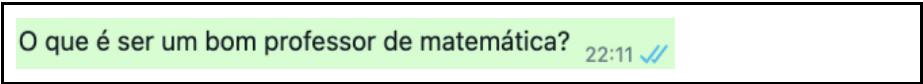
Antônia is a very participative student in *Mathematics Teaching Methodology* classes. Besides holding a degree in mathematics, Antônia teaches basic education, particularly early childhood education. She is very responsible and dedicated, and despite her difficulties, she is always proactive in class. In some of them, she seems tired due to her exhausting work day. However, as the activities progress, she gradually shows enthusiasm.

Technical description of Antônia

Figure 1 shows Antônia's written production answering the first question of the *Vaivém*.

Figure 1⁸⁹

Antônia's written production in the online Vaivém



O que é ser um bom professor de matemática? 22:11 ✓

⁷ In some parts of her answers, Antônia presented more than one type of reflection in a single written production. However, in this subsection, we will only focus on the moments when a technical description is provided on the subject being discussed.

⁸ We chose to explicitly include in a footnote the texts of the figures derived from the online vaivém. In the notes, the educator's questions will be in italics.

⁹ Figure text: *What does it mean to be a good math teacher?* Being a good math teacher is not about being content-oriented, but knowing how to spread questions and reflections. Being content-oriented does not mean being successful in class, because I may know math, but I [must know] how to transmit knowledge to my students. *What does it mean to be a content-oriented teacher?* Content-oriented teachers follow traditional pedagogy; they stand as the sole holders of knowledge.

Ser um bom professor de matemática não é ser conteudista, mas saber propagar questionamentos, reflexões. Porque ser conteudista não significa ter sucesso nas aulas, porque saber matemática posso saber, mas de que maneira irei transmitir o conhecimento para meu aluno.

22:21

1) o que é ser um professor conteudista?

Professor conteudista é aquele professor que segue a pedagogia tradicional, que se coloca como o único detentor do saber.

In the first part of her answer, “Being a good math teacher is not about being content-oriented,” Antônio gives a brief description of what it means to be a good math teacher, mentioning that just focusing on a content-oriented perspective, as a teacher, does not make a good teacher, in her view.

When asked what she understands by a “content-oriented teacher,” the student again gives a brief description without providing justifications or more profound reflections on the subject. Next, the teacher in charge questions the assumptions of traditional pedagogy to grasp what the student understands about being a good teacher.

Figure 2¹⁰

Antônia’ written production in the online Vaivém

1) Quais são os pressupostos de uma pedagogia tradicional?

A pedagogia tradicional apresenta o professor como o agente principal da transmissão do conhecimento. Tem o ensino baseado em verdades impostas.

1) existe alguma pedagogia em que o professor não transfira conhecimento?

Comente

Ao meu ver, não existe alguma pedagogia em que o professor não transfira seu conhecimento. Independente da forma que ele escolheu, ele está transferindo tal conhecimento.

¹⁰ Figure text: *What are the assumptions of a traditional pedagogy?* Traditional pedagogy presents the teacher as the main agent in the transmission of knowledge. It has teaching based on imposed truths. *Is there any pedagogy in which the teacher does not transfer knowledge? Give your thoughts.* In my opinion, there is no pedagogy in which the teacher does not transfer his/her knowledge. Regardless of the form he/she chose, he/she is transferring such knowledge.

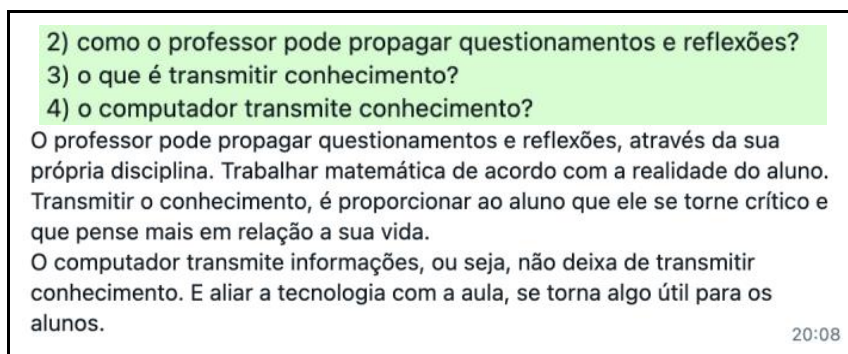
In all these arguments, Antônia presents direct and, in some cases, simplified statements. These responses are not intended to offer explanations or justifications for the questions but only to answer the question (Muir & Beswick, 2007). Such characteristics are descriptive and linear (Day, 1999).

Antônia's deliberate reflection

When we return to the preservice teacher's answer to the generating question (Figure 1), we see that, besides the technical description at the beginning, Antônia presents signs of deliberate reflection, explaining that being content-oriented does not guarantee success in classes. She believes the most important thing is how the content is worked with the students. By providing more contextualised explanations that justify her descriptions, Antônia presents some characteristics of this category, such as providing reasons for her descriptions (Day, 1999) and justifications for an action (Muir & Beswick, 2007). When answering some questions from the teacher in charge, Antônia presents the following written production:

Figure 3¹¹

Antônia's written production in the online Vaivém



¹¹ Figure text: *How can the teacher promote questions and reflections? What does transmitting knowledge mean? Does the computer transmit knowledge?* The teacher can spread questions and reflections through his/her own subject. Work mathematics according to the student's reality. Transmitting knowledge means enabling students to become critical and think more about their lives. The computer transmits information, that is, it does not stop transmitting knowledge. And combining technology with the classroom becomes something useful for students.

When answering the proposed questions, Antônia described information and explained how the teacher could use the subject to spark reflections and adapt the content to students' reality. When discussing knowledge transmission, she emphasises that the objective is to help the student develop, encouraging them to reflect, which depends not only on the teacher's participation but also on the student's willingness to grow.

Finally, in question 4, Antônia's argument is also considered deliberate reflection because she managed to describe what she thought and problematise technology for students' use, thereby enhancing their knowledge.

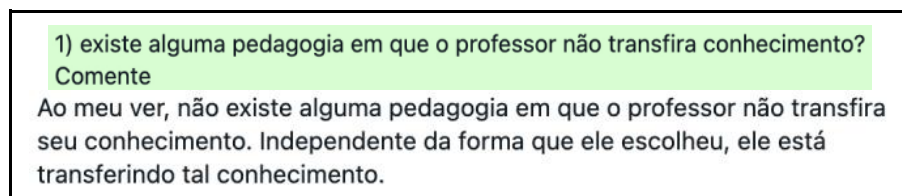
The aim of the head teacher in proposing these questions is to direct Antônia to reflect on the term "transmit." She interprets signs that expand this term, bringing possibilities associated with students, such as developing questions, reflections, and critical thinking.

Antônia's arguments, at this moment, seek to bring meaning to the descriptions she makes, interpreting them (Day, 1999). She presents signs of deliberate reflection, as evidenced by her exploration of justifications for her manifested understandings (Muir & Beswick, 2007).

In Figure 4, we present Antônia's written production for more questions that the professor in charge proposed.

Figure 4¹²

Antônia's written production in the online Vaivém



¹² Figure text: *Is there any pedagogy in which the teacher does not transfer knowledge? Give your thoughts.* In my opinion, there is no pedagogy in which the teacher does not transfer his/her knowledge. Regardless of the form he/she chose, he/she is transferring such knowledge. *How does the students' learning process work in this "transmission of knowledge"?* The teacher passes (content) on to his/her students, instigates, creates situations according to the content they are working on. Thus, students do not just learn in the usual way, but in a more dynamic way that often makes them more critical.

1) Como funciona o processo de aprendizagem dos estudantes nessa "transmissão de conhecimento"?

1) O professor passa para seus alunos, instiga, cria situações de acordo com tal conteúdo que ele está trabalhando. Dessa forma, o aluno não aprende apenas da forma comum, mas de uma forma mais dinâmica que muitas vezes o torna mais crítico.

21:51

Here, Antônia expresses her opinion that the teacher is always “transmitting knowledge” and explains that this occurs regardless of the process used. In her second answer, she argues that the teacher must instigate and create favourable and dynamic situations that enable learning development. Thus, according to Antônia, students have multiple ways to learn.

These justifications characterise her arguments as Level 2: Deliberate reflection, as they signify described processes (knowledge vision) (Day, 1999), providing evidence of how such a process occurs from her own perspectives. The head professor’s objective involved reflecting on the term “transmission.” Although Antônia, like other preservice teachers, uses this term, she demonstrates an understanding that the process of teacher-student interaction to promote learning is more complex and involves additional elements, such as instigation, creation, dynamism, and criticality. When considering such elements, Antônia justifies her own description (Muir & Beswick, 2007).

Antônia’s critical reflection

Figure 5 presents another written production by Antônia in the online Vaivém.

Figure 5¹³

Antônia’s written production in the online Vaivém

2) é possível ser detentor de conhecimento, então? Comente.

¹³ Figure text: *Is it possible to be the holder of knowledge, then? Give your thoughts.* I believe that it is possible to be the holder of knowledge, but as a teacher, I will use this as a way of showing my students that only I know what I am passing on to them and no one else does, or I will be the professional who will facilitate the students’ learning. As a teacher, do you consider yourself the holder of knowledge, and how did you express this to your students?

Considero que é possível ser detentor do conhecimento, mas eu enquanto professora irei usar isso como uma maneira de mostrar para meus alunos que só eu sei daquilo que estou passando para eles e ninguém mais sabe, ou serei a profissional que facilitará a aprendizagem dos alunos. Você, professor, se considera detentor do conhecimento e qual foi a maneira que externou isso para seus alunos?

In this answer, Antônia reflects on being a “holder of knowledge” and sees this “possession” as a resource for the classroom, but not as an absolute truth.

In Figure 6, she questions the teacher in charge about his vision. Antônia not only shared her idea but also sought to learn her teacher’s perspective, opening up space for exchanging ideas and showing herself to be vulnerable in an environment of problematisation (Muir & Beswick, 2007).

The critical incident in Antônia’s reflection is when she questions the head teacher about his vision of “holder of knowledge.”

Figure 6¹⁴

Antônia’s written production in the online Vaivém

3) eu não me considero detentor de conhecimento algum! Na minha visão o conhecimento é um processo, caracterizado por muitas coisas, que é produzido pelos alunos e por nós. Portanto, ou todos nós somos donos do conhecimento, ou ninguém é hehe. O que acha disso?

Acredito que não deixamos de ser dono do nosso conhecimento, porque podemos fazer dele o que quisermos. Como a bolsa, ela já existia antes de eu conhecê-la, mas a partir do momento que eu conheço e adquire, posso usá-la da maneira que for necessária, mesmo não parecendo correto na visão de outra pessoa.

13:09

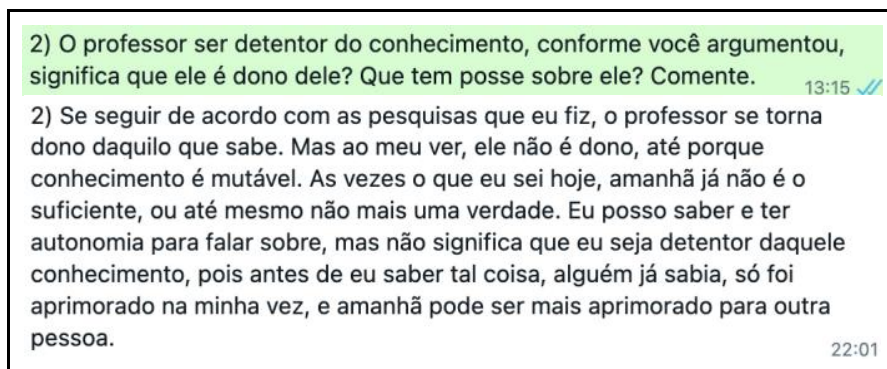
¹⁴ Figure text: *I don’t consider myself to have any knowledge! In my view, knowledge is a process, characterised by many things, which is produced by students and by us. Therefore, either we all own knowledge, or no one does, hehe. What do you think about it? I believe that we do not stop being the owners of our knowledge, because we can do with it whatever we want. Like a bag, it already existed before I knew about it, but from the moment I know about it and acquire it, I can use it in whatever way I need, even if it doesn’t seem right to someone else.*

Antônia's earlier questioning allowed her to establish an exchange of ideas with the professor and see his perspective on the vision of knowledge. Antônia's critical reflection provided an opportunity to contact another vision about being a "holder of knowledge." The professor thinks knowledge is a process under construction, something that both the teacher and students continually produce. He ends his reflection with an open question for Antônia, encouraging her to reflect further (Figure 7).

The critical incident in Antônia's reflection is when she establishes an exchange of ideas with the professor and begins to see his perspective on the vision of knowledge.

Figure 7¹⁵

Antônia's written production in the online Vaivém



Here, Antônia goes beyond justifying her vision, considering different interpretations of what it means to be a "holder of knowledge." Initially, she mentioned that, according to her research, the teacher could be seen as the "owner" of knowledge. After reflecting on it, she questions this idea. She says that knowledge is changeable and constantly evolving, highlighting that it

¹⁵ Figure text: *The fact that the teacher is the holder of knowledge, as you argued, does this mean that he/she owns it? Who has possession over it? Give your thoughts.* If you follow the research I did, the teacher becomes the owner of what he/she knows. But in my opinion, he/she is not the owner, because knowledge is changeable. Sometimes, what I know today is no longer enough tomorrow, or even no longer true. I may know and have the autonomy to talk about it, but it does not mean that I am the holder of that knowledge, because before I knew something, someone else already knew it, it was just improved on my turn, and tomorrow it may be further improved for someone else.

changes as it passes from person to person. Antônia's reflective path shows an interactive context of constant problematisation, characterised by the analysis of different perspectives (Muir & Beswick, 2007) and questioning various views of knowledge (Day, 1998). For these reasons, we understand that the arguments expressed here can be considered signs of critical reflection.

The critical incidents identified in Antônia's reflection involve the problematisation of being a "holder of knowledge" and how this notion can influence her practice as a future teacher. Another critical incident arises when Antônia seeks the perspective of the head teacher.

The following section presents the analysis for another preservice teacher: Claudia.

Claudia's reflective writing

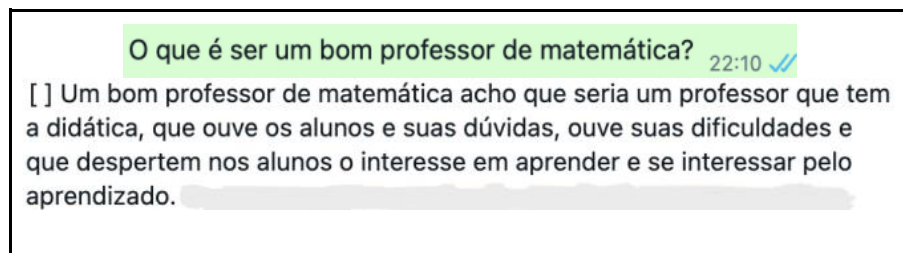
Claudia is a very critical and questioning preservice teacher. She always reacts almost instantly to situations that seem strange to her. Sometimes, she also arrives tired for classes because of her exhausting work. However, it always seeks to improve interactions and proposed activities.

Claudia's technical description

In Figure 8, we highlight Claudia's written production in the online *Vaivém*.

Figure 8¹⁶

Claudia's written production on the online Vaivém



¹⁶ Figure text: *What does it mean to be a good math teacher?* I think a good math teacher should have teaching skills, listen to students and their doubts, listen to their difficulties and awaken in students an interest in learning. *Is it possible for a teacher to have no teaching skills? Give your thoughts.* No, didactics is a technique for transmitting knowledge, so I believe that even if many teachers have a difficult or complicated technique, it is still a technique.

2) é possível que um professor não tenha didática? Comente.

2: Não, a didática é técnica de transmitir conhecimento então creio que mesmo que muitos professores tenham uma técnica difícil ou complicado mesmo assim é uma técnica.

When asked what it means to be a good mathematics teacher, Claudia provided a technical description in much of her answer, simply outlining a general theme (Muir & Beswick, 2007). As this is the head professor's first question to begin her reflection, we will focus only on her description.

Figure 9 describes the characteristics Claudia considers important in a good math teacher, noting that teaching is vital without further elaboration. When asked if there is a teacher without teaching methods, Claudia defines teaching methods as a way of transmitting knowledge, regardless of quality. Therefore, she described what she thought, but without reflecting further, she made a descriptive statement (Hampton, 2010; Muir & Beswick, 2007; Day, 1999).

Figure 9¹⁷

Claudia's written production

1) O que é transmitir conhecimento?

1: Acho que transmitir conhecimento é você transmitir aquilo que você sabe, ensinar algo, passar pra alguém aquilo que você sabe.

1) suponha que uma pessoa entre no seu trabalho para realizar exatamente sua função. Como você "passaria" o conhecimento do que deve ser realizado para ela?

1: Eu passaria como eu passo pros meus alunos, diria oque digo pra ele, daria a minha aula normal como é normalmente em sala de aula

¹⁷ Figure text: *What does it mean to transmit knowledge?* I think that transmitting knowledge involves sharing what you know, teaching something, and passing on your knowledge to someone. *Suppose a person comes into your workplace to perform exactly what you do. How would you convey to him/her the knowledge of what needs to be accomplished?* I would teach it as I teach it to my students, I would say what I say to them, I would give my normal class as it normally is in the classroom.

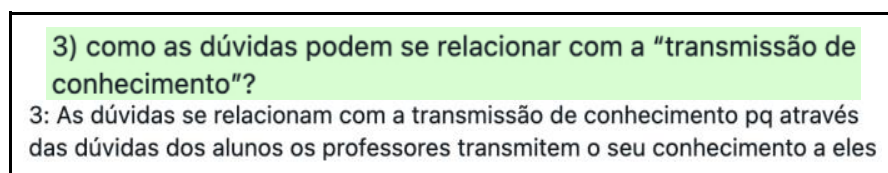
Claudia presented a simple and direct definition of what it means to “transmit knowledge” in her view; she did not delve into an analysis, that is, she just described questions (Hampton, 2010; Muir & Beswick, 2007; Day, 1999).

To always seek the reflections of his students, the professor asked a question to get Claudia to analyse the use of the expression “transmission of knowledge.” Again, Claudia answered straightforwardly how she would “pass on” the knowledge to another person without going further or providing justification.

In Figure 10, we present another of Claudia’s written productions, which involves clarifying the expression “transmission of knowledge.”

Figure 10¹⁸

Claudia’s written production on the online Vaivém



In this written production, Claudia describes the relationship between students’ doubts and the teacher’s actions in transmitting knowledge, without offering a more detailed analysis or exploring other possibilities, which reveals signs of a technical description (Muir & Beswick, 2007; Day, 1999).

Claudia’s deliberate reflection

Figure 11 illustrates another of Claudia’s written productions in the online *Vaivém*.

¹⁸ Figure text: *How can doubts relate to “transmission of knowledge”?* Doubts are related to the transmission of knowledge because, through students’ doubts, teachers transmit their knowledge to them.

Figure 11¹⁹

Claudia's written production on the online Vaivém

3) você parece valorizar bastante que o professor ouça os alunos. Por que?
3: Porque já tive professores que não souberam ouvir que apenas chegavam na sala dava a tarefa e só, que só chegava na sala passava o conteúdo e dava atividade sem ouvir as dúvidas sem ouvir as dificuldades dos alunos.
4) você diz que nem todos os alunos aprendem da mesma maneira. Como você aprende?
4: Pra eu aprender eu tenho que ouvir bastante e anotar o conteúdo porque as anotações me ajuda depois na hora de estudar pra provas e trabalhos.

Using a personal question based on Claudia's statements, the teacher directed the preservice teacher to reflect on an aspect that he considers crucial: the need for the teacher to listen to students in the classroom. In this way, Claudia did not find herself answering descriptively but rather reflecting on her own experience with teachers who did not know how to listen to students, bringing to light a critical incident from her own trajectory as a student and interpreting it (Muir & Beswick, 2007). She justifies the importance of teachers being open to dialogue with students.

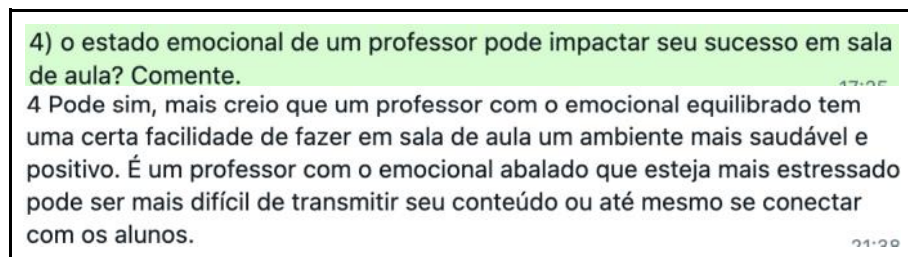
Following their interactions, the teacher again encourages the preservice teacher to reflect personally on how she learns. Claudia reflects on her way of learning, explaining that paying attention to the teacher's explanations and taking notes are essential tools for her learning process. At this point, she brings up critical incidents from her own trajectory, presenting elements of how learning happens for herself and explaining justifications for her actions (Muir & Beswick, 2007; Day, 1999).

In Figure 12, another of Claudia's written productions is represented in the online *Vaivém*.

¹⁹ Image text: *You seem to really value teachers listening to students. Why is that? Because I've had teachers who didn't know how to listen, who would simply enter the classroom, assign the task, and then leave, without addressing the students' doubts or difficulties. You say that not all students learn the same way. How do you learn? To learn, I have to listen attentively and write down the content, as the notes help me later when studying for tests and assignments.*

Figure 12²⁰

Claudia's written production on the online Vaivém

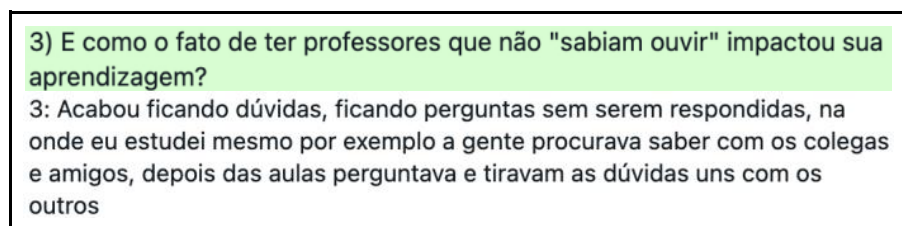


In this written production, Claudia seems to consider the impact of teachers' emotional state or shock on their work. This analysis provides evidence that Claudia explains a hypothetical teacher's actions and behaviours (Muir & Beswick, 2007). Therefore, we understand that Claudia presents signs of a deliberate reflection on teaching pedagogical practice in this production.

Claudia's critical reflection

Figure 13²¹

Claudia's written production on the online Vaivém



²⁰ Figure text: *Can teachers' emotional state impact their success in the classroom? Give your thoughts.* Yes, they can, but I believe that a teacher with balanced emotions has a certain ease in creating a healthier and more positive environment in the classroom. An emotionally shaken teacher who is more stressed may find it more difficult to convey their content or even connect with students.

²¹ Figure text: *And how did having teachers who didn't "know how to listen" impact your learning?* There were doubts and questions left unanswered. Where I studied, for example, we would ask our colleagues and friends questions, and after class we would ask and clear up our doubts with each other.

Here, Claudia reflects on the impact of teachers who fail to listen to their students, using her own experience as a reference. She explains how this issue affected her learning process, characterising it as a critical incident in her trajectory (Muir & Beswick, 2007). However, in the same incident, she argues that her colleagues, along with herself, sought support from each other after class, showing a reflection on how the lack of communication with the teacher led them to adopt alternative strategies to resolve their doubts. At this point, Claudia is seeking an alternative explanation for why teachers might not listen to students, so that they can seek support from each other. As Claudia does in this written production, offering alternatives to critical incidents reveals signs of critical reflection (Muir & Beswick, 2007; Day, 1999).

Claudia identifies the impact of teachers who do not know how to listen to students, using her own experience as a reference, which is why it is characterised as a critical incident.

Tiago's reflective writing

Tiago is one of the most participatory preservice teachers in *Mathematics Teaching Methodology* classes. He rarely misses classes, even though he does manual labour. Even when tired, he participates intensely in discussions, refusing to accept information from above. Tiago is highly critical and questioning.

He stands out for his critical and reflective stance in his written works, presenting a more in-depth view of the proposed themes. This approach highlights a greater critical sense, which sets him apart from other participants.

Furthermore, he rarely confines his reflections to a technical level of description. His answers often evolve into deliberate or critical reflections, showing an ability to analyse situations in greater depth. Given that his technical descriptions almost constantly evolve to higher levels of reflection, we began directly with the deliberate reflection section, recognising that Tiago constantly goes beyond the limits of descriptions and adopts a more contextualised and critical analysis stance.

Tiago's deliberate reflection

Figure 14 presents one of Tiago's written productions in the online Vaivém.

Figure 14²²

Tiago's written production in the online Vaivém.

O que é ser um bom professor de matemática?

Ser um bom professor vai além de ser um profissional na área, é ser um educador, ser competente entre outras qualidades. Porém quando se fala de matemática é diferente, tem todo um contexto para o ensino da matemática por ser uma matéria um pouco difícil de assimilar as ideias, com isso o aprendizado se torna complicado e um professor de matemática tem que abordar o conteúdo proposto de maneira que fique claro para o aluno e não apenas passe o conteúdo, mas que o explique de maneira que fique claro e assim possibilitando os alunos a ter um real aprendizado. A principal diferença entre ser professor e ser professor de matemática está aí, ensinar da maneira correta os alunos e também realizar um planejamento de modo que os conteúdos sejam lineares, de modo que esses conteúdos serão trabalhados na ordem correta para que os alunos não se sintam perdidos ou para que estão aprendendo aquele conteúdos em específico, pois ele sendo linear eles já deram uma breve noção do qual será o próximo conteúdo. Portanto, essas são minhas palavras do que é ser professor de matemática e sim não é a mesma coisa de ser professor de qualquer outra área, acredito que toda área tem sua metodologia específica para que seus alunos tenham um melhor aprendizado dos conteúdos dessa matéria.

12:55

²² Figure text: *What does it mean to be a good math teacher?* Being a good teacher goes beyond being a professional in the field, it is being an educator, being competent among other qualities. However, when it comes to mathematics, it is different. There is a whole context for teaching mathematics, as it is a subject in which it is a little difficult to assimilate ideas, which makes learning complicated and a mathematics teacher has to approach the proposed content clearly to the student and not just pass on the content, but explain it clearly and thus enabling students to have real learning. The main difference between being a teacher and being a math teacher is there, teaching students correctly and also planning so that the content is linear, so that this content will be worked on in the correct order so that students do not feel lost or so that they are learning that specific content, because being linear they already have a brief idea of what the next content will be. So, these are my words on what it is like to be a math teacher and yes, it is not the same as being a teacher in any other area, I believe that each area has its specific methodology so that its students learn better the contents of that subject.

Tiago's answer demonstrates deliberate reflection, possessing profound elements that justify his vision of being a good mathematics teacher (Muir & Beswick, 2007). Tiago not only describes what it means to be a good math teacher but also explains why he considers teaching math a special task, mentioning the difficulty students face when exploring mathematical concepts. He justifies that, to facilitate learning, the teacher must adapt the teaching method to make it clear and understandable.

He also reflects on the importance of adequate planning, highlighting that teaching mathematics requires a linear sequence so that students understand the content without feeling confused –a self-understanding of the role of the mathematics teacher. Tiago mentions all these needs in his view and justifies why he believes that following the content in a linear, sequential way is essential for students' learning and development.

All these aspects mentioned above reveal Tiago's need to justify critical incidents, giving explanations and meanings for the elements he describes (Muir & Beswick, 2007).

Figure 15 shows one of Tiago's written productions in the online *Vaivém*.

Figure 15²³

Tiago's written production in the online Vaivém.

Questão 2) o que é ser um educador?

Resposta da questão (2): Ser um educador em minha opinião é ser, profissional e coerente. Um educador tem uma função essencial na área da educação, ele é responsável pelo ensino dos alunos e isso é de certo modo desafiador pois é uma combinação de tanto dominar o conteúdo de modo que o professor saiba totalmente a matemática proposta para a aula quanto utilizar de práticas pedagógicas para que o aluno aprenda totalmente aquele conteúdo proposto.

Tiago's written production demonstrates that he believes an educator must thoroughly master the content to be addressed. Furthermore, Tiago

²³ Figure text: *What does it mean to be an educator?* Being an educator, in my opinion, is being professional and coherent. An educator has an essential role in the field of education, he/she is responsible for teaching students and this is somewhat challenging because it is a combination of both mastering the content so that the teacher fully knows the mathematics proposed for the class and using pedagogical practices so that the student fully learns the proposed content.

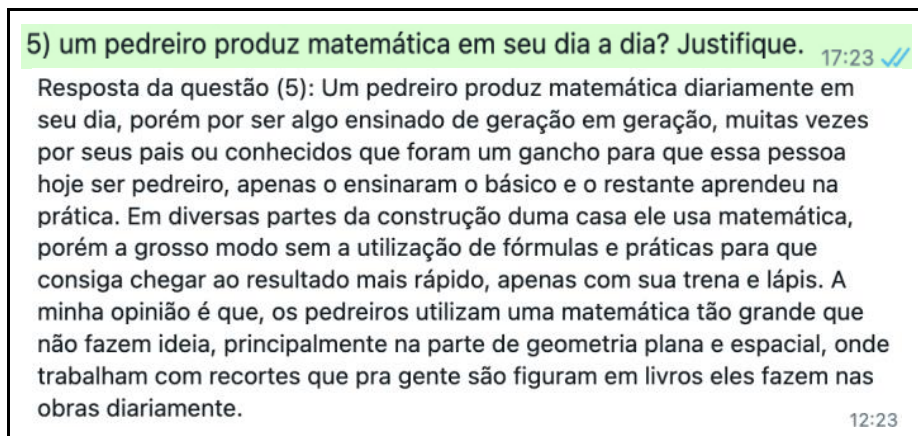
justifies the role of an educator, arguing that they must develop pedagogical practices that facilitate student learning. He also highlights the challenge of integrating theoretical knowledge with pedagogical practice, demonstrating a nuanced understanding of the profession's complex aspects.

In this written production, Tiago intends to explain his ideology regarding the teacher's professional practice (Day, 1999) and explain and justify the critical incidents he identified –characteristics of being an educator, and the teacher's role and its relationship with mathematical content.

Figure 16 presents one of Tiago's written productions in the online *Vaivém*.

Figure 16²⁴

Tiago's written production in the online Vaivém.



²⁴ Figure text: *Do bricklayers produce mathematics in their daily lives? Justify.* Bricklayers produce mathematics every day, but because it is something taught from generation to generation, often by their parents or acquaintances who were a hook for them to become bricklayers today, they only taught them the basics and the rest they learned in practice. In several parts of the construction of a house they use mathematics, but roughly without the use of formulas and practices so that they can reach the result faster, just with their tape measures and pencils. My opinion is that bricklayers use such a large amount of mathematics that they have no idea, especially in the part of plane and spatial geometry, where they work with cuts that for us are shown in books, they make on construction sites daily.

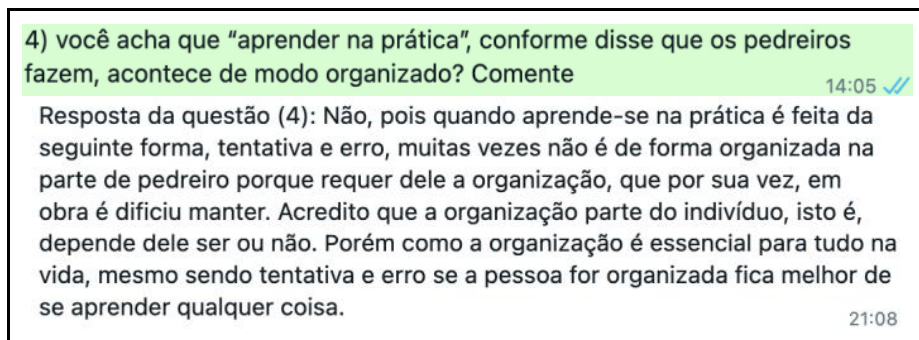
Although bricklayers do not use mathematical formulas, Tiago explains that they effectively employ mathematical concepts, particularly in geometry. He justifies this view by mentioning that bricklayers still use mathematical skills, such as a tape measure and a pencil, even without formal technical knowledge.

He also explains the everyday and informal mathematics practised by bricklayers, differentiating it from the formal mathematics taught in the classroom. Tiago considers how mathematical knowledge is explored from generation to generation, often without technical terms, demonstrating a more contextualised understanding of the role of mathematics outside of the academic context. At this point, Tiago presents an explanatory and justified analysis, which guarantees the mobilisation of evidence of deliberate reflection (Muir & Beswick, 2007). However, it does not explore alternatives beyond the critical incidents he describes.

Figure 17 shows another of Tiago's answers recorded in the online *Vaivém*.

Figure 17²⁵

Tiago's written production in the online Vaivém.



²⁵ Figure text: *Do you think that "learning by doing," as you said bricklayers do, happens in an organised way? Give your thoughts.* No, because when you learn –if in practice it is done in the following way, trial and error, it is often not in an organised way on the part of the bricklayer because it requires organization, which in turn, on site is difficult to maintain. I believe that the organisation starts with the individual, that is, it depends on whether they are organised or not. However, as organisation is essential for everything in life, even if it is trial and error, if the person is organised, it is easier to learn anything.

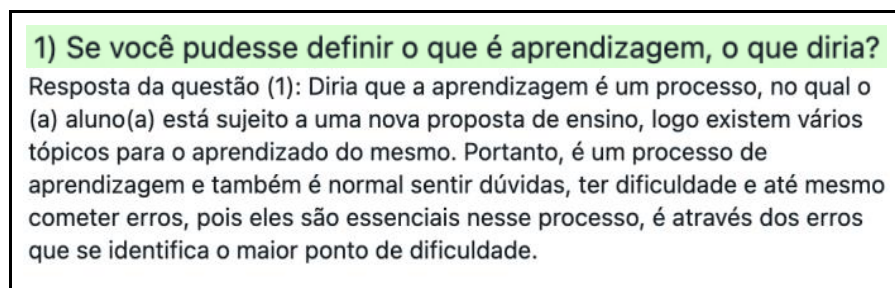
Tiago argues that practical learning for bricklayers does not necessarily occur in an organised manner; instead, it occurs through a process called “trial and error.” Often, bricklayers do not realise that there is mathematics involved; they only know it is something naturalised. They do it through experience and relationships with previous generations. Tiago argues that although mathematics can be developed through “trial and error” without organisation, planning, and structure can significantly facilitate learning. This reflection reinforces his understanding of the importance of planning and organisation in teaching mathematics.

In this written production, Tiago reveals ideological clues to his understanding of the role of mathematics (Day, 1999), while also providing convincing arguments that deliberately explain (Muir & Beswick, 2007) the manifestation of his critical incident: the bricklayers’ mathematical production.

Figure 18 shows another of Tiago’s written productions in the online *Vaivém*.

Figure 18²⁶

Tiago’s written production in the online Vaivém.



In this written production, Tiago not only defines learning as a process but also explains that it involves exposure to new teaching methods, addressing the elements of this process, such as doubts, difficulties, and mistakes. He argues that mistakes are essential for identifying areas of difficulty and allowing

²⁶ Figure text: *If you could define what learning is, what would you say?* I would say that learning is a process, in which the student is subject to a new teaching proposal, so there are several topics for learning the same. Therefore, it is a learning process and it is also normal to feel doubts, have difficulties and even make mistakes, as they are essential. In this process, it is through errors that the greatest point of difficulty is identified.

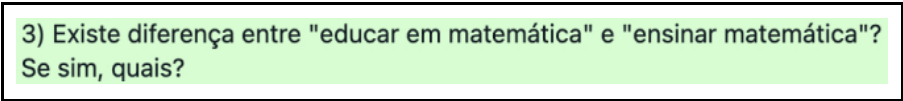
for growth opportunities, thereby demonstrating a deeper understanding of learning. In this argument, in addition to providing detailed explanations on a given topic, Tiago presents relevant information about his conception of the topic (Day, 1999). Such elements enable us to infer that, in this written production, he exhibits signs of deliberate reflection (Muir & Beswick, 2007).

Tiago's critical reflection

Figure 19 presents one of Tiago's written productions in the online *Vaivém*.

Figure 19²⁷

Tiago's written production in the online Vaivém.



3) Existe diferença entre "educar em matemática" e "ensinar matemática"?
Se sim, quais?

²⁷ Figure text: *Is there a difference between “educating in mathematics” and “teaching mathematics”? If positive, which ones?* Educating and teaching, although they are related through education, have different meanings. Educating comes from the sense of teaching values so that the student learns, for example, to respect, act, think, among other things that are the parents' responsibility. The teacher improves and completes them in the classroom, but the basis comes from home. Teaching already encompasses the technical term, specific content that will improve the student's knowledge in mathematics among other areas, but which is the teacher's responsibility. With this, the teacher can educate and teach even though they are not identical things, but they will be linked by mathematics. This can be done in the following way: mathematics education would deal with aspects related to how mathematics would be in this student's life, the reason he/she would learn mathematics and what it brings that is important in life and in everyday life. Therefore, teaching mathematics would be the teacher's area, where he/she would work teaching all the concepts of the area and scientific terms. So these are the differences between educating in mathematics and teaching mathematics.

Resposta da questão (3): Educar e ensinar embora estejam relacionados pela educação eles tem significados distintos. Educar vem do sentido de ensinar valores para que aquele aluno aprenda por exemplo a respeitar, agir, pensar entre outras coisas e que são de responsabilidade dos pais, o professor as aprimora e conclui em sala de aula, porém a base vem de casa. Ensinar já abrange o termo técnico, conteúdos específicos que vão aprimorar o conhecimento desse aluno em matemática entre outras áreas, mas que é de responsabilidade do professor. Com isso ele pode sim educar e ensinar mesmo não sendo coisas idênticas, mas que estarão ligadas pela matemática. Isso pode ser feito da seguinte maneira, a educação matemática lidaria com aspectos relacionados a como a matemática seria na vida desse aluno, o porque ele iria aprender a matemática e o que ela traz de importante na vida e no cotidiano. Logo ensinar matemática seria a área do professor, onde ele iria atuar ensinando todos os conceitos da área e termos científicos. Portanto, essas são as diferenças entre educar em matemática e ensinar matemática.

In this written work, Tiago reflects on the differences between “educating mathematics” and “teaching mathematics”, considering the teacher not only a “transmitter” of content but someone who contributes to the formation of students’ values and attitudes. He further extends his reflection, stating that the basis of education comes from home. Students who lack a solid educational foundation often struggle to collaborate and develop critical thinking both inside and outside the classroom.

He distinguishes “mathematics education” as a process that relates mathematics to the student’s daily life, whereas “teaching mathematics” refers to the technical and scientific aspects of the content, which we generally learn at school. Although “educating” and “teaching” are distinct, both can be integrated in a complementary way.

Tiago goes beyond justification and description, considering different aspects and revealing evidence of how approaches to mathematics education and mathematics teaching could occur from different intentions, showing alternative perspectives on the critical incidents he identifies (Muir & Beswick, 2007). For these reasons, Tiago’s written production reveals signs of critical reflection.

Figure 20 highlights another written production by Tiago in the online *Vaivém*.

Figure 20 ²⁸

Tiago's written production in the online Vaivém.

3) como utilizar a matemática em sala de aula de maneira que os alunos se tornem mais críticos?

Resposta da questão (3): Uma boa opção é abordar questões relacionadas à sociedade, uma vez que estão por dentro do que acontece a sua volta aprendem a ter uma ampla visão de sociedade e mundo. Com isso, os alunos vão tendo mais esse contato com questões cotidianas e com o passar do tempo vão aprendendo a julgar e ser críticos sobre assuntos importantes e polêmicos da sociedade.

Tiago argues that an effective way to use mathematics to develop students' critical thinking is by addressing issues related to society and their daily lives, which may create greater interest and some "identification." He mentions that exposing students to topics that involve the happenings around them broadens their view of the world and society. This connection with everyday life helps students develop the ability to critically evaluate and think about important societal issues. This approach that Tiago suggested reveals an alternative perspective (Muir & Beswick, 2007) for teaching mathematics in a way that the possibilities for learning are expanded and given meaning by the students. We consider that Tiago mobilises signs of critical reflection in this written production by providing significant explanations, presenting justifications, and proposing alternative perspectives.

FINAL CONSIDERATIONS

In this work, we analysed the reflective writings mobilised by preservice mathematics teachers using the online instrument *Vaivém*. For this, we used Muir and Beswick's (2007) categories of reflection. Through this theoretical framework, we highlighted the categories of reflection – technical description, deliberate reflection, and critical reflection – and the main themes addressed by each participant. This theoretical perspective enabled us to observe the process of developing each preservice teacher's reflections and

²⁸ Image text: *How to use mathematics in the classroom so that students become more critical?* A good option is to address issues related to society, since they are aware of what is happening around them and learn to have a broad view of society and the world. With this, students will have more contact with everyday issues and, over time, they will learn to judge and be critical about important and controversial issues in society.

identify how the online *Vaivém* contributed to the development of reflective and critical skills during the teacher education process.

Reflective writing has great potential for professional development, particularly in relation to mathematical content, and is a valuable tool for preservice teachers seeking to deepen their theoretical knowledge throughout their education. Reflecting on their experiences and practices enables preservice teachers to think more critically, expanding their opportunities for professional growth.

Reflective writing demonstrated by preservice teachers in the online *Vaivém* can be generalised to other formative contexts. In the case of **technical descriptions**, we observed that participants gave brief descriptions without providing justifications or in-depth reflections. The answers were usually straightforward, lacking details, and focused solely on answering the question without expanding the discussion. Furthermore, preservice teachers tend to describe characteristics they consider relevant without critically analysing them. They often define concepts superficially, without exploring broader implications or meanings.

In turn, through deliberate reflection, there is an advance toward a more technical description as participants begin to explain and contextualise their answers. They provide reasons for their descriptions, presenting justifications for their understandings. They also interpret the terms discussed more broadly, seeking to give meaning to the processes described. At this level, preservice teachers present their opinions and argue about what they believe to be relevant, connecting their experience with more in-depth explanations and seeking to justify their own descriptions.

Finally, in **critical reflection**, preservice teachers go beyond justification and description, considering different interpretations and questioning their own conceptions as well as those of others involved in the discussion. At this level, we observed that they not only share their ideas but also seek the perspective of the teacher educator, establishing an exchange of ideas that deepens the discussion. Critical reflection also involves questioning fundamental concepts, such as the idea of being a “holder of knowledge,” encouraging the analysis of different points of view. Furthermore, participants offer alternatives to the challenges encountered in teaching practice, reflecting on the impact of issues such as a lack of communication between teachers and students. This level stands out for providing significant explanations, presenting in-depth justifications, and proposing alternative perspectives, contributing to a more critical and conscious education of preservice teachers.

Among the themes addressed by the preservice teachers, in a logic of developing reflections, the following stand out: characteristics of a good teacher, epistemology of knowledge, alternative proposals for classroom practices, vision of learning, and emotional aspects that impact teacher education, among others. These themes are directly associated with the syllabus for *Mathematics Teaching Methodology*. Therefore, the online *Vaivém* enhanced the processes discussed in the subject and could be a teaching resource consistent with the curriculum proposal for the teaching degree in mathematics. Furthermore, such themes emerged naturally in the online *Vaivém* based on the wishes of the preservice teachers and the interventions of the teacher in charge.

In this sense, the head teacher's interventions were essential in encouraging reflective writing, enabling preservice teachers to expand their arguments and deepen their analyses. Among the intervention strategies employed, the following stand out: the use of open-ended questions, which led participants to reconsider their initial answers; the problematisation of concepts, encouraging the construction of new understandings; and the encouragement of the exchange of ideas between teacher and student. These interventions contributed to the preservice teachers' evolving reflections throughout the process.

Preservice teachers presented different levels and styles of reflection, showing that the online *Vaivém* is flexible and can adapt to the individual characteristics of each preservice teacher. We observed evolution in the participants' reflections, which initially included technical descriptions and, through the teacher's questions, advanced to more complex levels, such as deliberate and critical thinking.

The online *Vaivém*, as a new configuration of the traditional *Vaivém*, presented characteristics that, in our view, are potentially beneficial for the learning process of the preservice teacher. The fact that they were online and registered with WhatsApp allowed the graduates to get even closer to the instrument. From the educator's point of view, this new configuration made the instrument more practical, and interventions happen more quickly.

AUTHORSHIP CONTRIBUTION STATEMENT

JBS and PHR conceived the idea presented. PHR collected the data. JBS processed the data. LMCPO, WBT, JBS, and PHR analysed the data, discussed the results, and contributed to the final version of the manuscript.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study are under the responsibility of JBS and PHR. They may be made available upon reasonable request by other interested parties for five years upon signing a liability agreement.

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