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An Algebra as a historical object in technical vocational education: Perspectives and characterisation

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ABSTRACT

Background: This article addresses algebraic knowledge from a historical perspective, aiming to understand how algebra content was structured and organised for vocational education during Brazil's First Republic. This educational modality was marked by initiatives that sought to integrate schooling with preparation for work. **Objective:** To understand how the algebra content to be taught was structured and organised to meet the goals of vocational education. **Source:** The source of analysis is the work *Álgebra Elementar* by Guilherme Ivens Ferraz (n.d.), a textbook that is part of the *Library of Professional Instruction*. **Theoretical-methodological framework:** The study is grounded in theoretical and methodological contributions from the History of Mathematics Education, based on the theory of objectified knowledge, the concept of graded teaching constitutive of school mathematics, and textbook analysis criteria established by O'Keeffe (2013). **Results:** Among the findings, it stands out that the presentation of content through expository and instructional texts is a recurring feature throughout the work. It is also observed that the only didactic support used by the author is written language, without the inclusion of visual elements or other complementary resources. **Conclusions:** It is understood that the work reflects a didactic rationality anchored in the internal logic of mathematics, prioritising the development of operational skills. It is also suggested that future investigations could deepen the reflection on how the educational sciences might engage in dialogue with the structure and organisation of algebra content aimed at technical and vocational education.

Keywords: History of mathematical education; technical vocational education; textbook; álgebra.

Uma Álgebra como objeto histório no ensino profissional técnico: Perspectivas e caracterização

RESUMO

Contexto: Este artigo aborda os saberes algébricos sob uma perspectiva histórica, com o intuito de compreender como os conteúdos de álgebra estavam

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estruturados e organizados para a educação profissional no período da Primeira República. Uma modalidade de ensino cuja trajetória, no Brasil, foi marcada por iniciativas que buscavam integrar o ensino à formação para o trabalho. **Objetivo:** Compreender como os conteúdos de álgebra *a* ensinar estavam estruturados e organizados para atender às finalidades da educação profissional. **Fonte:** Toma-se como fonte de análise a obra *Álgebra Elementar*, de Guilherme Ivens Ferraz (s.d.), um livro didático pertencente à Biblioteca de Instrução Profissional. **Referencial teórico-metodológico:** O estudo fundamenta-se em aportes teórico-metodológicos da História da educação matemática, com base na teoria dos *saberes objetivados*, na *graduação* de ensino constitutiva da *matemática do ensino* e nos critérios de análise de livros didáticos estabelecidos por Okeeffe (2013). **Resultados:** Entre os resultados da análise, destaca-se que a apresentação dos *conteúdos* na forma de *textos expositivos* e *instrutivos* constitui um recurso recorrente na obra. Observa-se, ainda, que o único suporte didático utilizado pelo autor é a linguagem escrita, sem a incorporação de elementos visuais ou de outros recursos complementares. **Conclusões:** Compreende-se que a obra reflete uma racionalidade didática ancorada na lógica interna da matemática, priorizando o desenvolvimento de habilidades operatórias. Sugere-se, ainda, que investigações futuras possam aprofundar a reflexão sobre como as ciências da educação poderiam dialogar com a estrutura e organização dos *conteúdos* de uma álgebra, voltada para o ensino profissional técnico.

Palavras-chave: História da educação matemática; ensino profissional técnico; livro didático; álgebra.

INTRODUCTION

Vocational education in Brazil has its trajectory characterised by initiatives that sought to integrate teaching with work training. Among these initiatives, the Schools of Apprentice Craftspeople (Escolas de Aprendizes Artífices - EAA) stand out, created in 1909 by Decree No. 7.566, of September 23 (Brasil, 1909), as a reflection of the relationships between school, professionalisation, and the world of work. For Cunha (2000), the establishment of those schools represents "the most notable event in vocational education in the First Republic" (p. 63). Fonseca (1961), in turn, considers this creation as "the initial milestone of the federal government's activities in the field of vocational education" (p. 174). Decree N. 7.566/1909 drove the creation of nineteen EAA in state capitals. In Rio de Janeiro, the EAA was established in Campos, and Rio Grande do Sul was excluded from the list because it already had the Parobé Institute[1].

1 Created in 1906, under the name of Technical Vocational Institute of Porto Alegre [Instituto Técnico Profissional de Porto Alegre], it offered vocational technical

Teaching at the Schools of Apprentice Craftspeople (EAA), or Apprentice Programs, has undergone a process of reformulation throughout its history, driven by the creation of the *Technical Vocational Education Remodeling Service* [Serviço de Remodelação do Ensino Profissional Técnico]. This service was composed of a commission intended for "examining the functioning of schools and proposing measures that would remodel vocational education, making it more efficient" (Fonseca, 1961, p. 201). One of the primary outcomes of this process was the development of the *Documento de Consolidação dos Dispositivos Concernentes às Escolas de Aprendizes Artífices* [Document Consolidating the Provisions Concerning Apprenticeship Schools] in 1926, which unified and standardised the curriculum structure of those institutions. Furthermore, in the context of remodeling, Barbaresco (2019) highlights the work carried out by a committee formed by Heitor Lyra da Silva (rapporteur), Afrânio Peixoto, and Victor Vianna. This committee's role was to recommend books for teaching at EAA. Among the recommended works is the collection called *Professional Instruction Library* [Biblioteca de Instrução Profissional].

At the beginning of the 20th century, Thomaz Bordallo Pinheiro began the creation of the *Professional Instruction Library*, a collection of books published in Brazil and Portugal, which was reissued several times. According to Vale (2015), this collection brings together works aimed at vocational training and general education, filling the gap in technical manuals at the time. The collection comprises a series of books that include those aimed at teaching mathematics, such as: practical arithmetic, elementary algebra, plane geometry, and linear drawing. Considering that those books were part of a collection focused on vocational education, but addressed knowledge of general education, we question how the content to be taught in such works would be structured and organised to articulate vocational and general education. Due to textual limitations, in this article, we asks: **How were algebraic contents *to* teach structured and organised in order to meet the purpose of vocational education?**

Research in the field of history of mathematics education (HME) has focused on studying the objectification of mathematical knowledge for teaching, from a socio-historical perspective, in different contexts and periods. According to Vicent, Lahire, and Thin (2001), the school is conditioned to the

education for primary school. In 1917, it was renamed Instituto Parobé (Barbaresco, 2019).

existence of knowings that are objectified in their context. In this sense, Barbier (1996) develops the theoretical notion of *objectified knowings*, which are school knowings that gain notoriety from transmission and communication activities or even from teaching. For Valente (2019), such knowings are presented as a systematised discourse, and can circulate and be mobilised by different individuals and/or social groups; in other words, knowings formalised and legitimised by some social process, for example, the teaching activity. And yet, as pointed out by Hofstetter and Schneuwly (2017), considering *objectified knowings*, it is possible to determine two distinct types of knowings that make up the training and teaching profession: "the knowings *to* teach that are objects of their work; and the knowings *for* teaching, in other words, the knowledge that is the tools of their work" (Hofstetter & Schneuwly, 2017, p. 131).

Furthermore, the textbook, according to Barbaresco and Costa (2019), can be thought of as a support for *objectified knowings*, considering that this material expresses knowledge systematically and is aimed at teaching. Therefore, given what has been stated, we understand that algebra is an *objectified knowing* for technical vocational education. Thus, thought of as a teaching object, how can it be characterised based on the context for which it was created? The answer to this question involves analysis and interpretation of different elements linked to the process of objectification of knowings. Among them, the content to be taught and its structuring may indicate the formalisation of a given knowing, with particular characteristics.

According to Okeeffe (2013), mathematics and science textbooks exert a significant influence on classroom practice and are widely recognised as essential tools for implementing specific curricula. Its intentional organisation means that both the content and its structure play a fundamental role in promoting a certain curriculum vision. Thus, we intend to consider the textbook as a source for the research question, considering that it will be possible, from this document, to capture information that can be analysed and interpreted to enable a characterisation of an algebra aimed at technical vocational education.

THEORETICAL-METHODOLOGICAL CONSIDERATIONS

When analysing international research on textbooks, Choppin (2004) highlights that "The textbook, as Chris Stray observed in 1993, is a complex cultural product… [that] lies at the intersection of culture, pedagogy, publishing, and society" (Choppin, 2004, p. 563). Therefore, understanding the mathematics textbook as a multifaceted cultural artifact requires a broad view that goes beyond its mathematical content. Valente (2008) says that an approach exclusively focused on content cannot achieve the objectives of developing an

HME. Going beyond the simple analysis of the mathematical content contained in these books, the historian of mathematics education seeks to involve them in a broader context of meanings, allowing an analysis of their complexity as a cultural object. In this journey, a series of interconnected elements can emerge. From the authors' initial conception of the work, through the production process and the influence of publishers, to its use by students and teachers, mathematics textbooks can reveal legacies of pedagogical practices that still echo in the current teaching of this subject (Valente, 2008).

The *objectified knowings* for teaching mathematics constitute an object that Valente (2022) called the *mathematics of teaching*. This name is not information provided by empiricism, but rather a theoretical object constructed from a theory, which allows us to interpret how, in specific contexts and periods, mathematics was designed for teaching. According to Valente (2023), the *graduation*, understood as a "pace"[2] of teachings, constitutes this *mathematics of teaching*. The *graduation* can be analysed on different scales[3], a more external one, in which the role of the teaching object in the formative process is evaluated, for example, the *progression* of the objects that make up the *mathematics of teaching* within a curriculum framework. Moreover, the *graduation* can be thought of on an internal scale, in a sense of the *sequence* of the content to be taught, seeking to capture the meaning of the sequencing. The textbook facilitates the internal analysis of the *graduation*, as it expresses knowinge that is sequenced adequately from elements that re-establish a purpose for teaching, or even aimed at establishing a particular practice.

According to Okeeffe (2013), there is a consensus that both the content of textbooks and the way they are used impact teaching or teaching practice. Although the curriculum plays a central role in defining the topics covered in mathematics teaching, textbooks continue to be one of the main resources for its implementation in the classroom. Based on works by Halliday (1973), Morgan (2004), the Third International Mathematics and Science Study (TIMSS) (Valverde *et al.*, 2002), and Rivers (1990), Okeeffe (2013) established a framework for the analysis of mathematics textbooks, composed of four

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2 Subdivision of things into as many parts as necessary, showing that the sequence of this division refers to the same march towards understanding (Valente, 2023).

3 According to Valente (2023), *graduation* can be understood as *progression, programming, sequence,* or *pace of teaching*, and should be analysed according to the adopted observation scale. However, due to space limitations, this text adopts the scale of *sequence*.

elements: *content, structure, expectation, and language,* which will be addressed in this study.

Generally, Okeeffe (2013) says that the *content* of the textbook influences the choices and emphases adopted by teachers and students, directly impacting learning outcomes. The author highlights some essential aspects in the analysis of *content*, such as motivational factors, which include historical notes, biographies of scientists and mathematicians, information about careers, practical applications, and photographs. Furthermore, it mentions comprehension tips, which involve the use of colors and graphics to facilitate the understanding of concepts. In other words, we understand that the analysis of *content* should not be focused solely on the topics covered and the knowledge taught, but on all the elements that can help its transmission, such as visual resources and supplementary texts.

The textbook *structure* can either help or hinder its understanding, making it essential to carefully analyse the sequence and connections between the elements of the text. According to Okeeffe's (2013) studies, for the *structure* to have a positive impact, one must consider several aspects, including the physical organisation of the material. Although the *structure* of the knowing within the book is essential, the physical layout plays a determining role in how the target audience perceives and interacts with the *content*. This aspect involves multiple factors, such as formatting and arrangement of elements, use of images and text, graphic design, presence or absence of colours, levels of information, as well as strategies for unifying and separating *content*.

The *expectation* is the third element pointed out by Okeeffe (2013) in the analysis of textbooks. That is, we understand that performance *expectations* are embedded in those materials and significantly influence students' choice to deal with the topics presented. For example, if a math textbook emphasises repetition and practice, students are likely to subconsciously seek to simply reproduce previously learned methods when faced with a question, without exploring problem-solving strategies. The most fundamental consideration about the *expectation* is to ensure that both students and teachers can read, understand, and interpret the presented *content*.

Language is another element Okeeffe (2013) highlights in the analysis of textbooks. For an efficient understanding of the *contents*, students must be able to communicate mathematical concepts both verbally and in writing. When investigating mathematical language in these materials, the author identifies essential elements, such as the different types of discourse (narrative, descriptive, expository, etc.), the use of coordinators (connectors between sentences) and semantic structures. Among the key points of this analysis, Okeeffe (2013) highlights the *meaning of words*, which includes general vocabulary, technical and specialised mathematical terms, and abbreviations. Furthermore, she highlights the use of *notation signs*, such as Arabic and Hindu number systems and mathematical symbols, e.g., (>), as well as the presence of *graphic signs*, such as pictograms, diagrams, images, and graphs, which help in the representation and understanding of mathematical concepts.

Therefore, based on the theory of the *objectified knowings*, in the *graduation* in teaching, constitutive of the *mathematics of teaching*, and in the textbook analysis criteria established by Okeeffe (2013), we investigated the presence of these elements in the book *Álgebra Elementar* [Elementary Algebra].

THE CHARACTERISATION OF AN ALGEBRA BASED ON AN ANALYSIS OF THE BOOK *ÁLGEBRA ELEMENTAR* [ELEMENTARY ALGEBRA]

Initially, it is worth noting that the edition analysed does not have a publication date, which makes it difficult for researchers to place it accurately in its historical context of use. However, the work, consisting of almost 300 pages and published by Bertrand, is part of the *Professional Instruction Library*.

In the preface of the book *Álgebra Elementar* [Elementary Algebra], Guilherme Ivens Ferraz[4] highlights the importance of works that deepen mathematical knowings beyond arithmetic. In this sense, we understand that arithmetic knowings were taught before algebraic knowings, since the author himself mentions the importance of teaching algebra after arithmetic has been taught. Thus, it is possible to observe the existence of a *progression* of the teaching objects (arithmetic to algebra), one of the elements of *graduation* in the *mathematics of teaching.*

In line with some of the elements pointed out by Okeeffe (2013), Ferraz (n.d.) presents a concise structure of the work in just two pages, highlighting that it begins with an introductory section aimed at beginners, in which he explains, clearly and accessibly, nomenclature and algebraic notations. This aspect highlights the author's concern in addressing the *language* element for

4 The name is mentioned in the preface; therefore, for this analysis, the author of the work was considered to be Ferraz himself, since there is no further information available within the book about his authorship.

students to develop the ability to communicate mathematical concepts both verbally and in writing. In addition, the author provides a summary of the book, highlighting its *structure* in four mostly standardised parts.

The analysis of the overall *structure* reveals that, after the preface, Ferraz (n.d.) presents the introductory section "nomenclature and algebraic notation," which extends over 22 pages with 36 preliminary notions. The first preliminary notion is:

> 1 - Algebra is the part of mathematical sciences in which the reasoning required to resolve questions relating to numbers is abbreviated and generalised, with the help of letters representing numbers and specific signs indicating the operations to be carried out with them and their relationships. These questions are of two kinds: the theorem and the problem. The **theorem aims to demonstrate the existence of the generalisation of specific properties** in known and given numbers. The **problem aims to find numbers, and the knowledge of other numbers** with which they have connections **serves as a basis** (Ferraz, n.d., p. 1, our emphasis).

In the first preliminary notion, we observe that the author uses a mathematical language in an *expository text*[5], defining algebra as a branch of mathematical sciences focused on solving numerical questions through the use of letters, which are classified as theorems or problems. As highlighted in the quote above, Ferraz (n.d.) establishes a distinction between these two concepts: while the theorem aims to demonstrate the generalisation of certain properties of numbers, the problem seeks to determine numbers from previously known relationships.

The author demonstrates four theorems from the rule[6] of multiplication, for example: "the square of the sum of two quantities is equal to the square of

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5 Mathematical language is used in the form of propositional statements, generally numbered, with the purpose of presenting a notion or definition about the topic being addressed. Throughout this article, this form of writing will be referred to as *expository text.*

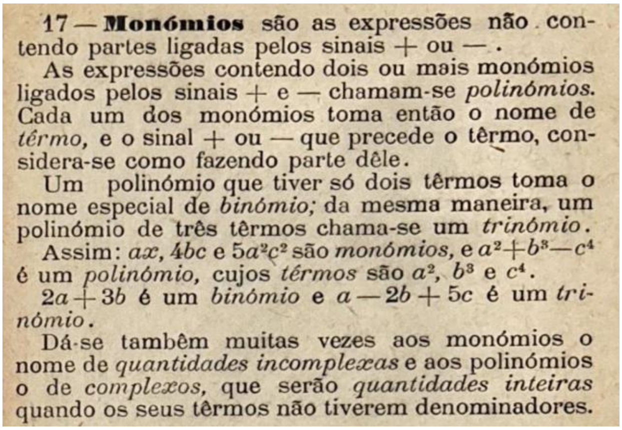
6 Used to guide the performance of algebraic operations. An example is the rule of signs, according to which, "in the multiplication of two terms, or monomials, the product is affected with the sign + when both factors have the same sign, positive or negative; and with the sign - when both factors have opposite signs." (Ferraz, n.d., p. 37).

the first quantity, plus twice the product of the first by the second, plus the square of the second," representing a formula7 algebraically: $(a + b)^2 = a^2 + 2ab + b2$ (Ferraz, n.d., p. 46). In this sense, one way to solve the problem of calculating the square of 127 would be to decompose this number into *120 + 7,* assigning *a = 120 and b = 7,* and applying the formula: $(a + b)^2$. I.e., $(120 + 7)^2 = 120^2 + 2.120.7 + 7^2 = 14\,400 + 1\,680 + 49 = 16\,129$.

However, in some of the 36 preliminary notions, this *expository* approach is enriched by *instructional texts*[8] that present solved examples or propose exercises with answers on particular concepts. Furthermore, they identify *notation signs* in some of these notions, but there is an absence of *graphic signs*, such as diagrams, images, and graphs, which could contribute to the visualisation and understanding of mathematical concepts. In Figure 1, for example, there is another preliminary notion called "monomials."

Figure 1

Image of the preliminary notion "monomials." Ferraz (n.d., p. 9)



17 — **Monómios** são as expressões não contendo partes ligadas pelos sinais + ou —.

As expressões contendo dois ou mais monómios ligados pelos sinais + e — chamam-se *polinómios.* Cada um dos monómios toma então o nome de *têrmo,* e o sinal + ou — que precede o têrmo, considera-se como fazendo parte dêle.

Um polinómio que tiver só dois têrmos toma o nome especial de *binómio;* da mesma maneira, um polinómio de três têrmos chama-se um *trinómio*.

Assim: ax, $4bc$ e $5a^2c^2$ são *monómios,* e $a^2 + b^3 - c^4$ é um *polinómio,* cujos *têrmos* são a^2, b^3 e c^4.

$2a + 3b$ é um *binómio* e $a - 2b + 5c$ é um *trinómio*.

Dá-se tambêm muitas vezes aos monómios o nome de *quantidades incomplexas* e aos polinómios o de *complexos,* que serão *quantidades inteiras* quando os seus têrmos não tiverem denominadores.

7 When using letters and signs to solve problems, algebraic expressions called formulas are used. These formulas indicate the operations to be performed to solve a specific type of question. "Once you know the algebraic formula to use to solve a specific order of questions, and if you want to apply it to a problem with particular numerical data, simply replace the letters with their numerical values and then carry out the indicated operations." (Ferraz, n.d., p. 5).

8 This approach in the form of *expository text* is enriched by intercalating *instructional texts*, understood as a series of instructive statements that aim to teach the reader how to do or use some notion taught.

To introduce this concept, Ferraz (n.d.) uses an *expository text*, complemented by the use of *notation signs*, with the addition (+) and subtraction (-) symbols, to explain that monomials are mathematical expressions that these operators do not connect. Still in the preliminary section "monomials," the author explains that expressions formed by two or more monomials are called "polynomials," which can be classified as binomials and trinomials. Thus, he expands the exposition of mathematical concepts, exemplifying with the expressions: monomial *(4ab)*, binomial *(2a + 3b)* and trinomial *(a - 2b + 5c)*. In summary, when analysing the preliminary notions of "algebra" and "monomials," we assume that the author wanted to present elementary notions (expressions, terms, signs, etc.), considered prerequisites for teaching the algebraic knowledge developed in the following sections of the book. It is worth noting that this type of approach can also be seen in other works designed for teaching algebra, such as the book *Álgebra Elementar*[9] [Elementary Algebra] by Antonio Trajano.

Only after the 36 preliminary notions (introductory section) does the first part of the work, entitled "Operações Algébricas" [Algebraic Operations], begin. Table 1 highlights the division of the book, as described by Ferraz (n.d.) in the preface. We can see that algebraic knowings were listed gradually, so that each section seems to serve as a basis for the next.

Table 1

Structure of the book Álgebra Elementar, adapted from the index[10] *(Ferraz, n.d., p. 295)*

Division	Section	Quantity		
		Examples	Exercises	Problems
Part 1 Algebraic operations	I - Addition	2	10	-
	II - Subtraction	5	12	-
	III - Multiplication	18	25	-

9 Trajano's work (1905) can be accessed at: https://repositorio.ufsc.br/handle/123456789/104463
10 The index of this work is located at the end of the book, unlike in current works, where it usually appears at the beginning.

(p. 23-110)	IV - Division	9	24	-
	V - Root extraction	7	13	-
	VI - Greatest common divisor	7	10	-
	VII - Lowest common multiple	2	12	-
	VIII - Algebraic fractions	18	51	-
Part 2 Algebraic operations (p. 111-175)	IX - First-degree equations with one unknown	9	15	-
	X - Continuation of first-degree equations with one unknown	6	20	-
	XI - First-degree problems with one unknown	5	-	18
	XII - Continuation of first-degree problems with an unknown	9	-	16
	XIII - Simultaneous first-degree equations with two unknowns	5	10	-
	XIV - Simultaneous first-degree equations with more than two unknowns	3	6	-
	XV - First-degree problems with more than one unknown	4	-	9
Part 3 Second-degree equations	XVI - Of negative, equal to zero, and fractional exponents	4	18	-
	XVII - Calculation of radicals	14	11	-
	XVIII - Second-degree equations with one unknown	15	14	-

(p. 176-234)	XIX - Equations that reduce to the second degree	6	13	-
	XX - Second-degree problems with one unknown	4	-	12
	XXI - Simultaneous quadratic equations	4	10	-
	XXII - Second-degree problems with more than one unknown	2	-	11
Part 4 (p. 235-294)	XXIII - Theories of logarithms	5	-	-
	XXIV - On the use of boards	12	10	-
	Table of logarithms of numbers 1 to 10,000	-	-	-

Table 1 provides an overview of the book, highlighting how the author *structured* its four parts. Each of them is made up of a set of sections that include an explanation of the *content*, a variety of solved examples, as well as suggested exercises[11] or problems, accompanied by their answers. In the sections that present solved examples, the author generally includes more than two, organised by levels of complexity[12]. It usually starts with equations involving one unknown (with or without parentheses), then addresses equations with two unknowns, and, later, those involving fractions, powers, and roots. A similar process occurs in the exercise suggestions that accompany each section:

11 It is worth noting that, as Souza (2017) points out, exercises appear as instruments for fixing and repeating previously taught procedures, with an emphasis on technique and the automation of algorithms. Problems require students to adopt an investigative approach, as they involve situations in which there is no previously established solution path, demanding the mobilisation of different knowledge and the construction of resolution strategies.

12 Different levels of complexity are considered, in which the simple is presented as initial data, an independent element, and the complex, the one resulting from the integration of multiple simple elements in interdependent relationships (Valente, 2015).

for the most part, they follow the pattern of the examples, both in *structure* as in complexity. However, only five sections present suggested problems.

In general, practically all sections follow a standard *structure*, i.e., explanation of the topic, examples, exercises or problems, and answers. Furthermore, the book is entirely composed of texts, letters, and symbols in black, without any illustrations, colours, or other visual elements that might attract attention.

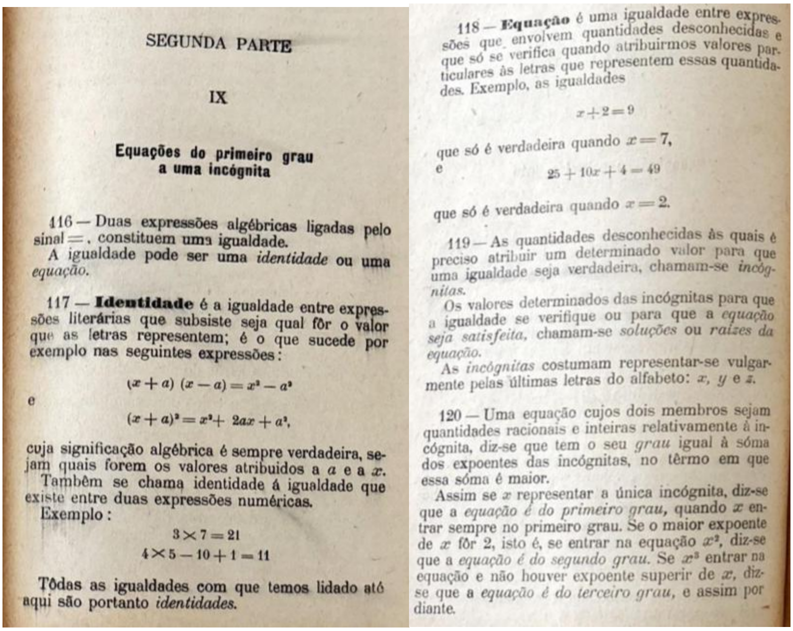
The *contents* presented in the sections constitute one of the central elements of this analysis. The distribution into four parts, carried out by Ferraz (n.d.), reveals an intentional *sequence* of algebraic knowings, *structured* sequentially, at different levels of complexity. An example of this can be seen in the second part of the book, dedicated to algebraic operations. This part begins with the section "first-degree equations with one unknown," goes through other sections such as "simultaneous first-degree equations with one and two variables," "first-degree problems with one unknown," and ends with the section "first-degree problems with more than one unknown."

In short, the work brings a *sequence* of *contents* that leads to the teaching of algebraic knowledge from the simplest to the most complex. Content organisation in this simple to complex logic implies a specific way of conducting teaching: it is assumed that the most elementary content should be taught first, and that, gradually, the articulation of this knowings will lead to the understanding of more complex topics. This perspective is part of the historical context of the First Republic, since, as Valente (2015) states, over time, the simple/complex dyad changed to a "new" duo: easy/difficult.

When analysing the section "first-degree equations with one unknown" in more depth, it is clear that the author uses an *expository text* when presenting the concept of equations. It establishes connections with terms already introduced in the preliminary section of the work, while also anticipating elements that will be explored in greater depth in subsequent sections. This organisational model is kept throughout the book, ensuring a cohesive and orderly *structure* for the *contents* addressed. The approach used in this section can be seen in Figure 2.

Figure 2

Conceptual image of the section "first-degree equations with one unknown" Ferraz (n.d., pp. 111-112)



SEGUNDA PARTE

IX

Equações do primeiro grau a uma incógnita

116 — Duas expressões algébricas ligadas pelo sinal $=$, constituem uma igualdade.

A igualdade pode ser uma *identidade* ou uma *equação*.

117 — **Identidade** é a igualdade entre expressões literárias que subsiste seja qual fôr o valor que as letras representem; é o que sucede por exemplo nas seguintes expressões:

$$(x + a)(x - a) = x^2 - a^2$$

e

$$(x + a)^2 = x^2 + 2ax + a^2,$$

cuja significação algébrica é sempre verdadeira, sejam quais forem os valores atribuidos a a e a x.

Também se chama identidade á igualdade que existe entre duas expressões numéricas.

Exemplo:

$$3 \times 7 = 21$$
$$4 \times 5 - 10 + 1 = 11$$

Tôdas as igualdades com que temos lidado até aqui são portanto *identidades*.

118 — **Equação** é uma igualdade entre expressões que envolvem quantidades desconhecidas e que só se verifica quando atribuirmos valores particulares às letras que representem essas quantidades. Exemplo, as igualdades

$$x + 2 = 9$$

que só é verdadeira quando $x = 7$,

e

$$25 + 10x + 4 = 49$$

que só é verdadeira quando $x = 2$.

119 — As quantidades desconhecidas às quais é preciso atribuir um determinado valor para que uma igualdade seja verdadeira, chamam-se *incógnitas*.

Os valores determinados das incógnitas para que a igualdade se verifique ou para que a *equação seja satisfeita*, chamam-se *soluções* ou *raízes da equação*.

As *incógnitas* costumam representar-se vulgarmente pelas últimas letras do alfabeto: x, y e z.

120 — Uma equação cujos dois membros sejam quantidades racionais e inteiras relativamente à incógnita, diz-se que tem o seu *grau* igual à sóma dos expoentes das incógnitas, no têrmo em que essa sóma é maior.

Assim se x representar a única incógnita, diz-se que a *equação é do primeiro grau*, quando x entrar sempre no primeiro grau. Se o maior expoente de x fôr 2, isto é, se entrar na equação x^2, diz-se que a *equação é do segundo grau*. Se x^3 entrar na equação e não houver expoente superir de x, diz-se que a *equação é do terceiro grau*, e assim por diante.

The author presents the *content* of this section through explanatory language, interspersing symbols, mathematical terms, and connectors. We note that, when introducing new terms, these are often accompanied by an explanation and an example, as in the case of the term "*igualdade*" [equality] (see Figure 2). The word "equality" has semantics that vary depending on the context in which it is used. In general, it can be defined as the quality or state of being equal. Its main meanings include the idea of equivalence between two or more things, without relevant differences. However, we note that, in this case, the author was careful to attribute a mathematical meaning to "equality", when describing and exemplifying the relationship between two linear expressions and two numerical expressions. This pattern is repeated in the other sections of the work.

It is also clear that Ferraz (n.d.) explains, in a *prescriptive text*, the steps to be followed to resolve certain *contents*, as, for example, in the case of algebraic equations. The presentation of the stages is given by:

> 1st - "*Desembaraçam-se*" [Clearing] the denominators, if any. 2nd - Passing the terms that contain the unknown to one member and the known terms to the other. 3rd - Carrying out the indicated operations. 4th - Dividing both members by the coefficient or sum of the coefficients of the unknown, thus obtaining the desired root (Ferraz, n.d., pp. 115-116).

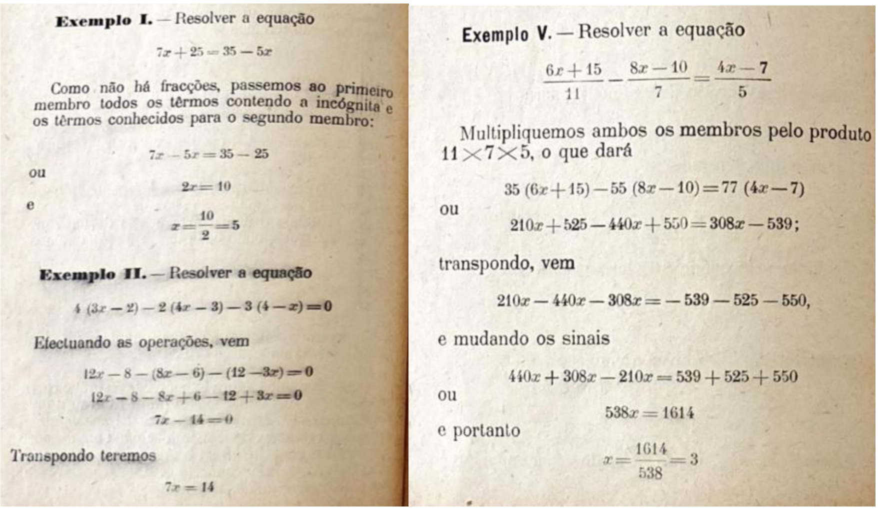
The author explains, in an *instructional text*, the steps to be followed in solving a given algebraic equation, organising the procedure into four well-defined stages. However, we observe the use of a term that is not so well-known. Nowadays, the term "*desembaraçar*" [clearing] indicates "eliminating" denominators. Furthermore, the prescribed step-by-step process can be seen in the resolution of the examples covered in the section. This structuring reinforces a conception of algebra as a resolution technique, in which understanding the procedures is subordinate to their correct application.

Throughout the book, the presence of numbered paragraphs can be observed, as illustrated in Figure 2. On pages 111 and 112, for example, numbers 116, 117, 118, 119, and 120 are found at the beginning of different paragraphs. This form of structuring indicates that each number introduces a proposition to be presented in an expository manner. We can therefore infer that the *content* of the book is organised into numbered propositions, each accompanied by an *expository and instructive text,* and in some cases, examples. From this perspective, the example serves as a resource to aid understanding of both *expository* and *instructive* texts.

In Figure 3, we note that the section "first-degree equations with one unknown" presents examples solved in an *explanatory* way. The author starts with simpler examples (Figure 3) and gradually progresses to more complex ones, approximately nine examples. That *structure* is repeated in the other sections, with variations in the number of solved examples, depending on the complexity of the *content* addressed in each section. Thus, it can be inferred that Ferraz (n.d.) uses this strategy to encourage both students and teachers to develop the suggested exercises or problems following the same resolution method. This approach reflects one of the elements highlighted by Okeeffe (2013) in the analysis of textbooks: *expectation*.

Figure 3

Image of examples from the section "first-degree equations with one unknown." Ferraz (n.d., pp. 116-118)



Exemplo I. — Resolver a equação

$$7x + 25 = 35 - 5x$$

Como não há fracções, passemos ao primeiro membro todos os têrmos contendo a incógnita e os têrmos conhecidos para o segundo membro:

$$7x - 5x = 35 - 25$$

ou

$$2x = 10$$

e

$$x = \frac{10}{2} = 5$$

Exemplo II. — Resolver a equação

$$4(3x - 2) - 2(4x - 3) - 3(4 - x) = 0$$

Efectuando as operações, vem

$$12x - 8 - (8x - 6) - (12 - 3x) = 0$$
$$12x - 8 - 8x + 6 - 12 + 3x = 0$$
$$7x - 14 = 0$$

Transpondo teremos

$$7x = 14$$

Exemplo V. — Resolver a equação

$$\frac{6x + 15}{11} - \frac{8x - 10}{7} = \frac{4x - 7}{5}$$

Multipliquemos ambos os membros pelo produto $11 \times 7 \times 5$, o que dará

$$35(6x + 15) - 55(8x - 10) = 77(4x - 7)$$

ou

$$210x + 525 - 440x + 550 = 308x - 539;$$

transpondo, vem

$$210x - 440x - 308x = -539 - 525 - 550,$$

e mudando os sinais

$$440x + 308x - 210x = 539 + 525 + 550$$

ou

$$538x = 1614$$

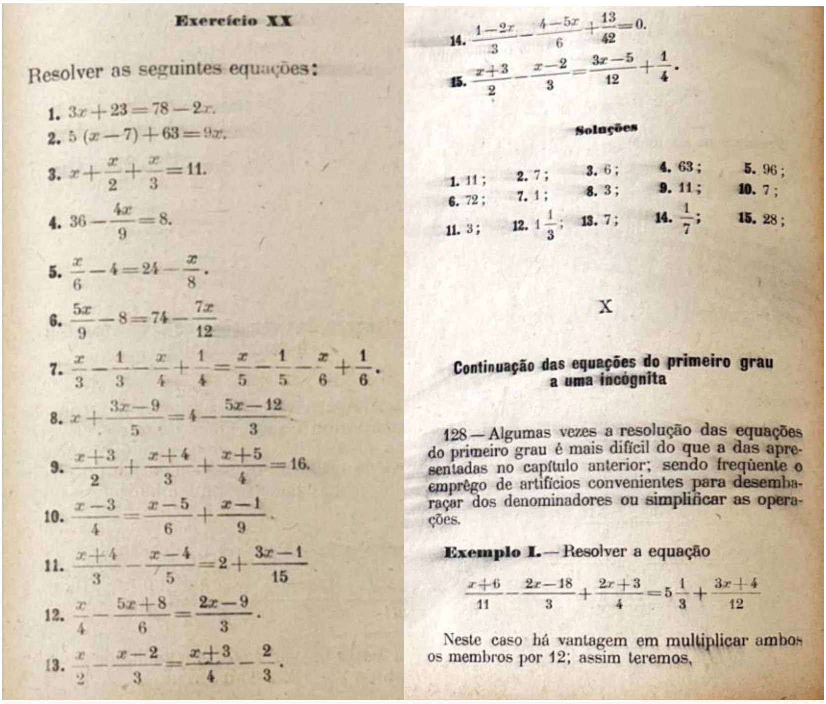
e portanto

$$x = \frac{1614}{538} = 3$$

Only after the theoretical introduction and the resolution of examples are exercises or problems presented, for the practice of the algebraic knowledge taught. At the end of many sections of the book, exercises are proposed that directly present the equation to be solved; that is, the emphasis is on the technical application of the algebraic procedures previously taught. In these cases, the student must simply follow the rules already established, with the respective answers provided directly. This approach predominates throughout practically all sections of the work. The exercise suggestions in the section "first-degree equations with one unknown" (Figure 4) exemplify this organisational pattern.

Figure 4

Image of the exercises in the section "first-degree equations with one unknown." Ferraz (n.d., pp. 119-120)



Exercício XX

Resolver as seguintes equações:

1. $3x + 23 = 78 - 2x.$
2. $5(x - 7) + 63 = 9x.$
3. $x + \frac{x}{2} + \frac{x}{3} = 11.$
4. $36 - \frac{4x}{9} = 8.$
5. $\frac{x}{6} - 4 = 24 - \frac{x}{8}.$
6. $\frac{5x}{9} - 8 = 74 - \frac{7x}{12}$
7. $\frac{x}{3} - \frac{1}{3} - \frac{x}{4} + \frac{1}{4} = \frac{x}{5} - \frac{1}{5} - \frac{x}{6} + \frac{1}{6}.$
8. $x + \frac{3x - 9}{5} = 4 - \frac{5x - 12}{3}.$
9. $\frac{x+3}{2} + \frac{x+4}{3} + \frac{x+5}{4} = 16.$
10. $\frac{x-3}{4} = \frac{x-5}{6} + \frac{x-1}{9}.$
11. $\frac{x+4}{3} - \frac{x-4}{5} = 2 + \frac{3x-1}{15}$
12. $\frac{x}{4} - \frac{5x+8}{6} = \frac{2x-9}{3}.$
13. $\frac{x}{2} - \frac{x-2}{3} = \frac{x+3}{4} - \frac{2}{3}.$
14. $\frac{1-2x}{3} - \frac{4-5x}{6} + \frac{13}{42} = 0.$
15. $\frac{x+3}{2} - \frac{x-2}{3} = \frac{3x-5}{12} + \frac{1}{4}.$

Soluções

1. 11; **2.** 7; **3.** 6; **4.** 63; **5.** 96;
6. 72; **7.** 1; **8.** 3; **9.** 11; **10.** 7;
11. 3; **12.** $1\frac{1}{3}$; **13.** 7; **14.** $\frac{1}{7}$; **15.** 28;

X

Continuação das equações do primeiro grau a uma incógnita

128 — Algumas vezes a resolução das equações do primeiro grau é mais difícil do que a das apresentadas no capítulo anterior; sendo freqüente o emprêgo de artifícios convenientes para desembaraçar dos denominadores ou simplificar as operações.

Exemplo I. — Resolver a equação

$$\frac{x+6}{11} - \frac{2x-18}{3} + \frac{2x+3}{4} = 5\frac{1}{3} + \frac{3x+4}{12}$$

Neste caso há vantagem em multiplicar ambos os membros por 12; assim teremos,

Regarding the exercises in Figure 4, the presence of fifteen proposals was observed, each suggesting the resolution of an algebraic equation. The *sequence* of equations follows a difficulty progression, from the simplest exercise, gradually advancing to the most complex one. The first exercise requires solving a simple algebraic equation, without parentheses. The second introduces parentheses, and the following ones involve equations with fractions, progressing from the simplest to the most elaborate equations. We found that this organisation agrees with the order of the examples previously solved by the author, something that is observed in almost all sections of the work. The proposed exercises revisit the same types of equations covered in *content* expository/descriptive, suggesting that the list of activities functions as a direct and systematic application of the *concepts* previously explored. This articulation between theory and practice demonstrates a concern with the *sequential* construction of the knowings *to* teach, ensuring continuity between theoretical explanation and practical resolution. In this way, the *sequence* of equations proposed can be understood as a structured application of the algebraic *content* presented, contributing to the consolidation of the operations involved in solving equations.

However, it is worth noting that only some sections of the work present problems. As identified in Table 1, only the sections dedicated to “first-degree problems” and “second-degree problems” contain statements with contextualised language. In these cases, the problems start with a statement from which the student must assemble the corresponding equation. This type of activity requires interpretation, mobilisation of knowings, and development of resolution strategies, therefore configuring itself as more open and investigative. When Ferraz (n.d.) presents the problems as situations that give meaning to algebra, he proposes, albeit implicitly, a more applied use of algebraic *contents*. We understand that such problems are not only intended to train the resolution procedure, but also to demonstrate where and how algebra can be used beyond abstraction.

Still about problems, we can characterise them into two types: *numerical problems*, in which, for example, “the sum of one number with another results in a certain value” (situations in which the focus remains close to the exercise, but the student is required to formulate the corresponding equation); and *contextualised problems*, which involve topics such as buying and selling, profit, capital, discount, and interest, and are, therefore, associated with commercial and financial situations. We observe that most of the problems present in the work are of the contextualised type, suggesting an attempt to bring the teaching of algebra closer to practical situations. For example: “A person bought 15 kilos of sugar of two different qualities, paying $2.28; the higher quality cost 20 cents per kilo and the lower quality cost 14 cents per kilo. How many kilos did you buy of each quality?” (Ferraz, n.d., p. 146). In this statement, the problem involves a purchasing operation with the calculation of the total cost, highlighting the link between algebraic knowledge and possible practical applications in the world of work.

Furthermore, the problem statement includes monetary values corresponding to the time, providing a historical contextualisation to the issues addressed. By relating algebraic knowledge to the historical and social context of students, during the First Republic and within the scope of technical professional education, we perceive an articulation between the contents and everyday situations, i.e., a connection between algebraic theory and its practical application. Thus, it can be said that the contextualised problems in the work function as a practical justification for the teaching of algebra. Even though this practice is built within a school model, it points to possible fields of professional application of the knowings *to* teach. Such situations make even more sense when considering the purpose of technical vocational education at the time, which was aimed at qualifying workers for administrative, commercial, or industrial roles.

FINAL CONSIDERATIONS

Analysis of the book *Álgebra Elementar* [Elementary Algebra], by Guilherme Ivens Ferraz, reveals the characterisation of an algebra focused on technical professional education. The *graduation* of the algebraic *contents*, the *structuring* given to them, the *language* used, and the didactic intention expressed in several passages of the work indicate the construction of a set of knowings *to* teach.

We observe that the presentation of the *contents* in the form of *expository and instructive texts* constitutes a recurring resource used by the author in all sections of the work. It is important to highlight that the only teaching resource used is writing, without the use of visual elements or other supports. Furthermore, we understand that the shape of the *instructional texts* seeks to develop in young people an instrumental skill of *content* from the establishment of a procedural *sequence* of actions aimed at obtaining a result.

The *sequence* of exercises respects a progression of complexity and revisits solved examples, indicating a didactic intention in the organisation of knowings *to* teach, demonstrating a teaching intentionality that values graduation in the teaching of algebra, approaching systematised instructional practices. Furthermore, it can be observed how this practice of organising teaching around lists of exercises with progression of difficulty constitutes a typical school form of transmitting knowings, characterised by repetition and systematisation, indicating that the teaching of algebra in this context is part of a specific school culture, linked to the purposes of technical education.

Ferraz (n.d.) prioritises the presence of exercises over problems in his work. While the exercises directly present the equation to be solved, most problems require the student, after reading the statement, to construct the equation that represents the proposed situation. In this sense, problems take on a distinct function: they give practical meaning to algebra by linking it to situations that simulate possible applications outside the purely academic context. These problems can be classified into two large groups: numerical problems, which involve abstract relationships between numbers, and contextualised problems, which describe “real” situations, generally associated with mercantile, commercial, or financial aspects. Thus, in the work, the characterisation of an algebra *to* teach can also be justified as a tool for dealing with practical situations in everyday life and the world of work, in line with the objectives of technical vocational education at the time.

Considering the results of the analysis, we observe that the organisation of the *contents* in Ferraz's work follows a logic of progression from the simple to the complex, which is based on the very mathematical structure of algebra as a reference science. This structure, in turn, seems little influenced by contributions from educational sciences, which, at the time, still offered little systematisation for the teaching of algebra in professional contexts. In this sense, we understand that the work reflects a didactic rationality anchored in the internal logic of mathematics, prioritising the development of operational skills. As a final note, we suggest that future research could deepen the reflection on how educational sciences could dialogue with the structure and organisation of *contents* of an algebra aimed at technical vocational education.

AUTHORSHIP CONTRIBUTION STATEMENT

RFS was responsible for designing the study, surveying and analysing the sources, and writing the article. CSB contributed to the theoretical deepening and the definition of methodological procedures. DAC participated in the data analysis and carried out the final review of the text. All authors actively participated in the preparation of the article and approved its final version.

DATA AVAILABILITY STATEMENT

The data sharing is not applicable to this article, as it is based on research using publicly available bibliography.

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