ISSN: 2178-7727
DOI: 10.64856/acta.scientiae.8385

Evaluate the Levels of Geometric Thinking in Student-teachers in Elementary Education Based on Van Hiele Theory

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ABSTRACT

Background: Geometry is an important concept in elementary school. The teachers' capabilities directly impact the quality of education. Hence, one must assess the capabilities of the student-teachers who will soon become elementary teachers. **Objectives**: Informed by the Van Hiele geometric thinking levels, this study investigates the levels of geometric thinking in student-teachers in elementary education in Iran. **Design**: Since the results of this study can improve the quality of elementary education, this article, purpose-wise, is considered an applied research study employing a descriptive-survey method that collects data through surveys completed by participants. **Setting and Participants**: The sample includes all 187 student-teachers studying elementary education at Farhangian University of Guilan in 2024. **Data collection and analysis**: The data collection tool was a questionnaire with 9 four-choice questions, and the validity and reliability of the questions were confirmed. The Skewness-Kurtosis test confirmed the data's normality. The T-test was employed to determine the access to the levels. **Results**: This descriptive survey concluded that second-year student-teachers are at the first level of the Van Hiele thinking, fourth-year student-teachers are at the second level, and none of the groups reached the third level. So, despite the increase in the level of student-teachers during university education, they are not yet at a desirable level in geometry and have not made significant gains in this domain. **Conclusions**: The results of this study will inform educational planners about the importance of enriching and improving the geometry curriculum for student-teachers and will ultimately help to improve the level of teachers' ability. It is also suggested to pay more attention to geometry in university courses for student-teachers of elementary education.

Keywords: geometry education; elementary education; student-teacher; Van Hiele theory.

Avaliar os Níveis de Pensamento Geométrico em Estudantes-Professores do Ensino Fundamental com Base na Teoria de Van Hiele

RESUMO

Contexto: Geometria é um conceito importante no ensino fundamental. As competências dos professores impactam diretamente a qualidade da educação. Portanto,

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é necessário avaliar as competências dos futuros professores que se tornarão professores do ensino fundamental. **Objetivos**: Com base nos níveis de pensamento geométrico de Van Hiele, este estudo investiga os níveis de pensamento geométrico em alunos-professores do ensino fundamental no Irã. **Design**: Como os resultados deste estudo podem melhorar a qualidade do ensino fundamental, este artigo, em termos de objetivo, é considerado um estudo de pesquisa aplicada que emprega um método descritivo-exploratório, coletando dados por meio de questionários respondidos pelos participantes. **Ambiente e participantes**: A amostra inclui todos os 187 alunos-professores que estudam educação fundamental na Universidade Farhangian de Guilan em 2024. **Coleta e análise de dados**: O instrumento de coleta de dados foi um questionário com 9 questões de quatro opções, cuja validade e confiabilidade foram confirmadas. O teste de Assimetria-Curtose confirmou a normalidade dos dados. O teste t foi utilizado para determinar o acesso aos níveis. **Resultados**: Esta pesquisa descritiva concluiu que os alunos-professores do segundo ano estão no primeiro nível do pensamento de Van Hiele, e os alunos-professores do quarto ano estão no segundo nível, e nenhum dos grupos atingiu o terceiro nível. Portanto, apesar do aumento no nível dos alunos-professores durante a formação universitária, eles ainda não atingiram um nível desejável em geometria e não obtiveram ganhos significativos neste domínio. **Conclusões**: Os resultados deste estudo informarão os planejadores educacionais sobre a importância de enriquecer e aprimorar o currículo de geometria para futuros professores e, em última análise, contribuirão para o aprimoramento da competência dos professores. Sugere-se também dar mais atenção à geometria nos cursos universitários para futuros professores do ensino fundamental.

Palavras-chave: educação em geometria; educação elementar; aluno-professor; teoria de Van Hiele.

INTRODUCTION

Historically, geometry has always been essential to the elementary mathematics curriculum (Armah et al., 2018; Pavlovicova et al., 2022; Watan & Sugiman, 2018). Geometry is deeply informed by the physical world, visual perceptions, logic, and reasoning; it is a powerful tool in the actualisation of human potential in the physical world (Yaftian & Safabakhsh Chekosari, 2020). Geometry is a fundamental area of mathematics which reinforces the students' problem-solving and critical-thinking capabilities, and helps them visualise mathematical concepts (Dereje et al., 2023). As a mathematical concept related to the physical world around us, geometry must be learned (Watan & Sugiman, 2018).

Engineers, architects, and interior designers make use of geometry on a daily basis (Pavlovicova et al., 2022). In addition, we make use of geometry in our day-to-day lives; we visualise the order and placement of our furniture, and calculate the amount of flooring needed for an area, such as a room or other parts of a building (Yaftian & Safabakhsh Chekosari, 2020). As a geometrical skill, spatial visualisation improves our positioning and orientation when encountering a jungle, city, or map (Pavlovicova et al., 2022). Geometry can act as a window through which the learner mathematically and systematically understands his or her environment. It is a recurrent field in mathematics as well as other fields (Alamian et al., 2018; Armah et al., 2018; Reyhani et al., 2010; Salifu et al., 2018). In addition, geometry enhances the reasoning and spatial visualisation capabilities of upcoming elementary teachers (Dereje et al., 2023).

Since natural phenomena echo geometry, they enable the learner to have a more critical and creative view of the relationship between natural phenomena and mathematics (Gorjipour Shoobi, 2018). In other words, the acquisition of geometry improves basic skills and establishes logical thinking, deductive reasoning, logical analysis, and problem-solving capabilities. That being said, geometry informs other mathematical fields such as measurement, algebra, calculus, and trigonometry, and is frequently employed by architects, engineers, planners, and other professionals. It also plays an important role in physics, astrophysics, astrology, and biology (Armah et al., 2018; Sudihartinih & Wahyudin, 2019).

Due to its multi-functionality, mathematicians call for the inclusion of geometry in the early stages of education (Yaftian & Gorjipour Shoobi, 2021). In this regard, geometry must be well-learned and well-taught. Although geometry maintained its importance in the modern world, students find it hard to grasp and have little to show for it (Lumbre et al., 2023; Salifu et al., 2018; Ural, 2016). Iranian students are no exception (Alamian et al., 2018; Sabzali and Tavanayi, 2020; Yaftian & Safabakhsh Chekosari, 2020). Also, high school students in Iran show difficulties in providing formal proofs in school geometry (Yaftian & Gorjipour Shoobi, 2021). Also, the mathematical and geometrical knowledgeability of elementary teachers and student-teachers is less than optimal (Kabiri, 2021; Qorbani et al., 2023; Shahbazi et al., 2024). Mathematic teachers have proved to be less effective (Reyhani et al., 2010).

These difficulties and misunderstandings problematise the inclusion of geometry in the school curriculum. Consequently, these problems necessitate the investigation of their root causes so the students can actualise their potential and reach higher levels of education and learning (Lumbre et al., 2023; Ural, 2016; Watan & Sugiman, 2018; Yaftian & Gorjipour Shoobi, 2021). In addition, to set an efficient method for student evaluation, it is necessary to check their level of thinking, especially in geometry (Gorjipour Shoobi, 2018).

There is a correlation between the knowledgeability of the teachers and student progress (Kabiri, 2021). Some studies show that pre-service teachers are limited in their conceptual understanding of geometrical concepts needed for effective teaching. In this regard, it is vital to find ways to improve their conceptual understanding of geometry (Mbatha & Bansilal, 2023). To make sure that elementary teachers have been properly trained during their student days, we need to measure their geometry knowledge level. Despite its importance, little to no research has been conducted on the levels of geometric thinking in elementary teachers in Iran. Through proper and adequate research, one can determine the strengths and weaknesses of present programs and act accordingly to better prepare student-teachers for their future careers. Considering the fact that mathematics is like an international language, other countries can benefit from the results and upgrade their curriculum accordingly.

The Van Hiele theory is among the most important theoretical frameworks for understanding the learning process in geometry students (Armah et al., 2018; Mahdian et al., 2017; Ural, 2016; Pavlovicova, et al., 2022; Salifu et al., 2018). This model of thinking is the best-known theoretical account for students' geometry learning. The model recommends that student's progress through a sequence of discrete, qualitatively different levels of geometric thinking (Wang & Kinzel, 2014). This model explicates the students' difficulties in learning geometry, in general, and in writing a proof, in particular. The Van Hiele theory is a learning model, which categorises the different thought processes of geometry students from visualisation to geometric proofs (Alamian et al., 2018).

Students' achievements in geometry, Van Hiele argues, are in accordance with three elements: the teacher, the student, and the concept. In this regard, to upgrade the quality of mathematics teaching, one can study the teachers (Lumbre et al., 2023). The interest in investigating teachers' mathematical knowledge can be traced to more than a century ago (Herbst et al., 2020). The teachers, who have no mastery of the concept and struggle with teaching, cannot guide the students to a profound understanding of mathematics (Mbatha & Bansilal, 2023). Due to the importance of elementary school in learning geometry, the teaching capabilities of the teachers become crucial. In this regard, in teaching the student-teachers, we must organise the curriculum to achieve optimum performativity in teaching mathematics and geometry.

Consequently, in light of the Van Hiele theory, this study investigates the teaching capabilities of elementary student-teachers who will soon become elementary geometry teachers. Informed by the Van Hiele method, this study aims to investigate the levels of geometric thinking in student-teachers in elementary education as well as the impact of their university education. Since their understanding of geometry correlates with their teaching capabilities, they have the potential to raise more mathematically and geometrically inclined students; in other words, this study helps us to have more effective teachers.

THEORETICAL BACKGROUND

The Van Hiele theory

Many researchers have verified the usefulness of the model for expressing the development of students, pre-service teachers, in-service teachers and teachers' geometric thinking (Arabaci & Kanbolat, 2023; Bonyah & Larbi, 2021; Lumbre et al., 2023; Ovez & Ozdemir, 2024; Wang & Kinzel, 2014).

The Van Hiele theory is important and functional in geometry. It categorises the levels of geometric thought and presents a structure for the learner's geometric thought process (Arabaci & Kanbolat, 2023; Mbatha & Bansilal, 2023; Yaftian & Safabakhsh Chekosari, 2020). Van Hiele formulates a structure for understanding the developments of the levels of geometric thinking, and provides valuable insights into how students conceptualise geometric proofs (Ovez & Ozdemir, 2024). It enables us to investigate the students' geometric thinking levels, which, in turn, allows us to provide adequate programs and tools accordingly (Ceilik & Yilmaz, 2022). Nowadays, the Van Hiele method is among the most frequent tools for assessing the levels of students' geometric thinking, their problems and shortcomings, and strategies for better understanding of geometric concepts (Mbatha & Bansilal, 2023; Sidelil et al., 2024; Watan & Sugiman, 2018).

The theory originated in 1957 in the doctoral dissertations of two Dutch teachers, Dina van Hiele-Geldof and Pierre van Hiele (Yaftian & Gorjipour Shoobi, 2021). A key characteristic in Van Hiele's levels of geometric thinking is their hierarchy and prerequisition which necessitate the completion of each level before progressing to the next. The progress in these levels of thought, according to Van Hiele, is more dependent on the quality of education a person receives as a learner than his or her age and developmental stages (Ceilik & Yilmaz, 2022; Yaftian & Safabakhsh Chekosari, 2020; Yaftian & Gorjipour Shoobi, 2021). The Van Hiele geometric thinking has five levels:

- Level 1, Visualisation: In which the learners correspond and identify geometric shapes in accordance with their similarity with a known shape. Also, they recognise and learn the names of figures (Pavlovicova et al., 2020; Wang

& Kinzel, 2014). At this level, the learner focuses on the general shape and rejects other namings (Yaftian & Gorjipour Shoobi, 2021). At this level, the learner is unable to analyse the shapes (Ovez & Ozdemir, 2024); for example, although s/he knows the shape of a rectangle, s/he is unable to determine its key characteristics (Mbatha & Bansilal, 2023).

● Level 2, Analysis: in which learners can catogorise geometric shapes in accordance with their corresponding similar characteristics. They define geometric shapes in light of their characteristics or components (Pavlovicova et al., 2022; Wang & Kinzel, 2014). At this level's mastery, the learner can analyse the characteristics of a geometric shape, but cannot make a connection among them (Armah et al., 2018; Ovez & Ozdemir, 2024); for instance, although the learner is able to distinguish a rectangle from a square, s/he does not realise that every square is a rectangle (Arabaci & Kanbolat, 2023).

● Level 3, Informal deduction: In which the learners are aware of the connections among geometric characteristics and are able to view them as a unified shape and can conclude one charactersitic from another (Pavlovicova et al., 2018). They argue informal reasons for their justification (Armah et al., 2018). At this level, the learners are able to follow the proof, but are unable to formulate one; to put it simply, they employ strategies along with their own deductive reasoning (Overz & Ozdemir, 2024). At the mastery of this level, the learner acquires different names for the shapes and can comprehend simple arguments, but his or her comprehension is not sharp (Yaftian & Gorjipour Shoobi, 2021).

● Level 4, Deduction: In which the learners comprehend the logical system of geometry deduction. They understand the necessity of the principles of the subject, theorems, and definitions, and they understand the meaning of necessary and sufficient conditions. Also, they can prove high-school-level theorems (Armah et al., 2018; Pavlovicova et al., 2020). For instance, informed by the characteristics of parallel lines, they are able to prove that the sum of all angles in a triangle equals 180 degrees (Arabaci & Kanbolat, 2023). In addition, they can prove and reason using induction (Ovez & Ozdemir, 2024). Of note here is that, at this stage, they are only familiar with Euclidean geometry (Erdogan, 2019).

● Level 5. Rigor: In which the learners compare different systems of thematic principles, know non-Euclidean geometry, and employ different proofs (Arabaci & Kanbolat, 2023; Pavlovicova et al., 2022). They can also analyse and compare theorems in light of different subject principle systems (Yaftian & Safabakhsh Chekosari, 2020).

Literature review

Literature provides a theoretical basis and context for research. It also introduces us to previous research on the topic and identifies potential research gaps. The literature in this paper is divided into three parts: (2.2.1) Validation of Van Hiele's theory, (2.2.2) Studies related to students, teachers, and preservice teachers.

The mathematical capabilities of teachers, university students, and elementary students are closely related to their levels of geometric thinking (Armah et al., 2018; Salifu et al., 2018). The academic performance of students is closely related to the knowledge and skills of their teachers (Ghorbani et al., 2023). The teachers, in kindergarten and elementary, are crucial for building a geometric foundation. In this regard, the teachers must possess great spatial visualisation in geometry and should be able to pass on the skills and information; however, the studies show that the teachers have low spatial visualtion when it comes to geometry (Pavlovicova et al., 2022).

The teacher, Van Hiele argues, is a guide who shows the path to the learners and systematically structures their learning so they can optimally learn mathematics (Alamian et al., 2018). The teacher is the main source of information. In this regard, if the teacher makes a mistake, then the students ought to repeat that same mistake in the class. Likewise, the capability of the teacher directly impacts the probability and the quality of learning geometry (Armah et al., 2018; Lumbre et al., 2023); for instance, the teachers' mastery of symmetry or rotation greatly impacts their teaching potential and results (Turgut et al., 2014); in other words, one can argue that the students' misunderstandings and challenges correlate to their teachers' shortcomings (Shahbazi et al., 2024).

A teacher who teaches geometry in today's world must do so according to the student's learning process (Gorjipour Shoobi, 2018). A teacher's geometric thinking directly impacts the quality of geometric teaching, for it serves as a basis, upon which, the teaching takes place (Bonyah & Lari, 2021).

Naturally, as the teachings become more complicated, the geometric comprehension of the teachings becomes more inclusive and multidimensional, which, in turn, deepens the teacher's geometric views and conceptualisations in accordance with the complexity of teaching material (Reyhani et al., 2010). Therefore, teachers must have a comprehensive understanding of geometry (Watan & Sugiman, 2018). In this regard, one must measure the geometric thinking levels of the teachers, especially student-teachers, so their capabilities can be repositioned to pave the way for their students.

By employing Van Hiele's levels of thinking, Lumbre et al. (2023) investigated the connection between 9th-grade mathematics teachers and their students. The results of their study showed that there is a direct connection between the teachers' levels of geometric thinking and their students. In other words, the students with a teacher in level 5 of Van Hiele geometric thinking had more improvements than those with a teacher in level 2.

In their research, echoing the Van Hiele method, Ovez and Ozdemir (2024) investigated the proofing capabilities of elementary student-teachers. The results of their study showed that most student-teachers are at the fourth level of the Van Hielean thinking levels. They also argued that the Van Hiele method directly impacts the students' thinking process.

Employing the Van Hiele method, Sidelil et al. (2024) explored secondary school mathematics teachers. None of the participants were able to reach the fourth or fifth level, but most achieved the first level; half achieved the second; and 4% achieved the third Van Hiele level.

Informed by the first and second Van Hiele levels, Ghorbani et al. (2023) investigated the geometrical thinking of symmetry in elementary student-teachers. The results showed that 34% of the student-teachers were at the first level and 18% at the second. Although the student-teachers were able to distinguish symmetry in different shapes, they were unable to pinpoint the type of symmetry, especially when the shape had a rotational, inclined axial, or compound symmetry.

Pavlovicova et al. (2022) studied the levels of geometric thinking in elementary student-teachers in light of Van Hiele's model. The research showed that elementary student-teachers face difficulties in the advanced stages of geometric thinking. According to their study, only 34.7 percent of the students were able to reach the third level of Van Hiele, abstraction. In 2021, Yaftian and Gorjipour Shoobi measured the Van Hiele levels of thinking and mastery of geometric argumentation in 9th-grade students. The results of their study showed that most 9th-grade students are at the first level of the Van Hiele model, and there were no signs of level 4 geometric thinking among them.

In their research, Bonyah and Lari (2021) investigated student-teachers who aimed to teach mathematics at an elementary level. They studied the level of achievement of male and female student-teachers in the first three levels of Van Hielean geometrical thinking. The results of their study showed that the participants were optimal at the first level, but female student-teachers performed better at the second and third levels.

In their research, Yaftian and Safabakhsh Chekosari (2021) investigated the levels of geometric thinking on quadrilaterals in 8th-grade students. The test consisted of five five-optioned questions, some of which required reasoning as well. The results of their study showed that although these students did better than school mathematics tests, these students were at the first or the second level on the Ven Heile model. In 2020, Sabzali and Tavanayi studied 8th-grade students in light of their level of geometric thinking in polygons. The results of the study showed that education in accordance with the Van Hiele method can upgrade level 1 students to level 2, but they could not surpass a level 3 understanding.

In their article, Sudihartinih and Wahyudin (2019) investigated the geometrical thinking in first-year biology university students in basic mathematics. They studied 90 students. Their study concluded that only 4 students were able to reach a level 2 Van Hiele geometrical thinking level whilst others were lower. It should be noted that female students performed better than the males. In 2018, Salifu et al. studied the Van Hiele geometric thinking levels of pre-service teachers in Ghana. Their study showed that among 473 pre-service teachers, 131 never reached any Ven Hiele level; 114 reached level 1; 73 reached level 2; only 24 were able to reach a level 4 Van Hiele geometric thinking level; and none were able to reach level 5. Thus, the pre-service teachers were not able to achieve elite Van Hiele thinking levels.

In their study, Watan and Sugiman (2018) studied the relationship between teaching methods and the 7th-grade students' Van Hiele level of geometric thinking in accordance with angle and line. In this study, 36.4 percent, 31.8 percent, and 18.2 percent of the students reached Van Hiele levels one to three, and only 4.5 percent were able to reach level 4. The result of their study showed that there is a meaningful connection between teaching methodology and Van Hiele's levels of geometric thinking.

Echoing the Van Hiele theory, Alamian et al. (2018) studied the 8th-grade geometry students – of Sannandaj, Kurdistan, Iran – to identify their misunderstandings and improve their skills. Their research showed that there are numerous errors in the students' geometrical understanding and skills but by employing the Van Hiele model, the teachers can reduce the students' misunderstanding and improve geometrical skills. In 2010, Reyhani et al. investigated Van Hiele's geometrical levels of thinking in mathematics teachers and last-year student-teachers of mathematics. Their study showed that the subjects were able to reach a level 3 Van Hiele geometric thinking but were unable to reach the next level. In other words, school and academic education were unable to prepare mathematics teachers and last-year student-teachers of mathematics for level 4 Van Hiele geometric thinking.

Investigating third-grade elementary teachers, Kabiri (2021) noted that these teachers have low educational subject knowledge. Investigating the knowledge of mathematics in elementary student-teachers in Iran, Shahbazi et al. (2024) argued that the professional and content knowledge of teaching mathematics was sub-optimal. Due to its importance, teaching geometry must be thoroughly studied. Despite numerous studies on the geometrical thinking levels of teachers and students all around the globe, there are few studies conducted on the geometrical thinking levels of elementary student-teachers in Iran. In this regard, there is a need for extensive investigation of the geometrical shortcomings faced by student-teachers and immediate action.

Teachers should be able to check the answers of students' exercises and guide them even when they have given incorrect answers or used incorrect methods (Mellone et al., 2020). The requirement for this work is that the teacher himself has sufficient knowledge in this field, including geometry. Numerous researchers have studied the geometrical thinking of students, but the literature on student-teachers, who will soon nurture and educate the next generation of students, is rather slim. Likewise, the literature on the measurement of the levels of geometric thinking in elementary student-teachers is pretty much non-existent. As mentioned by Lumbre et al. (2023), Moosapoor (2023), Schaeffer et al. (2021), and Watan and Sugiman (2018), the teachers' erudition is an important factor in teaching mathematics and, especially, geometry. In this regard, it is necessary to measure the levels of geometric thinking in elementary student-teachers. In addition, we will investigate the effectiveness of the current curriculum on student-teachers' geometric thinking. The purpose of this study is to evaluate the levels of geometric thinking in student-teachers in elementary education based on Van Hiele's theory.

This study aims to answer the following questions:

(1) Where on the Van Hiele geometric thinking theory are the second-year elementary education bachelor student-teachers located?

(2) Where on the Van Hiele geometric thinking model are the fourth-year elementary education bachelor student-teachers located?

(3) What is the difference, in terms of geometric levels of thought, between a second-year and a fourth-year elementary education bachelor student-teacher?

METHODOLOGY

Research approach

The purpose of this study is to investigate the levels of geometric thinking informed by the Van Hiele theory. Since the results of this study can improve the quality of elementary education, this article, purpose-wise, is considered an applied research that employs a descriptive-survey method which collects the data through a survey answered by the participants.

The participants

Farhangian University is among the most astounding teacher training facilities in Iran. In this regard, I selected the elementary education student-teachers as participants. The statistical population of this study was the elementary education bachelor student-teachers in Iran. I have conducted the present research on an available sample composed of all student-teachers who, at the time, were studying elementary education at Farhangian University of Guilan in 2024. Of these 187 people, 120 were second-year students and 67 were fourth-year students.

In one semester of each academic year, the elementary education bachelor student-teachers have a course in accordance with teaching mathematics and geometry. That being said, second-year students have two classes which relate to mathematics, and fourth-year students have four classes. Consequently, I have studied them separately.

Data collection tool

The data collection tool was a survey which entailed 9 four-choice questions regarding polygons and had no negative marks. Van Hiele's first level informed the first three questions of geometrical thinking, Van Hiele's second level informed the second three questions of geometrical thinking, and the third level informed the last three questions. The first question did not require the students to justify their choice, but other questions required the participants to justify their answers or hatch the shapes.

Questions 1, 2, 5, and 6 were informed by Yaftian and Safabakhsh Chekosari (2020), and questions 3, 4, 7, and 9 were informed by Gorjipour Shoobi (2018), and their validity and reliability were confirmed by mathematics professors and teachers. In addition, their reports calculated the Kronbakh Alpha coefficient as 0.7; Question 8 was designed by me, and, according to 5 mathematics professors and 5 mathematics teachers, it was a valid question with a hundred percent CVI and one CVR.

The grading process was as follows: for each correct answer (correct with correct justification, correct with no justification, and correct with wrong justification), the students would get one point, and for each wrong answer or blank answer, they would get zero points. The sum of the points of the first three questions measured their grade in Van Hiele's first level; the sum of the points of the second three questions measured their grade in Van Hiele's second level; and the sum of the points of the last three questions measured their grade in Van Hiele's third level. In this regard, each participant, for each Van Hielean level of geometric thinking, achieves a number from 0 to 3.

Data collection and analysis

Considering the fact that student-teachers were selected from various classes, I distributed the Questionnaire to each group separately and collected their answers. Although there was no time limit for answering the questions, the most time spent on answering the questionnaire was 20 minutes. The researcher supervised the process so that every student-teacher maintained his/her ethical conduct. Additionally, the researcher graded every participant's questionnaire separately.

To achieve each thinking level, according to the Van Hiele method, at least 2 out of 3 questions had to be answered correctly; for instance, if a participant answers two out of the first three questions, s/he has reached the first level of Van Hiele Thinking.

The normality of the data was also confirmed using the Skewness-Kurtosis test. To analyse the data, I employed SPSS (Version 26) informed by descriptive and inferential statistics methods. At the descriptive level, I employed indicators such as frequency distribution tables and their percentages. Also, since the data was scarce and normal, I employed the T-test to determine each group's access to the first, second, and third levels of Van Hiele's geometric thinking, as well as their correlating geometric thinking levels.

Ethical considerations

Ethical standards for the research were adhered to. Due to ethical considerations, I explained the purpose of the study, which was to measure the level of geometric thinking, to the participants. All student teachers participated in the study with consent. Participation had no psychological or physical effects on the participants. Finally, the data were protected and the questionnaires were anonymous.

RESULTS AND ANALISES

Table 1, presents the answers given to the first three questions of the study, in accordance with the first level of Van Hiele's geometric thinking, by the second-year and fourth-year bachelor students in elementary education.

Table 1

Distribution frequency of student-teachers' responses to the first three questions.

		Correct Answers				
		Right Justification	No Justification	Wrong Justification	Blanks	Wrong Answers
Second-year	First Question	91	0	0	0	29
	Second Question	52	12	0	1	55
	Third Question	93	15	0	2	10
Fourth-year	First Question	57	0	0	0	10
	Second question	49	2	0	3	13
	Third Question	57	2	1	1	6

According to Table 1, The most correct answers from the second-year students were to the third question with 108 correct answers, and the most incorrect answers were to the second question. Likewise, the most correct answers from the fourth-year students were to the third question with 60 correct answers, and the most incorrect answers were to the second question which asked the students to count and hatch trapezoidal shapes.

Figure 1

The second question of the study.

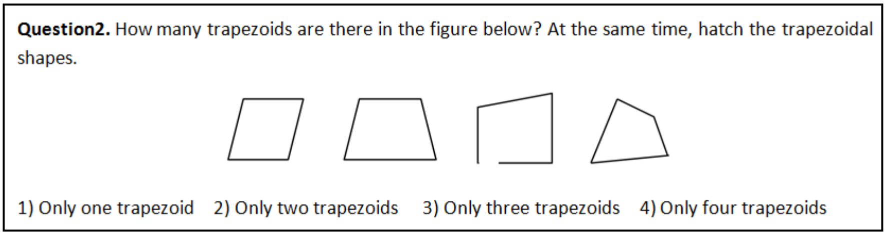


Figure 1 presents the second question of the study. Figure 2.a is a sample of student-teachers' correct answers to the second question, and Figure 2.b is a sample of student-teachers' wrong answers.

Figure 2

Figure 2.a. A sample of a correct answer to the second question.

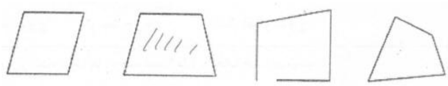


Figure 2.b. Samples of wrong answers to the second question.

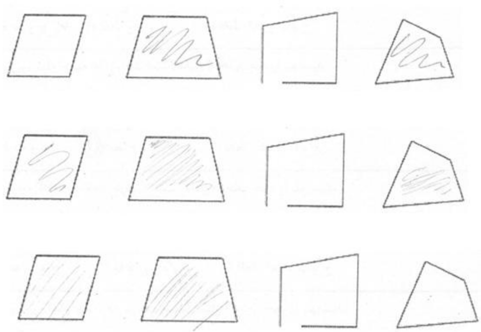
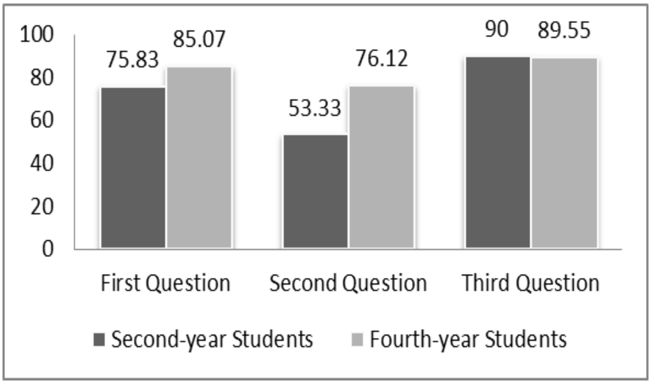


Figure 3 presents and compares the correct answers to the first three questions.

Figure 3

Comparing the percentage of correct answers to the first three questions.



The most correct answers from both groups, according to Figure 3, were to the third question, and the least correct answers from both groups were to the second question.

Table 2 presents the answers given to the second three questions of the study, in accordance with the second level of Van Hiele's geometric thinking, by the second and fourth-year bachelor student-teachers.

Table 2

Distribution frequency of student-teachers' responses to the second three questions.

		Correct Answers			Blanks	Wrong Answers
		Right Justification	No Justification	Wrong Justification		
Second-year	Fourth Question	82	11	1	1	25
	Fifth Question	92	0	0	5	23
	Sixth Question	31	0	0	9	80
Fourth-year	Fourth Question	51	1	1	0	14
	Fifth Question	56	0	0	4	7
	Sixth Question	39	0	0	4	24

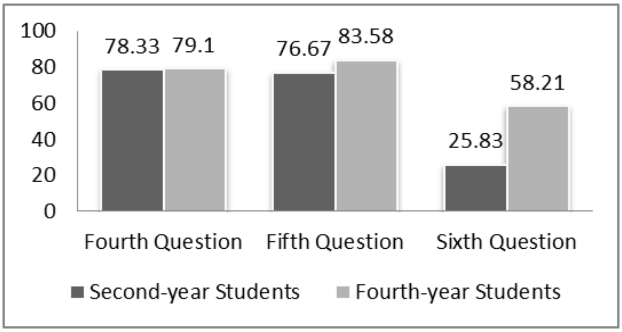
According to Table 2, the most correct answers from the second-year students were to the fourth question with 94 correct answers, and the most incorrect answers were to the sixth question. On the other hand, the most correct answers from the fourth-year students were to the fifth question with 56 correct answers, and the most incorrect answers were to the sixth question.

Figure 4 presents and compares the correct answers to the second three questions.

According to Figure 4, the most correct answers from the second-year students were to the fourth question, and the most correct answers from the fourth-year students were to the fifth question which was about the characteristics of a rectangle. The reason behind this success can be traced back to the fourth-year syllabus of the student-teacher curriculum; during their education, the professors present an in-depth analysis of the elementary mathematic books and their teaching methods, in which the rectangle has been thoroughly analized.

Figure 4

Comparing the percentage of correct answers to the second three questions of the questionnaire.



The least correct answers from both groups were to the sixth question which was about the characteristics of a parallelogram. The reason behind this statistic might be due to the reasoning skills needed to conclude the correct answer; in other words, the characteristics of a parallelogram were not directly mentioned in the books and required reasoning to be understood correctly.

The fourth question asked the students to identify the number of squares in the picture and hatch them (Figure 5).

Figure 5

The fourth question of the study

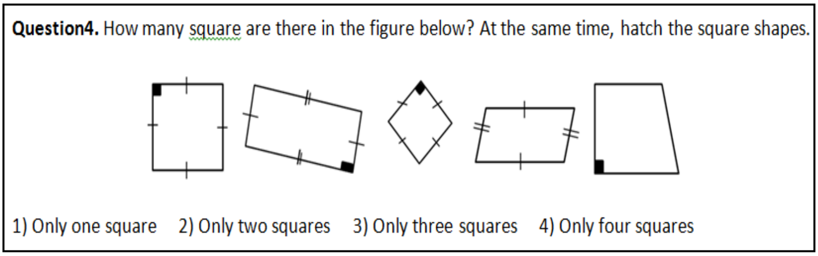


Figure 6.a is a sample of a correct answer to the fourth question. Though this question was mostly answered correctly, Figure 6.b is a sample of an incorrect answer.

Figure 6

Figure 6.a. A sample of a correct answer to the fourth question.

Figure 6.b. Samples of wrong answers to the fourth question.

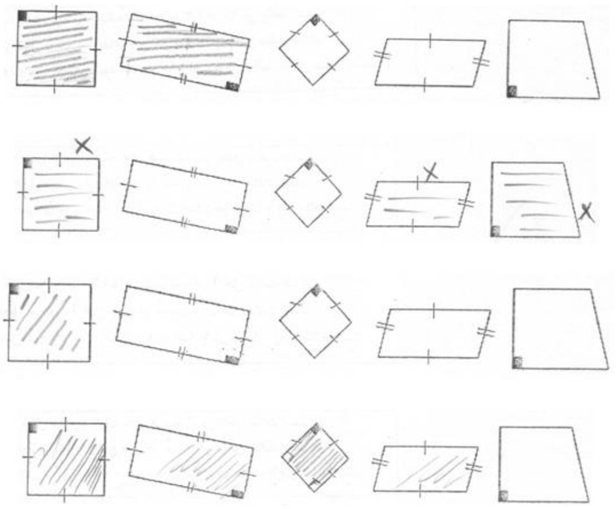


Table 3 presents the answers given to the last three questions of the study, in accordance with the third level of Van Hiele's geometric thinking, by the second and fourth-year bachelor student-teachers.

According to Table 3, the most correct answers from the second-year students were to the eighth question (Figure 7) with 78 correct answers, and the most incorrect answers were to the ninth question. On the other hand, the most correct answers from the fourth-year students were to the eighth question with 46 correct answers, and the most incorrect answers were to the seventh question.

Table 3

Distribution frequency of responses to the last three questions.

		Correct Answers				
		Right Justification	No Justification	Wrong Justification	Blanks	Wrong Answers
Second-year	Seventh Question	10	58	0	5	47
	Eighth Question	9	67	2	12	30
	Ninth Question	4	32	0	24	60
Fourth-year	Seventh Question	12	18	0	4	33
	Eighth Question	13	23	0	6	25
	Ninth Question	6	17	0	18	26

Figure 7

The eighth question of the study.

Question8. Which of the following sentences is correct? Write the reason for your choice.

A) Every rhombus is a parallelogram.
B) Every square is a parallelogram.
C) Every triangle is a parallelogram.
D) Every parallelogram is a rectangle.

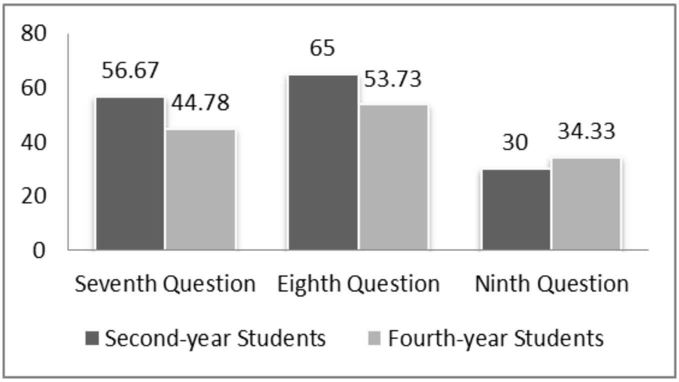
1) Only A and B 2) Only B and D 3) Only C and D 4) Only A, B and C

Figure 8 presents and compares the correct answers to the last three questions.

The most correct answers from both groups, according to Figure 8, were to the eighth question, and the least correct answers from both groups were to the ninth question. Since all three questions required reasoning and analysis, the student-teachers' sub-optimal results were partly due to their lack of reasoning skills and methodology.

Figure 8

Comparing the percentage of correct answers to the last three questions of the questionnaire.



By employing the Skewness-Kurtosis test, I prove the normality of the data (Table 4).

Table 4

The Results of the Skewness-Kurtosis Test for the Normality of the Data.

	N	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
level1year2	120	0.690	-0.271	0.221	-0.876	0.438
level1year4	67	0.683	-1.059	0.293	-0.097	0.578
level2year2	120	0.714	-0.125	0.221	-0.234	0.438
level2year4	67	0.789	-0.775	0.293	0.178	0.578
level3year2	120	0.961	0.010	0.221	-0.932	0.438
level3year4	67	1.021	0.262	0.293	-1.013	0.578

To answer the first research question, we must, firstly, figure out that have the second-year student-teachers reached the first level of Van Hielean thinking or not. According to Table 5, the obtained value is t=3.045 with a degree of freedom of 119 at the 95% confidence level. Considering the fact that Sig=0.003, so Sig<0.05 thus rejecting the null hypothesis and confirming the opposite hypothesis. That being said, since the value of t is positive, we can conclude that the second-year student-teachers have reached level one of Van Hielean thinking.

Table 5

The results of the T-test in Van Hiele's first level (μ=2) for second-year student-teachers.

	Test Value = 2, Mean=2.19					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
level2year2	3.045	119	0.003	0.192	0.07	0.32

Now I investigate that were the second-year student-teachers able to reach the second level of Van Hielean thinking or not. According to Table 6, the obtained value is t=-2.942 with a degree of freedom of 119 at the 95% confidence level. Considering the fact that Sig=0.004, so Sig<0.05 thus rejecting the null hypothesis and confirming the opposite hypothesis. That being said, since the value of t is negative, we can conclude that the second-year student-teachers could not reach the second level of Van Hielean thinking.

Table 6

The results of the T-test in Van Hiele's second level (μ=2) for second-year student-teachers.

	Test Value = 2, Mean=1.81					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
level2year2	-2.942	119	0.004	-0.192	-0.32	-0.06

I employ the results from Table 7 to investigate whether the second-year student-teachers reached the third level of Van Hielean thinking.

According to Table 7, the obtained value is t=-5.508 with a degree of freedom of 119 at the 95% confidence level. Considering the fact that Sig=0.000, so Sig<0.05 thus rejecting the null hypothesis and confirming the opposite hypothesis. That being said, since the value of t is negative, we can conclude that the second-year student-teachers have not reached the third level of Van Hielean thinking.

Table 7

The results of the T-test in Van Hiele's third level (μ=2) for second-year student-teachers.

	Test Value = 2, Mean=1.52					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
level3year2	-5.508	119	0.000	-0.483	-0.66	-0.31

In this regard, the second-year bachelor student-teachers in elementary education were able to reach the first level of Van Hielean geometric thinking, but could not reach the second or the third levels.

To answer the second research question, we must, firstly, investigate whether the fourth-year student-teachers have reached the first level of Van Hielean thinking or not. According to Table 8, the obtained value is t=6.086 with a degree of freedom of 66 at the 95% confidence level. Considering the fact that Sig=0.000, so Sig<0.05 thus rejecting the null hypothesis and confirming the opposite hypothesis. That being said, since the value of t is positive, we can conclude that the fourth-year student-teachers have reached the first level of Van Hielean thinking.

Table 8

The Results of the T-Test in Van Hiele's First Level (μ=2) for Fourth-year Student-teachers.

	Test Value = 2, Mean=2.51					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper

level1year4	6.086	66	0.000	0.507	0.34	0.67

Now I investigate that were the fourth-year student-teachers able to reach the second level of Van Hielean thinking or not. According to Table 9, the obtained value is t=2.168 with a degree of freedom of 66 at the 95% confidence level. Considering the fact that Sig=0.034, so Sig<0.05 thus rejecting the null hypothesis and confirming the opposite hypothesis. That being said, since the value of t is positive, we can conclude that the fourth-year student-teachers have reached the second level of Van Hielean thinking.

Table 9

The results of the T-test in Van Hiele's second level (μ=2) for fourth-year student-teachers.

	Test Value = 2, Mean=2.21					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
level2year4	2.168	66	0.034	0.209	0.02	0.40

I employ the results from Table 10 to investigate whether the fourth-year student-teachers reached the third level of Van Hielean thinking. According to Table 10, the obtained value is t=-5.386 with a degree of freedom of 66 at the 95% confidence level. Considering the fact that Sig=0.000, so Sig<0.05 thus rejecting the null hypothesis and confirming the opposite hypothesis. That being said, since the value of t is negative, we can conclude that the fourth-year student-teachers have not reached the third level of Van Hielean thinking.

Table 10

The results of the T-test in Van Hiele's third level (μ=2) for fourth-year student-teachers.

	Test Value = 2, Mean=1.33					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
level3year4	-5.386	66	0.000	-0.672	-0.92	-0.42

In this regard, the fourth-year bachelor student-teachers in elementary education were able to reach the first and second levels of Van Hielean geometric thinking, but could not reach the third level.

CONCLUSIONS

The teachers' knowledge of geometry directly impacts their teaching. This notion is important for elementary school teachers for they are influential in establishing the students' geometric knowledge. In this regard, we must assess the geometrical capabilities of would-be elementary teachers; one way is by employing the Van Hiele theory which I did in this study. In this study, I have investigated the success rate of second-year and fourth-year elementary student-teachers in achieving the first three levels of the Van Hielean geometric thinking regarding polygons. The results of this study show that the second-year student-teachers were at the first level, and the fourth-year student-teachers were at the second; to put it simply, one can argue that bachelor student-teachers in elementary education are at the first two levels of Van Hielean thought. Neither of the two groups was able to achieve the third level. Since the third Van Hielean geometric thinking levels correlate with informal reasoning, one can assume that the lack of attention to reasoning and proofs in the university curriculum plays an important role in the absence of reasoning and proofs in school books and curricula. Although, according to the findings, the student-teachers' levels progressed as they did in academia, their inability to reach the third level shows that the methodology and curriculum of geometry education need to be re-assessed.

Considering the few studies that have been done in Iran and other developing countries about the levels of geometric thinking of student-teachers in the field of elementary education, the results of this study are valuable to those who work in and impact higher education, for this research identifies the shortcomings that correlate with geometry and its quality in the student-teachers' curricula. By elevating the quality of the mathematics curriculum, especially geometry, we would have more potent teachers who will directly impact the quality of mathematics education as well as problem-solving and critical thinking skills.

As a result of this research, the following suggestions are presented:

(1) Considering that university education can be effective in elevating Van Hielean's thinking, I suggest that more mathematics-related courses be included in the elementary education bachelor's curriculum; courses which teach geometry as an independent course from elementary to advanced levels.

(2) Second-year student-teachers were at the first Van Hielean level, which indicates that their high school education was unable to prepare them for desirable geometric thinking. In this regard, schools should concentrate more on geometry and include problem-based or inquiry-based methods of teaching to reinforce effective inference and reasoning.

(3) Due to the sequentiality of Van Hiele's geometric thinking, the fourth and fifth-level geometric thinking of the two groups was not assessed. I suggest a call back in a few years for a re-run of the tests.

(4) The abundance of unjustified answers or informal reasoning, in the third Van Hielean level, proved the student-teachers' incapability to form or articulate justifications. In this regard, more educational activities should be designed and implemented in schools and universities on mathematics justification and reasoning articulation, such as organizing problem-solving or critical thinking workshops.

(5) This research evaluated student-teachers under questions informed by polygons. I suggest that the Van Hielean levels be assessed in accordance with other geometrical concepts.

(6) This research investigated the content knowledge of student-teachers in geometry. To analyse the effectiveness of the curricula, I suggest monitoring the content educational knowledge of student-teachers in geometry, for they should have optimal teaching capabilities. Also, I suggest to Evaluate the effects of new teaching methods.

(7) The present study has been conducted in Iran. I suggest that this research should be replicated singularly and comparatively in other countries.

(8) Evaluating the effects of new teaching methods.

AUTHORS' CONTRIBUTIONS STATEMENTS

Mansooreh Moosapoor conceived the presented idea, developed the theory, adapted the methodology to this context, performed the activities, collected the data, analysed the data, and reviewed and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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