ACTA SCIENTIAE

ISSN: 2178-7727
DOI: 10.64856/acta.scientiae.8387

Analysis of specialised knowledge of number sense in a pre-service mathematics teacher

Santiago Avilés Álvarez [a]
Paula Verdugo-Hernández[b]
Andrea Vergara Gómez [a]

[a] Centro de Investigación en Educación Matemática y Estadística, Universidad Católica del Maule, Facultad de Ciencias Básicas antes de Universidad Católica del Maule, Talca, Chile.
[b] Escuela de Pedagogía en Ciencias Naturales y Exactas, Facultad de Ciencias de la Educación, Universidad de Talca. Linares, Chile.

ABSTRACT

Background: Student learning is influenced by teacher knowledge, with initial education and practical experience playing a significant role. In this context, problem solving is a learning objective for which developing number sense is crucial. Little research examines number sense in pre-service mathematics teachers. **Objectives:** This article presents the results of a study about a pre-service mathematics teacher's mathematical knowledge as he implemented problem-solving activities that required number sense. **Methodology:** Using a qualitative approach, this study sought to characterise the pre-service mathematics teacher's number sense knowledge through a single case study as he solved problems. **Setting and Participants:** The case study involved an eighth-grade mathematics class taught by a pre-service mathematics teacher. The lesson focused on solving multiplication and division problems of fractions. **Data Collection and Analysis:** Data were collected from a video recording of the lesson and processed in a spreadsheet. The data were analysed using categories of mathematical knowledge and then number sense. **Results:** The results indicate greater use of number sense than algorithms. **Conclusion:** The analysis showed that both the promotion of number sense and the use of algorithms can be understood through the categories of the teacher's mathematical knowledge. Replication of the study is recommended to rule out the influence of the teacher's prior training.

Keywords: Teacher education; Problem solving; Arithmetic; Mathematics teaching; Number sense.

Análisis de conocimiento especializado sobre sentido numérico de un futuro profesor de matemáticas

RESUMEN

Contexto: El aprendizaje de los estudiantes está influenciado por el conocimiento del profesor, en el que juegan un rol importante la formación inicial y las prácticas formativas. En este contexto, la resolución de problemas es una meta de

Corresponding author: Paula Verdugo-Hernández. Email: pauverdugo@utalca.cl

aprendizaje para la cual es importante el desarrollo del sentido numérico. Hay escasa investigación sobre el sentido numérico en profesores en formación. **Objetivos:** En este artículo se presentan resultados que abordan el estudio del conocimiento matemático de un profesor en práctica profesional al implementar actividades de resolución de problemas que requieren sentido numérico. **Diseño:** Desde un enfoque de investigación cualitativo se buscó caracterizar el conocimiento relacionado con sentido numérico que despliega el profesor al resolver problemas, por medio de un estudio de caso único. **Escenario y participantes:** El caso corresponde a una clase de matemáticas de octavo básico de un profesor en práctica profesional. La clase fue de resolución de problemas asociados a la multiplicación y división de fracciones. **Recopilación y análisis de datos:** Los datos se recolectaron a partir de una grabación de vídeo de una clase y se trataron con una hoja de cálculo. Se analizaron a través de las categorías de conocimiento matemático y luego con las de sentido numérico. **Resultados:** Los resultados apuntan a una mayor presencia de sentido numérico que del uso de algoritmos. **Conclusión:** Se identifica que tanto la promoción del sentido numérico como el uso de algoritmos pueden comprenderse mediante un análisis a partir de las categorías del conocimiento matemático del profesor. Se recomienda replicar el estudio para descartar la influencia de la formación del profesor.

Palabras clave: Formación de profesores; Resolución de problemas; Aritmética; Enseñanza de las matemáticas; Sentido numérico.

Análise de conhecimento especializado sobre sentido numérico de um futuro professor de matemática

RESUMO

Contexto: A aprendizagem dos alunos é influenciada pelo conhecimento do professor, no qual a formação inicial e a experiência prática desempenham papéis significativos. Nesse contexto, a resolução de problemas é um objetivo de aprendizagem para o qual o desenvolvimento do senso numérico é crucial. Há poucas pesquisas sobre o senso numérico em professores em formação. **Objetivos:** Este artigo apresenta os resultados de um estudo sobre o conhecimento matemático de um professor estagiário ao implementar atividades de resolução de problemas que exigem senso numérico. **Metodologia:** Adotando uma abordagem de pesquisa qualitativa, este estudo buscou caracterizar o conhecimento de senso numérico demonstrado pelo professor estagiário ao resolver problemas, por meio de um estudo de caso único. **Local e Participantes:** O estudo de caso envolveu uma turma de matemática do oitavo ano ministrada por um professor estagiário. A aula focou na resolução de problemas de multiplicação e divisão de frações. **Coleta e Análise de Dados:** Os dados foram coletados a partir de uma gravação em vídeo da aula e processados em uma planilha. Os dados foram analisados utilizando categorias de conhecimento matemático e, em seguida, de senso numérico. **Resultados:** Os resultados indicam uma maior presença de senso numérico do que a de uso de algoritmos. **Conclusão:** Constatou-se que tanto a promoção do senso numérico quanto o uso de algoritmos podem ser compreendidos por meio da análise das categorias do conhecimento matemático do professor.

Recomenda-se a replicação do estudo para descartar a influência da formação prévia do professor.

Palavras-chave: Formação de professores; Resolução de problemas; Aritmética; Ensino de matemática; Senso numérico.

INTRODUCTION

Various studies show that teachers' knowledge is crucial to their performance and students' learning of mathematics (e.g., Ball et al., 2008; Carrillo et al., 2013; Rowland et al., 2005; Zakaryan et al., 2018). In this regard, early formative placements in initial teacher education (ITE) are crucial, as they prevent the adoption of teaching methods dictated by school pressure, thereby fostering educational innovation (Brouwer & Korthagen, 2005). These placements offer opportunities to develop skills through various teaching experiences, such as planning, teaching, and assessment (Caires & Almeida, 2005).

Indeed, Chilean universities responsible for teacher education face the challenge of ensuring the quality and effectiveness of early formative practices, focusing on how pre-service mathematics teachers promote the development of fundamental mathematical skills (Verdugo et al., 2023). Research on teaching problem solving (PS) in mathematics education has highlighted the specific knowledge teachers need to implement this key skill effectively in the classroom (Piñeiro et al., 2021; Rodríguez & Parraguez, 2014).

Among the pioneers of PS, Polya (May Cen, 2015) stands out, as he encouraged students to imitate mathematicians' methodologies using heuristics (Schoenfeld, 1992) to develop general strategies. Schoenfeld underscores the importance of reflecting on how specific heuristics are used to solve mathematical problems. It also highlights the importance of metacognition and beliefs in problem solving (henceforth PS), emphasising elements of mathematical thinking such as conjectures, patterns, generalisations, and justifications.

Chapman (2015), for his part, highlights the need to investigate knowledge of PS from the classroom perspective, with the aim of providing practical guidance for effectively engaging teachers. Other studies have attempted to characterise this knowledge necessary for teaching PS (e.g., Depaepe et al., 2010; Piñeiro et al., 2016; Piñeiro et al., 2021). For example, Depaepe et al. (2010) explored the heuristics and metacognitive strategies teachers use when teaching PS. Meanwhile, Piñeiro et al. (2016) proposed a model describing the knowledge primary school teachers need to teach PS, based on an analysis of curriculum documents from different countries.

Among these types, two stand out: routine problems, which are mechanically repetitive and quickly solved by children, and non-routine problems, which are genuine (Baroody, 1994) and challenging, requiring heuristics for their analysis (Baroody, 1994; Mousoulides & Sriraman, 2020). Non-routine problems are key to enabling students to explore different ways of representing and applying their solution methods to other problems (Santos-Trigo, 2007).

According to the National Council of Teachers of Mathematics (NCTM, 2000), number sense (NS) is crucial to developing problem-solving skills and is a learning goal in mathematics. NS streamlines reasoning by reducing the burden of minor mental operations (Sharma, 2015) and has been linked to success in mathematics, especially in problem solving (Levine et al., 1992; Krajewski & Schneider, 2009). It is therefore essential to foster the development of NS in students rather than focusing solely on memorising methods, to boost their mathematical performance (Boaler, 2015).

For the purposes of this study, we have adopted Sowder's (1992) definition of number sense. The author highlights the relationship between NS and PS, describing NS as a well-organised network of concepts that enables creative and flexible problem solving, as well as relating numbers to operations and their properties. McIntosh et al.'s (1992) perspective is also considered, which highlights the role of flexibility and creativity in solving mathematical problems.

Indeed, in primary and secondary education, it is common to focus on tasks with single solutions, limiting students' perspective on PS (Almeida & Bruno, 2014). For this reason, we recommend introducing NS in classrooms and encouraging different strategies rather than a single correct procedure (Veloo, 2010; Clement & Sarama, 2020). It is also noted that, at the beginning of number learning, children use creative and effective approaches to calculations, which can be hindered by the exclusive use of formal algorithms (McIntosh et al., 1992).

Given the importance of NS in PS, numerous studies have been dedicated to this area, both with primary school pupils (e.g. Ben-Yehuda & Sharoni, 2021; Jordan et al., 2010) and secondary school pupils (e.g. Almeida et al. 2014; Cheung & Yang, 2018; Santos et al., 2022; Şengül & Zora, 2023). Secondary school teachers tend to use the NS more than primary school teachers, possibly because of their more extensive mathematical schooling (Almeida et al., 2014; Veloo, 2010). Although there is little information on NS research among pre-service mathematics teachers (Noviyanti & Suryadi, 2019)

or in-service teachers, the results of previous research indicate that the former (PTs) of mathematics have shortcomings in the use of the NS (Almeida et al., 2014; Almeida et al., 2015; Mawaddah et al., 2021).

Given the scarcity of studies that seek to characterise teachers' knowledge regarding the use of NS in PS, we asked ourselves: What knowledge does a mathematics teacher deploy when teaching classes focused on problem solving and the development of number sense? This study therefore analyses the subdomains of mathematical knowledge (Carrillo et al., 2014) that a secondary education pre-service mathematics teacher demonstrates while delivering a PS lesson in his final teaching practicum, paying particular attention to how the use of NS and its components is promoted (Almeida & Bruno, 2016; Bruno & Almeida, 2017). To this end, our research is based on a qualitative approach through content analysis of the discursive aspects of the lesson, within the framework of a single case study.

CONCEPTUAL FRAMEWORK

Mathematics teacher's specialised knowledge

The mathematics teacher's specialised knowledge (MTSK) model proposes an intrinsic division of mathematical knowledge, referring to knowledge of mathematical objects, focusing on how relationships between mathematical objects are structured and the ways of proceeding required by each field. It also recognises a body of knowledge that emerges from the didactics of mathematics and work with teachers (Flores et al., 2016). MTSK refers to the specialised knowledge of the mathematics teacher (Carrillo et al., 2013), so it concerns the teacher's knowledge, which is neither exclusively mathematical nor purely didactic. Its specialised nature lies in the combination of its six subdomains (Flores et al., 2014). Figure 1 illustrates these six MTSK subdomains.

Figure 1

MTSK domains and subdomains (Escudero-Ávila et al., 2015)



As shown in Figure 1, MTSK comprises two broad domains: mathematical knowledge (MK) and pedagogical content knowledge (PCK), which encompass conceptions of mathematics that, in turn, permeate them (Liñan et al., 2016). The MK domain and its subdomains are defined below, as they underpin this research.

MK domain

MK encompasses the entire body of mathematical knowledge, including concepts, procedures, the structuring of ideas, connections between concepts, the etymology of procedures, and means of proof (Carillo et al., 2013).

Knowledge of topics (KoT)

The KoT subdomain refers to in-depth, well-founded knowledge characteristic of a maths teacher's knowledge when planning teaching (Vasco et al., 2017), and it incorporates the formal, rigorous knowledge of what students are expected to learn. It also considers the complexity of the mathematical objects that may appear in the classroom (Carillo *et al*., 2018). In other words, KoT describes what and how the teacher knows the mathematical topics they will teach, referring to the traditionally considered blocks of

knowledge: numbers and operations, algebra, measurement and data analysis, and probability (Flores et al., 2014).

Knowledge of the structure of mathematics (KSM)

KSM refers to knowing the connections with previous and subsequent content (Carrillo et al., 2013). This subdomain addresses the internal relationships in mathematics (Montes et al., 2013), which enable the teacher to build practice-based reasoning based on the knowledge of its structure (Vasco et al., 2017). It describes the connections between mathematical topics (Flores-Medrano, 2022), considers the delimitation of the mathematical object, which allows us to have an idea of the boundary of the concept and to understand the inter-conceptual relationships; and temporality, viewed from the curriculum perspective, which allows us to establish relationships between mathematical topics, depending on the stage of schooling (Flores-Medrano et al., 2014).

Knowledge of the practice of mathematics (KPM)

The focus of KPM is the means of producing and implementing mathematics. It enables the teacher to recognise mathematical reasoning, allowing them to manage students' reasoning by refuting or refining it appropriately (Carrillo et al., 2018). For KPM, the teacher must know both the mathematical results (an element covered by KoT) and the procedures for reaching them, as well as the specific features of mathematical work (Flores et al., 2014). This includes knowledge of the structure of mathematics, mathematical communication, reasoning, verification, definitions and their application, establishing relationships between concepts and properties, understanding correspondences, appropriate representations, reasoning, generalising, and exploring (Carrillo et al., 2013). The difference between KSM and KPM is that the former takes into account mathematical knowledge of the relationships between the different timescales of the curriculum elements. Second, institutions regulate the order (Flores-Medrano, 2022).

Characterisation of number sense

Although NS has become difficult to define, some researchers have described its components (Almeida et al., 2015). These related elements are presented separately for analytical reasons. We can find those proposed by Almeida et al. (2014): (1) Understanding the meaning of numbers, (2) Recognising the relative and absolute size of the magnitudes of numbers, (3) Using benchmarks, (4) Using the composition and decomposition of numbers, (5) Using multiple representations of numbers and operations, (6)

Understanding the relative effect of operations, and (7) Developing appropriate strategies and assessing the reasonableness of an answer.

The components of the NS are dynamic, with proposals for their characterisation that group or divide elements to facilitate research (Bruno & Almeida, 2017). For example, Almeida and Bruno (2016) remove component 4 and add representations of numbers and operations. They also divide component 7 into understanding the problem context and the required operation, awareness of multiple strategies, and recognition of reasonable results.

Another interesting proposal is Bruno and Almeida's (2017), which divides the components into two large blocks. The first, called "Components of number sense", is related to number knowledge and fluency; and the second, called "Applying knowledge and fluency in the use of numbers and operations to solve problems with numbers", is related to the application of the NS components from the first block and fluency in numerical PS. The components included in the second block are "Understanding the relationship between the problem context and the operation required", "Realising that there are multiple strategies", and "Recognising the reasonableness of the problem".

Other authors, for example, Mawaddah et al. (2021), have used components such as "Counting and calculation", "Effect of operations", "Equivalent expression", "Multiple representations", and "Number concepts". These suggestions show how the components of the NS have been proposed in flexible and diverse ways through research.

METHODOLOGY

This research falls within the interpretative paradigm, which aims to explore a subject of study in greater depth to facilitate understanding (Ricoy, 2006). A qualitative approach is used to analyse the data, as the aim is to characterise a teacher's knowledge of PS, including the use of the NS, in depth. To this end, a single case study (Yin, 2018) is conducted, taking advantage of the case's conditions to generate specific knowledge (Vasilachis, 2006). Its purpose is particularisation, not generalisation, as it is expected to contribute to similar future research (Stake, 1999).

The case concerns the teaching of an eighth-grade class (13 to 14 years old) by a PT in his final teaching practice. He is being trained to be a secondary school mathematics teacher qualified to teach grades 7 (middle school) to grade 4 of secondary school. The lesson included PS activities related to multiplying and dividing fractions, aligned with his stated objective. The PT's lesson

planning and delivery consisted of summarising previous content and presenting routine problems.

Furthermore, following Carrillo et al. (2014), teacher knowledge is assumed to be unique and inseparable; consequently, the division proposed by the MTSK is merely a framework for organising the analysis. Furthermore, the components of the NS are related, so more than one could arise when tackling the same task (Almeida & Bruno, 2016).

Data collection and analysis

The data were collected through a class video recording that lasted two teaching hours (1 hour and 30 minutes). The lead researcher stored the video recordings and processed them using a spreadsheet. All data derived from the research were, and will continue to be, protected to ensure absolute confidentiality, as well as the consent of the parents and the assent of the students.

They were analysed using content analysis (Bernete, 2014), taking into account reference categories. The units of analysis were all the teacher's discursive interventions in the written transcript. Where necessary, the video was edited to provide context, such as whiteboard use. The analysis covered both general interventions and specific episodes.

> Each intervention taken from the recorded class transcript involves only the teacher (T) and the students (S) to maintain anonymity. The study did not individualise student interventions, as they are not part of the research objectives. The grid in Table 1 was used, based on the MTSK, specifically on the sub-dimensions of the MK, adapted from categories validated by Delgado-Rebolledo and Espinoza-Vásquez (2021).

The category "The practice of problem solving and modelling" emerged from the data after an initial application of the analysis grid. In a second and final analysis, this emerging category was included in the codes. In addition, each classified category of knowledge was, in turn, identified as "Evidence" or "Indication": it was classified as evidence when the intervention clearly corresponded to one of the indicators of the MK categories; and as an indication when the elements did not show a direct correspondence with the indicators of the knowledge categories, but it was possible to infer links. If any teaching intervention, identifiable under a subdomain, cannot be classified in a specific category, it is assigned the code NI, short for "Not identified".

Table 1

MK subdomains and categories

Subdomains	Categories of knowledge
Knowledge of topics (KoT)	Procedures (P) Definitions, properties and their foundations (DPF) Registers of representation (RP) Phenomenology and applications (P&A)
Knowledge of the structure of mathematics (KSM)	Complexification connections (CC) Simplification connections (SC) Cross-cutting connections (CCC) Auxiliary connections (AC)
Knowledge of the practice of mathematics (KPM)	The practice of demonstration (PD) The practice of definition (Pdef) *The practice of problem solving and modelling (PPSM) The role of mathematical language (RML)

*Note: *Category that emerges from the data*

Once all units of analysis had been categorised according to the MK subdomains and categories, they were analysed again, taking into account the presence of the NS. To analyse the components of the NS, structured categories and indicators adapted from the proposal by Bruno and Almeida (2017) are used as a protocol (Table 2). Some indicators in the grid were taken from the capacities related to the NS described by Castro and Segovia (2015). Finally, to analyse how the use of NS components is manifested, we use the following criteria adapted from Almeida et al. (2014): (a) Number sense (NS), uses one or some NS components; (b) Partial number sense (PNS), combines the use of NS components and memorised or algorithmic rules, or uses NS elements with the intention of applying algorithms and memorised rules; (c) Rule-based (RB): Does not use NS components, using only algorithms and memorised rules.

Table 2

NS analysis grid

Category	Indicators
Understanding the meaning of the numbers	Perceiving and handling number structures Recognising or using equivalence relationships between numerical expressions and quantities

Category	Indicators
Recognising the relative and absolute size of the magnitudes of numbers	Understanding the decimal positional number system Grasping the magnitude of numbers, using estimates or numerical properties to make comparisons
Using reference points	Using a reference system or quantity to approach a solution Establishing relationships and comparisons between numbers or quantities to perform calculations
Using multiple representations of numbers and operations	Learning about different representations (graphical, numerical, pictorial, or manipulative) of numbers and using the most appropriate ones Combining or articulating different representations to work with numbers or quantities
Understanding the relative effect of operations and their properties	Perceiving, understanding, and using numerical patterns Estimating the result of operations on numbers according to their relative magnitude Inferring or justifying results based on the effect of operations
Assessing the reasonableness of strategies based on the problem	Recognising errors with numbers made by others or by oneself Discerning when an exact value is needed or when an approximate value will do Assessing the relevance of procedures, results, and operations according to the problem context.
Developing appropriate strategies	Realising that there are different strategies Recognising and using the simplest procedure among several possible ones Simplifying or combining calculation procedures Designing or using procedures other than the standard ones

RESULTS AND ANALYSIS

The results are shown in three main groups: MK results, separated into indications and evidence (Figure 2); NS results (Figure 3); and the results of

cross-referencing the MK analysis grid and the NS grid (Figures 4 and 5). Although the in-service teacher explicitly announces a problem-solving lesson, these problems are routine. Although all the situations include contextual elements, the aim is for the student to apply prior knowledge of operations with fractions.

Table 3 shows that, of the three subdomains, KoT recorded the highest frequency, with a total of 64 (63.4%), of which 42 correspond to the Procedures (P) category (42%), which is the knowledge category with the highest total in the MK. Another category within the KoT that stands out is Phenomenology and applications (P&A), which records a moderate frequency of knowledge (10.9%), explained by the fact that various context-related problems were addressed during the class.

Table 3

MK results

Knowledge categories	KPM		KSM		KoT	
	Indication	Evidence	Indication	Evidence	Indication	Evidence
NI	0	0	0	0	0	1
RML	4	1	-	-	-	-
PPSM	14	10	-	-	-	-
Pdef	0	0	-	-	-	-
PD	0	0	-	-	-	-
AC	-	-	1	0	-	-
CCC	-	.-	0	0	-	-
SC	-	-	4	3	-	-
CC	-	-	0	0	-	-
P&A	-	-	-	-	8	3
RR	-	-	-	-	5	0
DPF	-	-	-	-	4	0
P	-	-	-	-	4	39

Note. Procedure (P); Definitions, properties and their foundations (DPF); Registers of representation (RR); Phenomenology and applications (P&A);

Complexification connections (CC); Simplification connections (SC); Cross-cutting connections (CCC); The practice of demonstration (PD); The practice of defining (PDef); The practice of problem solving and modelling (PPSM); The role of mathematical language (RML); Not identified (NI).

Below is an example in which we can see knowledge that may form part of evidence classified in category P. In this episode, the pre-service mathematics teacher recalls the classic cross-multiplication algorithm to teach the division of fractions.

> Teacher (T) - OK, to solve a division of fractions, do you remember what you have to do?
>
> Student (S) - Cross.
>
> T- Cross-multiply, or another simpler way to remember it, so you don't get confused, you invert the second fraction, and it becomes multiplication. I'll show you.

The following excerpt presents an example of evidence from the P&A category, in which the teacher tries to contextualise the problem. On several occasions, the teacher's intention to give the problems a realistic slant was evident.

> T - Our dog, Firulais, eats ⅔ of a cup a day, right? One cup of dog food, and we have a container, or a bag, that says it holds 30 cups of dog food. Then he asked us, "How many days' worth of food is there for Firulais?"

Another category with relevant results (Table 3) is the "Practice of problem-solving and modelling" (PPSM) category, belonging to the KPM subdomain, not only because of the high frequency recorded (accounting for 23.8% of the total), but also because this category was defined inductively based on the characteristics of the case. The following excerpt shows an example of an indication in the PPSM category, where the teacher organises the information available to help his students solve the problem:

> T - So, one cup. In other words, to make a cake I need 3/5 of a cup of flour, right (he writes the information on the board). I do want to make 15/4 of a cake. How many cups of flour do I need? How many cups of flour would I need?

Table 4 shows the results obtained using the NS analysis grid, where the components are identified according to the criteria "Number sense" (NS)

and "Partial number sense" (PNS). In addition, the results for the "Rule-based" (RB) criterion are recorded; this criterion is not associated with a specific component, as it indicates the use of traditional procedures and algorithms in the absence of NS.

Table 4

Number of interventions classified under the NS components

Category	PNS	NS	RB
NO use of NS	-	-	23
(7) Developing appropriate strategies	8	2	-
(6) Assessing the reasonableness of the responses	1	16	-
(5) Understanding the relative effect of operations and their properties	1	3	-
(4) Using multiple representations of numbers and operations	1	9	-
(3) Using reference points	0	1	-
(2) Recognising the relative and absolute size of the magnitudes of numbers	2	2	-
(1) Understanding the meaning of the numbers	9	4	-

A higher frequency of the RB criterion is observed. However, if we take all the insights recorded in the NS category (37) into account, we can see that they predominate over the RB criterion. On the other hand, if we consider the PNS criterion as a hybrid between NS and RB, it takes elements from both criteria, so the latter category cannot be ruled out as an expression of the use of NS. In this regard, grouping the NS and PNS criteria (59) shows an even greater difference from interventions classified under the RB criterion.

The component that stands out most in the results of the classification in the NS grid is "Assessing the reasonableness of strategies based on the problem" (component 6), as it is the most prevalent and is also associated with the Number Sense (NS) category in almost all cases (94.1% of cases). Below is an example of how component 6 manifests itself in the teacher's practice. In

this episode, the teacher checks the result by replicating the procedure. This can be observed on several occasions during the class sessions.

> T - How much will I be charged?
>
> S - 7,850.
>
> T - It is 7,850. Is that OK? You take out your trusty calculator and say, "1,000 pesos for tomatoes, 5,600 for avocados, plus 1,250 for potatoes; I owe you 7,850 pesos, right?" (referring to adding up the amounts).

Another component that stands out is "Using multiple representations of numbers and operations" (component 4), as, like the previous component, its use is mainly associated with the NS criterion (90%). This component was activated in most cases when the teacher presented the data on the board, using drawings and symbols to represent the variables. The following excerpt shows an example of component 4, in which we can see how the teacher poses an equation in two variables to model the problem:

> T - OK, now, I don't know how much either of them costs. I know the total, but I do know that the suit costs 2/5 of the hat. So, what I can do is: take away the suit and replace it with the hat, using the same hat from there, OK? Multiplied by 2/5.

Another element to consider is that the second most prevalent component was "Understanding the meaning of numbers" (component 1), which is mostly concentrated under the PNS criterion (69.2%). This component was generally used to facilitate the application of traditional procedures and algorithms. In the excerpt below, you can see how the teacher indicates that an integer can be expressed as a fraction by writing a 1 in the denominator, with the intention of being able to apply the algorithm for multiplying fractions:

> T - And what about this magic trick, then? We use this magic trick to multiply and divide fractions. When we multiply a number without a denominator by a fraction, we can use this trick, OK? (Put a denominator of 1).

It is striking how little use was made of the "Using benchmarks" component, even though the lesson revolved around the PS that incorporates fractions, for which this component is recommended (Almeida & Bruno, 2016). Table 5 shows the results of cross-referencing the MK and NS analysis grids. The possible differences with the counts in Figures 2 and 3 are explained by the fact that not all MK knowledge can be classified under the NS components, and

that more than one piece of MK knowledge was used for classification through the NS components.

Table 5 shows an overview of the results of cross-referencing the MK with the NS, separating indications from evidence. The knowledge category that records the most indications of co-occurrence with the NS components is PPSM (KPM subdomain), especially in the NS criterion. In turn, the category of knowledge that records the most evidence of co-occurrences with the NS components is P (KoT subdomain), in which the PNS and RB criteria stand out. This is because most procedures (P) were geared toward using, or facilitating the use of, traditional procedures and algorithms. P&A presents only one registered piece of knowledge, unlike what is observed in Table 3, because this piece of knowledge is hardly associated with elements of the NS, PNS, or RB.

Table 5

Cross-reference findings of indications and evidence between MK categories and NS criteria

Categories of knowledge	PNS		NS		RB	
	Indications	Evidence	Indications	Evidence	Indications	Evidence
RML	0	0	1	0	1	0
PPSM	1	0	8	5	0	1
Pdef	0	0	0	0	0	0
PD	0	0	0	0	0	0
AC	0	0	0	0	0	0
CCC	0	0	0	0	0	0
SC	0	0	2	2	1	0
CC	0	0	0	0	0	0
P&A	0	0	0	1	0	0
RR	1	0	1	0	0	1
DPF	2	0	1	0	0	0
P	1	12	1	9	2	13

The small amount of knowledge recorded in the "Registers of representation" (RR) category is striking, as one would expect more in a class that addresses problem solving, especially with fractions. In principle, nevertheless, the RR category could be considered directly associated with the NS component "Using multiple representations of numbers and operations".

However, we could record the presence of the NS components in only two of the interventions categorised under this knowledge.

CONCLUSIONS

In this research, we examine a pre-service mathematics mathematics teacher's mathematical knowledge (MK) while he teaches a lesson as part of his professional practice. The Grade 8 class engaged in problem-solving (PS) activities that required using number sense (NS) in the context of rational numbers. The analysis takes a qualitative approach, considering the crossing between the MK categories and the NS components. The MK dimension of mathematics teachers' specialised knowledge (MTSK) enabled an initial classification of the pre-service teacher's interventions.

Initially, when the teacher proposes a lesson focused on problem-solving, all the problems presented are routine and realistic because the context in which they arise simulates reality or represents it (Díaz, 2022). For this reason, in a preliminary observation of the class, without a clear frame of reference, the presence of the use of NS could be considered poor. However, detailed analysis of each intervention revealed that the teacher demonstrated both explicit and implicit mastery of number sense as part of his specialist knowledge.

In this regard, the partial number sense (PNS) category was crucial, as it revealed elements of NS even when the teacher intended to help students apply or remember a traditional calculation algorithm to solve a problem. In the case observed, even using the traditional algorithm required NS to support students' understanding. Therefore, although the pre-service mathematics teacher's practice did not explicitly promote NS among his students, he used it consistently to facilitate rule-based (RB) procedures and thus support progress in PS.

For this reason, we believe that the results do not conclusively match those obtained by Almeida et al. (2014). This finding raises the possibility that the pre-service mathematics teacher's education may have influenced these results. It is therefore necessary to replicate this study with a larger sample of teachers to compare and verify the consistency of the results. This extension would also be useful for pre-service mathematics primary school teachers, as it would help verify the results of Yang and Hsu (2009), who also show that teachers at this educational level can help children develop number sense with fractions without relying on a rule-based method. Furthermore, we believe our

results are consistent with those of Mawaddah et al. (2021), as teachers' use of NS is at least moderate.

Regarding PS, we believe that although it is recommended that students tackle non-routine problems (Santos-Trigo, 2007), routine PS in the mathematics classroom can also be beneficial, especially when linked to NS. We therefore agree with Camacho and Santos (2006), who argue that, regardless of problem type, routine or non-routine, what matters is the willingness to explore solutions and support mathematical relationships.

From a theoretical-methodological point of view, although not all manifestations of mathematical knowledge can be classified under the categories of NS, the subdomains of MK proved to be a relevant analytical tool for addressing the classification of knowledge under the components and criteria of NS. In this way, we could identify conceptual relationships between the MK and NS categories. Among them, the relationship between the procedure (P) category and the NS criterion stands out. However, we cannot infer that NS is exclusively procedural, as the practice of problem solving and modelling (PPSM) category also co-occurred with NS. Likewise, the P category also showed relationships with the PNS and RB criteria, as both categories include using traditional procedures and algorithms.

Future studies could investigate the scope of using NS in PS, involving algebraic notions or modelling. Similarly, it would be interesting to study the role of PNS in facilitating calculations and procedures in a broader study, considering in-service teachers. One limitation of the study is the difficulty of perceiving and interpreting knowledge through video observation alone.

AUTHORSHIP CONTRIBUTION STATEMENT

This article is based on a thesis for a master's degree in didactic of mathematics. For this article, SAA wrote the first draft, and all authors, SAA, AVG, and PVH, contributed to the conceptualisation, methodology, data analysis, writing, review, and editing of this paper. Furthermore, all authors agreed with the results and conclusions obtained.

DATA AVAILABILITY STATEMENT

The data associated with this document will be available through the corresponding author. Furthermore, the authors declare that the study was approved by the Reaccredited Scientific Ethics Committee of the Regional Health Ministry of Maule (approval number 1976/28.09.22) and that they obtained written informed consent from the participants.

SPONSORSHIP

The corresponding author, PVH, received funding from the National Research and Development Agency of Chile through the National Fund for Scientific and Technological Development for Initiation 2023, Project Folio 11230240. AVG thanks the National Research and Development Agency of Chile through the National Fund for Scientific and Technological Development for the 2024 Start-up Grant, Project Folio 11240150.

REFERENCES

Almeida, R., Bruno, A., & Perdomo-Díaz, J. (2014). Estrategias de sentido numérico en estudiantes del Grado en Matemáticas. *Enseñanza de las Ciencias, 32*(2), 9-34. https://doi.org/10.5565/rev/ensciencias.997

Almeida, Rut, Bruno, A, & Perdomo-Díaz, J. (2015). Strategies of Number Sense in Pre-service Secondary Mathematics Teachers. *International Journal of Science and Mathematics Education, 14*(5), 959-978. https://doi.org/10.1007/s10763-014-9601-6

Almeida, R., Bruno, A., & Perdomo-Diaz, J. (2014). Respuestas de estudiantes de secundaria a tareas de sentido numérico, en M. T. González, M. Codes, D. Arnau & T. Ortega (Eds.), *Investigación en Educación Matemática XVIII,* (pp. 127-136). SEIEM.

Almeida, R., & Bruno, A. (2016). Uso de puntos de referencia y de representaciones gráficas para resolver tareas numéricas en secundaria. *PNA, 10*(3), 191-217. https://doi.org/10.30827/pna.v10i3.6088

Baroody, A. (1994). *El Pensamiento Matemático de los Niños*. Aprendizaje Visor.

Ben-Yehuda, M., & Sharoni, V. (2021). Number Sense Makes All the Difference: Calculation Using Number Sense by Pupils With and Without Learning Difficulties in Math. *Journal of Cognitive Education and Psychology, 20*(1), 47–67. https://doi.org/10.1891/jcep-d-20-00029

Bernete, F. (2014). Análisis de contenido. En A. Lucas & A. Noboa (eds.), *Conocer lo social: estrategias y técnicas de construcción y análisis de datos* (pp. 221-261). Fragua.

Boaler, J., Williams, C., & Confer, A. (2015). Fluency without fear: Research evidence on the best ways to learn math facts". *Reflections, 40*(2), 7-12.

Brouwer, N., & Korthagen, F. (2005). Can Teacher Education Make a Difference? *American Educational Research Journal, 42*(1), 153–224. https://doi.org/10.3102/00028312042001153

Bruno, A. & Almeida, R. (2017). Establishing profiles on the use of number sense". *Journal of Research in Mathematics Education, 6*(1), 56. https://doi.org/10.17583/redimat.2017.1910

Caires, S., & Almeida, LS., (2005). Teaching practice in Initial Teacher Education: its impact on student teachers' professional skills and development. *Journal of Education for Teaching, 31*(2), 111–120. https://doi.org/10.1080/02607470500127236

Camacho, M., & Santos, L. (2006). La relevancia de los problemas en el aprendizaje de las matemáticas a través de la resolución de problemas, *Números. Revista de didáctica de las matemáticas, 58*, 45-60.

Carrillo, J., Climent, N., Contreras, L. & Muñoz-Catalán, M. (2013). Determining specialised knowledge for mathematics teaching. En B. Ubuz, Ç. Haser & M. A. Mariotti (Eds.), *Proceedings of the 8th Congress of the European Society for Research in Mathematics Education* (pp. 2985-2994). CERME.

Carrillo, J., Escudero-Ávila, D., & Flores, E. (2014). El uso del MTSK en la formación inicial de profesores de matemáticas de primaria. *Revista de Análisis Matemático-Didáctico para profesores, 1*, 16-26.

Carrillo, J., Climent, N., Montes, M., Contreras, L., Flores-Medrano, E., Escudero-Ávila, D., Vasco, D., Rojas, N., Flores, P., Aguilar-González, Á., Ribeiro, M. & Muñoz-Catalán, M. (2018). The mathematics teacher's specialised knowledge (MTSK) model.

Research in Mathematics Education, 20(3), 236–253. https://doi.org/10.1080/14794802.2018.147998

Castro, E. & Segovia, I. (2015). Sentido numérico. En P. Flores, & L. Rico (eds), *Enseñanza y aprendizaje de las matemáticas en Educación Primaria,* Salamanca (pp. 109-126). Edición Pirámide.

Chapman, O. (2015). Mathematics teacher´s knowledge for teaching problem l solving. *LUMAT: International Journal of Math, Science and Technology Education, 3*(1), pp. 19–36. https://doi.org/10.31129/lumat.v3i1.1049

Clements, D., & Sarama, J. (2020). *Learning and Teaching Early Math: The Learning Trajectories Approach* (3rd ed.). Routledge. https://doi.org/10.4324/9781003083528

Delgado-Rebolledo, R. & Espinoza-Vásquez, G., (2021). ¿Cómo se relacionan los subdominios del conocimiento especializado del profesor de matemáticas? En J. G. Moriel-Junior (Ed.), *Anais do V Congreso Iberoamericano sobre Conocimiento Especializado del Profesor de Matemátic*as (pp. 288-295). V CIMTSK.

Depaepe, F., De Corte, E., & Verschaffel, L. (2010). Teachers' metacognitive and heuristic approaches to word problem solving: analysis and impact on students' beliefs and performance. *ZDM-Mathematics Education, 42*(2), 205-218. https://doi.org/10.1007/s11858-009-0221-5

Díaz, V. (2022). Design and Validation of a Test for the Types of Mathematical Problems Associated with Reading Comprehension. *Education Sciences, 12*(11), 795. https://doi.org/10.3390/educsci12110795

Escudero-Ávila, D., Carrillo, J., Flores-Medrano, E., Climent, N., Contreras, L., & Montes, M. (2015). El conocimiento especializado del profesor de matemáticas detectado en la resolución del problema de las cuerdas. *PNA, 10*(1), 53-77. https://doi.org/10.30827/pna.v10i1.6095

Flores-Medrano, E., Escudero-Avila, D., Montes, M., Aguilar, Á. & Carrillo, J. (2014). Nuestra modelación del conocimiento especializado del profesor de matemáticas, el MTSK. En A. Aguilar, E. Carmona, J. Carrillo, L. Contreras, N. Climent, D. Escudero, E. Flores, J. Huitrado, M. Montes, M. Muñoz, N. Rojas, L. Sosa, D. Vasco & D. Zakaryan (Eds.) *Un Marco Teórico para el Conocimiento Especializado del Profesor de Matemáticas* (pp. 57-72). Universidad de Huelva Publicaciones.

Flores-Medrano, E., Sosa, L. & Ribeiro, M. (2016). Tránsito del MKT al MTSK. En J. Carrillo, L.C. Contreras & M. Montes. En J. Carrillo, L. C. Contreras & M. A. Montes (coord.), *Reflexionando sobre el conocimiento del profesor. Actas de las II Jornadas del Seminario de Investigación de Didáctica de la Matemática de la Universidad de Huelva*, (7-11). SGSE.

Flores-Medrano, E. (2022). Conocimiento de la estructura de las matemáticas, en J. Carrillo, M. Montes & N. Climent (eds.), *Investigación sobre conocimiento especializado del profesor de matemáticas (MTSK),* (pp.47-56) Dykinson.

Galleguillos, J., Ribeiro, M., & Montes, M. (2022). El conocimiento matemático que despliega & desarrolla un grupo de profesores en la resolución del problema del corral. *Educación Matemática, 34*(3), 194–217. https://doi.org/10.24844/em3403.07

Hallman-Thrasher, A. (2015). Prospective elementary teachers' responses to unanticipated incorrect solutions to problem-solving tasks. *Journal of Mathematics Teacher Education, 20*(6), 519–555. https://doi.org/10.1007/s10857-015-9330-y

Henríquez-Rivas, C., Guerrero-Ortiz, C., & Ávila-Barrera, A. (2021). Trabajo matemático de profesores universitarios: Heurísticas de solución de una tarea. *Educación Matemática, 33*(3), 233–262. https://doi.org/10.24844/em3303.09

Jordan, N., Glutting, J., & Ramineni, C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning*

and Individual Differences 20(2), 82–88. https://doi.org/10.1016/j.lindif.2009.07.004

Krajewski, K. & Schneider, W. (2009). Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties: Findings from a four-year longitudinal study, *Learning and Instruction, 19*(6), 513–526. https://doi.org/10.1016/j.learninstruc.2008.10.002

Lera, C., & Deulofeu, J. (2014). Conocimientos y creencias entorno a la resolución de problemas de profesores y estudiantes de profesor de matemáticas. *Bolema, 28*(48), 191–208. https://doi.org/10.1590/1980-4415v28n48a10

Lester, F. (2013). Thoughts About Research On Mathematical Problem-Solving Instruction. *The Mathematics Enthusiast, 10*(1-2), 245–278. https://doi.org/10.54870/1551-3440.1267

Levine, S., Jordan, N., & Huttenlocher, J. (1992). Development of calculation abilities in young children. *Journal of Experimental Child Psychology, 53*(1), 72–103. https://doi.org/10.1016/s0022-0965(05)80005-0

Liljedahl, P., Santos-Trigo, M., Malaspina, U. & Bruder, R. (2016). Problem solving Problem Solving in Mathematics Education (pp. 1-39). Springer. https://doi.org/10.1007/978-3-319-40730-2

Mallart S., Font M., & Malaspina, U. (2016). Reflexión sobre el significado de qué es un buen problema en la formación inicial de maestros. *Perfiles educativos, 38*(152), 14-30. https://doi.org/10.22201/iisue.24486167e.2016.152.5758

Liñan, M., Contreras, L. & Barrera, V. (2016). Conocimiento de los Temas (KoT). En J. Carrillo, L. C. Contreras & M. Montes (Eds.). *Reflexionando sobre el conocimiento del profesor, Actas de las II Jornadas del Seminario de Investigación de Didáctica de la Matemática de la Universidad de Huelva* (pp. 12 -20) SGSE.

Ball, D., Thames, M., & Phelps, G. (2008). Content Knowledge for Teaching. *Journal of Teacher Education, 59*(5), 389–407. https://doi.org/10.1177/0022487108324554

May Cen, Iván. de J. (2015). George Polya (1965). *Cómo plantear y resolver problemas* [título original: *How To Solve It*?] México: Trillas. 215 pp. *Entreciencias: Diálogos en la Sociedad del Conocimiento*, *3*(8), 419-420. https://doi.org/10.21933/J.EDSC.2015.08.005

Mawaddah, S., Noorbaiti, R., & Ulfah, A. (2021). Analysis of number sense capabilities of prospective mathematics teachers. *Journal of Physics: Conference Series, 1760*(1), 012042. https://doi.org/10.1088/1742-6596/1760/1/012042

McIntosh, A., Reys, B., & Reys, R. (1992). A proposed framework for examining basic number sense. *For the learning of mathematics, 12*(3), 2-8.

Mesa, V., & Leckrone, L. (2020) Assessment of mathematics teacher knowledge. En Lerman.S (Ed.). *Encyclopedia of Mathematics Education* (2nd ed., pp. 66-68). Springer. https://doi.org/10.1007/978-3-030-15789-0_13

Montes, M., Aguilar-González, Á., Carrillo, J. & Muñoz-Catalán, M. (2013). MTSK: From common and horizon knowledge to knowledge of topics and structures. En B. Ubuz, C. Haser & M.A. Mariotti (Eds.), *Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education (CERME 8)* (pp. 2055–3064). ERME.

Mousoulides, N. & Sriraman, B. (2020). Heuristics in Mathematics Education. En S. Lerman (Ed), *Encyclopedia of Mathematics Education* (2nd ed., ‘p. 649), Springer. https://doi.org/10.1007/978-3-030-15789-0

National Research Council (2000). *Principles and Standards for School Mathematics: Final Report.* The National Academies Press. https://doi.org/10.17226/9870

Noviyanti, M., Suryadi, D., Fatimah, S. & Adriany, V. (2019). Mathematics and children's construction in early childhood education. En W. Harun, S. Jaafar & N. Abdullah (Eds.), *Early Childhood Education in the 21st Century* (pp. 73-78) Routledge. https://doi.org/10.1201/9780429434914-13

Piñeiro, J., Castro-Rodíguez, E. & Castro-Rodríguez, E. (2016). Conocimiento profesional para la enseñanza de la resolución de problemas en primaria: una perspectiva curricular. En J. A. Macías, A. Jiménez, J. L. González, M. T. Sánchez, P. Hernández, C. Fernández, F. J. Ruiz, T. Fernández & A. Berciano (Eds.), *Investigación en Educación Matemática XX* (pp. 346-427). SEIEM.

Piñeiro, J., Castro-Rodríguez, E., & Castro, E. (2021). Conocimiento sobre la resolución de problemas de matemáticas manifestado por estudiantes para profesor. *Bolema, 35*(71), 1416–1437. https://doi.org/10.1590/1980-4415v35n71a09

Pino-Fan, L., Castro, W., & Moll, V. (2022). A Macro Tool to Characterize and Develop Key Competencies for the Mathematics Teacher' Practice. *International Journal of Science and Mathematics Education, 2*(5), 1407–1432. https://doi.org/10.1007/s10763-022-10301-6

Ricoy, M. (2006). Contribución sobre los paradigmas de investigación. *Educação. Revista do Centro de Educação, 31*(1), 11-22.

Rodríguez, M., & Parraguez, M. (2014). Interpretando estrategias en Resolución de Problemas desde dos constructos teóricos: Un estudio de caso. *Revista Electrónica de Investigación En Educación en Ciencias, 9*(2),1–12. https://doi.org/10.54343/reiec.v9i2.175

Rowland, T. (2005). The knowledge quartet: A tool for developing mathematics teaching. En A. Gagatsis (Ed.), *Proceedings of the 4th Mediterranean Conference on Mathematics Education* (Vol. 2, pp. 69-81). Cyprus Mathematical Society.

Santos, M. (2020). Problem-Solving in Mathematics Education. En Lerman. S. (Ed.), *Encyclopedia of mathematics education* (pp. 567-587). Springer. https://doi.org/10.1007/978-3-030-15789-0_129

Santos, R., Collantes, L., Ibañez, E., Ibarra, F., & Pentang, J. (2022). Innate Mathematical Characteristics and Number Sense Competencies of Junior High School Students. *International Journal of Learning, Teaching and Educational Research 21*(10), 325–340. https://doi.org/10.26803/ijlter.21.10.18

Santos-Trigo, M. (2007). Mathematical problem solving: an evolving research and practice domain. *ZDM Int J Math Educ, 39*(5-6), 523–536. https://doi.org/10.1007/s11858-007-0044-1

Schoenfeld, A. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. En D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–370). Macmillan.

Şengül, S. & Zora, L. (2023). Sınıf Öğrencilerinin Kesirler Konusuyla İlgili Şekil, İşlem ve Senaryo Formundaki Sorularda Kullandıkları Sayı Hissi Stratejilerinin Belirlenmesi. *Eğitim ve Bilim, 48*(214) https://doi.org/10.15390/eb.2023.11339

Sepúlveda, A., Medina, C., & Sepúlveda, D. (2009). La resolución de problemas y el uso de tareas en la enseñanza de las matemáticas. *Educación Matemática*, *21*(2), 79-115.

Sharma, M. (2015). Number sense: a window into dyscalculia and other mathematics difficulties. En S. Chinn (Ed.), *The Routledge international handbook of dyscalculia and mathematical learning difficulties* (pp. 277-291). Routledge.

Sowder, J. (1992). Estimation and number sense. En D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 371–389). MacMillan.

Stake, R. (1999). *Investigación con estudio de casos* (2° ed.). Ediciones Morata.

Ukobizaba, F., Nizeyimana, G., & Mukuka, A. (2021). Assessment Strategies for Enhancing Student´s Mathematical Problem-solving Skills: A Review of Literatur. *Eurasia Journal of Mathematics, Science and Technology Education*, *17*(3), 1945. https://doi.org/10.29333/ejmste/9728

Vasco, D., Gómez, J. & Contreras, L. (2017). Subominios del Mathematic´s Theacher`s Specialised knowledge (MTSK). En J. Carrillo & L.C. Contreras (Eds), *Avances, utilidades y retos del modelo MTSK. Actas de las III jornadas del Seminario de Investigación de Didáctica de la Matemática de la Universidad de Huelva* (pp. 29-37). SGSE.

Vasilachis, I. (2019*). Estrategias de investigación cualitativa*. Buenos Aires: Gedisa.

Verdugo-Hernández, Paula y Espinoza-Vásquez, Gonzalo (2023). "El desarrollo de habilidades en el trabajo matemático: el caso de Lucas, un futuro profesor de matemática, en su práctica profesional", *Uniciencia,* vol. 37, núm. 1, pp. 1-23. http://dx.doi.org/10.15359/ru.37-1.10

Veloo, P. (2010). *The development of number sense and mental computation proficiencies: An intervention study with secondary one students in Brunei Darussalam* (Doctoral dissertation). University of Otago.

Cheung, K. L., & Yang, D.-C. (2018). Examining the differences of Hong Kong and Taiwan students' performance on the number sense three-tier test. *EURASIA Journal of Mathematics, Science and Technology Education*, *14*(7). https://doi.org/10.29333/ejmste/91682

Yang, D-C., & Hsu, C.-J. (2009). Teaching Number Sense for 6th Graders in Taiwan. *International Electronic Journal of Mathematics Education 4*(2), 92–109. https://doi.org/10.29333/iejme/232

Yin, Robert E. (2018). *Case study research and applications. Design and methods* (6th ed.). Thousand Oaks, CA: SAGE Publications.

Zakaryan, D., Estrella, S., Espinoza-Vásquez, G., Morales, S., Olfos, R., Flores-Medrano, E. & Carrillo, J. (2018). Relaciones entre el

conocimiento de la enseñanza y el conocimiento de las características del aprendizaje de las matemáticas: caso de una profesora de secundaria. *Enseñanza de las Ciencias, 36*(2), 105–123. https://doi.org/10.5565/rev/ensciencias.2260