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Development of didactic analysis and intervention competency among teachers of Brazilian Basic Education

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ABSTRACT

Background: Competency-based education has become an international trend, highlighting the need to train teachers in the field of professional competencies. **Objective:** To analyse the level of the didactic analysis and intervention competency developed by third-grade elementary school teachers. **Design:** Qualitative descriptive-interpretative research. **Setting and participants:** Two third-grade elementary school teachers from the municipality of Canoas, RS, Brazil, participants in a continuing education cycle focused on the four operations with natural numbers, based on the competencies and didactic-mathematical knowledge model of the onto-semiotic approach. **Data collection and analysis:** Discursive textual analysis using a priori categories grounded in the didactic suitability criteria and in the levels of development of the didactic analysis and intervention competency. The analysis was carried out through classroom observations and records of the participants before and during the training cycle. **Results:** The findings indicate a significant improvement in the participants' didactic analysis and intervention competency, as evidenced by advances in their teaching practices, with clearer explanations adapted to students' levels and supported by strategies such as mental calculations and symbolic representations. The training also encouraged the use of concrete materials and visual resources, fostering interaction between peers, collaboration, and student autonomy. In addition, the teachers reached higher levels of critical reflection on lesson planning, enhancing their practices through the didactic suitability criteria. **Conclusions:** Continuing professional development of teachers proved essential for strengthening their competencies, underscoring the need to expand and sustain such formative initiatives to ensure pedagogical practices align with the demands of contemporary education.

Keywords: Continuing professional development of teachers; Mathematics teaching; Didactic analysis and intervention competency; Didactic suitability criteria.

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Desenvolvimento da competência de análise e intervenção didática desenvolvida por professoras da Educação Básica brasileira

RESUMO

Contexto: O ensino baseado em competências tornou-se uma tendência internacional, o que evidencia a necessidade de formar professores no âmbito das competências profissionais. **Objetivo**: Analisar o nível da competência de análise e intervenção didática desenvolvido por professoras do terceiro ano do Ensino Fundamental. **Design**: Pesquisa qualitativa descritivo-interpretativa. **Ambiente e participantes**: Duas professoras do terceiro ano do Ensino Fundamental do município de Canoas, RS, Brasil, participantes de um ciclo de formação continuada relacionado às quatro operações com números naturais, pautado no modelo de competências e conhecimentos didático-matemáticos da Abordagem Ontossemiótica. **Coleta e análise de dados**: Análise textual discursiva a partir de categorias *a priori* baseadas nos critérios de adequação didática e nos níveis de desenvolvimento da competência de análise e intervenção didática, realizada a partir das observações e registros das aulas das participantes antes e durante o ciclo formativo. **Resultados**: Os resultados apontam que houve uma melhora significativa na competência de análise e intervenção didática das participantes, evidenciada pelos avanços nas práticas docentes, com explicações mais claras, adequadas ao nível dos alunos e apoiadas no uso de estratégias como cálculos mentais e representações simbólicas. A formação também fomentou o uso de materiais concretos e recursos visuais, favorecendo as interações entre os pares, colaboração e autonomia dos estudantes. Além disso, as professoras alcançaram níveis elevados de reflexão crítica sobre o planejamento, aprimorando suas práticas a partir dos critérios de adequação didática. **Conclusões**: A formação continuada se mostrou fundamental para o fortalecimento das competências profissionais docentes, indicando a necessidade de ampliar e manter iniciativas formativas dessa natureza, de modo a garantir práticas pedagógicas cada vez mais alinhadas às demandas do ensino contemporâneo.

Palavras-chave: Formação continuada de professores; Ensino de matemática; Competência de análise e intervenção didática; Critérios de adequação didática.

INTRODUCTION

Competency-based teaching has become an international trend and a widely discussed topic in curriculum reforms in several countries (Breda et al., 2016). An example of a competency-based curriculum in basic education (K-1 through K-12) is the National Common Curriculum Base (BNCC) (Brasil, 2018), which ensures that essential learning is aligned with local realities and describes the regulations to be approached within the scope of this school level in Brazil. In particular, it highlights eight specific competencies in mathematics for elementary education, in addition to the general competencies of basic education.

This scenario justifies the production of materials and the need for continuing education for mathematics teachers to develop students' mathematical skills. For this to occur, teachers must have formative preparation that helps them address the challenges of the competency model (Pereira et al., 2019).

Currently, Brazilian legislation for teaching degree courses, including the teaching degree in pedagogy, requires pre-service teachers to reflect on their teaching practice and articulate the connection between theory and practice during their initial training through their pedagogical actions (Silva et al., 2022). This requirement aligns with the specific competencies outlined in the Common National Base for Initial Training of Basic Education Teachers (BNC-Formação) (Brasil, 2019). However, the few subjects on the didactics of mathematics offered in pedagogy courses are insufficient to guarantee mastery of knowledge and professional practice or engagement with mathematics teaching.

From this perspective, there is a need to invest in the continuing education of mathematics teachers, create and make available guidance materials, and maintain permanent teacher education processes that enable the continuous improvement of mathematics teaching and learning processes (Brasil, 2018). In particular, we emphasise the importance of training teachers within the scope of professional skills, who are reflective on their own practice and capable of understanding and justifying their pedagogical decisions, mobilising, for this purpose, the knowledge of their specialty (Mello, 2000).

The onto-semiotic pproach to mathematical knowledge and instruction (OSA) argues that mathematics teaching requires teachers to hold didactic-mathematical knowledge to analyse and interpret the processes of mathematical instruction (planning, implementation, and replanning), as well as the essential professional competencies to act adequately in these processes. One of the tools proposed by the OSA to address this challenge is mathematical competency, as well as didactic analysis and intervention competency (Godino et al., 2017).

A literature review study on Brazilian dissertations and theses produced between 2016 and 2020, focusing on teacher training within the scope of the OSA, pointed out that the elements present in the continuing education of teachers who teach mathematics include the mobilisation of didactic-mathematical knowledge and changes and reflections on teaching practice (Breda et al., 2021; Silva et al., 2023a). The analysis of these texts enabled us to understand that the OSA, in contexts of continuing education, promotes both the teacher's critical and reflective action, as well as the development of didactic and mathematical skills and knowledge.

This article emphasises the development of the didactic analysis and intervention competency, which consists of "designing, applying, and evaluating learning sequences through didactic analysis techniques and didactic adequacy criteria, to establish planning, implementation, evaluation, and proposals for improvement cycles" (Breda et al., 2017, p. 1897). In particular, we seek to analyse the development of the sub-competencies in the participating teachers (Breda et al., 2017; Pino-Fan et al., 2023).

In this sense, this study aims to assess the didactic analysis and intervention competency of two third-grade elementary school teachers in the city of Canoas, RS, participants in a training course based on the model of didactic-mathematical competency and knowledge of the onto-semiotic approach.

THEORETICAL APPROACH

The onto-semiotic approach to mathematical knowledge and instruction is a comprehensive theoretical system that integrates other theoretical research models in the field of mathematics education. At its core, it seeks to analyse the processes of mathematics teaching and learning, as well as identify ways to improve them (Godino, 2012). Its theoretical bases, recently presented by Godino et al. (2019), aim to respond to several problems that have arisen in the field of mathematics education.

One of the issues addressed concerns teacher training, specifically: What are the key competencies that teachers should develop to enhance mathematics teaching and learning processes? One of the tools proposed by the OSA to address this challenge is the assessment of dimensions and components of mathematical competency and didactic analysis and intervention competency (Godino et al., 2017).

The choice to consider didactic analysis and intervention competency as central is consistent with what several researchers have said about the importance of reflection on practice in teacher education (Alsina et al., 2009). This is because the teacher who teaches mathematics develops a complex practice that requires the combination of different types of knowledge, competencies, and skills; reflection contributes to understanding the complexity of educational processes.

This competency is composed of four sub-competencies: sub-competency in the analysis of mathematical activity; sub-competency in the analysis and management of interactions; sub-competency in the use and management of resources; and sub-competency in the analysis and valuation of didactic adequacy. Each presents increasing levels of development on a discrete scale, ranging from zero to three.

The subcompetency of mathematical activity analysis refers to the teacher's ability to examine the mathematical activities they develop, encompassing the techniques, tools, and ways of applying the meanings of mathematical objects. In other words, it is the analytical capacity that the teacher develops from the epistemic dimension of mathematics and its teaching. For example, to measure the development of this sub-competency, the following chart serves as a reference (Chart 1).

Chart 1

*Levels of development of the subcompetency of the mathematical activity analysis. (*Pino-Fan et al., 2023, p. 1419).

L_n	Level Description
L_0	At this level, no characteristics are observed that can be associated with the analysis of mathematical activity. In turn, in the teacher's mathematical activity, we observe certain practices (operational or discursive) that are considered incorrect from a mathematical perspective. There are also ambiguities (in the formulation of definitions, procedures, or properties) that can confuse students. In particular, metaphors are misused, which can lead to learning conflicts.
L_1	No characteristics have yet been observed that can be associated with the didactic analysis of mathematical activity; however, regarding didactic intervention, the teacher no longer experiences what is indicated in L_0. No mathematical practice is considered incorrect, nor are there ambiguities (in definitions, explanations, procedures, properties) that could confuse students. Metaphors are used in a more controlled way, although explanations, verifications, or demonstrations are not adapted to the educational level being taught.

L_2	In addition to L_1, the teacher promotes the use of various forms of representation of the mathematical object under study and encourages discussion with students. The definitions, procedures, properties, and explanations are clear and correct, as are the verifications or demonstrations appropriate to the educational level for which they are intended. The teacher assesses the students' prior knowledge to advance on the subject (either by checking whether they have already learned it or by teaching it himself). In the mathematical activity carried out, however, the representativeness of the meanings of the mathematical object to be taught is not yet observed (Pino-Fan, Godino, & Font, 2011, 2018). The characteristics of the didactic analysis of mathematical activity are observed at certain moments, in which the teacher reflects on his/her activity (or the activity of his/her colleague). In particular, he/she recognises errors and mathematical ambiguities in the explanations, definitions, propositions, or metaphors used (this analysis is based on knowledge of some theoretical-methodological tool not yet mastered, for example, the epistemic configuration, or with the experience acquired over years of teaching service).
L_3	The teacher knows and systematises the use of some theoretical-methodological tools (for example, epistemic configuration) to carry out his/her intervention and perform the didactic analysis of the mathematical activity carried out. In addition to what is described in L_2, he/she foresees and uses different procedures and arguments when faced with the same problematic situation. He/she suggests tasks and explanations that promote different mathematical meanings of the object under study and uses intra and extra-mathematical contexts that promote a richness of meanings. When applying didactic analysis, the teacher identifies both the key elements present in the mathematical activity – representations, concepts/definitions, properties/propositions, procedures, and arguments – regarding the meanings used by the students and identifies conflicts of meaning among them. It helps, however, to propose alternatives to overcome these conflicts. Furthermore, it promotes and identifies mathematical and cognitive processes relevant to mathematical activity (e.g., generalisation, modelling, argumentation, problem-solving, intra- and extra-mathematical

connections, representational changes, and conjectures). This ensures curriculum adaptation to student differences.

The sub-competency of analysis and management of interactions is one in which the teacher must be familiar with various didactic (dialogical) configurations in order to utilise them, understanding their consequences on student learning to design and manage them effectively in the teaching and learning processes. For example, in the context of teaching natural numbers, it is necessary to establish “rules” and “norms” to carry out fundamental operations. To understand what a number means, for example, teaching and learning processes must create a dynamic in which knowledge emerges from interaction. This complex system requires that the mathematics teacher be competent in planning and managing didactic settings. In other words, it is the capacity the teacher develops to manage interactions between teacher and student, interactions between students, and student autonomy, so that conflicts of meaning are addressed and learning is effective (Pino-Fan et al., 2023).

The subcompetency of analysis, use, and management of resources is related to the teacher’s ability to evaluate material and temporal resources. For example, whether the time allocated to studying mathematical content is adequate, which resources should be used to promote the learning of certain content, what are the advantages and disadvantages of the chosen resources, how to create suitable collaborative work environments through the use of material or technological resources and what obstacles to teaching and learning may arise with the use of these resources.

Finally, the sub-competency of analysis and valuation of didactic suitability, which is considered the most complex to develop, involves the teacher’s reflective competency, which unfolds in three distinct moments: *a priori*, *in situ*, and *a posteriori*. To develop this sub-competency, it is essential to use the didactic suitability criteria (DSC) tool, which allows organising and systematising the teacher’s reflection on their practice when evaluating and improving the instructional process. Thus, the teacher can use the criteria and respective components during reflection, becoming capable of identifying important elements in the development of a mathematics class, valuing them and adapting them didactically (Breda, 2020; Breda et al., 2018; Breda et al., 2017; Font et al., 2010; Godino et al., 2019).

Epistemic DSC enables the reflection and appropriation of mathematical content to assess whether the mathematics taught is “good mathematics" (Font et al., 2010). Epistemic suitability considers whether the teacher seeks to improve knowledge of mathematical content by incorporating the representativeness of mathematical objects and the processes that make them emerge (representation, definition, procedure, argument, proposition, problem-solving, etc.) that make up the dimension of mathematics established by the scientific community. This approach reinforces the principles and standards for school mathematics presented by the National Council of Teachers of Mathematics (NCTM, 2000), which serve as a global reference.

Cognitive DSC enables us to assess, prior to the beginning of the instruction process, whether mathematics teaching is aligned with the content, skills, and competencies that students already possess, and whether the acquired learning aligns with what was intended to be taught at the end of the process. This DSC also allows the teacher to adapt their planning, when necessary, to the peculiarities of each student, including, for example, students with special needs or high skills.

The interactive DSC evaluates whether the interactions that occur during the teaching and learning processes effectively address students' doubts, difficulties, and suggestions. The mediational DSC enables us to reflect on whether the use of material resources (teaching resources) and physical resources (classroom environment) is appropriate for the teaching and learning processes, as well as to assess whether the time allocated for the pedagogical process was sufficient to implement the instructional process.

The affective DSC makes it possible to evaluate the involvement between teacher and student, verifying whether the pedagogical process favours the expansion or inclusion of students' interests and motivations and whether there is exploration of socio-emotional skills throughout instruction. Finally, the ecological DSC assesses the suitability of the instructional process in relation to the school's curriculum, particularly regarding necessary adaptations to environmental issues, the institution's educational project, and national, state, or municipal curriculum guidelines, as well as the social and professional context. Each DSC has its respective components (Chart 2).

Chart 2

DSC and its components. Adapted from Breda and Rosário Lima (2016, pp. 80-83).

Criteria	Components
Epistemic	Errors; ambiguities; richness of processes (problem solving, mathematical modelling, intra and extra-mathematical connections, argumentation, communication, representation, conjecture-making, etc.); representativeness of the complexity of the mathematical object (variety of problem situations, procedures, languages, etc.).
Cognitive	Prior knowledge; curriculum adaptation to individual differences; learning; high cognitive demand.
Interactional	Teacher-student interaction; interaction between students; autonomy; formative assessment.
Mediational	Material resources; number of students, class schedule, and conditions; time.
Affective	Interests and needs; attitudes; emotions
Ecological	Adaptation to the curriculum; intra and interdisciplinary connections; socio-labour utility; didactic innovation.

The operationalisation of the didactic suitability criteria requires the definition of a set of observable indicators that allow the assessment of the degree of suitability of each of these criteria. For example, there is a consensus that it is necessary to do “good” mathematics, but this concept can be interpreted in very different ways. Godino (2013), updated by Breda et al. (2017), established a system of indicators that serves as a guide for analysing and evaluating didactic suitability, applicable to instructional processes at any educational level.

METHODOLOGY

Research context and subjects

The approach of this research is qualitative and follows a descriptive-interpretative design. As a data collection instrument, we observed the classes of two teachers who teach third-grade elementary school students in Canoas, RS, as participants in an extension course in continuing education entitled *"Who Tells a Tale Adds a Tail?" Recontextualising science through stories* belonging to an umbrella project called *Recontextualizar as Ciências e a Contação de Histórias para os Processos de Ensino e de Aprendizagem da Educação Básica à Formação de Professores* [Recontextualising Science and Storytelling for Teaching and Learning Processes from Basic Education to Teacher Training] (Felicetti & Backes, 2023; Silva et al., 2023b, Backes et al., 2024), approved by the National Research Ethics Commission (CONEP), under CAAE registration: 98789018.5.0000.5307.

The course was offered to teachers from the municipal schools network of Canoas, who worked in the 3rd grade of elementary school, and took place in three stages. The second stage, considered in this research, corresponded to the formative cycle entitled "*The four operations – Who wants to increase the tail?"* One of the goals of this second stage was to develop the teachers' key competencies in preparation for stage 3. Seventeen of the 29 registered teachers participated in the second stage. Pre-tests were applied to groups of students from the 17 teachers, and the following were selected as research subjects: a) the students presented the lowest number of correct answers in the mathematics test (pre-test); b) they agreed to participate in the research by signing the Free and Informed Consent Form. Thus, the selected teachers were teacher K and teacher L, from EMEF 8, in the municipality of Canoas, located in a region of economic and social vulnerability (Brasil, 2021).

Twenty of the participants' classes were observed and recorded, with ten classes observed before the training cycle and ten during the training cycle. For the observation, systematic observation based on the DMKC model was used. The recordings were made using the camera and sound of a notebook in all observed classes, allowing researchers to review, when necessary, the speeches and actions worked on in classroom practices, ensuring greater reliability in data collection and minimising subjectivity, which is frequently questioned in qualitative research (Kenski, 2003).

The training cycle "*The four operations – Who wants to increase the tail?*" carried out in 2022 in a blended modality and organised in five threehour meetings, aimed to understand mathematical notions to plan classes that included pedagogical practices involving the four operations with natural numbers. This phase of the course was structured to meet the previous categories of the DMKC model (Godino et al., 2017), recognising the need to develop teaching competencies that covered not only mathematics teaching practices, but also mathematical content. Specifically, it aimed to promote pedagogical practices that integrated common sense knowledge and scientific knowledge through storytelling and the development of teaching materials, to enhance the teaching and learning processes of mathematics in the third grade of elementary school. Chart 3 presents a summary of the topics addressed with the participants during the training cycle.

Chart 3

Summary of the training cycle: The four operations – Who wants to increase the tail? Prepared by the authors.

Meeting	Description of activities carried out
First meeting: storytelling	The activity "Alice in the Land of Numbers" aimed to develop mathematical, didactic and analytical intervention competencies through the integration of children's literature into teaching practice. Eighteen teachers participated, who, after the researcher's presentation and schools' introductions, received printed or PDF materials and followed the playful and adapted reading of the initial chapter of Frabetti's book (2021) using scenery and finger puppets. Based on this narrative, ways of applying storytelling as a resource for exploring mathematical objects and processes were discussed, encouraging the creation of problem situations aligned with the realities of the classes. The thematic units "Quantities and Measurements" and "Probability and Statistics" were addressed through practical examples, and, at the end, the teachers developed teaching plans in groups for application in the classroom.
Second meeting: mathematical literacy	The meeting on mathematical literacy, focusing on numbers and operations, used the story "O Conto da Conta" [The Tale of the Account] (Centurión, 1994) to explore the representativeness of numbers and their operations, connecting literature and teaching practice.

	The activity was planned considering the DMKC’s didactic analysis and intervention competency, aiming to understand the meanings and mathematical objects linked to the concept of number. The teachers discussed definitions, properties, and procedures they had been using, reflecting on their representativeness and suitability. Materials were presented to encourage the use of different forms of communication (verbal, graphic, symbolic) and conversions between them. The meeting included the history of numbers, symbolic and verbal representations, reading, writing, comparing and ordering natural numbers, as well as fundamental facts and calculation procedures with addition and subtraction. Throughout the activity, the teachers revised their plans to address the symbolic construction and understanding of numbers, relating content, materials, and representations, and reflecting on epistemic suitability criteria and the complexity of the mathematical objects being addressed.
Third meeting: the decimal number system	The meeting on the decimal number system, inspired by the theme “A manobra dos números” [The manoeuvre of the numbers] (Lobato, 1935), began with the sharing of the teachers’ experiences in applying the previous class on number construction. The discussion highlighted positional meaning as a central element for understanding the system, also addressing didactic adaptations aligned with the competency of analysing and valuing the didactic suitability of the DMKC. To explore the topic, the teachers used abacuses—including the cup and stick abacus, combined with the golden interlocking material—to recognise units, tens, and hundreds and perform addition. Teaching strategies were suggested, such as the use of the monetary system and storytelling based on the chapter “Manobra dos Números” in *Aritmética da Emília* [Emilia’s Arithmetic]. The activity ended with the playful game “*amarradinho*” [tied up], in which dice-rolling should form tens, making learning more interactive and meaningful.

Fourth meeting: four basic operations	The meeting focused on the meanings of the mathematical objects involved in the four fundamental operations, aligning with the competency of analysing global meanings (Godino et al., 2017). The teachers began by sharing experiences from the previous class about the decimal system and the use of resources such as the "*amarradinho*" game, bills, and number lines. They then revisited the numbering system using the stick abacus, relating it to operations, particularly exploring the properties of addition and subtraction, as well as multiplication and division problem situations. The golden material and different types of abacuses were used to visualise values and operations, while the introduction of the concept of area allowed us to approach multiplication as counting units and their relationship with ordered pairs. The activities, which integrated visual and practical representations, encouraged reflection on didactic adequacy and planning, promoting contextualised strategies that connect multiple meanings and representations, making teaching more meaningful and applied.
Fifth meeting: what makes a good math class?	The fifth meeting was dedicated to reflecting on didactic suitability in mathematics teaching, highlighting how different contexts influence teaching and learning. When discussing the properties and meanings of addition and subtraction operations, the importance of considering local realities was emphasised to make the content more relevant. A video exemplified how valuing students' reasoning and procedures can transform teaching practices, creating an environment of safety and expression. Principles and criteria for didactic suitability were presented, summarised in a hexagon, and the teachers, organised into groups, redesigned the course's lesson plans applying these criteria. At the end, they shared their reflections and adaptations with the larger group, ending the meeting in a collaborative way.

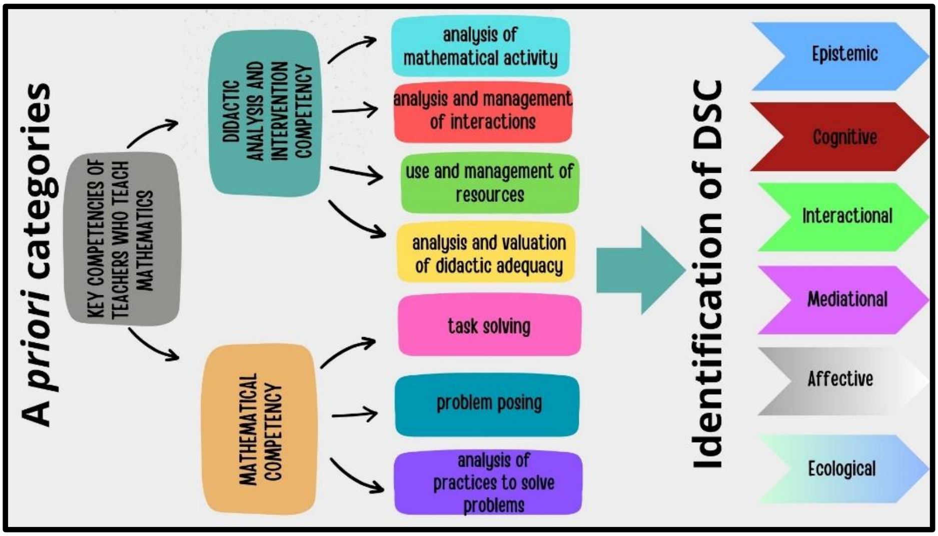
Data analysis

The data were analysed qualitatively, through discursive textual analysis (Moraes & Galiazzi, 2007), through categories *a priori* established grounded in the didactic analysis and intervention competency and the didactic suitability criteria (DSC): epistemic, cognitive, interactional, mediational, affective, and ecological (Breda et al., 2017; Godino et al., 2017). The purpose of these categories was to make the corpus of analysis more descriptive and interpretative, bringing meaning to the actions analysed in the process.

The analysis of the teachers' teaching practices based on the increasing levels of development of the sub-competencies of the didactic analysis and intervention competency (discrete scale ranging from zero to three, see Chart 1), before and during the training cycle, made it possible to highlight the levels of development of the competency achieved by them. Those levels were established according to the DSC identified in the excerpts of the corpus of analysis. In Figure 1, we present the form resulting from the *a priori* analysis, along with its categories.

Figure 1

A priori categories. Own preparation.



To establish the configuration level of each sub-competency, we present an example of how the tool for configuring the levels of the didactic analysis and intervention competency for the two participants was designed and applied before and during the training cycle, based on observations and recordings of their classes. This example (Chart 4) refers to the level of the sub-competency of analysis of the mathematical skills of participants L and K before the training cycle based on the DMKC model.

Chart 4

Example of a tool for assessing the levels of the mathematical activity analysis sub-competency. Own preparation.

	DSC		**Indicators**	**Characteristic**	**Evidence**	**Level**
ANALYSIS AND DIDACTIC INTERVENTION COMPETENCY	Epistemic	**Errors and ambiguities**	The definitions of the mathematical object to be taught (place value of numbers) are unclear or inaccessible to students. The definitions and procedures are neither clear nor correct, and are not adapted to the educational level for which they are intended. The different representations and their connections (concrete, pictorial, and symbolic) for the given educational	At this level, no characteristics are observed that can be associated with the analysis of mathematical activity. In turn, in the teacher's mathematical activity, we observe certain practices (operational or discursive) that are considered incorrect from a mathematical perspective. There are also ambiguities (in the formulation of definitions, procedures, or properties) that can confuse	*Teacher L begins to explain what addition sums are, calling them "continhas de mais" [adding accounts]. It does not cover the fundamental concepts of the decimal system, including units, tens, and hundreds. When adding, teacher L uses terms like "catching" the line, since she is drawing on the board.*	L_0

	level are not presented. Situations in which students need to generate or negotiate definitions, representations, propositions, or procedures are not proposed.	students. In particular, metaphors are misused, which can lead to learning conflicts.	*Teacher K uses the golden material, but calls each piece a cube, a bar, and a plate, without relating them to their numerical positional value.*

Each sub-competency was analysed before and during the instruction of the training cycle, based on the observation of the classes of teachers K and L. The definition of the levels underwent an *a priori* analysis by the researchers, later triangulated with experts from the DMKC model of the onto-semiotic approach.

RESULTS AND ANALYSIS

The configuration of the level of development of the didactic analysis and intervention competency, before and during the instruction of the training cycle, based on the didactic and mathematical competencies and kowledge model (DMKC), was structured in the levels of sub-competencies: analysis of mathematical activity, analysis and management of interactions, analysis and use and management of resources and analysis and valuation of didactic suitability (Pino-Fan et al., 2023).

Analysis of mathematical activity before and during the training cycle

The configuration of the mathematical activity analysis competency of the participants, before the training cycle based on the DMKC model, was presented at level L_0 (see Chart 4). The data showed that the teachers did not clearly work on the meanings of the mathematical objects (positional value of numbers and basic operations), with definitions adapted to the cognitive level of the students (3rd-grade elementary school students).

In the tasks prepared by the teachers using the golden material, they were expected to explain that every ten cubes form a bar, and every ten bars form a plate, establishing relationships between units, tens, and hundreds. In this way, there would be evidence of the teachers' understanding of the positional value of digits in the decimal system. This definition of number and its representation was addressed in the second meeting of the training cycle. In this meeting, the participating teachers related the meaning of numbers through the connection between their definitions and representations. As the aforementioned meeting had not yet taken place, the teachers demonstrated a lack of knowledge of definitions, properties, procedures, or techniques explored during the construction of the notion of number.

When guiding the students: "*The teacher will show a number here in front and you will assemble it with golden material; today we will only use the bar,*" Teacher K assumed that her students had already consolidated the construction of natural numbers. The pedagogical action "*How many numbers does the bar have*? [...]" (Teacher K) could avoid ambiguities in understanding this definition, minimising incoherent answers between the representation of the number made using the golden material and its corresponding positional value.

This difficulty was observed when one of Teacher K's students misinterpreted the ten bar of the golden material, relating it to 12 units, showing a disconnect between the representation made using the concrete material and the expected positional value. This aspect was also evident in teacher L's classes. We found that 50% of her students had difficulty with addition operations and had not yet constructed the positional meaning of the number, as they were unable to distinguish the unit from the ten, according to Centurión (1994).

When operating $27 + 39$, for example, the teacher began the explanation by adding the units: "*The teacher crosses out seven lines and nine lines below and then uses the words 'get together' and 'pick up'.*" Then she ran the algorithm with the sentence "*pretend the ten doesn't exist,*" without establishing the relationship between the pictographic representation and the ten (that is, that ten lines correspond to a ten).

With operations involving tens, the teacher interacted with the students, adding the values directly: "*two plus three,*" and the students answered "*five.*" When adding "*another one that's up there,*" However, he did not explain that this "one" represented a ten resulting from the sum of ten units. Furthermore, it did not clarify that "two" equals twenty and "three" equals thirty.

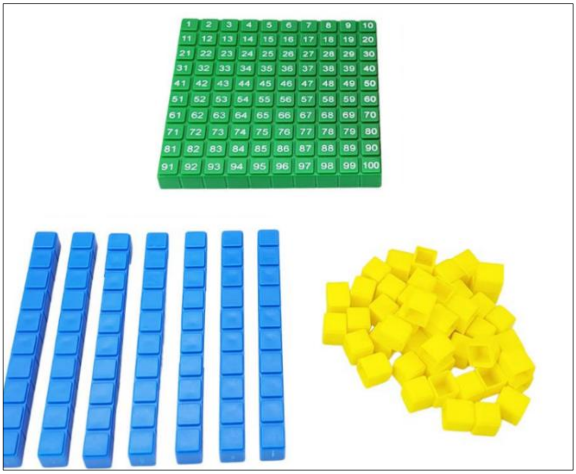
Furthermore, the participants did not demonstrate a level of mathematical activity capable of connecting the mathematical object with intra- and extra-mathematical aspects, without enabling the relationship between what is intended to be taught and the student's prior knowledge, a principle considered in cognitive DSC. One piece of evidence of this last aspect was that the teachers did not conduct planning that considered the competencies and skills previously consolidated by the students.

To develop this sub-competency, the teacher requires more in-depth training in the epistemic suitability criterion, particularly in the components of process richness and the representativeness of the complexity of the mathematical object. Once the training cycle was completed, the participants had made significant progress in developing their competency in analysing mathematical activities. The teachers started at the initial level (L_0) and, by the end of the course, they had reached a higher level (L_2), presenting clearer, more accurate definitions, procedures, and explanations tailored to the students' level. Working on the process of exchanging representation regularly.

Teacher L, for example, used the golden beads material to promote a dialogue with her students, helping them understand the relationship between units, tens, and hundreds —fundamental elements for grasping the decimal number system. When showing the numbered green plate, she asked the students how many units the blue bar had, to which they immediately responded that the bar had ten units. Then, she continued the dialogue: "*What about the green plate?*" Students answered "*100*". The teacher then asked: "*Why is the plate worth 100?*" The students promptly replied that the plate contained 100 cubes, relating tens to ten units (cubes) and hundreds to 100 units (cubes) to different values to the unit (Figure 2).

Figure 2

Golden beads material. Prepared by the authors



This interaction demonstrated Teacher L's understanding of the importance of working with concrete representations and building a progressive and logical understanding for students. The teacher used strategies that brought students' knowledge of the number system closer, showing how values can be grouped and manipulated. By asking why the plate is worth 100 and encouraging the students to answer, she confirmed their understanding of the relationships between numbers. Furthermore, Professor L made appropriate use of the golden material, using it to illustrate addition and subtraction operations in a concrete manner.

Finally, they began to consider students' prior knowledge and the meanings of the positional value of numbers and the first operations in a representative way, promoting mathematical literacy. The results at the end of the training cycle demonstrated the teachers' understanding of the representativeness of the mathematical objects taught in third-grade elementary school, contextualising them with didactic aspects that lead to the appropriate teaching and learning of mathematics.

Analysis and management of interactions before and during the training cycle

Before the training cycle, the participating teachers maintained their usual interaction management practices, without provoking significant reflections or contextualising mathematics in the students' daily lives, although they identified some suitable interactional dynamics, a zero level predominated (L_0) and, to a greater extent, level one (L_1) of the analysis and management of interactions sub-competency. The classes showed differences: in teacher L's class, the students sought out the teacher for corrections; in teacher K's class, it was the teacher who went to the students. For example, in one of Teacher K's speeches, "*The teacher will 'show' a number here in front, and you will assemble it*." When using the word "show," students expected a clear, understandable explanation with concrete examples, as they were in a literacy period.

Despite going around the room and proposing some strategies, such as providing cards with the names of the numbers, the actions lacked approximation, contextualisation, and appreciation of the connections between the mother tongue and mathematics, as Felicetti (2010, p. 36) rightly writes: "understanding in mathematics depends on the way in which language establishes communication". The lack of interdisciplinary proposals, specific planning to resolve difficulties, and exploration of multiple languages highlighted limitations in managing interactions, reinforcing the need for training to develop competencies in dialogue, contextualisation, and the promotion of autonomy in mathematics teaching. The training cycle facilitated an improvement in the level of this sub-competency among the participating teachers. Initially, this sub-competency was located at level 2 (L_2) and, by the end of the training cycle, had reached level 3 (L_3).

At the beginning of the cycle, the participating teachers demonstrated efforts to include extra-mathematical contexts that would encourage greater student interaction during the proposed activities. Furthermore, they adopted actions that promoted student autonomy, such as avoiding providing immediate answers to their questions. This space allowed students to develop their own solutions and explanations.

However, it was only at the end of the DMKC model instruction that the teachers began to reflect more deeply on the management of interactions, structuring strategies that integrated the following dimensions: student-teacher interaction, student-student interaction, and the development of student autonomy. From these interactive practices, learning the meaning of the mathematical objects taught – such as the notion of natural numbers and the definition of the decimal system – became more evident.

Teachers also began to develop more intentional actions to encourage active student participation, promoting emotional adaptation in the classroom, as students felt included and motivated. One example of this was providing opportunities for students to solve operations using concrete materials, such as an abacus or golden material, and to present the resolution processes to the large group. These dynamics not only reinforced understanding of the content but also promoted collaboration, dialogue, and student involvement in the teaching and learning processes.

> The students built the numbers with the golden beads material. One of the students chose the pieces and placed them in their respective decimal places on the board. The teacher asked the student to write the number formed and pointed out that it was necessary to start with the units. The student counted four units together with the class, counting by ones. The teacher continued, "Now, the student will count the tens." The students answered four, but the teacher asked the student to count by one so that everyone could see that there were four tens. When counting the hundreds, all the students responded: "one", and at the same time said the number constructed, all excitedly reciting "one hundred and forty-four". The teacher called another student, and the student built another. The teacher is excited, along with the students, wanting to know what number student L will form. (Researcher's diary).

Analysis, use, and management of resources before and during the training cycle

Before the training cycle, the teachers' sub-competency in analysis, use, and management of resources showed a lack of suitability in managing material and temporal resources. The teachers' statements revealed this limitation, such as in Teacher K's record: "The teacher continues with the explanation very quickly, not respecting the student's time" (Researcher's diary), or in Teacher L's observation: "The teacher then begins to explain, even though she had recently asked the student to wait" (Researcher's diary). The concern with completing activities quickly and prioritising compliance with planning resulted in a lack of attention to the individual pace of learning. As for resources, xerographic copies taken from the Internet and recurring use of the board for corrections predominated —as described in the record: "The teacher hands out another photocopy of the activity [...] when correcting the questions, she corrects them by writing the answers on the board" (Researcher's diary). Although Teacher K introduced the golden material, she did not plan its use in a way that would promote understanding, answering hurriedly to a student who asked: "*Can I use this one? We won't use it with the plates*" (Teacher K). The lack of contextualisation of tasks, combined with the absence of technological resources and the lack of appreciation for the handling of concrete materials, resulted in activities that were distant from the students' reality and with little potential to promote mathematical literacy, keeping the practice at a zero level (L_0) of this sub-competency.

During the training cycle, the teachers progressed in this sub-competency, reaching level 3 (L_3), as time was no longer a limiting factor. They began to use materials such as number lines, graphs, and golden material to work on the addition and subtraction of natural numbers. Their statements demonstrate this progress: Teacher L, for example, when introducing the number line, tells the students that "*In addition to the number line on the sheet, you can use the individual number line that each one of you has*" (Teacher L) and relates the resource to the decimal system using familiar terms such as "up" and "down", reinforcing that they must keep *"*unit under unit*"* in the algorithm. It also broadens students' understanding by observing that the number line "*goes up to the number 30, but [...] it can be compared with natural numbers that can be much larger, giving an idea of infinity*" (Teacher L). Teacher K, although she did not explicitly explore the relationship with the decimal system, used graphs constructed with the class and the golden material, promoting an understanding of quantities and relationships. Both used the poster abacus and the golden material to reinforce the biunivocal relationships between units, tens, and hundreds. However, limitations persisted, such as the recurrent use of photocopied sheets delivered as activities were completed and the lack of more precise resource forecasts, as in the case of the use of the graph without measurement on the vertical axis, indicating that the consolidation of this sub-competency, even though classified at level L_3, could still be improved.

Analysis and assessment of didactic suitability

In the context of this research, it is essential to note that no specific moments were allocated for the group participants to engage in explicit reflection on their practice prior to the training cycle. Therefore, there is no available evidence that allows us to identify the level of reflective competency of teachers at the initial stage.

The analysis of the level of this sub-competency was only possible throughout the training cycle, when participants began to reflect on their practice, allowing us to observe how the instruction of the DMKC model influenced the development of their capacity to analyse and evaluate the didactic suitability of their practices.

At this stage, the teachers reached level two (L_2) of the sub-competency of analysis and assessment of didactic suitability. They jointly chose Teacher L's plan, which involved creating graphs based on the students' favourite games and applying them using different questions, all with the same objective. Reflection based on the didactic suitability criteria (DSC) allowed teachers to attribute personal and institutional meanings to the mathematical object, promoting improvements in planning. The participants' statements demonstrate this reflection, as when Teacher L commented: "*I don't know if I'm doing it right, if I'm understanding it right... I fear math, I have a lot of doubts, you know?*" highlighting her insecurity, but also her openness to critical analysis. Furthermore, the teachers reflected on the meanings that the students attributed to the activity, improving their understanding and ability to value the teaching and learning processes. Table 5 presents the analysis and assessment of the didactic suitability conducted by the participants.

Chart 5

Analysis and assessment of the didactic suitability carried out by the participants. Prepared by the authors

Criteria	**Didactic reflection carried out by the participants**
Epistemic	Teacher K states that the planning carried out by Teacher L includes the manipulative, graphic (iconic), and symbolic (utilisation of algorithms) phases. That is, there is a variety of representations of the mathematical objects being addressed (for example, the mathematical object "sum"). Teacher K: "*Ah, and we also use the number line when doing the math, how many more, how many less, visualising it on the number line. I made a large number line on the wall and a small one for them to use individually, so we want to see... was there manipulation?*" Teacher K: "*... epistemic is the representation that was the graph itself, which you also created, okay?*" Teacher K: "*It is the construction of the graph itself, their thoughts, they represented through the graph*".

Cognitive	Teacher L returns to the issue of prior knowledge: "*I think that prior knowledge is taken into account when we think about quantities, that he has to know how to relate, number-quantity, know how to count, I think that this can improve. Can the symbolic phase be considered when we ask people to make numerical representations? And the calculations? Is it, using the algorithm, doing the written calculations?*" Teacher L says: "*Knowing how to count does not always mean that the child knows how to make numerical representations*." Teacher K: "*Teacher L, I think it's because of the numbers on the side of the graph*" (referring to the vertical axis). Teacher L: "*This is to see the graph representation, that's right!"* Teacher K: "*Which is growing, and as you look, you can see that one is bigger than the other."* Teacher L: "*And then associating it with the number line can show what they already knew, something about numbers, right. So we did the symbolic phase; in the symbolic phase, we symbolised with the little squares, with the chips that represented each person's favourite game. Because then you could count, how many chips, how many people. I understood that this is it*." Teacher K: "*No, that's it, Teacher L. Cognitive too, right, because it makes them think, reflect, logical reasoning*."
Interactional	Teacher K resumes Teacher L's lesson plan and says: "*I observed that the interactional criterion contemplated, as it is the interaction, Teacher L.*" Teacher L clarifies what she understood about the interaction criterion: "*It's the same thing as manipulation, interacting with students about the activity, clarifying the activity and interacting in an affective way.*" Teacher K: "*That's it and a little more [...] because in what you planned, the interaction is in the way they will carry*

	out the construction of the graph, giving protagonism to the student, valuing and enabling the participation of everyone, thinking about their realities."
Mediational	Teacher L: "*And when manipulating the graph, they were the ones who were putting it together; I called them and each one put it together, pasting a little square of paper in the colour of their favorite game, and once it was ready, we started counting, doing the math, who prefers to jump rope the most, how many prefer to jump rope the most, how much less do they prefer this, do they prefer that. The students used the paper to identify the quantity. I explained how the graph works, they could visualise it. I said that Teacher K did that too. The mediational, did it mediate technical resources?*" Teacher K: "*I think so*."
Affective	Teacher L: "*This affective is here*." Teacher K: "*Because emotion is what he likes*." Teacher L: "*And also precariousness, and I understand that it is precariousness*." Teacher L takes notes down: interaction, mediation, ... Teacher K: "*And respect for other people's preferences, right, which, sometimes, are not the same as mine. I think Teacher L worked on this, too*." Teacher L: "*I think this has to do with how he understands the strategy he uses to do it, it must be; I think so. The strategy he uses to execute what to do an operation, the motivation, cognitive attitude, yes, participation appropriation*."

Ecological	Teacher K: “*I don’t know this ecological thing here.*” Teacher L: “*Ecological is talking about adaptation.*” Teacher K: “*I think it was contemplated, because the graph goes to smaller numbers, the graph we made has smaller numbers, right? I think it goes in, then.*” Teacher L: “It *contemplates the curriculum.*” Teacher L: “*Yes; there is the construction of the graph, placing them within a context, to make meaningful what is meaningful to them*.” Teacher K: “*Yes, from the preference of understanding that the other person doesn’t have the same preferences as me, right?*” Teacher L: “*It has to do with putting it, for example, when I proposed: let’s see four games, four games that each person prefers. This put them into context. If you chose other games that were from their realities. This would also be a way to contextualise and stay within reality*.” Teacher K: “*That is true.*” Teacher L: “*In a contextual way.*” Teacher L: “*The ecological DSC talks about the curriculum, we were also fulfilling it.*” Teacher K: “*Is it about students who have less understanding, special students? Through your graph, everyone was able to participate, but the ecological aspect is related to this, to adapting the activity.*” Teacher L: “*..., but look at this arrow here, the coexistence and adaptation that made it possible for everyone to participate, thinking about their realities.*”

The teachers reflected, from an epistemic point of view, that the planning contemplated different forms of representation of mathematical objects—manipulative, graphic, and symbolic—, highlighting the use of the number line and the construction of graphs as resources that favour students’ understanding. From the cognitive criterion, they emphasised the importance of prior knowledge in the number-quantity relationship and in numerical representation, as well as the use of graphs, number lines, and concrete materials, which facilitate contemplation of the symbolic phase and stimulate students' logical reasoning.

Regarding the interactional criterion, the teachers valued that the planning favoured dialogue with the students, highlighting student protagonism, collective participation, and the appreciation of the students' realities. Regarding the mediational criterion, the participants noted that the students actively participated by pasting colored papers that represented their preferences, which enabled them to quantify and compare data in a visual and symbolic manner. Furthermore, they emphasise the affective and social dimension of classroom work, highlighting the appreciation of students' preferences, respect for differences, as well as the importance of strategies, motivation, and participation for the appropriation of knowledge.

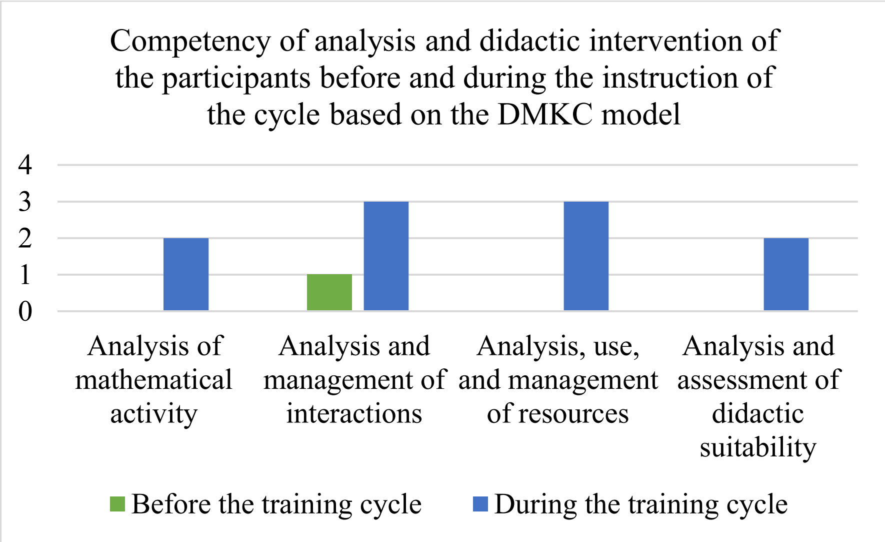
Finally, they discussed the ecological criterion, understanding it as the adaptation of planning to the needs and realities of students, highlighting that the construction of the graph, for example, took into account the curriculum, enabled everyone's participation, and promoted a significant contextualisation of the activities.

Collaborative reflection facilitated by the teacher educator enabled participants to identify both the negative and positive aspects of their approaches, which contributed to the improvement of their practices. In the end, the teachers were able to fully assess the suitability of the teaching and learning processes carried out, making a complete reflection on the instruction process using DSC and its components.

As a general result, it can be stated that the teachers participating in the training cycle demonstrated an evolution in their development of didactic analysis and intervention competency, as evidenced by improvements in each of the sub-competencies, as shown in Figure 3.

Figure 3

Comparison of the levels of didactic analysis and intervention competency of the participants before and during the training cycle. Prepared by the authors



CONCLUSIONS

This study aimed to assess the didactic analysis and intervention competency of two third-grade elementary school teachers in the city of Canoas, RS, participants in a training course based on the model of competencies and didactic and mathematical knowledge of the onto-semiotic approach.

Before the cycle, the teachers identified limitations in the sub-competency of analysing mathematical activity, particularly in understanding mathematical objects and processes, as well as in the appropriate use of epistemic criteria. Before training, errors in definition, procedures, and representations were frequent, in addition to a weak connection between students' prior knowledge and the content taught. This result aligns with international findings that teachers struggle to interpret epistemic aspects, resulting in various types of mathematical errors (Font et al., 2024; Stahnke et al., 2016).

With the application of the DMKC model in the training cycle, there was an evolution of the initial level, resulting in more precise explanations that were appropriate to the students' level, considering their prior knowledge, in addition to the use of strategies that promoted advances in learning, such as mental calculations and symbolic representations. This result is related to the importance that teachers attributed to checking students' prior knowledge to determine whether the content and teaching approach are at a reasonable distance from what students already know (Gunduz & Hursen, 2015; Maximin, 2010).

In the sub-competency of analysis and management of interactions, teachers initially showed low performance, with little integration of content in an interdisciplinary manner and a lack of adequate strategies to assist students with difficulties. Training using the DMKC model facilitated progress, leading to the adoption of more reflective and integrated practices, such as the use of concrete materials (an abacus and golden material) to encourage participation and collaboration among students. These changes resulted in richer interactions and the development of student autonomy.

Regarding resource analysis and use, initial practices indicated a low level, characterised by a lack of innovation, inadequate use of materials, and little planning to adapt the learning environment to students' needs. After training, the teachers reached the highest level, using resources such as number lines, graphs, and golden material to facilitate mathematical understanding.

Finally, in the sub-competency of analysis and valuation of didactic suitability, the teachers reached a high level, developing critical reflection on planning based on the didactic suitability criteria (DSC). The collaboration between them, mediated by the teacher educator, favoured the exchange of experiences and the improvement of practices, especially in the use of manipulative, graphic, and symbolic representations. This result coincides with other investigations that used the DSC tool to organise and deepen teachers' reflection for the development of the analysis and valuation of didactic suitability competency in different training contexts in different countries (among others, Garcés et al., 2021; Giacomone et al., 2018; Morales-López et al., 2024; Seckel & Font, 2020).

This study presents contributions to education, especially in the training of mathematics teachers in the early years of elementary school. On the one hand, we highlight the creation and implementation of a training cycle based on the model of didactic and mathematical knowledge and competencies (DMKC), aligned with the BNCC and BNC-Formação, which promoted the integration between mathematical and linguistic literacy, strengthening pedagogical practices. In the second, when defining levels of the didactic analysis and intervention competency for teachers in the initial years, the positive impact of training on the evolution of the sub-competencies of analysis of mathematical activity, analysis and management of interactions, analysis and Uuse of Rresources and analysis and valuation of didactic suitability was highlighted.

According to the conclusions, the study has limitations and offers opportunities for future research. The first limitation refers to the lack of an in-depth analysis of the development of the teachers' didactic analysis and intervention competency after the training cycle, in their actual practices in the municipal network, which could offer a more complete and continuous view of the results; the second concerns the limited presence of children's stories in the teachers' planning during the training cycle, an aspect that could have been better emphasised and worked on; the third concerns the difficulty of establishing competency levels in the DMKC model, due to the complexity inherent in this type of categorisation.

As a perspective, on the one hand, the aim is to continue the analyses, configuring the development of the participants' didactic analysis and intervention competency in their professional practices, years after completing the training cycle. On the other hand, it is necessary to promote other training cycles in partnership with Education Departments, both to strengthen the key competencies of teachers who teach mathematics—whose fragility has been highlighted—and to expand the study to other stages of basic education. As a sequential action, we are working on the development of the didactic analysis and intervention competency and mathematical competency of mathematics teachers in the final years of elementary school in the municipality of Sapucaia do Sul, RS, through a development project financed by the ITAÚ Foundation (2025), which has been presenting fruitful results in the scope of mathematics teacher education.

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AUTHORSHIP CONTRIBUTION STATEMENT

AB and RAS conceived the idea presented. AB developed the theory. RAS, AB, and VLF adapted the methodology to this context, created the models, carried out the activities, and collected the data. RAS, AB, and VLF analysed the data, RPGD review the manuscript. All authors actively participated in the discussion of the results, reviewed, and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author upon reasonable request.

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