




# Professional Learning of Elementary School Teachers in Lesson Study

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## ABSTRACT

**Background:** Professional learning, conceived as a dimension of teacher professional development, has gained momentum in the last decade due to the global dissemination of a Japanese-origin training process, *jogyō kenkyū*, translated into Portuguese as *estudo de aula* (lesson study). **Objective:** To highlight and discuss the professional learning of elementary school teachers developed through lesson study. **Design:** Interpretative qualitative research based on content analysis. **Setting and Participants:** The study involved a lesson study conducted with five early elementary school teachers from a public school in southern Brazil. **Data Collection and Analysis:** Data were collected through a questionnaire administered to the teachers at the beginning of the lesson study, audio recordings of the sessions, and interviews at the end of the process. The empirical material consisted of researchers' field notes and transcriptions of the interviews and sessions. **Results:** Lesson study provides a context for teachers to engage in professional learning regarding the connection between theory and practice and creative mathematics. **Conclusion:** By promoting professional learning, lesson study contributes to teachers' professional development, enables changes in classroom practice, and thus fosters student learning.

**Keywords:** Professional learning; Lesson study; Teacher professional development; Mathematics teaching; Elementary School.

## Aprendizagens Profissionais de Professores dos Anos Iniciais em Estudos de Aula

## RESUMO

**Contexto.** A aprendizagem profissional, concebida como uma dimensão do desenvolvimento profissional, ganhou impulso na última década em face da divulgação global de um processo formativo, de origem japonesa, *o jogyō kenkyū*, traduzido para o português como estudo de aula. **Objetivo.** Evidenciar e discutir as aprendizagens profissionais de professores dos anos iniciais desenvolvidas em estudos de aula. **Design.** Pesquisa qualitativa interpretativa, baseada na análise conteúdo. **Ambiente e participantes.** A pesquisa envolveu um estudo de aula,

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realizado com cinco professores dos anos iniciais do Ensino Fundamental de uma escola da rede pública de ensino do sul do Brasil. **Coleta e análise de dados.** Os dados foram constituídos a partir de um questionário aplicado aos professores no início do estudo de aula, gravações em áudio das sessões e entrevista ao final do processo. O material empírico constitui-se das notas de campo dos pesquisadores e das transcrições das entrevistas e das sessões. **Resultados.** O estudo de aula oferece um contexto para o professor realizar aprendizagem profissional relativamente à articulação entre teoria e prática e sobre a matemática criativa. **Conclusão.** Ao promover a aprendizagem profissional, o estudo de aula contribui para o desenvolvimento profissional do professor, possibilita mudanças na prática e favorece a aprendizagem dos alunos.

**Palavras-chave:** Aprendizagens profissionais; Estudo de Aula; Desenvolvimento profissional; Ensino de Matemática; Ensino Fundamental.

## INTRODUCTION

Although not recent in education, the discussion about teachers' professional learning has gained momentum in the last decade in the wake of the professional development research movement. In mathematics education, the trend was manifested by the global dissemination of a model of teacher education and professional development of Japanese origin, widely practised in that country: *jugyou kenkyu*, translated into English as lesson study.

Lesson study is a process of teacher professional development that emphasises collaboration and reflection, with a focus on student learning through teacher development (Murata, 2011; Richit, 2023). In Portuguese, this process has been translated mostly as *estudo de aula*, a name adopted in Portuguese-speaking groups. Research results on the contributions of the lesson study to the development of the teacher who teaches mathematics (Ponte et al., 2014; Richit, 2016; Bezerra, 2017; Crecci et al., 2019; Batista & Paulo, 2019; Quaresma & Ponte, 2019; Martins, 2020; Richit, 2023; Richit et al., 2024; Sol et al., 2025) show that this process provides learning both about mathematics and about the ways of teaching it, promoting gradual changes in teaching practices.

Professional learning characterises the process of resignification and expansion of knowledge, values, practices and cultures, through which teachers develop competencies and knowledge of teaching and the profession, establishing new ways of working with colleagues and new practices, improving teaching and promoting student learning (Flores, 2004; Richit et al., 2021).

The article highlights and discusses aspects of teachers' professional learning in the early years of elementary school, developed through lesson studies. Data from a lesson study conducted with five teachers working on the

early years of elementary school at a public school in Rio Grande do Sul are analysed. The results of the investigation add to the knowledge base on professional learning developed in lesson studies, as they reveal aspects of classroom mathematics teaching and clarify the nature and context in which these learnings occur.

The research can contribute to discussions on professional development in the context of mathematics education by highlighting the centrality of professional learning to teacher growth and the implementation of teaching changes. It also provides subsidies to understand how teachers learn to teach and to analyse the elements of a personal nature and the contexts that influence professional development (Richit & Tomkelski, 2022; Huang, 2023).

## **TEACHER LEARNING AND PROFESSIONAL DEVELOPMENT**

The nature and specificity of teachers' activity, whose central dimension is teaching, requires teachers to engage in a process of growth throughout their careers (Day, 2001; Guskey, 2002), based on the resignification of professional knowledge, the strengthening of moral commitment to teaching, the modification of cultures and professional practices, as well as the realisation of professional learning (Richit, 2021). This dialectical process, conceived as professional development, involves different experiences "of learning and consciously planned activities, carried out for the direct or indirect benefit of the individual, the group or the school and which contribute, through them, to the improvement of the quality of education in the classroom" (Day, 2001, p. 20). It therefore presupposes professional learning opportunities through which teachers work collaboratively and develop specialised knowledge about teaching in the classroom (Ribeiro & Llinares, 2025).

In a literature review of the conceptualisation of professional development, Webster-Wright (2009) associates it with professional learning, which develops from experience and reflection on practice. The author emphasises the need and relevance of promoting reflections on authentic experiences in teacher education, taking into account local realities and professional responsibilities. In addition, she argues for a change in the discourse and focus of actions, moving from the transmission and evaluation of professional development programs to the support of authentic professional learning (Webster-Wright, 2009).

Professional learning, conceived as a dynamic, permanent, personal and socially constituted phenomenon in the interaction between teachers and

in the confrontation and modification of ideas, as well as in the reinterpretation of experiences (Flores, 2004), involves learning related to the disciplinary field, classroom teaching, teaching management, the specificities of the professional context, and other processes inherent to the profession (Richit et al., 2021). It therefore involves changes in teacher knowledge and practice, as well as in the dispositions and beliefs that can influence practice (Goldsmith et al., 2014; Richit, 2020; Richit, 2023).

In a study on the contributions of professional learning to teacher development, Desimone (2009) highlighted five aspects to be prioritised in teacher education actions and processes: focus on disciplinary content, active learning, coherence<sup>1</sup> of formative programs, adequate duration, and collective participation. The author points out that the tensions between these elements are one reason changes in large-scale teaching processes are difficult. To improve teaching, promoting changes in practice, professional learning needs to overcome the gap between the teacher's knowledge and beliefs and the knowledge from the research (Desimone, 2009). In addition, collective participation can add to this process a collective sense of the difficulties related to content due to changes in norms about desirable teaching (Spillane, 2000).

Fiorentini (2013) clarifies that professional learning is a process situated in social practice, developed through active participation in social communities and the constitution of identity in relation to these communities. In this way, it materialises and manifests itself in shared ways of acting “within the community, which result from negotiation dynamics, involving full or legitimate peripheral participation and reification in (or from) the community” (Fiorentini, 2013, p. 157). In this sense, professional learning refers to the process of participation and negotiation of meanings within a community, and to the transformation of the subject's identity, evidenced in “belonging to” or “being a member of” a community (Crecci & Fiorentini, 2018).

This perspective corroborates David Kolb's theory of professional learning, which considers that people, by their social nature, learn from

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<sup>1</sup> Regarding coherence, Garet et al. (2001) emphasize that the professional learning process needs to consider several aspects, such as: public policies for teacher education; curriculum guidelines, curriculum and education assessment (internal and external); context and characteristics of the school, as well as its needs and objectives; teachers' knowledge, experiences and needs; results of research on teacher education and educational changes; and recommendations and guidance from professional associations.

experience, especially through conscious reflection on professional experiences (Kolb, 1984). Kolb affirms that learning presupposes processes of reflection that allow the development of professionalism, constituting learning cycles sustained by concrete experience, reflective observation, abstract conceptualisation, and active experimentation. Reflection allows the teacher to develop professional knowledge (Korthagen & Nuijten, 2022) and promotes the integration between theory and practice, in which theory depends on practice, which is the purpose of theory. This understanding is supported by Candau and Lelis (1999), who argue that the dialectical experience of theory and practice is necessary for man's action in the world (Candau, 2013).

In summary, teacher professional development is achieved through professional learning, which is promoted dynamically and continuously at the personal and social levels through interactions among professionals within a specific group, in which ideas are transformed, and experiences reinterpreted (Flores, 2004; Richit, 2021).

## **LESSON STUDY AND PROFESSIONAL LEARNING**

The *lesson study* consists of a process of teacher professional development, sustained and guided by collaboration and reflection, focused on student learning (Richit, 2016). Originally called *jugyou kenkyu*, this process emerged and has been practised in Japan for more than a century (Akiba et al., 2019; Lewis, 2016; Garay, 2018) and is deeply intertwined with the fabric of Japanese educational culture (Robutti et al., 2016).

In Japan, teachers, especially elementary school teachers, participate in multiple lesson study cycles (Lewis & Perry, 2017; Pozzobon & Ponte, 2024), usually including four stages: definition of objectives for the research lesson; collaborative class planning; research lesson teaching, observed by team members; and reflection on the class, based on the records produced by the observers (Lewis, 2002; Richit & Ponte, 2017).

Experiences developed in contexts outside Japan exhibit particularities. For example, in China, lesson study has been practised since the beginning of the twentieth century (Huang & Shimizu, 2016) with a dual focus: on the development of a product (an exemplary lesson plan) and on the improvement of teaching through repeated teaching cycles and the continuous involvement of specialists (Huang et al., 2017). Chinese teachers generally participate in several cycles of lesson study, depending on the level of experience; for example, classes of proposals under construction are conducted by beginning teachers, while demonstrations of exemplary classes are conducted by experienced teachers (Huang et al., 2017).

In the specialised literature, the lesson study model structured in four stages predominates: interrogate, study and plan, execute, and reflect (Huang & Shimizu, 2016; Ponte et al., 2016; Richit, 2016; Lewis & Perry, 2017; Bussi & Ramploud, 2018; Richit et al., 2024), characterising what we call the global lesson study model. The first stage consists of formulating a research problem about the practice and defining objectives for the research lesson, while considering students' learning difficulties in the curriculum topic chosen to be deepened in the cycle. In the second stage, called planning, teachers study the curriculum and discuss curriculum documents and research results on the curriculum topic addressed and, from this, they collaboratively plan a class to achieve the objectives formulated. While the third stage consists of conducting the research lesson, taught voluntarily by a teacher participating in the cycle, the other teachers observe and collect data on the students' actions and learning (Richit, Ponte, & Tomkelski, 2019). Depending on the results of the class and the reflection, and if it is in the participants' interest, a fifth stage can be added, called follow-up, characterised by the improvement of the class, its adaptation to another class and the promotion of a new cycle of lesson study (Ponte et al., 2014; Richit & Ponte, 2017; Fujii, 2018; Tomasi, 2020).

The Japanese lesson study involves a specific pedagogical approach, *structured problem solving*, organised into four main actions: teacher's presentation of a problem, students' resolution of the problem, discussion of the resolutions, and synthesis of the class (Fujii, 2018). Next to the structured problem-solving approach, the exploratory approach has been reported in the literature on lesson study conducted outside Japan (Richit, 2020). This approach allows students to work with problems for which they do not have an immediately applicable resolution method, leading them to build or deepen mathematical concepts, representations, and ideas (Ponte et al., 2014).

Huang (2023) defines lesson study as a path of collaborative professional development focused on promoting teacher learning and improving teacher competence. Arcavi and Mena-Lorca (2009) conceive of lesson study as a permanent activity that involves several actors in the Japanese educational system, including teachers, who, when working collaboratively, share knowledge and learn from each other.

Ponte et al. (2012) investigated 2nd-cycle teachers' understanding of the concept of direct proportionality. The results indicate professional learning regarding the selection of tasks to propose in class, students' reasoning processes, and communication in the classroom, especially during collective discussion. Participation in the lesson study provided teachers with an opportunity to reflect on the practice. This aspect shows implications for the

organisation of lesson studies, showing that the results are closely related to how they are streamlined and conducted (Ponte et al., 2012).

In a research with high school mathematics teachers, Richit and Tomkelski (2020) point out that lesson study provides teachers with professional learning, highlighting the deepening of curriculum content, strategies and resources for teaching mathematics topics, as well as the development of skills for collaborative work and reflection on practice. The authors conclude that lesson study is a promising professional development process, both for mathematics teaching and for professional culture.

When analysing the professional learning opportunities of prospective teachers in class studies, Neves and Fiorentini (2021) highlight that participants developed the ability to identify students' difficulties and propose strategies to overcome them. From this perspective, Sol et al. (2025) conducted a study focused on teachers' practical arguments as a tool for identifying students' difficulties, reinforcing the relevance of reflection on teaching practice in lesson study cycles. In addition, Neves and Fiorentini (2021, p. 24) observed that the proper exploration of teaching materials, such as the *Geoplano*, based on "well-planned tasks and teachers' mediating action, plays a fundamental role in the management of student learning, especially in the construction of geometric concepts." This aspect is corroborated by Sol et al. (2024), who investigated teachers' practical argumentation in selecting the most appropriate concrete material for working with geometry with children in early childhood education.

By conducting a bibliographic study centred on the specialised international literature, Richit et al. (2021) show that participation in lesson studies provides participants with professional learning in three domains: mathematics, mathematics teaching, and professional culture. These learnings promote teacher professional development and favour changes in teachers' professional practices, dispositions, values, and knowledge.

In a survey of teachers in the early years of elementary school, Castro (2023) highlights aspects of professional learning achieved through lesson study. Teachers began to value collaborative planning as a space to deepen studies and seek new approaches to the classroom, began to recognise the importance of students' autonomy in carrying out investigative tasks, as well as the relevance of the content for the development of mathematical reasoning through manipulative materials, and identified the need to be better prepared to develop mathematical topics in the classroom.

When analysing the experience of participation of a teacher-researcher in a blended lesson study (BLS) context with a focus on the meaning of the number zero, Paula and Fiorentini (2023) highlight learning

organised into three categories: learning as doing, learning as belonging, and learning as transformation. Regarding learning as doing, the teacher-researcher learned to explore, for example, data from different sources of evidence (audio and video recordings, written narratives, interviews, among others), with emphasis on the meaning of zero, and to triangulate them in the analysis. As for learning as belonging, he took on greater responsibility in the Community of Collaborative Practice, having organised an event and sometimes acting as an educator. Finally, in the domain of learning as transformation, he began to understand and recognise himself as a researcher-educator in the process of transformation (Paula & Fiorentini, 2023).

The research by Andrioli (2024), conducted with 9th-grade mathematics teachers in public schools in Santa Catarina, shows that participants in the lesson study learned in three dimensions: planning as a teaching dimension, mathematical tasks, and changes in teaching practice. The results indicate that the lesson study enables participants to develop professional learning of mathematics teaching, promoting changes in professional practice and culture, and contributing to professional development.

In addition, the lesson study concretises the approximation between theory and practice from a perspective of complementarity and unity. Theory and practice are articulated to the extent that the theoretical deepening provided in this process clarifies, grounds, and guides practice, while practice enables the resignification of theoretical knowledge (Richit, 2023). In this sense, lesson study enables the teacher to develop professional learning related to teaching, appropriating new perspectives on mathematics, rather than the reproductive model (Ernest, 1996), in which students' actions are limited to the reproduction of procedures and resolution strategies previously presented by the teacher.

## **METHODOLOGY**

The investigation is based on a qualitative and interpretive perspective (Erickson, 2011), focusing on the objective of highlighting and discussing the professional learning of teachers of the early years developed through lesson studies. Data collection took place in a lesson study carried out in the second half of 2023 in a public school in Rio Grande do Sul, involving five elementary school teachers (1st to 5th grade), who worked in different school grades in that school: Maria (1st grade teacher), Antônia (2nd grade), Joana (3rd grade), Lúcia (4th grade) and Beatriz (5th grade). A master's student (Gabi), a doctoral student (Laís) and a pedagogy student (Bianca) also

participated, in addition to the cycle facilitators (Alice and Mateus). All names are fictitious.

The lesson study was organised into twelve meetings. In the first two, the curriculum topic to be addressed (volumetric capacity), the research question that guided the cycle, the objectives for the research lesson, and the detailed schedule of activities were jointly defined. In the following meetings, from the third to the ninth, readings and discussions were held on the origin and dynamics of the lesson study, the national curriculum guidelines, research results on the teaching and learning of mathematics (including the concept of volume), exploratory teaching and its stages, and strategies to favour student learning. During this period, the participants also planned the research lesson, outlined strategies and elaborated resources and materials, prepared the observation script and organised each moment of the lesson, following the exploratory approach. After the research lesson, in sessions 11 and 12, the team met to reflect on the records produced and evaluate the entire process. The task prepared for the research lesson was structured as follows:

### Figure 1

*Task prepared for the research lesson*

#### **TAREFA 1**

*O Chafariz da Praça da Bandeira da cidade de Erechim possui forma elíptica (oval) e suas dimensões são: 12 metros de comprimento, 10 metros de largura e 70 centímetros (0,70 m) de altura, conforme indicado na figura abaixo. A capacidade de água é de aproximadamente  $63 \text{ m}^3$  (metros cúbicos).*



Sabendo que  $1 \text{ m}^3$  equivale a 1.000 litros.

a) Qual a capacidade do chafariz, em litros? Explique a sua estratégia de resolução.

The research lesson involved a 4th-grade class comprising 25 children aged between nine and eleven years. Four meetings were held on different days, which constituted the research lesson. The first meeting was dedicated to the contextualisation of the topic ‘volumetric capacity and units of measurement’, conducted voluntarily by Gabi. The second and third meetings

were voluntarily conducted by Lucia, the class teacher, and focused on solving the task. In these meetings, the students, organised in pairs or threesomes, worked on the carefully prepared task and, in the end, the teacher promoted collective discussion and the systematisation of mathematical ideas. The last meeting consisted of a visit to a public space in the municipality (the central square), where the city's fountain is located, which is considered one of the local historical monuments. On this visit, whose objective was to deepen the historical and mathematical aspects related to the research lesson theme (fountain), the students attended a public class taught by a municipal historian. The research was approved by the Research Ethics Committee (CAAAE n°. 88232325.4.0000.5564).

The analysis focused on the collected data, which were examined from an interpretative qualitative perspective (Erickson, 2011), leading to the identification of the categories: theory-practice articulation and creative mathematics.

## RESULTS

### *Articulating theory and practice*

The dynamics of the lesson study, especially during the planning stage, constitute a favourable context for strengthening the relationship between theory and practice from an integrative and interdependent perspective. Through reflection on students' difficulties in mathematics and the strategies often mobilised to help them, teachers highlight the possibility of modifying practice based on the study of theories that act as catalysts for new strategies and resources.

*Antônia:* This coming back, studying, is very important. Analyse your practice and say: Look, I could have done it like this. Here are several thinking heads that take different things from the theory and see different things when applied in practice.

*Beatriz:* [I try to understand] how the child learns, and this is a difficulty for the teacher, and now, in fifth-grade mathematics, I am finding it very difficult. [This] is difficult. [Including] my research topic [of the master's degree] is also related to these difficulties and [I seek in theory] another methodology to teach mathematics differently for them (Session 1, Sep. 2023).

Planning, the second stage of the lesson study, is characterised by its guidance by theory. During the cycle, the participants delved deeper into the principles of the exploratory approach and the characteristics of open tasks, as opposed to conventional exercises.

*Lúcia:* In this type of [task], they will conclude by themselves.

*Gabi:* [This type of task changes the class], how they react when someone raises a hypothesis, how much someone in the group raises a hypothesis, whether they reject or consider themselves.

*Láís:* [Changes the] justification they use to justify [the results].

*Alice:* And that corroborates the theory. The exploratory activities, more open, are more interesting, more thought-provoking for them (Session 6, Oct. 2023).

After the theoretical study, the teachers planned the research lesson guided by the theory. They also had the opportunity to reframe aspects of the theory of mathematical reasoning, understanding it as a tool to explain the students' difficulties and learning.

After reading and discussing exploratory teaching, in which the team analysed the structure of the task to be developed and its potential to highlight students' strategies and learning, the participants reported that they had recently adopted approaches with more open didactic methodologies.

*Lúcia:* I applied an activity for the class to do, and they loved it. [I selected] a situation, a problem, [...] from the test that the fourth-grade classes did. Wow! Some thought a lot, talked a lot, and some obtained the result, but did not know why they had obtained it. It was very interesting.

*Alice:* Yes, because you can understand how they think.

*Lúcia:* Normally, I don't go through those paths, but now I'm starting to do it precisely because I'm learning, and I think it's very interesting. But, I usually do not let them decide the path to the results (Session 8, Nov. 2023).

The choice of the task type to be proposed for the research lesson was guided by theory, particularly the principles of the exploratory approach, and

supported by the use of open tasks, as illustrated in Figure 1. The theoretical deepening provided encouraged participants to try new practices in their classes, always guided by theory.

Through discussions on the curriculum guidelines and research results on mathematics teaching, the team reflected on the Singapore bar model<sup>2</sup>, confronting the Singaporean curriculum guidelines with the Brazilian ones. The participants, in addition to highlighting the potential of the bar model for mathematics learning, present insights in which they re-signify classroom experiences.

*Gabi:* I want to comment on the bars of Singapore [...]. This decision-making, this attitude that we are going to face. This model of the bars was created, which is revolutionary because learning from this perspective, learning understanding that there is nothing wrong with the concrete material, quite the opposite, that this does not need to have a final parameter, there, in the early grades of elementary school, it is possible to be lifelong. I think that if I had learned fractions in this way, a period of disaffection, of “disaffection” with mathematics, would have been overcome much earlier in my life.

*Beatriz:* [I want to tell you something that] happened yesterday in my mathematics class. [When solving an activity about fractions] two students went down different paths than the ones I showed, and they really took to what Alice explained. [They did it their way and explained it like this]: Teacher, I didn't do it like that, I did it like this [showing the paper]. On the whole, he took the parts indicated in the problem, and he went another way, just like the bar model. So, he developed this bar system in his head without knowing it (Session 6, Oct 2023).

In addition to being guided by the theoretical basis of the exploratory approach and respecting its four phases — introduction, students' autonomous work on the task, collective discussion, and systematisation of learning —the

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<sup>2</sup> It is a strategy for problem solving that uses pictorial bars, present in the curricula and curriculum guidelines of Singapore. This model has been gaining prominence in mathematics education (Richit & Richit, 2022).

research lesson incorporated the principles of interdisciplinarity, which shaped both the task and the structure of the class.

*Mateus:* The research lesson will be structured in [four meetings with the students], right? In the first, the introduction, the class begins with the drawing of the fountain. Alternatively, it can be all together, do the round of conversation and do the drawing. That would be the first moment. The second is with the [city historian]. He will teach a class and [...] we will learn a lot from him. The third is the class [in which students will solve the task]. And the fourth is closure. [...]. For this lesson structure, we will survey the historical context of the fountain and put it here, right? Put this information at the beginning of the task?

*Joana:* Yes.

*Mateus:* In the conversation circle, we will address something of that, of the story, to introduce the task, in these four or five initial lines, to remember some things from the context that were presented, for example, it was built in such-and-such year, something like that. And then falls into the [task]?

*Gabi:* What we had developed, what we have: after the questions, knowing the measures of the fountain, the capacity. Here comes the task, the historical context that we will elaborate. The lesson itself is planned around a conversation circle, a presentation of the book [on the history of the fountain], a drawing of the fountain, and the systematisation of the hypotheses of capacity, with a worksheet for the students with the task. [I remembered that I need to] organise the students' form.

*Lúcia:* We thought so far. But, the drawing, I do not know if we should let it.

*Gabi:* Would you take away the drawing part? The drawing I don't know. Here, in the sequence of the introduction, were the puzzle, the projection of the fountain image, the conversation circle with the [historian], and the book of the history of the fountain and the systematisation of history.

*Lúcia:* Without the drawing, how is it?

*Gabi*: Then they would do the puzzle, and we would project the image. At this moment of the projection, you can already fit the historical contextualisation. Then there is the conversation circle about the book, and then there is the systematisation of the hypotheses. We can deliver a form and then compare the hypotheses (Session 7, Nov. 2023).

After this discussion, the lesson structure was designed to promote the interdisciplinary approach to the topic, culminating in four meetings: one for the historical contextualisation of *volumetric capacity and units of measurement*; two for students' resolution of the task; and one for the public class taught by the city historian. The relationship between theory and practice materialised in an exploratory task on volume and volumetric capacity, elaborated from an interdisciplinary perspective, which allowed students to understand and deepen the historical aspects, capacity, functioning, and characteristics of the city's fountain. In addition, they discussed environmental issues, sustainability, and the preservation of water resources.

*Antônia*: There is the issue of Corsan<sup>3</sup>, which would perhaps be interesting for them to have this notion of water. And we work on the issue of the litre measurement, we work on [water consumption] here in the neighbourhood.

*Alice*: An approach like that, for example, is this a waste of water? How much water does the city use?

*Maria*: [Reflects with them]: If there is so much water on the planet, why are there so many campaigns to save water? Then it goes into the problematisation [of consumption]. Then there would be the question of how much water you normally use to bathe. When we work on the environment, we ask: if I brush my teeth with the tap running, how many litres of water are used? And in the shower, when we stay a long time?

*Joana*: Usually, they don't have this notion where the water comes from, and they say: 'It comes from the pipes'. But how

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<sup>3</sup> Companhia Riograndense de Saneamento (Corsan), founded in 1966, is responsible for performing studies, projects, constructions, operations, exploration, and expansion of public services for the supply of drinking water and sanitary sewage in the state of Rio Grande do Sul.

is the water captured? Where is it supplied? (Session 7, Oct. 2023).

The lesson study also provided students with mathematical discoveries about units of measurement across different cultures, exploration of ratios between units, and analysis of the coherence of the measurement strategies chosen by the groups. This process allowed teachers to establish relationships between theory and practice and to reflect on the curriculum. In the research lesson, the materialisation of the articulation between theory and practice led the participants to observe a sustained, theory-oriented practice.

*Antônia:* The three students were well engaged [to solve the task]. They [...] related the question of the measure of capacity very well with what they could use mathematics to solve, because we usually consider the measure of capacity something that students cannot perceive. [But, when] you can bring that into their reality, they understand.

*Mateus:* [During class, while they solved the task, we identified] How many and which representations were used (Session 11, Dec 2023).

Participants highlighted that the interdisciplinary approach to the topic acted as a catalyst for prior knowledge in mathematics, allowing teachers to identify the strategies and representations students mobilised and, consequently, the learning they carried out. The articulation between theory and practice emerged in the post-class reflection, during which the participants analysed and discussed the students' actions and learning in relation to the established objectives. In this context, teachers produced new understandings about theory through reflection on the practice they experienced.

*Antônia:* The class managed, from what I realised, to achieve the objective of the lesson and obtain the answer to the three questions and with good arguments that they had in the notion of volume. I think the class with the [historian] was very interesting, as they could combine practice and theory (Session 12, Dec. 2023).

The theoretical deepening provided insights into the practice, especially the importance of sensitive listening to students and their issues, functioning as a device for didactic decision-making. Thus, theory, by informing and guiding each stage of the lesson study, translates into reflected

and reflective practices, providing the conceptual basis for decisions regarding the strategies adopted in the research lesson, as well as for the elaboration of resources and teaching materials.

Similarly, in the initial phase of the lesson study, theory, intertwined with practice and used as a lens for reflection on shared professional experiences, favoured the emergence of the research question that guided the process and the definition of the objectives of the research lesson. This particularity highlights the perspective of unity between theory and practice.

In addition, the theory allows the teacher to understand different teaching approaches and their impacts on student learning, analyse students' difficulties and learning processes, and outline classroom strategies that are appropriate and consistent with their characteristics and needs. In an interview, the participants emphasised that the complementarity between theory and practice, materialised in the process of planning, delivering, and reflecting on the lesson, is fundamental to professional learning.

*Lais:* The lesson study reinforced the importance of contextualising the problems of units of measurement and volumetric capacity within the students' reality, making learning more relevant and engaging. By bringing the fountain situation to the square, the students were able to better visualise [...] the content. It highlighted the need to carefully plan the questions on the activity sheet, anticipating potential difficulties and organising them in a sequence that allows students to progress gradually and autonomously (Interview, Dec. 2023).

*Lúcia:* It contributed to the improvement of the quality of planning the lessons, greater understanding of the students' learning process, development of the ability to reflect on the practice and greater participation and engagement of students (Interview, Dec. 2023).

Therefore, the lesson study articulates theory and practice from the perspectives of unity and complementarity, insofar as participants formulate a problem to investigate the practice and outline the objectives of the research lesson through reflection on students' difficulties. In addition, based on the deepening of theoretical aspects about the exploratory approach and mathematical reasoning, they plan the lesson and the task. The lesson, in turn, was developed as an object of reflection, guided by the theoretical lenses that have sustained the process from the beginning.

### *Creative mathematics*

The lesson study offered a context for teachers to understand and experience mathematics from a perspective opposite that of the reproductive model, in which students reproduce models, strategies, and procedures previously presented by the teacher. In the first stage of the cycle, the participants expressed the expectation of experiencing a classroom practice that would involve students and provide a motivating experience. In addition, during the planning, participants suggested different strategies to engage and instigate students.

*Joana:* They like curiosities, different things, and we could explore something, like, about water.

*Antônia:* The question of the liquid: why does the liquid rise in space [instead of falling]? They have this curiosity.

*Lúcia:* You can experiment with containers they have at home – measuring cups, bottles, jugs – and they can compare them.

*Maria:* We can make a table and insert several utensils, and they can estimate on the side. [Thus, they can experiment and compare] in the same place.

*Alice:* This idea of comparison is genius, Lúcia. It is very simple for children and for us to say: in a litre there are 1,000 ml [millilitres]. Now, when I say what has a thousand ml, it means that I'm comparing that container with that other unit, which is one ml, which is more or less a drop, right? But, they have to understand that measurement, in fact, is when you compare something with other things. I can compare with a lot of other objects; for example, in the case of length units, I could, instead of cm, establish that the unit of measurement is this folder.

*Mateus:* In that part, in the comparison between the units, there is already a mathematical relationship. And the comparison between the different containers, the issue of the representation [of the unit], which will represent a liquid inside a glass, etc. So, we have already managed to involve up to two different strategies. And in the relationship with the calculation now, which we have in relation to the proportions, to the measures. (Session 7, Oct. 2023).

Lesson planning, especially the intention to promote an interdisciplinary approach to the topic of volume and capacity from the city's fountain, allowed the design of a creative (productive) mathematical practice. In this practice, students can compare units of measurement using bottles, jugs, buckets, and cups, formulate hypotheses about the ratios between these measurements, and establish conclusions about the time required to empty the fountain depending on the container used to remove the water. In this context, mathematical learning was constituted as a process of exploration and discovery.

The participants also highlighted aspects of the lesson study that favour the emergence of new practices, especially the proposal of coherent and appropriate approaches to the school context, the flexible and dialogical nature of this process, and the emphasis on valuing the professional experience of all involved.

*Lucia:* I [highlight] this issue of taking into account the context of schools to choose the topic and the theme, which is not that static thing. This is not a method, something that you have to work like that and that's it. [On the contrary], it's a different proposal, and that's what's cool. The classes are always in groups, the question of listening and being heard and promoting discussion as a means of communicating learning, the question that is always done in a way that the theme, in short, the proposal involves the exploration of things. It is not only the abstract, it also goes into practice; it is a process that, for us, the teachers, we experiment with new practices and expand knowledge (Session 9, Nov 2023).

In the discussion about the distinction between exercises, problems and exploratory tasks, the participants highlighted each one's characteristics and purposes, and emphasised the aspects that corroborate the potential of mathematics as a space for creation, through students' proposals of new strategies and the formulation of coherent conclusions.

*Lais:* The exercise is great for the teacher to check whether students have acquired the knowledge. This is an interesting goal for an exercise. Now, another problem arises: the possibility that students are the protagonists. They develop knowledge [...], but it is also important to have exercises for some things, for some objectives. For example, the teacher can identify students who have doubts, problems, or

difficulties. [...] it is to diversify, not simply to say that they should not do it.

*Beatriz:* And it serves for the teacher to reflect on where the student is making mistakes and what they have to [emphasise in the exercises], reinforce, so that they do not make errors anymore (Session 9, Nov 2023).

The lesson study was constituted as a context in which teachers could reflect on reproductive mathematics, predominant in school cultures, highlighting the difficulty of accepting resolutions that are not supported by classical procedures.

*Antônia:* And how much we are conditioned in the question of calculation, and when we see a child who can do the mental calculation, [we say] I want the reasoning, I want the calculation, because [we] do not trust the answer.

*Lúcia:* I think this group now, what we are doing, does not only influence mathematics, because you start to think differently about things (Session 9, Nov. 2023).

Participants also reflected on the dynamics of the reproductive class, highlighting ways to foster student learning, especially when this perspective is combined with other strategies that enhance student communication.

*Maria:* Sometimes I think that one student will copy from the other, that only one will do the exercise while the other copies. But we often observe that they explain things to the colleague differently than we do, and maybe that explanation is better for the colleague to understand. But we have this thing (Session 7, Oct. 2023).

In planning the research lesson, the teachers adopted a creative (productive) perspective on mathematics, enabling students to experience it as a context for discoveries. In this sense, mathematical learning emerges from a process of dialogue, the negotiation of meanings, the discussion of hypotheses, the formulation of conclusions, and the validation of results.

*Alice:* When we deal with these issues of understanding and exploring the students' strategies, we learn that there are many ways to reach [resolution], that these ways are correct and need to be valued, and that sometimes the longest way is much more elaborate than the shortest way, for example. And

sometimes the shortest path is well elaborated, with a much higher level of abstraction, while others are more creative. So, the student's ability and learning are not associated with reproducing exactly the teacher's mode of resolution, but rather with increasing or even proposing others. It is from this perspective that we work, and the exploratory approach goes in this direction of valuing these different paths (Session 6, Oct. 2023).

The lesson study provided a context for teachers to reflect on the specificities underlying the students' resolution processes, highlighting both the abstraction capacity present in more succinct paths and the richness and ability to establish relationships in more detailed paths. Such paths show distinct mathematical learning, characterising a creative perspective of mathematics.

In addition, they highlight the relationship between creative mathematics within students' contexts and the appreciation of the knowledge they have.

*Gabi:* And this issue of contributions makes us recall that the issue of specificities is according to students' reality and the material that these contributions are building, and then to the extent that they are doing all this, they are building this reasoning, all these ideas arise, these conclusions [...] It also involves our sensitivity to students' knowledge, how they learn, and what sensitises them. They bring their issues, their experiences, their daily lives, and we should use them as [context], as an incentive to introduce content [...]. (Session 11, Dec. 2023).

In the lesson study, through an exploratory approach and carefully elaborated tasks, mathematics learning assumes a perspective of creation and discovery. Productive mathematics prioritises deep conceptual understanding, flexibility in problem solving and the stimulation of creativity in the paths of resolution, critical thinking, and innovation. As a result, it favours the emergence of new ideas, deepens understanding, enriches the repertoire of mathematical strategies and procedures, refines justifications and conclusions, and thus promotes mathematical learning.

*Gabi:* [...] the secret lies in the strategies they will find to solve. And even if they do not reach the objectives initially thought, but that this path taken in the elaboration of the

strategy leads to other and other learnings [...] (Session 12, Dec. 2023).

Lesson study provides participants with a creative approach to mathematics, through which students can explore, investigate, conjecture, solve problems, justify, represent, formulate hypotheses, discover, construct, verify, explain, predict, develop, describe, and apply mathematical knowledge.

## DISCUSSION

*Articulating theory and practice.* Lesson study promotes professional learning regarding the integration of theory and practice, enabling teachers to improve lesson planning and pedagogical practice while feeling better prepared to face classroom challenges and unforeseen events (Lent & Ponte, 2019; Huang, 2023; Richit, 2023; Andrioli, 2024). The unity between theory and practice is concretised to the extent that the lesson study mobilises these dimensions inseparably, and it is at the intersection between both that the objectives of the research lesson and the question that guides the process are forged, allowing the participants to experiment with new practices supported by theory.

The relationship between theory and practice also manifests in the modification of how students' participation in the learning process is conceived (Castro, 2023; Andrioli, 2024). This aspect indicates that theoretical deepening favours the transformation of teachers' beliefs, dispositions, and predispositions (Desimone, 2009; Richit, 2023), thereby promoting changes in pedagogical practice in favour of student learning.

In summary, the perspective of unity between theory and practice, evidenced in the interdependence in which both constitute an indissoluble unity, in a dialectical movement marked by autonomy and mutual dependence (Candau & Lelis, 1999), is configured as an intrinsic principle of lesson study. In this context, the objectives and research question that guide the cycle are outlined. Likewise, the lesson study consolidates the complementary relationship between theory and practice, in that both continuously feed back into each other.

*Creative mathematics.* Creative, or productive, mathematics involves the search and negotiation of paths, as well as the creation of strategies that lead students to mathematical exploration and discovery (Ernest, 1996). The lesson study, informed and supported by theoretical references – especially the exploratory and interdisciplinary approach – allowed teachers to carry out professional learning (Ponte et al., 2012; Richit & Tomkelski, 2020) related to

the promotion of practices centred on discovery and, consequently, on mathematical creation, concretising a productive (creative) approach to mathematics.

Productive – or creative – mathematics expands students' argumentation skills by challenging them to outline their own strategies and procedures for solving tasks, intentionally designed to address their difficulties (Richit & Richit, 2022). In addition, it favours conceptual understanding and the abstraction of mathematical concepts and properties. Practice in mathematics must articulate, in complementary ways, the productive and reproductive perspectives, combining them in a balanced manner. The reproduction of procedures is necessary for the development of the skills recommended by the curricula, but should be complemented by a productive understanding to overcome superficial learning.

In the research by Sánchez et al. (2022), mathematics teachers relate creativity and its development to several elements of the teaching and learning process. Most comments are associated with tasks that allow students to develop mathematical processes, such as exploration, argumentation, problem solving, and modelling. In addition, the participants highlight the use of technologies or manipulative materials, as well as cooperative work, as factors that can favour students' creativity, a result corroborated by the studies by Seckel et al. (2019).

It is evident, therefore, that lesson study promotes professional learning to the extent that it provides teachers with successful professional experiences, with the potential to mobilise the transformation of professional beliefs and dispositions (Guskey, 2002; Richit, 2023). It also provides the teacher with immediate and comprehensive feedback on the results of pedagogical practice and student performance, fundamental aspects for strengthening the willingness to change and basic for professional development.

## **CONCLUSIONS**

The results corroborate the literature on lesson study by enabling the approximation between theory and practice; however, they advance by pointing out that this approximation assumes two perspectives: complementarity and unity. To some extent, theory and practice can be considered autonomous and independent dimensions; however, the perspective of their uniqueness is exclusive in lesson study, and it is at the intersection of the two that the research questions and the objectives that guide the cycle arise.

In addition, the results corroborate the potential of the lesson study to promote gradual changes in practice, advancing by clarifying the nature of these changes, with emphasis on the possibility of teachers and students experiencing a productive and creative approach to mathematics. Finally, professional learning involves both the competence to do something new and the ability to attribute meaning to new ways of doing things.

Two limitations of the research stand out. The first refers to the team's profile: the first is that, due to the specificity of teachers' initial education (a degree in pedagogy), some theoretical aspects of mathematics teaching were less well known, which may have limited professional learning, and the second limitation concerns the categories of analysis generated by the theoretical framework adopted (an exploratory approach). We believe that a different theoretical basis could yield different results regarding the changes observed in classroom teaching (Breda et al., 2021).

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### **AUTHORSHIP CONTRIBUTION STATEMENT**

AR and MLT conceived the idea presented. AR developed the theory. AR and MLT adapted the methodology to this context, built the models, conducted the activities, and collected the data. AR, MLT, and AB analysed the data and revised the text. All authors actively participated in the discussion of the results, reviewed, and approved the final version of the work.

### **DATA AVAILABILITY STATEMENT**

The data supporting the results of this study will be made available by the corresponding author, AR, upon reasonable request.

### **REFERENCES**

- Akiba, M., Murata, A., Howard, C. C., & Wilkinson, B. (2019). Lesson study design features for supporting collaborative teacher learning. *Teaching and Teacher Education*, 77(1), 352-365.  
<https://doi.org/10.1016/j.tate.2018.10.012>
- Andrioli, A. A. (2024). *Aprendizagens profissionais de professores que ensinam matemática no ensino fundamental – anos finais a partir dos*

*estudos de aula*. Dissertação (Mestrado em Educação), Universidade Federal da Fronteira Sul, Chapecó.

- Arcavi, A., & Mena-Lorca, A. (2009). El estudio de clases japonés en matemáticas. *Colección Digital Eudoxus*, 18.  
<https://doi.org/10.4067/S0718-50062013000400005>
- Batista, C. C., & Paulo, R. M. (2019). A formação de professores de matemática: compreensões da experiência vivida com estudo de aula. *Educere et Educare*, 14(32).  
<https://doi.org/10.17648/educare.v14i32.22535>
- Bezerra, R. C. (2017). *Aprendizagens e desenvolvimento profissional de professores que ensinam Matemática nos anos iniciais do Ensino Fundamental no contexto da Lesson Study*. Tese (Doutorado em Educação) - Universidade Estadual Paulista, Presidente Prudente.
- Breda, A., Hummes, V. B., Silva, R. S., & Sánchez, A. (2021). The role of the phase of teaching and observation in the lesson study methodology. *Bolema - Boletim de Educação Matemática*, 35(69), 263-288.  
<https://doi.org/10.1590/1980-4415v35n69a13>
- Bussi, M. G. B. & Ramploud, A. (2018). *Il lesson study per la formazione degli insegnanti*. Carocci Faber
- Candau, V. M. (2013). *Rumo a uma nova didática*. Vozes.
- Candau, V. M., & Lelis, I.A. (1999). A Relação Teoria-Prática na Formação do educador. In V. M. Candau (Org.). *Rumo a uma Nova Didática*. (pp. 56-72). Vozes.
- Castro, P. C. S. dos S. (2023). *Estudos de Aula e aprendizagens relativas ao Conhecimento Pedagógico do Conteúdo de professores dos anos iniciais do Ensino Fundamental*. Dissertação (Mestrado em Educação em Ciências e em Matemática) - Universidade Federal do Paraná.
- Crecci, V. M., & Fiorentini, D. (2018). Desenvolvimento Profissional em comunidades de aprendizagem docente. *Educação em Revista*, 34, 1-18.
- Crecci, V. M., Paula, A. P. M., & Fiorentini, D. (2019). Desenvolvimento profissional de uma professora dos anos iniciais que participa de um Lesson Study Híbrido. *Educere et Educare*, 14(32), 1-21.

- Day, C. (2001). *Desenvolvimento profissional de professores: os desafios da aprendizagem permanente*. (11ª ed.). Porto Editora.
- Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualisations and measures. *Educational Researcher*, 38(3), 181–199.
- Erickson, F. (2011). Qualitative research methods for science education. In Fraser et al. (eds.), *Second international handbook of science education* (pp. 1451-1469). Springer Netherlands. [https://doi.org/10.1007/978-1-4020-9041-7\\_93](https://doi.org/10.1007/978-1-4020-9041-7_93)
- Ernest, P. (1996). Investigações, Resolução de Problemas e Pedagogia. In P. Abrantes, P., L. Leal, & J. P. Ponte (Eds.), *Investigar para aprender Matemática: Textos selecionados* (pp. 25-48). APM.
- Fiorentini, D. (2013). Learning and professional development of the mathematics teacher in research communities. *Sisyphus - Journal of Education*, 1(3), 152-181. 10.25749/sis.3710
- Flores, M. A. (2004). *The early years of teaching: Issues of learning, development and change*. Porto, Portugal: Rés..
- Fujii, T. (2018). Lesson Study and Teaching Mathematics Through Problem Solving: The Two Wheels of a Cart. In.: Quaresma, M.A. et al. (Eds). *Mathematics Lesson Study Around the World: Theoretical and Methodological Issues* (pp. 1-22). Springer.
- Garay, S. (2018). De Japón a Puente Alto. El estudio de la clase: una innovación en la enseñanza de las matemáticas. *Revista de Gestión de la Innovación en Educación Superior*, 3, 94-114.
- Garet, M., Porter, A., Desimone, L., Birman, B., & Yoon, K. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research Journal*, 38(4), 915-945.
- Goldsmith, L. T., Doerr, H. M., & Lewis, C. C. (2014). Mathematics teachers' learning: A conceptual framework and synthesis of research. *Journal of Mathematics Teacher Education*, 17, 5–36.
- Guskey, T. R. (2002). Professional Development and Teacher Change. *Teachers and Teaching*, 8(3), 381–391. <https://doi.org/10.1080/135406002100000512>

- Huang, R. (2023). Lesson Study China: su historia, desarrollo e implicaciones. *Paradigma*, 44(2), 61-79.
- Huang, R., & Shimizu, Y. (2016). Improving teaching, developing teachers and teacher educators, and linking theory and practice through lesson study in mathematics: an international perspective. *ZDM Mathematics Education*, 48(4), 393-409.
- Huang, R., Fang, Y., & Chen, X. (2017). Chinese lesson study: An improvement science, a deliberate practice, and a research methodology. *International Journal for Lesson and Learning Studies*, 6(4), 270-282.
- Kolb, D. (1984). *Experiential learning: Englewood Cliffs*. Prentice Hall.
- Korthagen, F., & Nuijten, E. (2022). *The power of reflection in teacher education and professional development: Strategies for in-depth teacher learning*. Routledge.
- Lewis, C. (2002). *Lesson study: A handbook of teacher-led instructional change*. Research for Better Schools.
- Lewis, C. (2016). How does lesson study improve mathematics instruction? *ZDM Mathematics Education*, 48, 571–580.  
<https://doi.org/10.1007/s11858-016-0792-x>
- Lewis, C. C., & Perry, R. R. (2017). Lesson study to scale-up research-based knowledge: A randomised, controlled trial of fractions learning. *Journal for Research in Mathematics Education*, 48(3), 262–300.
- Martins, P. B. M. (2020). *Potencialidades dos estudos de aula para a formação continuada de um grupo de professores que ensinam matemática na rede municipal de São Paulo no contexto de uma pesquisa envolvendo implementação curricular*. Tese (Doutorado em Educação), Universidade Cruzeiro do Sul, São Paulo, Brasil.
- Murata, A. (2011). Introduction: Conceptual Overview of Lesson Study. In: Hart, L., Alston, A., Murata, A. (eds) *Lesson Study Research and Practice in Mathematics Education*. Springer.  
[https://doi.org/10.1007/978-90-481-9941-9\\_1](https://doi.org/10.1007/978-90-481-9941-9_1)
- Neves, R. D. S. P., & Fiorentini, D. (2021). Aprendizagens de futuros professores de matemática em um estágio curricular supervisionado em processo de Lesson Study. *Perspectivas da Educação Matemática*, 14(34), 1-30.

- Paula, A. P. M., & Fiorentini, D. (2023). Aprendizagens e aprendizados de um professor-pesquisador ao participar de uma experiência de Lesson Study sobre o sentido de zero. *Revista Paranaense de Educação Matemática*, 12(29), 386–405. <https://doi.org/10.33871/22385800.2023.12.29.386-405>
- Ponte, J. P., Baptista, M., Velez, I., & Costa, E. (2012). Aprendizagens profissionais dos professores de Matemática através dos estudos de aula. *Perspectivas da Educação Matemática*, 5, 7-24.
- Ponte, J. P., Quaresma, M., Baptista, M., & Mata-Pereira, J. (2014). Os estudos de aula como processo colaborativo e reflexivo de desenvolvimento profissional. In J. Sousa & I. Cevallos (Eds.), *A formação, os saberes e os desafios do professor que ensina Matemática* (pp. 61-82). Editora CRV.
- Pozzobon, M. C. C., & Ponte, J. P. M. (2024). Relações entre o Planejamento da Aula e as Aprendizagens Matemáticas em um Estudo de Aula no Sul do Brasil. *Bolema – Boletim de Educação Matemática*, 38, e230115. <https://doi.org/10.1590/1980-4415v38a230115>
- Quaresma, M., & Ponte, J. P. (2019). Dinâmicas de reflexão e colaboração entre professores do 1º. Ciclo num estudo de aula em Matemática. *Bolema*, 33(63), 368-388. <https://doi.org/10.1590/1980-4415v33n63a18>
- Ribeiro, A. J. & Llinares, S. (2025). Becoming a mathematics teacher educator: Identifying characteristics of learning opportunities. *Acta Scientiae*, 27(3), 1-37. <https://doi.org/10.64856/acta.scientiae.8401>
- Richit, A. (2016). Teacher professional development from the point of view lesson studies. *ICME 13*, Hamburgo, Alemanha
- Richit, A. (2020). Estudos de aula na perspectiva de professores formadores. *Revista Brasileira de Educação*, 25. <https://doi.org/10.1590/S1413-24782020250044>
- Richit, A. (2021). Desenvolvimento profissional docente: um referencial teórico. *Research, Society and Development*, [S. 1.], 10(14), 1-19. <https://doi.org/10.33448/rsd-v10i14.22247>

- Richit, A. (2023). Professional Development of Professors in Lesson Study. *Educação Unisinos*, 27(2023), 1-17. <https://doi.org/10.4013/edu.2023.271.20>
- Richit, A., & Ponte, J. P. (2017). Teachers' perspectives about Lesson Study. *Acta Scientiae*, 19(1), 20-30. <http://www.periodicos.ulbra.br/index.php/acta/article/view/2808>
- Richit, A., Ponte, J. P., & Quaresma, M. (2021). Aprendizagens profissionais de professores evidenciadas em pesquisas sobre estudos de aula. *Bolema – Boletim de Educação Matemática*, 35(70), 1107–1137. <https://doi.org/10.1590/1980-4415v35n70a26>
- Richit, A., Ponte, J. P., & Tomkelski, M. L. (2019). Estudos de aula na formação de professores de matemática do ensino médio. *Revista Brasileira de Estudos Pedagógicos*, 100(254), 54-81. <https://doi.org/10.24109/2176-6681.rbep.100i254.3961>
- Richit, A., Ponte, J. P., & Tomkelski, M. L. (2024). Professional Collaboration among Elementary School Teachers in Lesson Study. *Journal of Research in Mathematics Education*, 13(2), 111-131. <https://doi.org/10.17583/redimat.14337>
- Richit, L. A., & Richit, A. (2022). O Modelo de Barras de Singapura na Resolução de Problemas Aritméticos e Algébricos. *Bolema – Boletim de Educação Matemática*, 36(73), 697-727. <https://doi.org/10.1590/1980-4415v36n73a05>
- Richit, A., & Tomkelski, M. L. (2020). Aprendizagens profissionais de professores de matemática do ensino médio no contexto dos estudos de aula. *Acta Scientiae*, 22(3), 2-27. <https://doi.org/10.17648/acta.scientiae.5067>
- Richit, A., & Tomkelski, M. L. (2022). Meanings of mathematics teaching forged through reflection in a lesson study. *Eurasia Journal of Mathematics Science and Technology Education*, 18(9), em2151, 1-15. <https://doi.org/10.29333/ejmste/12325>
- Robutti, O., Cusi, A., & Clark-Wilson, A. et al. (2016). ICME international survey on teachers working and learning through collaboration: *ZDM Mathematics Education*, 48, 651-690. <https://doi.org/10.1007/s11858-016-0797-5>

- Sánchez, A., Font, V., & Breda, A. (2022). Significance of creativity and its development in mathematics classes for preservice teachers who are not trained to develop students' creativity. *Mathematics Education Research Journal*, 34, 863–885. <https://doi.org/10.1007/s13394-021-00367-w>
- Seckel, M. J., Breda, A., Sánchez, A., & Font, V. (2019). Criterios asumidos por profesores cuando argumentan sobre la creatividad matemática. *Educação e Pesquisa*, 45, e211926. <https://doi.org/10.1590/S1678-4634201945211926>
- Sol, T., Breda, A., Richit, A., & Sala-Sebastià, G. (2024). Critérios de adequação didática emergentes na argumentação prática de futuros pedagogos. *Zetetike*, 32(00), e024004. <https://doi.org/10.20396/zet.v32i00.8676234>
- Sol, T., Breda, A., Richit, A., Tomkelski, M. L., & Sala-Sebastià, G. (2025). Argumentação prática de professores sobre as dificuldades dos alunos em noções de volume e unidades de medida em um contexto de Estudo de Aula. *Educação Matemática em Revista - RS*, 3(26). <https://doi.org/10.37001/t2gsak89>
- Spillane, J. P. (2000). A fifth-grade teacher's reconstruction of mathematics and literacy teaching: Exploring interactions among identity, learning, and subject matter. *The Elementary School Journal*, 100(4), 307-330.
- Tomasi, A. P. (2020). *Aspectos da colaboração profissional docente mobilizados em um estudo de aula (Lesson Study) no contexto brasileiro*. Dissertação (Mestrado em Educação), Universidade Federal do Rio Grande do Sul, Chapecó.
- Webster-Wright, A. (2009). Reframing professional development through understanding authentic professional learning. *Review of Educational Research*, 79(2), 702–739. <https://doi.org/10.3102/0034654308330970>